Critical fluctuations in heavy-ion collisions

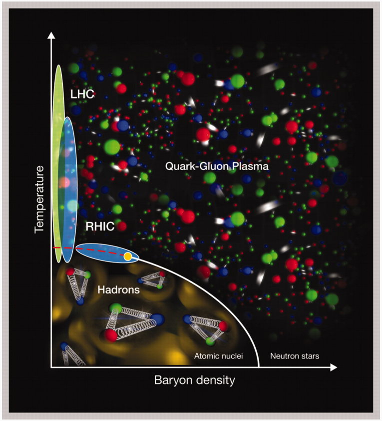
Marlene Nahrgang

From correlation functions to QCD phenomenology, Bad Honnef 05-04-2018

SUBATECH, IMT Atlantique, Nantes, France

Ideas about the QCD phase diagram

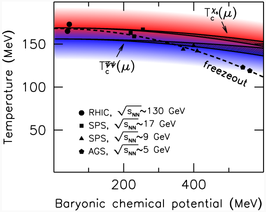
- Properties of strongly interacting many-body systems.
- Phases of hot and dense nuclear matter.
- Phase transition from the quark-gluon plasma (QGP) to a hadron gas.
- Is there a critical point in the phase diagram of QCD?



B. Jacak and B. Müller Science 337 (2012)

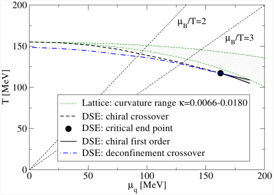
Lattice QCD calculations

- Crossover at $\mu_B = 0$ and $T = [145, 165]$ MeV
  WB JHEP1009 (2010), HotQCD PoS LATTICE2010 (2010)
- Fermionic sign problem at $\mu_B \neq 0 \Rightarrow$ usual importance sampling fails.
- Methods to extend to finite μ_B, e.g. **Taylor expansion**, etc.
  $\Rightarrow$ no critical point for small $\mu_B/T < 1$.

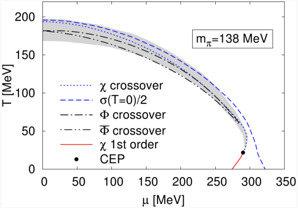


G. Endrodi et al. JHEP1104 (2011)

Functional methods (DSE/FRG)



C. Fischer, J. Luecker, C. Welzbacher, PRD90 (2014)

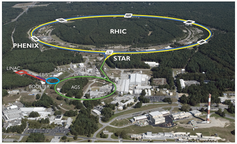


T. Herbst, J. Pawlowski, BJ. Schaefer PRD88 (2013)

- $\Rightarrow$ a critical point exists for large μ_B/T.

QCD phase diagram: the experimental perspective

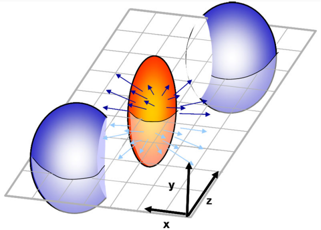
- Highest energies at LHC, CERN: PbPb at $\sqrt{s_{\mathrm{NN}}} = 2.76,\ 5$ TeV
  $\Rightarrow$ Energy deposition at the highest beam energies $\rightarrow$ temperature.
- Beam energy scan at RHIC, BNL: AuAu at $\sqrt{s_{\mathrm{NN}}} = 200 - 7.7$ GeV
  $\Rightarrow$ Baryon stopping at lower beam energies $\rightarrow$ baryochemical potential.
- Measure particle species at chemical freeze-out (instance where inelastic collisions become rare) $\rightarrow$ success of statistical hadronization models
- Measure particle spectra at kinetic freeze-out (instance where elastic collisions become rare) $\rightarrow$ success of fluid dynamics



Fluid dynamical description of heavy-ion collisions

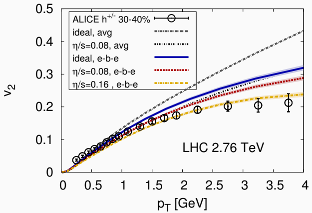
- The discovery of RHIC: The QGP is an almost ideal strongly coupled fluid.
- Early fluid dynamical calculations reproduce spectra and elliptic flow.

  P. Kolb, U. Heinz, QGP (2003)
- Long road of improvements during the last decade:
  $(3+1d)$, viscosity, initial conditions, initial state fluctuations, hybrid models



Spatial eccentricity $\Rightarrow$ momentum anisotropy
via fluid dynamical pressure

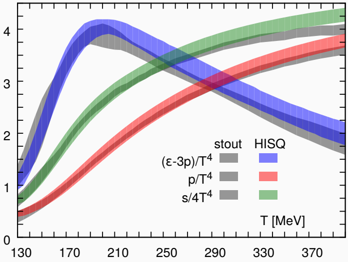
elliptic flow at LHC



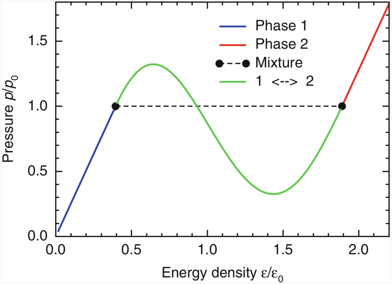
MUSIC by B. Schenke, S. Jeon, C. Gale PLB702 (2011)

Equation of state and phase transitions

HotQCD Coll. PRD90 (2014); WB Coll. PLB730 (2014)



CBM physics book, Lecture Notes in Physics 814 (2011)

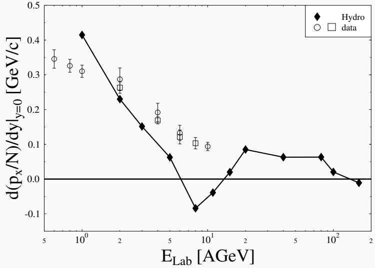


- Thermodynamic quantities change characteristically at the phase transition.
- Speed of sound $c_s^2 = (\partial p/\partial e)_S \rightarrow$ minimum at the phase transition/crossover.
- Compressibility $\kappa_S = -1/V(\partial V/\partial p)_S \rightarrow$ maximum at the phase transition/crossover.

"softest point"
anomaly in the pressure

Phase transitions in fluid dynamics

- Describing a phase transition fluid dynamically is simple!
- Need to know the equation of state and transport coefficients!



H. Stöcker, NPA780 (2005)

- A pronounced minimum in the slope of the directed flow v_1 is observed in a first-order phase transition.

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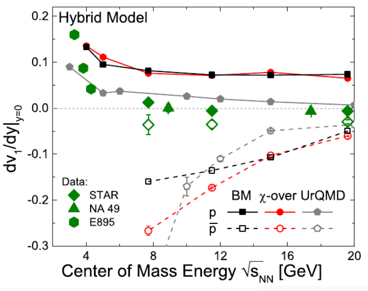


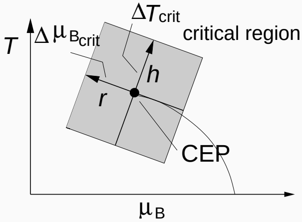
J. Steinheimer, J. Auvinen, H. Petersen, M. Bleicher, H. Stöcker, PRC89 (2014)

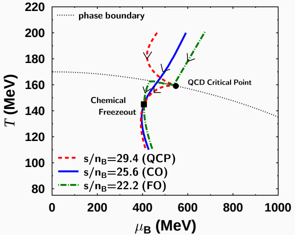
- A pronounced minimum in the slope of the directed flow v_1 is not observed in a first-order phase transition?
- In dynamical simulations: no clear sensitivity on a phase transition in the equation of state yet...

