Domain wall networks and hadron properties

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G.V. Efimov, , Ya. Burdanov, M. Deka, B. Galilo, A. Kalloniatis, SN, S. Solunin, V. Tainov, V. Voronin

April 6, 2018

Mean field approach to QCD vacuum and hadron phenomenology motivated by the quantum effective action of QCD.

Almost everywhere Abelian (anti-)selfdual homogeneous gauge fields

R. Feynman *et al* PRD 3 1971; H. Pagels, and E. Tomboulis, NPB 143. 1978;
H. Leutwyler, J. Stern, PLB 73, 1978; PLB 77, 1978;
P. Minkowski, NPB 177, 203 (1981); H. Leutwyler, PLB**96**, 154 (1980);
NPB**179**, 129 (1981);
A. Eichhorn, H. Gies, J. M. Pawlowski, Phys. Rev. D83 (2011).

- Quantum effective action and physical QCD vacuum
- Gluon condensates and domain wall network as QCD vacuum
- The domain model - static characteristics of QCD vacuum
- Effective meson action – bosonization
- Properties of mesons: mass spectrum, decay constants, formfactors
- Strong electromagnetic field as a trigger of deconfinement
- Confinement-deconfinement: heterophase fluctuations

- **Confinement of both static and dynamical quarks** $\longrightarrow$

$$W(C) = \langle \mathrm{Tr}\ \mathrm{P}\ e^{i\int_C dz_\mu \hat{A}_\mu} \rangle$$
$$S(x,y) = \langle \psi(y)\bar{\psi}(x) \rangle$$

- **Dynamical Breaking of chiral** $SU_L(N_f) \times SU_R(N_f)$ **symmetry** $\longrightarrow \langle \bar{\psi}(x)\psi(x) \rangle$

- $U_A(1)$ **Problem** $\longrightarrow \eta'$ (χ, Axial Anomaly)

- **Strong CP Problem** $\longrightarrow Z(\theta)$

- **Colorless Hadron Formation:** $\longrightarrow$ Effective action for colorless collective modes:
  hadron masses,
  form factors, scattering

  Light mesons and baryons, **Regge spectrum** of excited states of light hadrons, **heavy-light** hadrons, **heavy quarkonia**

Deconfinement, chiral symmetry restoration under "extreme" conditions, including strong electromagnetic fields

QCD effective action and vacuum gluon configurations

$$Z = N \int_{\mathcal{F}_B} DA \int_{\Psi} D\psi D\bar{\psi} \exp\{-S[A,\psi,\bar{\psi}]\}$$

$$\mathcal{F}_B = \left\{ A : \lim_{V\to\infty} \frac{1}{V} \int_V d^4x g^2 F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) = B^2 \right\}.$$

H. Leutwyler,
Nucl. Phys. B 179 (1981) 129

L. D. Faddeev,
[arXiv:0911.1013 [math-ph]]

B.V. Galilo and S.N. ,
Phys. Rev. D84 (2011) 094017

$$A^a_\mu = B^a_\mu + Q^a_\mu, \quad D(B)Q = 0$$

$$1 = \int_{\mathcal{B}} DB\Phi[A,B] \int_{\mathcal{Q}} DQ \int_{\Omega} D\omega \delta[A^\omega - Q^\omega - B^\omega]\delta[D(B^\omega)Q^\omega]$$

Q^a_μ – perturbative fluctuations of gluon field in the background vacuum configurations B^a_μ - carriers of nonzero condensates.

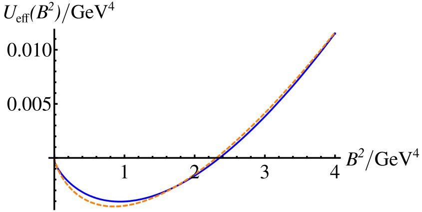
$$Z = N' \int_{\mathcal{B}} DB \int_{\mathcal{Q}} DQ \int_{\Psi} D\psi D\bar{\psi} \det[D(B)D(B+Q)]\delta[D(B)Q] \exp\{-S[B+Q,\psi,\bar{\psi}]\}$$

Particular features of background fields B have yet to be identified by the dynamics of fluctuations:

$$Z = N' \int_{\mathcal{B}} DB \int_{\Psi} D\psi D\bar{\psi} \int_{\mathcal{Q}} DQ \det[D(B)D(B+Q)]\delta[D(B)Q] \exp\{-S[B+Q,\psi,\bar{\psi}]\}$$
$$= N'' \int_{\mathcal{B}} DB \exp\{-S_{\rm eff}[B]\}$$

Global minima of $S_{\rm eff}[B]$ – field configurations that are dominant in the limit $V \to \infty$. Homogeneous Abelian (anti-)self-dual fields are of particular interest.

$$B_\mu = -\frac{1}{2} n B_{\mu\nu} x_\nu, \; \tilde{B}_{\mu\nu} = \pm B_{\mu\nu}$$
$$n = T^3 \ \cos\xi + T^8 \ \sin\xi.$$



H. Pagels, and E. Tomboulis, Nucl. Phys. B 143 (1978) 485
P. Minkowski, Nucl. Phys. B177 (1981) 203
H. Leutwyler, Nucl. Phys. B 179 (1981) 129

A. Eichhorn, H. Gies and J. M. Pawlowski, Phys. Rev. D 83, 045014 (2011)

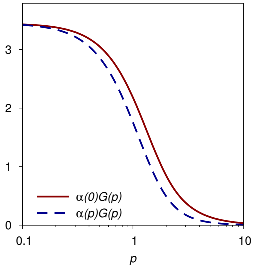
$$G\left(z^2\right) \sim \frac{e^{-Bz^2}}{z^2}, \quad \tilde{G}\left(p^2\right) \sim \frac{1}{p^2}\left(1 - e^{-p^2/B}\right)$$

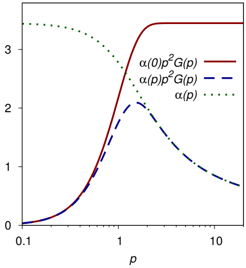
$\Rightarrow$ confinement

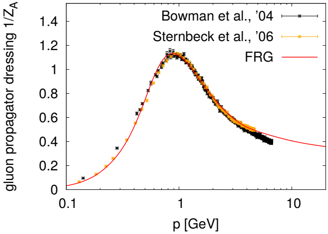
$\Rightarrow$ Regge trajectories

H. Leutwyler, Phys. Lett. B 96 (1980) 154
G.V. Efimov, and S.N. , Phys. Rev. D 51 (1995)

$$\tilde{G}\left(p^2\right) \sim \frac{1}{p^2}\left(1 - e^{-p^2/B}\right)$$







Functional RG, DSE, Lattice QCD

Pure gluodynamics - Ginzburg-Landau approach:

$$\mathcal{L}_{\rm eff} = -\frac{1}{4\Lambda^2}\left(D^{ab}_\nu F^b_{\rho\mu} D^{ac}_\nu F^c_{\rho\mu} + D^{ab}_\mu F^b_{\mu\nu} D^{ac}_\rho F^c_{\rho\nu}\right) - U_{\rm eff}$$
$$U_{\rm eff} = \frac{\Lambda^4}{12}{\rm Tr}\left(C_1 F^2 + \frac{4}{3} C_2 F^4 - \frac{16}{9} C_3 F^6\right),$$