Critical point in fluid dynamics

- construct an eos with CP from the universality class of the 3d Ising model
- map the temperature and the external magnetic field (r, h) onto the (T, μ)-plane $\Rightarrow$ critical part of the entropy density S_c
- match with nonsingular entropy density from QGP and the hadron phase:

$$s = 1/2(1 - \tanh S_c) s_H + 1/2(1 + \tanh S_c) s_{QGP}$$





- focussing of trajectories ... or not? Strongly depends on mapping & matching!

C. Nonaka, M. Asakawa PRC71 (2005); M. Asakawa et al. PRL101 (2008); M. Bluhm, B. Kampfer PoS CPOD2006 (2006)

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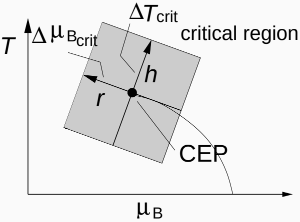

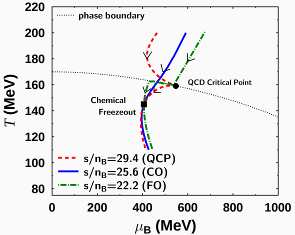

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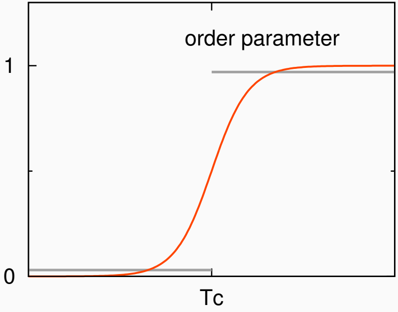
Fluctuations matter at the phase transition!

Phase transitions: order parameter & derivatives

- An order parameter changes characteristically at the phase transition - discontinuously or continuously.



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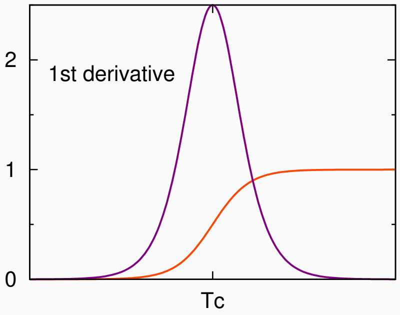

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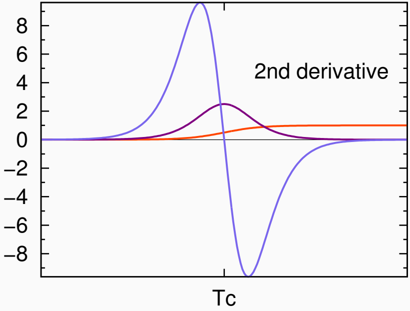

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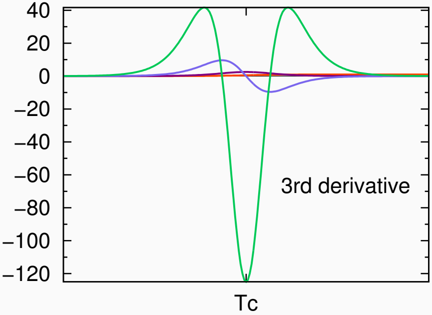

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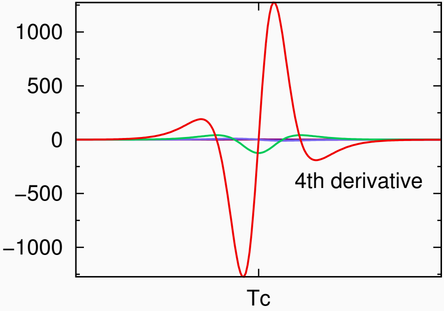

- Derivatives reveal more details!
- Derivatives of thermodynamic quantities are related to fluctuations!

What are fluctuation observables?

- Susceptibilities $\chi_n = \frac{\partial^n (P/T^4)}{\partial(\mu/T)^n}\Big|_T$ relate to fluctuations in multiplicity

$$\chi_1 = \frac{1}{VT^3}\langle N\rangle\,, \quad \chi_2 = \frac{1}{VT^3}\langle(\Delta N)^2\rangle\,, \quad \chi_3 = \frac{1}{VT^3}\langle(\Delta N)^3\rangle\,,$$

$$\chi_4 = \frac{1}{VT^3}\langle(\Delta N)^4\rangle_c \equiv \frac{1}{VT^3}\left(\langle(\Delta N)^4\rangle - 3\langle(\Delta N)^2\rangle^2\right)\,.$$

- To zeroth-order in volume fluctuations:

$$\frac{\chi_2}{\chi_1} = \frac{\sigma^2}{M} \qquad \frac{\chi_3}{\chi_2} = S\sigma \qquad \frac{\chi_4}{\chi_2} = \kappa\sigma^2$$

variance   Skewness   Kurtosis

- M, σ^2, S and κ are obtained from measured event-by-event multiplicity distributions.

STAR Coll. PRL112 (2014), PRL113 (2014); PHENIX Coll. arxiv:1506.07834

Fluctuations at a CP vs first-order

at a critical point:

- Correlation length diverges $\xi \to \infty \Rightarrow$ Fluctuations of the critical mode σ diverge.
- Higher-order cumulants more sensitive to ξ:

$$\langle \Delta\sigma^2 \rangle \propto \xi^2 , \quad \langle \Delta\sigma^3 \rangle \propto \xi^{9/2} , \quad \langle \Delta\sigma^4 \rangle_c \propto \xi^7 .$$

- Relaxation time $\tau_{\rm rel} \propto \xi^z$ diverges $\Rightarrow$ critical slowing down!

at a first-order phase transition:

- Coexistence of two stable thermodynamic phases at $T = T_c$.
- Metastable states above and below $T_c \Rightarrow$ supercooling and -heating.
- Nucleation & spinodal decomposition.

$\Rightarrow$ Domain formation and large inhomogeneities.

P. Hohenberg, B. Halperin, RMP49 (1977); T. Hatsuda, T. Kunihiro, PRL55 (1985); L Csernai, I Mishustin, PRL74 (1995); M. Stephanov, K. Rajagopal, E. Shuryak, PRL81 (1998), PRD60 (1999); S. Jeon, V. Koch, PRL83 (1999); B. Berdnikov and K. Rajagopal, PRD61 (2000); Y. Hatta, T. Ikeda, PRD67 (2003); M. Stephanov, PRL102 (2009); J. Randrup, PRC79 (2009), PRC82 (2010); M. Stephanov, PRL107 (2011)

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Large ensemble fluctuations in equilibrated systems!