B.V. Galilo, S.N. , Phys. Part. Nucl. Lett., 8 (2011) 67

D. P. George, A. Ram, J. E. Thompson and R. R. Volkas, Phys. Rev. D 87, 105009 (2013) [arXiv:1203.1048 [hep-th]]

where

$$D^{ab}_\mu = \delta^{ab}\partial_\mu - iA^{ab}_\mu = \partial_\mu - iA^c_\mu (T^c)^{ab},$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - i f^{abc} A^b_\mu A^c_\nu,$$
$$F_{\mu\nu} = F^a_{\mu\nu} T^a, \quad T^a_{bc} = -i f^{abc}$$
$$C_1 > 0,\ C_2 > 0,\ C_3 > 0.$$

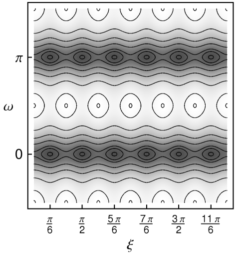
$U_{\rm eff}$ possesses degenerate discrete minima:

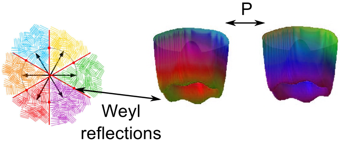
$$B_\mu = -\frac{1}{2} n_k B_{\mu\nu} x_\nu,\ \tilde{B}_{\mu\nu} = \pm B_{\mu\nu},$$

matrix n_k belongs to the Cartan subalgebra of $su(3)$

$$n_k = T^3 \ \cos\left(\xi_k\right) + T^8 \ \sin\left(\xi_k\right), \quad \xi_k = \frac{2k+1}{6}\pi,\ k = 0, 1, \ldots, 5,$$

$$\vec{E}\vec{H} = B^2 \cos(\omega)$$





Domain wall network

ξ, $\langle g^2 F^2 \rangle \longrightarrow$ vacuum values

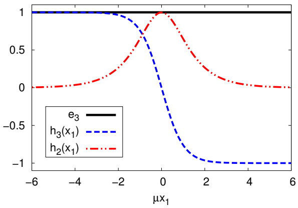
$$\mathcal{L}_{\rm eff} = -\frac{1}{2}\Lambda^2 b_{\rm vac}^2 \partial_\mu \omega \partial_\mu \omega - b_{\rm vac}^4 \Lambda^4 \left(C_2 + 3C_3 b_{\rm vac}^2\right) \sin^2 \omega,$$

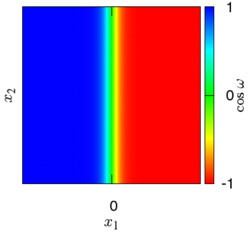
leads to sine-Gordon equation

$$\partial^2 \omega = m_\omega^2 \sin 2\omega, \quad m_\omega^2 = b_{\rm vac}^2 \Lambda^2 \left(C_2 + 3C_3 b_{\rm vac}^2\right),$$

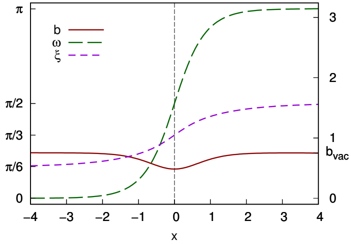
and the standard kink solution

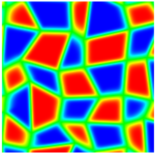
$$\omega(x_\nu) = 2 \,\mathrm{arctg}\left(\exp(\mu x_\nu)\right)$$

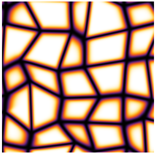




"Domain wall involving the topological charge density (in units of $\langle g^2F^2\rangle$), $su(3)$ angle ξ and the background action density simultaneously"







The general kink configuration can be parametrized as

$$\zeta(\mu_i, \eta^i_\nu x_\nu - q^i) = \frac{2}{\pi} \arctan \exp(\mu_i(\eta^i_\nu x_\nu - q^i)).$$

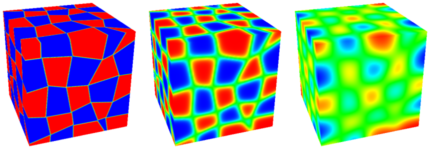
A single lump in two, three and four dimensions is given by

$$\omega(x) = \pi \prod_{i=1}^{k} \zeta(\mu_i, \eta^i_\nu x_\nu - q^i).$$

for $k = 4, 6, 8$, respectively. The general kink network is then given by the additive superposition of lumps

$$\omega = \pi \sum_{j=1}^{\infty} \prod_{i=1}^{k} \zeta(\mu_{ij}, \eta^{ij}_\nu x_\nu - q^{ij})$$

S.N., V.E. Voronin, Eur.Phys.J. A51 (2015) 4



$\langle F^2 \rangle = B^2$
$\langle |F\widetilde{F}| \rangle = B^2$

$\langle F^2 \rangle = B^2$
$\langle |F\widetilde{F}| \rangle \ll B^2$

"Phase transitions and heterophase fluctuations" V. I. Yukalov, Phys. Rep. 208, 396 (1991)

What could stabilize a finite mean size of the domains?

Lower dimensional defects?

Quark (quasi-)zero modes?

Domain bulk - harmonic confinement

Elementary color charged excitations - fluctuations, eigenmodes decay in all four directions.

Eigenvalue problem for scalar field in $\mathbb{R}^4$: *H. Leutwyler, Nucl. Phys. B 179 (1981);*

$$B_\mu = B_{\mu\nu} x_\nu, \widetilde{B}_{\mu\nu} = \pm B_{\mu\nu}, B_{\mu\alpha} B_{\nu\alpha} = B^2 \delta_{\mu\nu}.$$

$$-\left(\partial_\mu - iB_\mu\right)^2 G = \delta \quad \longrightarrow \quad G(x-y) \sim \frac{e^{-B(x-y)^2/4}}{(x-y)^2}$$

$$-\left(\partial_\mu - i\breve{B}_\mu\right)^2 \Phi = \lambda\Phi \quad \longrightarrow \quad \left[\beta_\pm^+ \beta_\pm + \gamma_+^+ \gamma_+ + 1\right] \Phi = \frac{\lambda}{4B}\Phi,$$

$$\beta_\pm = \frac{1}{2}(\alpha_1 \mp i\alpha_2), \quad \gamma_\pm = \frac{1}{2}(\alpha_3 \mp i\alpha_4), \quad \alpha_\mu = \frac{1}{\sqrt{B}} x_\mu + \partial_\mu,$$

$$\beta_\pm^+ = \frac{1}{2}(\alpha_1^+ \pm i\alpha_2^+), \quad \gamma_\pm^+ = \frac{1}{2}(\alpha_3^+ \pm i\alpha_4^+), \quad \alpha_\mu^+ = \frac{1}{\sqrt{B}} x_\mu - \partial_\mu.$$

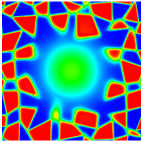
The eigenfunctions and eigenvalues - 4-dim. harmonic oscillator

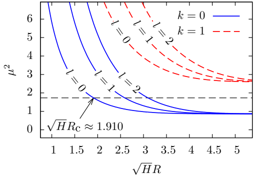
$$\Phi_{nmkl}(x) = \frac{1}{\pi^2 \sqrt{n!m!k!l!}} \left(\beta_+^+\right)^k \left(\beta_-^+\right)^l \left(\gamma_+^+\right)^n \left(\gamma_-^+\right)^m \Phi_{0000}, \ \Phi_{0000} = e^{-\frac{1}{2}Bx^2}$$

$$\lambda_r = 4B(r+1), \ r = k+n \text{ self-dual field}, \ r = l+n \text{ anti-self-dual field}$$

Domain wall junctions - deconfinement

S.N. , V.E. Voronin, Eur.Phys.J. A51 (2015) 4





The color charged scalar field inside junction:

$$-\left(\partial_\mu - i\breve{B}_\mu\right)^2 \Phi = 0, \quad \Phi(x) = 0, \quad x \in \partial\mathcal{T}, \quad \mathcal{T} = \left\{x_1^2 + x_2^2 < R^2, \ (x_3, x_4) \in \mathrm{R}^2\right\}$$

The solutions are quasi-particle excitations

$$\phi^a(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[a^+_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3x_3} + b_{akl}(p_3) e^{-ix_0\omega_{akl} + ip_3x_3}\right] e^{il\vartheta} \phi_{alk}(r),$$

$$\phi^{a\dagger}(x) = \sum_{lk} \int_{-\infty}^{+\infty} \frac{dp_3}{2\pi} \frac{1}{\sqrt{2\omega_{alk}}} \left[b^+_{akl}(p_3) e^{-ix_0\omega_{akl} + ip_3x_3} + a_{akl}(p_3) e^{ix_0\omega_{akl} - ip_3x_3}\right] e^{-il\vartheta} \phi_{alk}(r),$$

$$p_0^2 = p_3^2 + \mu_{akl}^2, \quad p_0 = \pm\omega_{akl}(p_3), \quad \omega_{akl} = \sqrt{p_3^2 + \mu_{akl}^2},$$

$$k = 0, 1, \ldots, \infty, \quad l \in \mathbb{Z},$$

In general near the boundaries

$$\mathrm{div}\vec{H} \neq 0, \quad \mathrm{div}\vec{E} \neq 0$$

The description of the domain walls as well as separation of the Abelian part in the general network in terms of the vector potential requires application of the gauge field parametrization suggested by L.D. Faddeev, A. J. Niemi (2007); K.-I. Kondo, T. Shinohara, T. Murakami(2008); Y.M. Cho (1980, 1981); L.Prokhorov, S.V. Shabanov (1989,1999)
The Abelian part $\hat{V}_\mu(x)$ of the gauge field $\hat{A}_\mu(x)$ is separated manifestly,

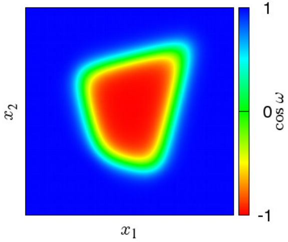
$$\begin{aligned}
\hat{A}_\mu(x) &= \hat{V}_\mu(x) + \hat{X}_\mu(x), \, \hat{V}_\mu(x) = \hat{B}_\mu(x) + \hat{C}_\mu(x), \\
\hat{B}_\mu(x) &= [n^a A^a_\mu(x)]\hat{n}(x) = B_\mu(x)\hat{n}(x), \\
\hat{C}_\mu(x) &= g^{-1}\partial_\mu \hat{n}(x) \times \hat{n}(x), \\
\hat{X}_\mu(x) &= g^{-1}\hat{n}(x) \times \left(\partial_\mu \hat{n}(x) + g\hat{A}_\mu(x) \times \hat{n}(x)\right),
\end{aligned}$$

where $\hat{A}_\mu(x) = A^a_\mu(x)t^a$, $\hat{n}(x) = n_a(x)t^a$, $n^a n^a = 1$, and

$$\partial_\mu \hat{n} \times \hat{n} = i f^{abc}\partial_\mu n^a n^b t^c, \; [t^a, t^b] = i f^{abc} t^c.$$

$$[\hat{V}_\mu(x), \hat{V}_\nu(x)] = 0$$

Both the color and space orientation of the field can become frustrated at the junction location and, thus, develop the singularities in the vector potential. The potential singularities cover the whole range of defects – vortex-like, dyon-like and zero-dimensional instanton-like defects.



Rough estimation!

To mimic finite size of the region of homogeneity of the background field, let us introduce infra-red cutoff s_{IR} both to the quark and glue potentials,

$$U_{\rm eff}(B, s_{IR}) = \Lambda^4 \left\{ B^2 a \ln(bB^2 + s_{IR}^{-2}) + \frac{N_f}{8\pi^2} \int\limits_0^{s_{IR}} \frac{ds}{s^3} \mathrm{Tr}_n \left[s^2 \coth^2(s\hat{n}B) - 1 - \frac{2}{3} s^2 \hat{n}^2 B^2 \right] \right\}.$$

with $a = .00528$ and $b = .433$.