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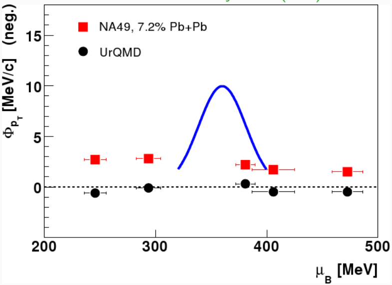
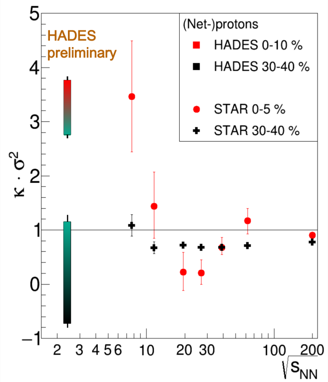
Large inhomogeneities/fluctuations in nonequilibrium systems!

P. Hohenberg, B. Halperin, RMP49 (1977); T. Hatsuda, T. Kunihiro, PRL55 (1985); L Csernai, I Mishustin, PRL74 (1995); M. Stephanov, K. Rajagopal, E. Shuryak, PRL81 (1998), PRD60 (1999); S. Jeon, V. Koch, PRL83 (1999); B. Berdnikov and K. Rajagopal, PRD61 (2000); Y. Hatta, T. Ikeda, PRD67 (2003); M. Stephanov, PRL102 (2009); J. Randrup, PRC79 (2009), PRC82 (2010); M. Stephanov, PRL107 (2011)

Critical Point: What has been seen in HIC?





HADES collab. QM 2017 talk

- first fluctuation measurements at the CERN-SPS did not see signals of the critical point
- confirmed by BES phase I: no criticality seen in σ^2/M
- interesting deviations from the "baseline" observed in Skewness and Kurtosis measurements during BES → is it related to the critical point?

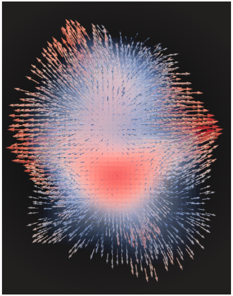
Why is dynamical modeling important?

In a grand-canonical ensemble, the system is

- in thermal equilibrium (= long-lived),
- in equilibrium with a particle heat bath,
- spatially infinite
- and static.

Systems created in heavy-ion collisions are

- short-lived,
- spatially small,
- inhomogeneous,
- and highly dynamical!



plot by H. Petersen, madai.us

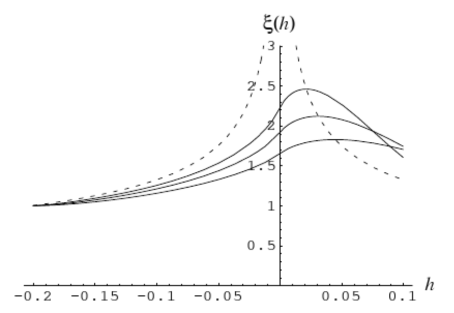
Critical slowing down

long relaxation times near a critical point $\Rightarrow$ critical slowing down
$\Rightarrow$ the system is driven out of equilibrium

$$\frac{\mathrm{d}}{\mathrm{d}t} m_\sigma(t) = -\Gamma[m_\sigma(t)] \left(m_\sigma(t) - \frac{1}{\xi_{\mathrm{eq}}(t)} \right)$$

with $\Gamma(m_\sigma) = \frac{A}{\xi_0}(m_\sigma \xi_0)^z$
$z = 3$
dynamical critical exponent (model H)

$\Rightarrow \xi \sim 1.5 - 2$ fm



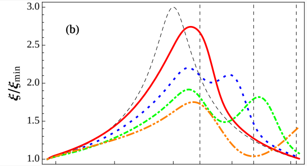
B. Berdnikov and K. Rajagopal, PRD 61 (2000)); D.T.Son, M.Stephanov, PRD 70 (2004);
M.Asakawa, C.Nonaka, Nucl. Phys. A774 (2006))

Real-time evolution of cumulants

real-time evolution of non-Gaussian cumulants in the scaling regime:

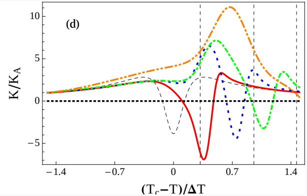
$$L_{\mathrm{micro}} \ll \xi \ll L_{\mathrm{sys}}$$

- leading-order expansion in ξ/L_{sys} of dynamics
- memory effects are important
- magnitude and sign can be different in non-equilibrium compared to equilibrium expectations
- different trajectories, chemical freeze-out conditions and τ_{rel} can give similar results
- Kibble-Zurek scaling of the nonequilibrium dynamics of (non)-Gaussian correlation functions
- needs full dynamical space-time evolution!



black dashed: equilibrium

colors: different τ_{rel}



S. Mukherjee, R. Venugopalan, Y. Yin, PRC92 (2015), PRL117 (2016)

Nonequilibrium chiral fluid dynamics (NχFD)

Propagate the critical mode σ coupled to a fluid dynamical expansion

- Relaxational equation for the critical mode: damping and noise from the interaction with the fermions/fast modes

$$\partial_\mu \partial^\mu \sigma + \frac{\delta V_{\mathrm{eff}}(\sigma)}{\delta \sigma} + \eta \partial_t \sigma = \xi$$

- Phenomenological dynamics for the Polyakov-loop

$$\eta_\ell \partial_t \ell T^2 + \frac{\partial V_{\mathrm{eff}}(\ell)}{\partial \ell} = \xi_\ell$$

- Fluid dynamical expansion = heat bath, including energy-momentum exchange

$$\partial_\mu T^{\mu\nu}_{\mathrm{fluid}} = S^\nu = -\partial_\mu T^{\mu\nu}_\sigma \,, \quad \partial_\mu N^\mu_{\mathrm{q}} = 0$$

$\Rightarrow$ includes a stochastic source term!

MN, S. Leupold, I. Mishustin, C. Herold, M. Bleicher, PRC 84 (2011); PLB 711 (2012); JPG 40 (2013); C. Herold, MN, I. Mishustin, M. Bleicher PRC 87 (2013); NPA925 (2014), C. Herold, MN, Y. Yan, C. Kobdaj JPG 41 (2014); MN and C. Herold, 1602.07223; C. Herold, MN, Y. Yan and C. Kobdaj, PRC93 (2016) no.2

NχFD - equation of motion

Calculate the equation of motion for the sigma field via the 2PI effective action on the Keldysh contour:

$$\begin{aligned}\Gamma[\sigma, S] =& S_{\mathrm{cl}}[\sigma] - i\mathrm{Tr}\ln S^{-1} - i\mathrm{Tr}S_0^{-1}S + \Gamma_2[\sigma, S] \\ =& g\mathrm{tr}S_{\mathrm{th}}^{++}(x,x)\Delta\sigma(x) - \frac{T}{V}\ln Z_{\mathrm{th}} \\ &+ \int \mathrm{d}^4 x D[\bar{\sigma}](x)\Delta\sigma(x) + \frac{i}{2}\int \mathrm{d}^4 x \int \mathrm{d}^4 y \Delta\sigma(x)\mathcal{N}[\bar{\sigma}](x,y)\Delta\sigma(y)\end{aligned}$$