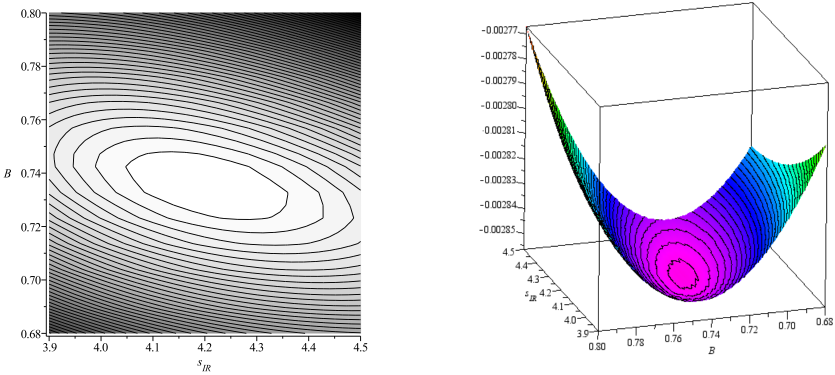


Figure: Effective potential (in units of Λ^4) as a function of angles B (in units of Λ^2) and s_{IR} (in units Λ^{-2}) for $N_f = 3$. One can see that the minimum of the potential is achieved at $B \approx .74$, $s_{IR} \approx 4.2$.

To summarize, integration over background field B in the partition function

$$Z = N' \int\limits_{\mathcal{B}} DB \int\limits_{\Psi} D\psi D\bar{\psi} \int\limits_{\mathcal{Q}} DQ \det[D(B)D(B+Q)]\delta[D(B)Q] \exp\{-S[B+Q,\psi,\bar{\psi}]\}$$

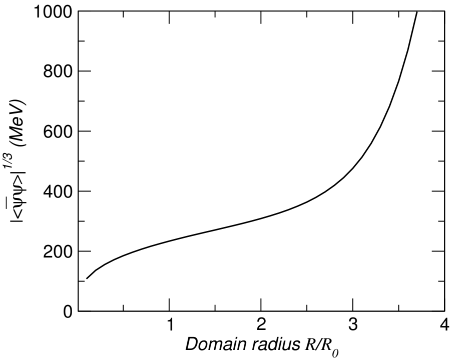
may be defined as the integral over ensemble of specific domain wall networks representing domains with certain mean size and mean gluon field strength.

Testing the model: characteristics of the domain wall network ensemble

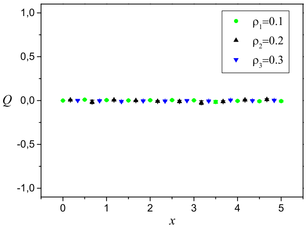
Spherical domains

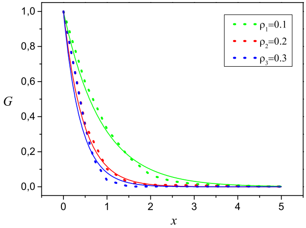
A.C. Kalloniatis and S.N. , Phys. Rev. D 64 (2001); Phys. Rev. D 69 (2004); Phys. Rev. D 71 (2005); Phys. Rev. D 73 (2006), Eur.Phys.J. A51 (2015), arXiv:1603.01447 [hep-ph] (2016)

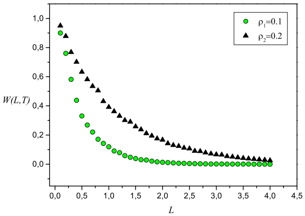
Area law
Spontaneous chiral symmetry breaking
$U_A(1)$ is broken by anomaly
There is no strong CP violation

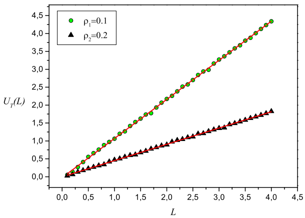


PRELIMINARY!: pure glue, domains - tubes with two finite dimensions (mean topological charge, two-point correlator of top. charge density, Wilson loop and static potential)









P. Olesen,"Confinement and random fluxes", Nucl. Phys. B, Volume 200 (1982) 381-390.

Hadronization

G.V. Efimov and S.N. , Phys. Rev. D 51 (1995); Phys. Rev. D 54 (1996)

A.C. Kalloniatis and S.N. , Phys. Rev. D 64 (2001); Phys. Rev. D 69 (2004); Phys. Rev. D 71 (2005); Phys. Rev. D 73 (2006)

$$\mathcal{Z} = \int dB \int_{\Psi} \mathcal{D}\psi \mathcal{D}\bar{\psi} \int_{\mathcal{Q}} \mathcal{D}Q \delta[D(B)Q] \Delta_{\mathrm{FP}}[B, Q] e^{-S^{\mathrm{QCD}}[Q+B,\psi,\bar{\psi}]} =$$

$$\int dB \int_{\Psi} \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left\{ \int dx \bar{\psi} \left(i \not\partial + g \not{B} - m \right) \psi \right\} W[j]$$

$$W[j] = \exp\left\{ \sum_n \frac{g^n}{n!} \int dx_1 \cdots \int dx_n j^{a_1}_{\mu_1}(x_1) \dots j^{a_n}_{\mu_n}(x_n) G^{a_1 \cdots a_n}_{\mu_1 \cdots \mu_n}(x_1, \dots, x_n | B) \right\}$$

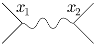
$$j^a_\mu = \bar{\psi} \gamma_\mu t^a \psi,$$

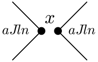
Next step: $W[j]$ is truncated up to the term including two-point gluon correlation function.

$$\mathcal{Z} = \int dB \int_{\Psi} \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left\{ \int dx \bar{\psi} \left(i\partial\!\!\!/ + gB\!\!\!/ - m \right) \psi + \frac{g^2}{2} \int dx_1 dx_2 G^{a_1 a_2}_{\mu_1 \mu_2}(x_1, x_2|B) j^{a_1}_{\mu_1}(x_1) j^{a_2}_{\mu_2}(x_2) \right\}$$

Fierz transform, center of mass coordinates $\longrightarrow \int dz dx G(z|B) J^{aJ}(x,z) J^{aJ}(x,z)$

α_s  $= \alpha_s(0)$  $\left[1 + \Pi^{\mathrm{R}}(p^2)\right]; \ \Pi^{\mathrm{R}}(0) = 0$

0  $z \ \rightarrow \dfrac{e^{-\frac{1}{4}Bz^2}}{4\pi^2 z^2}$ $\qquad \int dx_1\, dx_2$ 

$= \int dx \sum_{aJln}$ 

$\rightarrow \alpha_s(p) \dfrac{1 - \exp(-p^2/B)}{p^2}$

$$J^{aJ}(x,z) = \sum_{nl} \left(z^2\right)^{l/2} f^{nl}_{\mu_1 \ldots \mu_l}(z) J^{aJln}_{\mu_1 \ldots \mu_l}(x), \quad J^{aJln}_{\mu_1 \ldots \mu_l}(x) = \bar{q}(x) V^{aJln}_{\mu_1 \ldots \mu_l} \left(\frac{\overleftrightarrow{D}(x)}{B} \right) q(x),$$

$$f^{nl}_{\mu_1 \ldots \mu_l} = L_{nl}\left(z^2\right) T^{(l)}_{\mu_1 \ldots \mu_l}(n_z), \quad n_z = \frac{z}{\sqrt{z}}.$$

$T^{(l)}_{\mu_1 \ldots \mu_l}$ are irreducible tensors of four-dimensional rotational group

$$\int_0^{\infty} du \rho_l(u) L_{nl}(u) L_{n'l}(u) = \delta_{nn'}, \quad \rho_l(u) = u^l e^{-u} \leftrightarrow \frac{e^{-Bz^2}}{z^2} \quad \text{gluon propagator}$$

Effective meson action for composite colorless fields:

$$Z = \mathcal{N} \lim_{V\to\infty} \int D\Phi_{\mathcal{Q}} \exp\left\{ -\frac{B}{2}\frac{h^2_{\mathcal{Q}}}{g^2 C_{\mathcal{Q}}} \int dx \Phi^2_{\mathcal{Q}}(x) - \sum_k \frac{1}{k} W_k[\Phi] \right\}, \quad \mathcal{Q} = (aJln)$$

$$1 = \frac{g^2 C_{\mathcal{Q}}}{B} \tilde{\Gamma}^{(2)}_{\mathcal{Q}\mathcal{Q}}(-M^2_{\mathcal{Q}}|B), \quad h^{-2}_{\mathcal{Q}} = \frac{d}{dp^2} \tilde{\Gamma}^{(2)}_{\mathcal{Q}\mathcal{Q}}(p^2)|_{p^2=-M^2_{\mathcal{Q}}}.$$

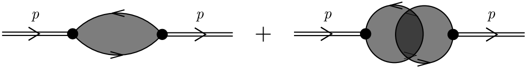
$$W_k[\Phi] = \sum_{\mathcal{Q}_1\dots\mathcal{Q}_k} h_{\mathcal{Q}_1}\dots h_{\mathcal{Q}_k} \int dx_1 \cdots \int dx_k \Phi_{\mathcal{Q}_1}(x_1)\dots\Phi_{\mathcal{Q}_k}(x_k)\Gamma^{(k)}_{\mathcal{Q}_1\dots\mathcal{Q}_k}(x_1,\dots,x_k|B)$$

$$\Gamma^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2} = \overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)} - \Xi_2(x_1-x_2)\overline{G^{(1)}_{\mathcal{Q}_1}G^{(1)}_{\mathcal{Q}_2}},$$

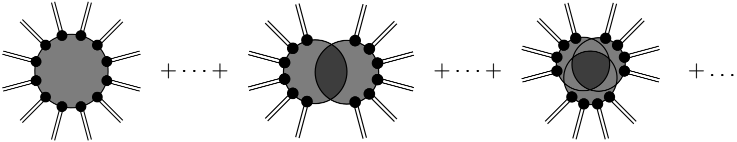
$$\Gamma^{(3)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3} = \overline{G^{(3)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3}(x_1,x_2,x_3)} - \frac{3}{2}\Xi_2(x_1-x_3)\overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)G^{(1)}_{\mathcal{Q}_3}(x_3)}$$
$$+\frac{1}{2}\Xi_3(x_1,x_2,x_3)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(1)}_{\mathcal{Q}_2}(x_2)G^{(1)}_{\mathcal{Q}_3}(x_3)},$$

$$\Gamma^{(4)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4} = \overline{G^{(4)}_{\mathcal{Q}_1\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4}(x_1,x_2,x_3,x_4)} - \frac{4}{3}\Xi_2(x_1-x_2)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(3)}_{\mathcal{Q}_2\mathcal{Q}_3\mathcal{Q}_4}(x_2,x_3,x_4)}$$
$$-\frac{1}{2}\Xi_2(x_1-x_3)\overline{G^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2}(x_1,x_2)G^{(2)}_{\mathcal{Q}_3\mathcal{Q}_4}(x_3,x_4)}$$
$$+\Xi_3(x_1,x_2,x_3)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(1)}_{\mathcal{Q}_2}(x_2)G^{(2)}_{\mathcal{Q}_3\mathcal{Q}_4}(x_3,x_4)}$$
$$-\frac{1}{6}\Xi_4(x_1,x_2,x_3,x_4)\overline{G^{(1)}_{\mathcal{Q}_1}(x_1)G^{(1)}_{\mathcal{Q}_2}(x_2)G^{(1)}_{\mathcal{Q}_3}(x_3)G^{(1)}_{\mathcal{Q}_4}(x_4)}.$$

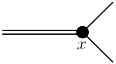
$\Gamma^{(2)}_{\mathcal{Q}_1\mathcal{Q}_2} =$



$\Gamma^{(n)}_{\mathcal{Q}_1\mathcal{Q}_2\ldots\mathcal{Q}_n} =$



Meson-quark vertex operators $\Leftarrow J^{aJln}_{\mu_1\dots\mu_l} = \bar{q}(x)V^{aJln}_{\mu_1\dots\mu_l}q(x)$



$$V^{aJln}_{\mu_1\dots\mu_l}(x) = M^a\Gamma^J \left\{\left\{F_{nl}\left(\frac{\overleftrightarrow{D}^2(x)}{B^2}\right)T^{(l)}_{\mu_1\dots\mu_l}\left(\frac{1}{i}\frac{\overleftrightarrow{D}(x)}{B}\right)\right\}\right\},$$

$$F_{nl}(s) = s^n\int_0^1 dt t^{n+l}\exp(st) = \int_0^1 dt t^{n+l}\frac{\partial^n}{\partial t^n}\exp(st),$$

$$\overleftrightarrow{D}=\overleftarrow{D}\,\xi_{f'} - \overrightarrow{D}\,\xi_f, \xi_f = \frac{m_f}{m_f+m_{f'}}$$