EoM: $-\frac{\delta S_{\mathrm{cl}}[\sigma]}{\delta\Delta\sigma} = \frac{\delta\Gamma_2[\sigma,S]}{\delta\Delta\sigma}$

lowest order in the eq. of motion for the σ field: $g\mathrm{tr}S_{\mathrm{th}}^{++}(x,x)\Delta\sigma(x)$

equilibrium properties, equation of state: $-\frac{T}{V}\ln Z_{\mathrm{th}}$

dissipative processes: $\int \mathrm{d}^4 x D[\bar{\sigma}](x)\Delta\sigma(x)$

origin of fluctuations: $\frac{i}{2}\int \mathrm{d}^4 x \int \mathrm{d}^4 y \Delta\sigma(x)\mathcal{N}[\bar{\sigma}](x,y)\Delta\sigma(y)$

NχFD - damping and noise

imaginary part of Γ is interpreted as stochastic fluctuations

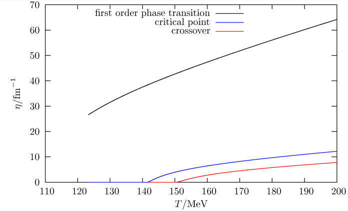
$$\exp\left[-\frac{1}{2}\int \mathrm{d}^4x \int \mathrm{d}^4y \Delta\sigma(x)\mathcal{N}(x,y)\Delta\sigma(y)\right] = \int \mathcal{D}\xi P[\xi] \exp\left[i\int \mathrm{d}^4x\xi(x)\Delta\sigma(x)\right]$$

with $P[\xi]$ Gaussian measure: $\langle\xi\rangle = 0$ and $\langle\xi(x)\xi(x')\rangle = \mathcal{N}(x,x')$

damping term η and noise ξ in the Markovian limit for $\mathbf{k} = 0$

$$\eta = g^2\frac{d_q}{\pi}\left(1 - 2n_{\mathrm{F}}\left(\frac{m_\sigma}{2}\right)\right)\frac{\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{\frac{3}{2}}}{m_\sigma^2}$$

$$\langle\xi_a(t)\xi_{a'}(t')\rangle = \frac{\delta_{a,a'}}{V}\delta(t-t')m_\sigma\eta\coth\left(\frac{m_\sigma}{2T}\right)$$



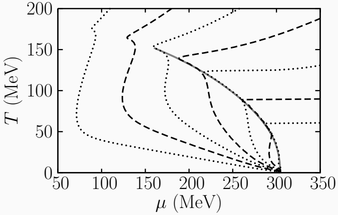
generalized dissipation-fluctuation relation holds

see also: M. Morikawa, PRD33 (1986); D. Boedecker et al, PRD52 (1995); C. Greiner et al, PRD55 (1997); D. Rischke PRC58 (1998); C. Greiner et al, Annals Phys. 270 (1998); F. Gautier et al. PRD86 (2012);...

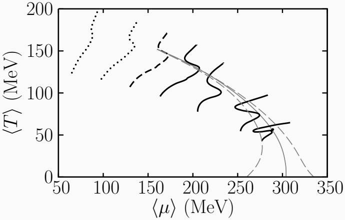
Trajectories and isentropes at finite μ_B

- Solve coupled system of fluctuating sigma field and fluid dynamics + stochastic source term, for various initial conditions.

Isentropes in the PQM model



Fluid dynamical trajectories

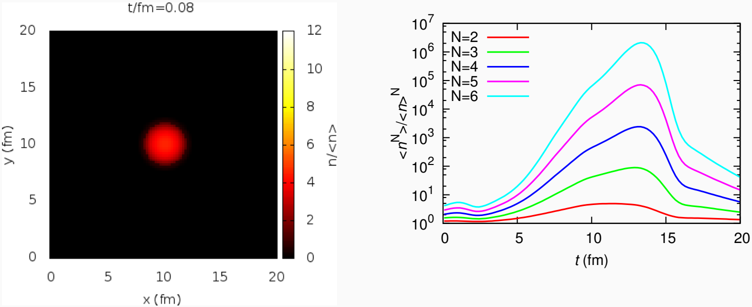


- Fluid dynamical trajectories similar to the isentropes in the crossover region.
- No significant features in the trajectories left of the critical point.
- Right of the critical point: trajectories differ from isentropes and the system spends significant time in the spinodal region! $\Rightarrow$ possibility of spinodal decomposition!

C. Herold, MN, I. Mishustin, M. Bleicher NPA**925** (2014)

Domain formation & decay at the QH phase transition

- use a chiral effective model with correct low-temperature degrees of freedom in NχFD! V. Dexheimer, S. Schramm, PRC81 (2010); M. Hempel, V. Dexheimer, S. Schramm, I. Iosilevskiy PRC88 (2013)

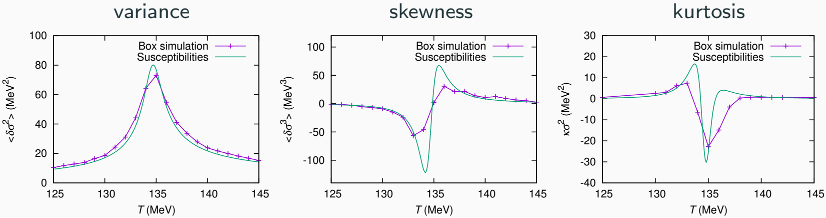


- droplets of quark density decay in the hadronic phase due to non-vanishing large pressure (cf. also J. Steinheimer, J. Randrup, V. Koch PRC89 (2014))
- future: combine initial and dynamical fluctuations, include particlization and late hadronic interactions

MN and C. Herold, 1602.07223

Sigma field Fluctuations/Susceptibilities in the critical region

- static, finite-size medium, periodic boundary conditions
- fixed temperature (no back-coupling) at $\mu_q = 100$ MeV

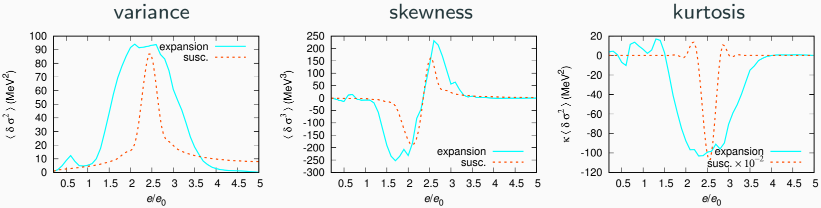


- Shape is well reproduced, some discrepancy for higher-order cumulants (longer equilibration times needed)!

QM 2017 talk by C. Herold

Sigma field fluctuations - broadening of critical region

- evolution of temperature according to fluid dynamics ($T_{\text{ini}} \sim 160$ MeV, $\mu_{\text{ini}} = 160$ MeV)
- inhomogeneous medium evolution
- average fluctuations over a hypersurface of constant energydensity e/e_0



- critical region is broadened by the dynamical evolution of fluctuations and medium ⇒ enhances chances to be seen experimentally!

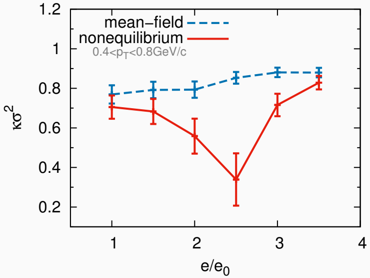
C. Herold et al, EPJA54 (2018)

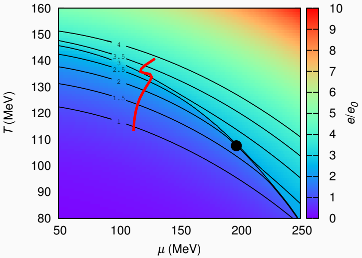
Net-Proton fluctuations near the critical point

- From densities to particle via Cooper-Frye particlization:

$$E\frac{\mathrm{d}N_i}{\mathrm{d}^3p} = \int \mathrm{d}\sigma^\mu p_\mu (f_i^{\mathrm{eq}}(p) + \delta f)$$

- Here: couple the densities of the order parameter field with the fluid dynamical densities





C. Herold, MN, Y. Yan and C. Kobdaj, PRC93 (2016) no.2

- Future: include δf in the particlization and perform calculations for BES!