Quark propagator in homogeneous Abelian (anti-)self-dual field

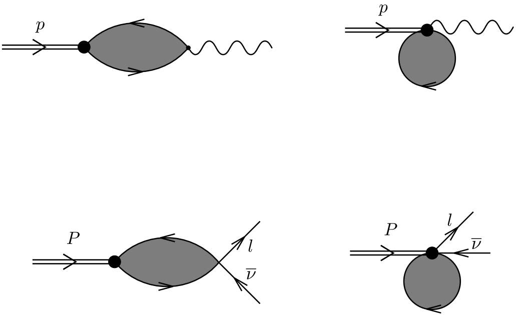
 $=$ $\dfrac{}{m(0)}\left[1+\Sigma^{\mathrm{R}}(p^2)\right]$; $\Sigma^{\mathrm{R}}(0)=0$

$$S(x,y) = \exp\left(-\frac{i}{2}x_\mu B_{\mu\nu}y_\nu\right)H(x-y),$$

$$\widetilde{H}_f(p|B) = \frac{1}{\upsilon B^2}\int_0^1 ds e^{(-p^2/\upsilon B^2)s}\left(\frac{1-s}{1+s}\right)^{m_f^2/2\upsilon B^2}\left[p_\alpha\gamma_\alpha \pm is\gamma_5\gamma_\alpha\frac{B_{\alpha\beta}}{\upsilon B^2}p_\beta + \right.$$
$$\left. + m_f\left(P_\pm + P_\mp\frac{1+s^2}{1-s^2} - \frac{i}{2}\gamma_\alpha\frac{B_{\alpha\beta}}{\upsilon B^2}\gamma_\beta\frac{s}{1-s^2}\right)\right]$$

$$\tilde{H}_f(p|B) = \frac{m}{2\upsilon\Lambda^2}\mathcal{H}_S(p^2) \mp \gamma_5\frac{m}{2\upsilon\Lambda^2}\mathcal{H}_P(p^2) + \gamma_\alpha\frac{p_\alpha}{2\upsilon\Lambda^2}\mathcal{H}_V(p^2) \pm i\gamma_5\gamma_\alpha\frac{f_{\alpha\beta}p_\beta}{2\upsilon\Lambda^2}\mathcal{H}_A(p^2) \qquad (2)$$
$$+\sigma_{\alpha\beta}\frac{mf_{\alpha\beta}}{4\upsilon\Lambda^2}\mathcal{H}_T(p^2).$$

Weak and electromagnetic interactions



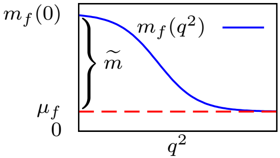
Masses of radially excited mesons

The parameters of the model are

$$\alpha_s\,(0) \quad m_{u/d}(0) \quad m_s(0) \quad m_c(0) \quad m_b(0) \quad B \quad R$$

$$\langle \alpha_s F^2 \rangle = \frac{B^2}{\pi} \quad \chi_{\rm YM} = \frac{B^4 R^4}{128\pi^2}$$

Dynamical chiral symmetry breaking:



$$\widetilde{m} = 136 \text{ MeV}$$

$$\mu_{u/d} = m_{u/d} - \widetilde{m}$$

$$\mu_s = m_s - \widetilde{m}$$

$$\frac{\mu_s}{\mu_{u/d}} = 26.7$$

$$\Lambda^2 \Phi^{(0)}_{\mathcal{Q}_1} = \sum_{k=1}^{\infty} \frac{g^k}{k} \sum_{\mathcal{Q}_1 \ldots \mathcal{Q}_k} \Phi^{(0)}_{\mathcal{Q}_2} \ldots \Phi^{(0)}_{\mathcal{Q}_k} \Gamma^{(k)}_{\mathcal{Q}_1 \ldots \mathcal{Q}_k},$$

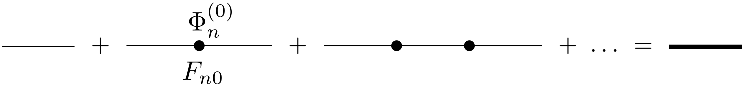


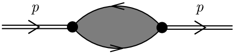
Figure: Mass corrections to the quark propagator due to the constant scalar condensates $\Phi_n^{(0)}$ coupled to nonlocal form factor F_{n0}. Summation over the radial number n is assumed.

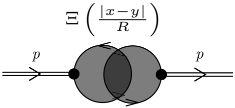
Asymptotic Regge spectrum :

$$M_n^2 \sim Bn,\ n \gg 1$$
$$M_l^2 \sim Bl,\ l \gg 1$$

G.V. Efimov and S.N. , Phys. Rev. D 51 (1995)

η and η'!





Polarization operator

Polarization operation for $l=0$:

$$\Pi_J^{nn'}\left(-M^2; m_f, m_{f'}; B\right) =$$

$$\frac{B}{4\pi^2}\mathrm{Tr}_\upsilon \int\limits_0^1 dt_1 \int\limits_0^1 dt_2 \int\limits_0^1 ds_1 \int\limits_0^1 ds_2 \left(\frac{1-s_1}{1+s_1}\right)^{m_f^2/4\upsilon B} \left(\frac{1-s_2}{1+s_2}\right)^{m_{f'}^2/4\upsilon B} \times$$

$$\times\, t_1^n t_2^{n'} \frac{\partial^n}{\partial t_1^n}\frac{\partial^{n'}}{\partial t_2^{n'}}\frac{1}{\Phi_2^2}\left[\frac{M^2}{B}\frac{F_1^{(J)}}{\Phi_2^2} + \frac{m_f m_{f'}}{B}\frac{F_2^{(J)}}{(1-s_1^2)(1-s_2^2)} + \frac{F_3^{(J)}}{\Phi_2}\right]\exp\left\{\frac{M^2}{2\upsilon B}\frac{\Phi_1}{\Phi_2}\right\}.$$

$$\Phi_1 = s_1 s_2 + 2\left(\xi_1^2 s_1 + \xi_2^2 s_2\right)(t_1+t_2)\upsilon,$$

$$\Phi_2 = s_1 + s_2 + 2(1+s_1 s_2)(t_1+t_2)\upsilon + 16(\xi_1^2 s_1 + \xi_2^2 s_2)t_1 t_2 \upsilon^2,$$

$$F_1^{(P)} = (1+s_1 s_2)\left[2(\xi_1 s_1 + \xi_2 s_2)(t_1+t_2)\upsilon + \right.$$
$$\left. 4\xi_1\xi_2(1+s_1 s_2)(t_1+t_2)^2\upsilon^2 + s_1 s_2(1-16\xi_1\xi_2 t_1 t_2\upsilon^2)\right],$$

$$F_1^{(V)} = \left(1-\frac{1}{3}s_1 s_2\right)\left[s_1 s_2 + 16\xi_1\xi_2 t_1 t_2 \upsilon^2 + 2(\xi_1 s_1 + \xi_2 s_2)(t_1+t_2)\upsilon\right] +$$
$$4\xi_1\xi_2(1-s_1^2 s_2^2)(t_1-t_2)^2\upsilon^2,$$

$$F_2^{(P)} = (1+s_1 s_2)^2, \quad F_2^{(V)} = (1-s_1^2 s_2^2),$$

$$F_3^{(P)} = 4\upsilon(1+s_1 s_2)(1-16\xi_1\xi_2 t_1 t_2\upsilon^2),\; F_3^{(V)} = 2\upsilon(1-s_1 s_2)(1-16\xi_1\xi_2 t_1 t_2\upsilon^2).$$

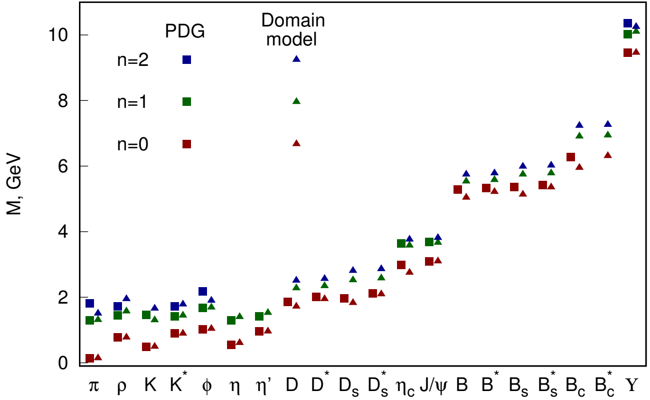

M, GeV
PDG
Domain model
n=2
n=1
n=0
0
2
4
6
8
10
π ρ K K* ϕ η η' D D* Ds Ds* ηc J/ψ B B* Bs Bs* Bc Bc* Υ

Table: Model parameters fitted to the masses of $\pi, \rho, K, K^*, \eta', J/\psi, \Upsilon$ and used in calculation of all other quantities.

$m_{u/d}$, MeV	m_s, MeV	m_c, MeV	m_b, MeV	Λ, MeV	α_s	R, fm
145	376	1566	4879	416	3.45	1.12

Table: Masses of light mesons. $\tilde{M}$ denotes the value in the chiral limit.

Meson	n	$M_{\rm exp}$ (MeV)	M (MeV)	$\widetilde{M}$ (MeV)	Meson	n	$M_{\rm exp}$ (MeV)	M (MeV)	$\widetilde{M}$ (MeV)
π	0	140	140	0	ρ	0	775	775	769
$\pi(1300)$	1	1300	1310	1301	$\rho(1450)$	1	1450	1571	1576
$\pi(1800)$	1	1812	1503	1466	ρ	2	1720	1946	2098
K	0	494	494	0	K^*	0	892	892	769
$K(1460)$	1	1460	1302	1301	$K^*(1410)$	1	1410	1443	1576
K	2		1655	1466	$K^*(1717)$	1	1717	1781	2098
η	0	548	621	0	ω	0	775	775	769
η'	0	958	958	872	ϕ	0	1019	1039	769
$\eta(1295)$	1	1294	1138	1361	$\phi(1680)$	1	1680	1686	1576
$\eta(1475)$	1	1476	1297	1516	ϕ	2	2175	1897	2098

Table: Masses of heavy-light mesons and their lowest radial excitations .

Meson	n	M_{exp} (MeV)	M (MeV)	Meson	n	M_{exp} (MeV)	M (MeV)
D	0	1864	1715	D^*	0	2010	1944
D	1		2274	D^*	1		2341
D	2		2508	D^*	2		2564
D_s	0	1968	1827	D_s^*	0	2112	2092
D_s	1		2521	D_s^*	1		2578
D_s	2		2808	D_s^*	2		2859
B	0	5279	5041	B^*	0	5325	5215
B	1		5535	B^*	1		5578
B	2		5746	B^*	2		5781
B_s	0	5366	5135	B_s^*	0	5415	5355
B_s	1		5746	B_s^*	1		5783
B_s	2		5988	B_s^*	2		6021
B_c	0	6277	5952	B_c^*	0		6310
B_c	1		6904	B_c^*	1		6938
B_c	2		7233	B_c^*	2		7260

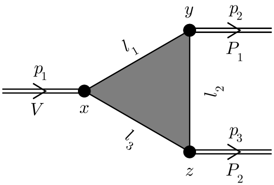
Table: Masses of heavy quarkonia.

Meson	n	M_{exp} (MeV)	M (MeV)
$\eta_c(1S)$	0	2981	2751
$\eta_c(2S)$	1	3639	3620
η_c	2		3882
$J/\psi(1S)$	0	3097	3097
$\psi(2S)$	1	3686	3665
$\psi(3770)$	2	3773	3810
$\Upsilon(1S)$	0	9460	9460
$\Upsilon(2S)$	1	10023	10102
$\Upsilon(3S)$	2	10355	10249

Table: Decay and transition constants of various mesons

Meson	n	$f_P^{\exp}$ (MeV)	f_P (MeV)	Meson	n	$g_{V\gamma}^{\exp}$	$g_{V\gamma}$
π	0	130	140	ρ	0	0.2	0.2
$\pi(1300)$	1	—	29	ρ	1		0.034
K	0	156	175	ω	0	0.059	0.067
$K(1460)$	1	—	27	ω	1		0.011
D	0	205	212	ϕ	0	0.074	0.069
D	1	—	51	ϕ	1		0.025
D_s	0	258	274	J/ψ	0	0.09	0.057
D_s	1	—	57	J/ψ	1		0.024
B	0	191	187	Υ	0	0.025	0.011
B	1	—	55	Υ	1		0.0039
B_s	0	253	248				
B_s	1	—	68				
B_c	0	489	434				
B_c	1		135				

Stong decays: g_{VPP}



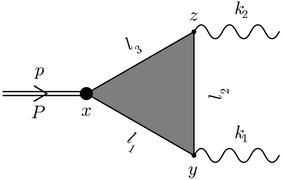
Decay	g_{VPP} [*]	g_{VPP}
$\rho^0 \to \pi^+\pi^-$	5.95	7.58
$\omega \to \pi^+\pi^-$	0.17	0
$K^{*\pm} \to K^\pm\pi^0$	3.23	3.54
$K^{*\pm} \to K^0\pi^\pm$	4.57	5.01
$\varphi \to K^+K^-$	4.47	5.02
$D^{*\pm} \to D^0\pi^\pm$	8.41	7.9
$D^{*\pm} \to D^\pm\pi^0$	5.66	5.59

Background field color gauge invariance

[*] K.A. Olive et al. (Particle Data Group) Chinese Phys. C 38,090001, 2014

Pion transition form factor

$$T_a^{\mu\nu}(x,y,z) = h_P \sum_n u_n^a \int d\sigma_B \mathrm{Tr}\ t_a e_f^2 V^n(x)\gamma_5 S(x,y|B)\gamma_\mu S(y,z|B)\gamma_\nu S(z,x|B),$$