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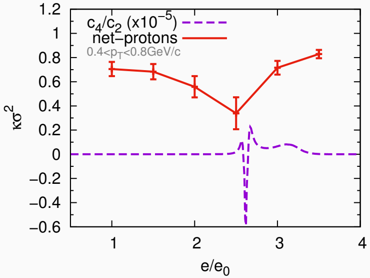
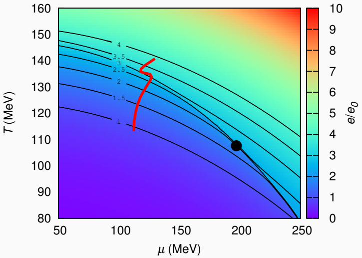


C. Herold, MN, Y. Yan and C. Kobdaj, PRC93 (2016) no.2



- Future: include δf in the particlization and perform calculations for BES!

Fluid dynamical fluctuations

Conventional fluid dynamics propagates thermal averages of the energy density, pressure, velocities, charge densities, etc.

However, ...

- ... already in equilibrium there are thermal fluctuations
- ... the fast processes, which lead to local equilibration also lead to noise!

Conventional ideal fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}}$$
$$N^{\mu} = N^{\mu}_{\mathrm{eq}}$$

Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC85 (2012); C. Chafin, T. Schäfer, PRA87 (2013); J. Kapusta, C. Young, PRC90 (2014); P. Romatschke, R. Young, PRA87 (2013); P. Kovtun, G. Moore, P. Romatschke, JHEP1407 (2014); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015)

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Conventional viscous fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}_{\rm visc}$$
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Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC85 (2012); C. Chafin, T. Schäfer, PRA87 (2013); J. Kapusta, C. Young, PRC90 (2014); P. Romatschke, R. Young, PRA87 (2013); P. Kovtun, G. Moore, P. Romatschke, JHEP1407 (2014); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015)

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Fluctuating viscous fluid dynamics:

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$$N^{\mu} = N^{\mu}_{\mathrm{eq}} + \Delta N^{\mu}_{\mathrm{visc}} + \mathrm{I}^{\mu}$$

Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC85 (2012); C. Chafin, T. Schäfer, PRA87 (2013); J. Kapusta, C. Young, PRC90 (2014); P. Romatschke, R. Young, PRA87 (2013); P. Kovtun, G. Moore, P. Romatschke, JHEP1407 (2014); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015)

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- $\langle T^{\mu\nu} T^{\mu\nu} \rangle$ give viscosities (Kubo-formula), consistently with dissipation-fluctuation theorem fluctuations need to be included as well!
- This is especially important at the critical point, because the true critical mode is the net-baryon density!

Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC85 (2012); C. Chafin, T. Schäfer, PRA87 (2013); J. Kapusta, C. Young, PRC90 (2014); P. Romatschke, R. Young, PRA87 (2013); P. Kovtun, G. Moore, P. Romatschke, JHEP1407 (2014); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015)

Fluid dynamical fluctuations - nonlinearities

- correlation functions from linearized fluctuations describe noninteracting modes
- if nonlinearities are included $\rightarrow$ interaction of modes
  $\rightarrow$ modification of correlations
  $\rightarrow$ contributions to transport coefficients
- symmetrized correlator:

$$G_S^{xyxy}(\omega, \mathbf{0}) = \int \mathrm{d}^3x\mathrm{d}t\, e^{i(\omega t - \mathbf{k}\cdot\mathbf{x})} \left\langle \frac{1}{2}\{T^{xy}(t,\mathbf{x}), T^{xy}(0,\mathbf{0})\} \right\rangle$$

- for the shear-shear contribution $\Rightarrow$

$$G_{R,\mathrm{shear-shear}}^{xyxy}(\omega, \mathbf{0}) = -\frac{7T}{90\pi^2}\Lambda^3 - i\omega\frac{7T}{60\pi^2}\frac{\Lambda}{\gamma_\eta} + (i+1)\omega^{3/2}\frac{7T}{90\pi^2}\frac{1}{\gamma_\eta^{3/2}}$$

P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); C. Chafin, T. Schfer, PRA87 (2013); P. Romatschke, R. Young, PRA87 (2013)

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cutoff-dependent
fluctuation contribution
to the pressure

P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); C. Chafin, T. Schfer, PRA87 (2013); P. Romatschke, R. Young, PRA87 (2013)

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cutoff-dependent fluctuation contribution to the pressure

cutoff-dependent correction to η

P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); C. Chafin, T. Schfer, PRA87 (2013); P. Romatschke, R. Young, PRA87 (2013)

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cutoff-dependent fluctuation contribution to the pressure	cutoff-dependent correction to η	frequency-dependent contribution to η and τ_π

P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); C. Chafin, T. Schfer, PRA87 (2013); P. Romatschke, R. Young, PRA87 (2013)

Diffusive dynamics of net-baryon density

remember talk by M. Bluhm

$\partial_\mu N_B^\mu = 0$ net-baryon number conservation

The diffusive dynamics occurs such as to minimize the free energy $\mathcal{F}$:

$$\partial_t n_B = \kappa \nabla^2 \left(\frac{\delta \mathcal{F}}{\Delta n_B} [\Delta n_B] \right)$$

MN, M. Bluhm, T. Schaefer, S. Bass, work in progress

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For the study of intrinsic fluctuations include a stochastic current:

$$J(t,x) = \sqrt{2T\kappa}\,\zeta(t,x)$$

MN, M. Bluhm, T. Schaefer, S. Bass, work in progress

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$\zeta(t,x)$ is Gaussian: $\langle \zeta^{(2n+1)} \rangle = 0$ and $\langle \zeta^{(2n)} \rangle = \sum_{i \neq j = 0}^{2n} \langle \zeta_i \zeta_j \rangle^n$

$\zeta(t,x)$ is uncorrelated (white noise): $\langle \zeta(x,t) \zeta(0,0) \rangle = \delta(x)\delta(t)$

MN, M. Bluhm, T. Schaefer, S. Bass, work in progress

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Fluctuation-dissipation theorem $\Rightarrow P_{\mathrm{eq}}[\Delta n_B] = \frac{1}{\mathcal{Z}} \exp \left(\frac{-\mathcal{F}[\delta n_B]}{T} \right)$

MN, M. Bluhm, T. Schaefer, S. Bass, work in progress

Net-baryon diffusion with 3d Ising model couplings

The diffusion equation:

$$\partial_t n_B = \frac{D}{n_c}\left(m^2 - K\nabla^2\right)\nabla^2 n_B + D\nabla^2\left(\frac{\lambda_3}{n_c^2}\,\Delta n_B^2 + \frac{\lambda_4}{n_c^3}\,\Delta n_B^3 + \frac{\lambda_6}{n_c^5}\,\Delta n_B^5\right) + \sqrt{2Dn_c}\nabla\zeta$$

The couplings depend on temperature via the correlation length $\xi(T)$:

$$m^2 = \frac{\tilde{m}^2}{\xi_0^3}\,, \quad \tilde{m} = \frac{1}{\xi/\xi_0}$$

$$K = \tilde{K}/\xi_0$$

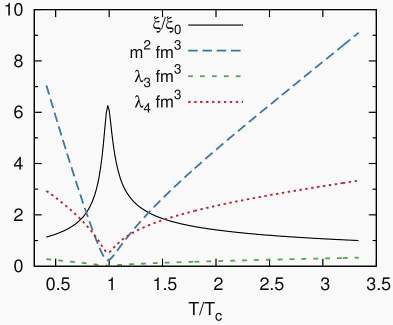
$$\lambda_3 = n_c\,\tilde{\lambda}_3\,(\xi/\xi_0)^{-3/2}$$

$$\lambda_4 = n_c\,\tilde{\lambda}_4\,(\xi/\xi_0)^{-1}$$

$$\lambda_6 = n_c\,\tilde{\lambda}_6$$

M. Tsypin PRL73 (1994); PRB55 (1997)

parameter choice:
$\xi_0 \sim 0.5$ fm, $n_c = 1/3$ fm^{-3}
$K = 1$ (surface tension)
$\tilde{\lambda}_3$, $\tilde{\lambda}_4$, $\tilde{\lambda}_6$ (universal but mapping to QCD)



in this Fig: $\tilde{\lambda}_3 = 1$, $\tilde{\lambda}_4 = 10$

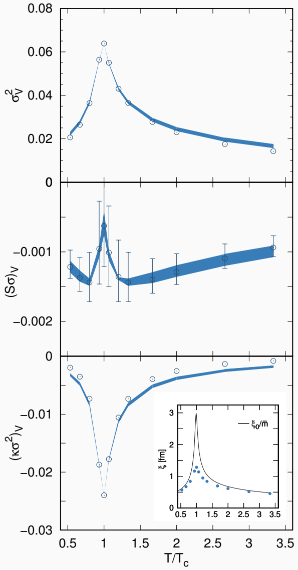
Scaling of equilibrium cumulants

Expected scaling in an infinite system ($\xi \ll V$): M. Stephanov, PRL102 (2009)

$$\sigma_V^2 \propto \xi^2$$
$$(S\sigma)_V \propto \xi^{2.5}$$
$$(\kappa\sigma^2)_V \propto \xi^5$$

Here, a finite system with exact baryon conservation ($\xi \lesssim V$)! Can be systematically studied in $\xi/V \Rightarrow$ affects equilibrium scaling! E.g. for the skewness terms $\propto \lambda_3\lambda_4$ and $\propto \lambda_3\lambda_6$ contribute with opposite sign.

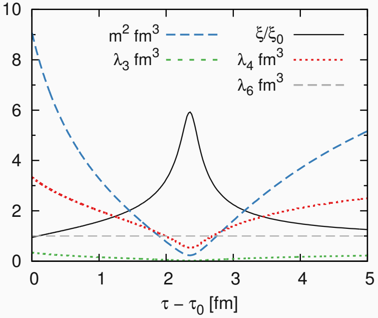
$$\sigma_V^2 \propto \xi^{1.3\pm0.05}$$
$$(S\sigma)_V \propto -\#\xi^{1.47\pm0.05} + \#\xi^{2.4\pm0.05}$$
$$(\kappa\sigma^2)_V \propto \xi^{2.5\pm0.1}$$

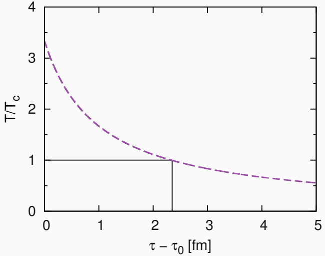


Dynamics: time-dependent couplings

time-dependent temperature:

$$T(\tau) = T_0 \left(\frac{\tau_0}{\tau}\right)^{dc_s^2}$$

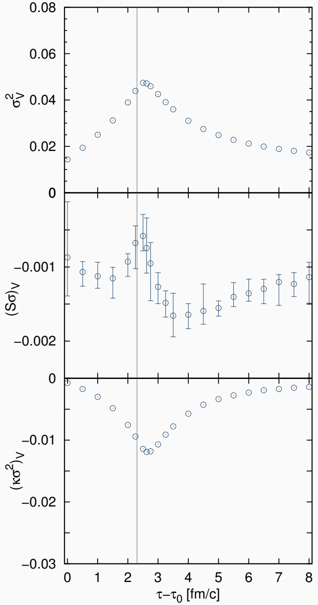




- choose $c_s^2 = 1/3$

  (should be $c_s^2 = c_s^2(T)$)
- initialize system at $T_0 = 0.5$ GeV, $D(T_0) = 1$ fm
- T_c reached at $\tau - \tau_0 = 2.33$ fm

Dynamics: time evolution of critical fluctuations



- shift of extrema for variance and kurtosis (retardation effects) to later times corresponding to $T(\tau) < T_c$
- |extremal values| in dynamical simulations below equilibrium values (nonequilibrium effects):

$$(\sigma^2_V)^{\rm max}_{\rm dyn} \approx 0.75\, (\sigma^2_V)^{\rm max}_{\rm eq}$$
$$((\kappa\sigma^2)_V)^{\rm min}_{\rm dyn} \approx 0.5\, (\sigma^2_V)^{\rm min}_{\rm eq}$$

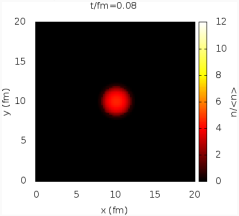
- expected behavior with varying D and c_s^2 (expansion rate)

MN, M. Bluhm, T. Schaefer, S. Bass, work in progress

What's next?

1.)

NχFD framework



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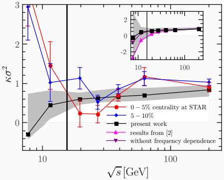
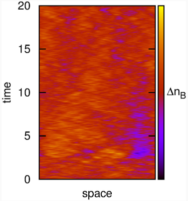
FRG input



collaboration with M. Bluhm, Y. Jiang, M. Mitter, J. Pawlowski, F. Rennecke, N. Wink and EMMI

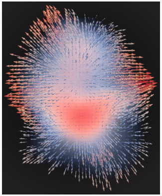
2.)

critical net-baryon diffusion



+

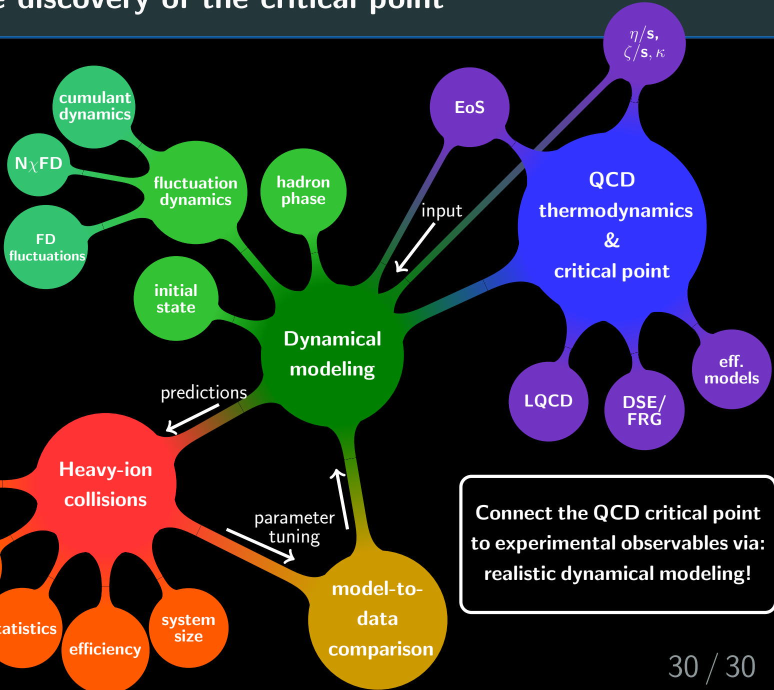
3+1d fluctuating fluid dynamics



collaboration with M. Bluhm and supported by "Etoiles Montantes" from Région Pays de la Loire

= quantitative signatures of the critical point in HIC

Toward the discovery of the critical point



RHIC BES I&II

NICA

FAIR CBM

JPARC

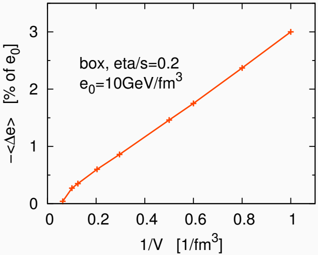
NA61/SHINE

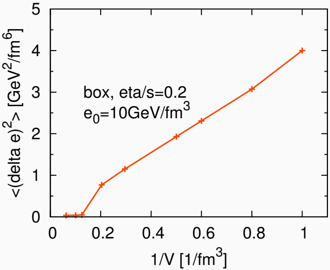
Connect the QCD critical point to experimental observables via: realistic dynamical modeling!

more interesting stuff

Numerical realization

$$\partial_\mu T^{\mu\nu} = \partial_\mu \left(T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}_{\rm visc} + \Xi^{\mu\nu} \right) = 0$$





- Proportionality to $1/V$ reproduced for the correction to the average and the variance of energy density in the local rest frame.
- Implementing fluid dynamical fluctuations is important,

  but requires a sustained and systematic effort!

MN, M. Bluhm, T. Schaefer, S. Bass, CPOD 2016, arXiv:1704.03553

Non-critical effects on fluctuation observables

- Limited acceptance & detector efficiency. A. Bzdak, V. Koch, PRC86 (2012); PRC91 (2015)
- Isospin randomization. M. Kitazawa, M. Asakawa, PRC85, PRC86 (2012)
- Volume fluctuations V. Skokov, B. Friman, K. Redlich, PRC88 (2013)
  (→ strongly intensive measures).
  E. Sangaline, arxiv:1505.00261; M. Gorenstein, M. Gazdzicki, PRC84 (2011)
- Global net-baryon number conservation.
  MN, T. Schuster, M. Mitrovski, R. Stock, M. Bleicher, EPJC72 (2012); A. Bzdak, V. Koch, V. Skokov, PRC87 (2013)

⇒ These effects are or can be included in microscopic transport models, e.g. UrQMD, (P)HSD, or hybrid models = valuable baseline studies!

- Initial fluctuations due to baryon stopping.

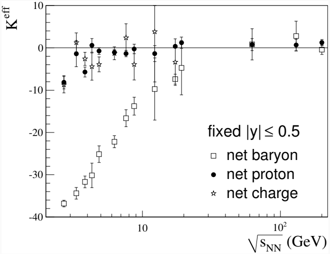
⇒ Need to be well understood!

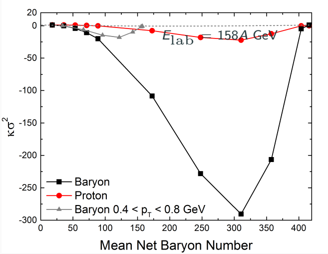
Non-critical effects on fluctuation observables

- Global net-baryon number conservation.

  MN, T. Schuster, M. Mitrovski, R. Stock, M. Bleicher, EPJC72 (2012); A. Bzdak, V. Koch, V. Skokov, PRC87 (2013)

- In a microscopic transport model the microcanonical nature of individual scatterings is preserved.
- Strongly negative kurtosis of net-baryon number due to global conservation and volume fluctuations.
- Net-proton fluctuations follow this trend slightly.





by J. Steinheimer

MN, T. Schuster, M. Mitrovski, R. Stock, M. Bleicher, EPJC72 (2012)

From order parameter fluctuations to net-proton fluctuations

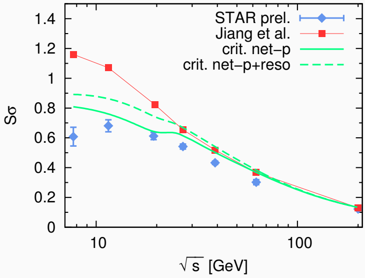
Couple order parameter to measurable particles: $g_p \bar{p} \sigma p$

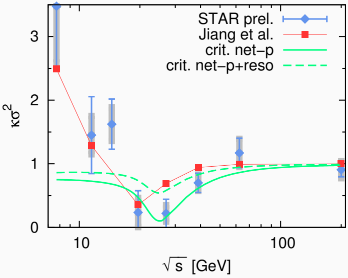
M. Stephanov, K. Rajagopal, E. Shuryak, PRL81 (1998), PRD60 (1999); C. Athanasiou, K. Rajagopal, M. Stephanov, PRD82 (2010)

- Finite expectation value of σ in the chirally broken phase contributes to the mass of the proton
- Fluctuations $\Delta\sigma$ lead to fluctuations in the proton mass $m_p \to m_p + g\Delta\sigma$,
- Modification of flucs (statistical + critical) in the distrib. function:

$$\delta f = \delta f^0 + g \frac{\partial f^0}{\partial m_p} \Delta\sigma$$

Critical net-proton fluctuations - phenomenology





MN, QM2015 proceedings, 1601.07437

- Equilibrium 3d Ising model assumptions for $\Delta\sigma$
- Fluctuations in net-protons at chemical freeze-out
- Critical fluctuations are reduced but survive when resonance decays are included

  M. Bluhm, MN, S. Bass, T. Schaefer work in progress
- Particle emission during Cooper-Frye freeze-out over a hypersurface from fluid dynamical evolution

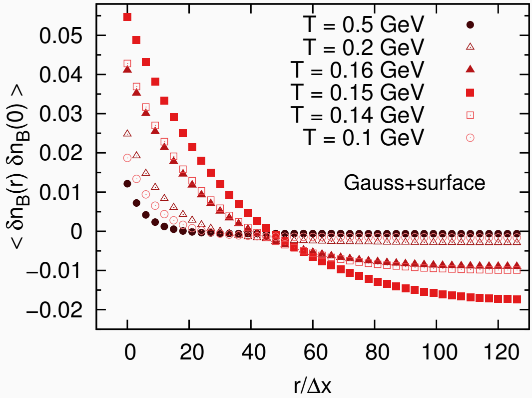
  L. Jiang, P. Li and H. Song, arXiv:1512.06164

Still no dynamical fluctuations...

Correlation function and - length

For $K = 0$ fluctuations are delta-correlated, finite surface tension leads to a finite correlation length with $\xi > \Delta x$.

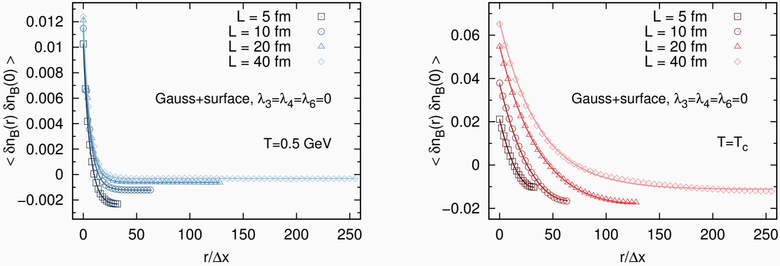
Thermodynamic correlation function: $\langle \delta n_B(r) \delta n_B(0) \rangle = \frac{n_c^2}{2m^2\xi} \exp\left(-\frac{|r|}{\xi}\right)$



Broader spatial correlations for temperatures near $\mathbf{T_c = 0.15}$ GeV!

Correlation function and baryon conservation

Local fluctuations need to be balanced within L in order to conserve net-baryon density exactly.

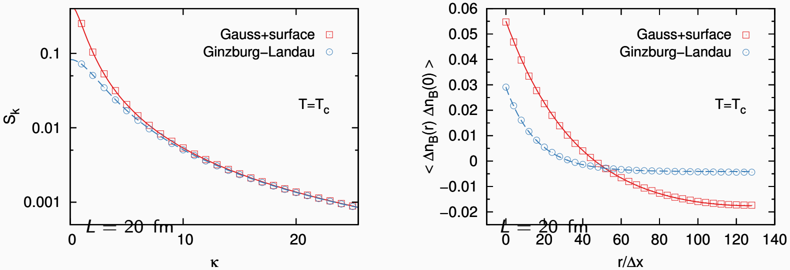


net-B conservation: $\int_L \mathrm{d}r \langle \delta n_B(r) \delta n_B(0) \rangle = 0$: $\rightarrow$ correction term < 0!

$\Rightarrow$ perfectly reproduced by the numerical result!

note: very large equilibration times needed for $L \rightarrow \infty$

Ginzburg-Landau model in equilibrium

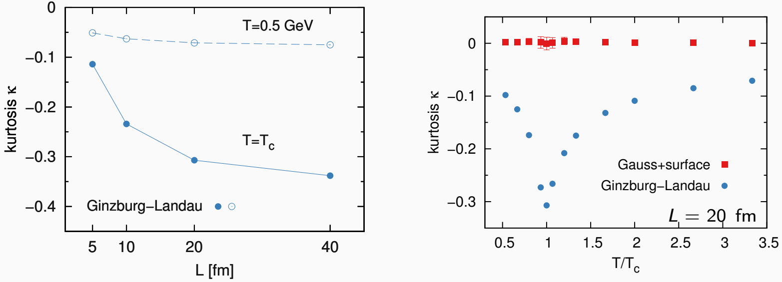


- nonlinear interactions reduce S_k for long-wavelength fluctuations!
- spatial correlations are significantly smaller!

$\Rightarrow$ at the level of 2-point correlations, results of Ginzburg-Landau model can be described by a **renormalized** Gauss+surface model (with m^2 modified, K essentially unaffected)!

T-dependence of non-Gaussian fluctuations

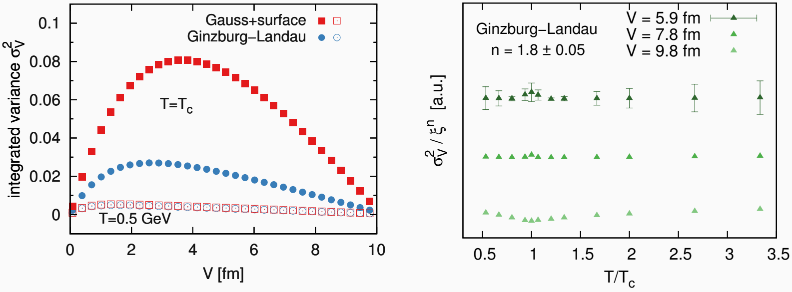
Kurtosis vanishes in the absence of nonlinear interactions!



- negative kurtosis observed for Ginzburg-Landau model with a pronounced signal near T_c!

Note: we choose $L = 20$ fm ($N_x = 256$) for the following studies!

Integrated variance



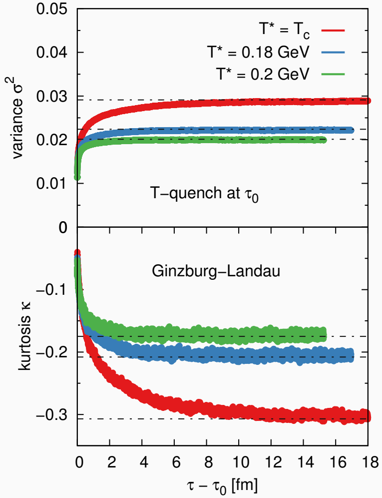
- in this talk: study of local variance, taken over $V = \Delta x$, where $\sigma^2 \propto \xi$!
- integrated variance: $\sigma_V^2 = \frac{1}{V^2} \int dx \int dy \, \langle \Delta n_B(x) \Delta n_B(y) \rangle \propto \xi^2$

M. Stephanov et al. PRL81 (1998); PRD60 (1999); PRL102 (2009)

- including finite-size and $\langle N_B \rangle$-conservation we find:

$$\sigma_V^2 \propto \xi^n \text{ with } n \sim 1.80 \pm 0.05\,!$$

Dynamics: temperature quench and equilibration



- temperature quench: at τ_0 temperature drops from $T_0 = 0.5$ GeV to T^*
- fast initial relaxation
- variance approaches equilibrium value faster than kurtosis
- long relaxation times near T_c

B. Berdnikov, K. Rajagopal PRD61 (2000)

About the critical mode

- At $\mu_B \neq 0$ σ mixes with the net-baryon density n (and e and $\vec{m}$)
- In a Ginzburg-Landau formalism:

$$V(\sigma, n) = \int \mathrm{d}^3x (\sum_m (a_m \sigma^m + b_m n^m) + \sum_{m,l} c_{m,l} \sigma^m n^l) - h\sigma - jn$$

- $V(\sigma, n)$ has a flat direction: $(a\sigma, bn)$ with vanishing curvature $D \to 0$ at the CP
- Equations of motion (including symmetries in $V(\sigma, n)$):

$$\partial_t \sigma = -\Gamma \delta V / \delta \sigma + \ldots \qquad \partial_t n = \gamma \vec{\nabla}^2 \delta V / \delta n + \ldots$$

- eigenfrequencies

$$\omega_1 \propto -i\Gamma a \qquad \to \text{short time scale}$$
$$\omega_2 \propto -i\gamma D / a\, \vec{q}^2 \qquad \to \text{long time scale}$$

- The diffusive mode becomes the critical mode in the long-time dynamics. These fluctuations need to be included at the critical point!

H. Fuji, M. Ohtani PRD70 (2004); M. Stephanov, D. Son PRD70 (2004)