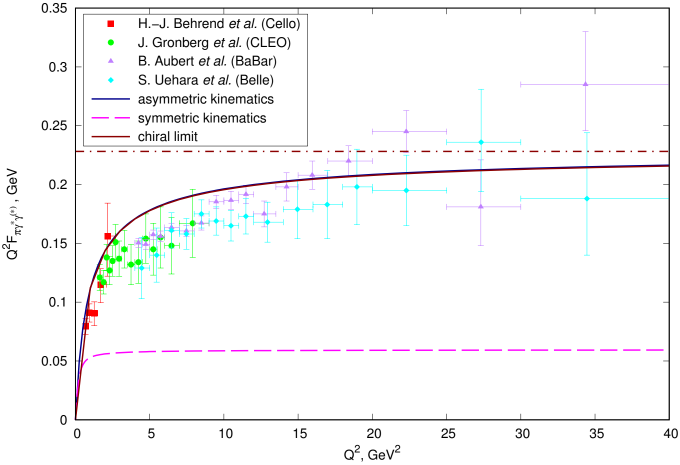
In momentum representation, the diagram has the following structure:

$$T_a^{\mu\nu}(p^2,k_1^2,k_2^2) = ie^2\delta^{(4)}(p-k_1-k_2)\varepsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}T_a(p^2,k_1^2,k_2^2).$$

$$F_{P\gamma}\left(Q^2\right) = T\left(-M_P^2, Q^2, 0\right).$$

$$\Gamma(P\to\gamma\gamma) = \frac{\pi}{4}\alpha^2 M_P^3 g_{P\gamma\gamma}^2$$

$$g_{P\gamma\gamma} = T(-M_P^2,0,0) = F_{P\gamma}(0).$$



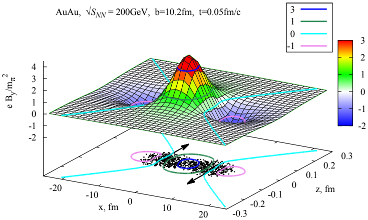
$$g_{\pi\gamma\gamma} = 0.272\mathrm{GeV}^{-1} \qquad (g^{\mathrm{exp}}_{\pi\gamma\gamma} = 0.274\mathrm{GeV}^{-1}).$$

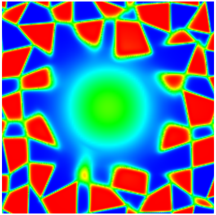
$$F_{\pi\gamma^*\gamma^*}\left(Q^2\right) = T\left(-M_P^2, Q^2, Q^2\right).$$

"Polarization of QCD vacuum by the strong electromagnetic fields"

- **Relativistic heavy ion collisions - strong electromagnetic fields**

V. Skokov, A. Y. Illarionov and V. Toneev, Int. J. Mod. Phys. A **24** *(2009) 5925*
V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya,
V. P. Konchakovski and S. A. Voloshin, Phys. Rev C 84 (2011)





Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies!

One-loop quark contribution to the effective potential in the presence of arbitrary homogenous Abelian fields

$$U_{\rm eff}(G) = -\frac{1}{V}\ln\frac{\det(i\not{D}-m)}{\det(i\not{\partial}-m)} = \frac{1}{V}\int\limits_V d^4x\mathtt{Tr}\int\limits_m^\infty dm'\left[S(x,x|m') - S_0(x,x|m')\right]|$$

$$U_{\rm eff}^{\rm ren}(G) = \frac{B^2}{8\pi^2}\int\limits_0^\infty \frac{ds}{s^3}\mathtt{Tr}_n\left[s\varkappa_+\coth(s\varkappa_+)s\varkappa_-\coth(s\varkappa_-) - \mathbf{1} - \frac{s^2}{3}(\varkappa_+^2+\varkappa_-^2)\right]e^{-\frac{m^2}{B}s},$$

$$\varkappa_\pm = \frac{1}{2B}\sqrt{\mathcal{Q}\sigma_\pm} = \frac{1}{2B}\left(\sqrt{2(\mathcal{R}+\mathcal{Q})}\pm\sqrt{2(\mathcal{R}-\mathcal{Q})}\right),$$

$$\mathcal{R} = (H^2-E^2)/2 + \hat{n}^2B^2 + \hat{n}B(H\cos(\theta) + iE\cos(\chi)\sin(\xi))$$

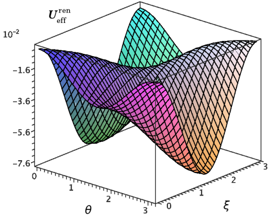
$$\mathcal{Q} = \hat{n}BH\cos(\xi) + i\hat{n}BE\sin(\theta)\cos(\phi) + \hat{n}^2B^2(\sin(\theta)\sin(\xi)cos(\phi-\chi) + cos(\theta)cos(\xi))$$

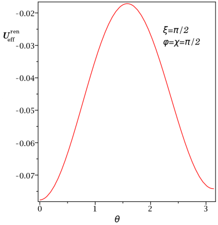
Y. M. Cho and D. G. Pak, Phys.Rev. Lett., 6 (2001) 1047

$H \neq 0$, $E \neq 0$ and arbitrary gluon field

$$\Im(U_{\rm eff}) = 0 \Longrightarrow \cos(\chi)\sin(\xi) = 0,\ \sin(\theta)\cos(\phi) = 0$$

Effective potential (in units of $B^2/8\pi^2$) for the electric $E = .5B$ and the magnetic $H = .9B$ fields as functions of angles θ and ξ ($\phi = \chi = \pi/2$)





Minimum is at $\theta = \pi$ and $\xi = \pi/2$:
orthogonal to each other chromomagnetic and chromoelectric fields: $\mathcal{Q} = 0$.
Strong electro-magnetic field plays catalyzing role for deconfinement and anisotropies?!

B.V. Galilo and S.N., Phys. Rev. D84 (2011) 094017.
M. D'Elia, M. Mariti and F. Negro, Phys. Rev. Lett. **110**, 082002 (2013)
G. S. Bali, F. Bruckmann, G. Endrodi, F. Gruber and A. Schaefer, JHEP **1304**, 130 (2013)

Summary

$\langle g^2F^2\rangle \neq 0 \longrightarrow$ domain wall network, almost everywhere abelian (anti-)self-dual gluon fields.

An ensemble of almost everywhere Abelian homogeneous (anti-)self-dual gluon fields represented by the domain wall networks looks like a suitable framework for studying mechanisms of confinement, chiral symmetry realisation and hadronization.

Background of domain wall networks - harmonic confinement.

(Anti-)self-duality - quark zero mode driven realization of chiral symmetry.

Quark and gluon propagators - qualitative agreement with FRG and DSE.

Meson effective action - quantitatively correct phenomenology both with respect to confinement and chiral symmetry.

Polarization effects in QCD vacuum due to the strong electromagnetic fields, deconfinement, chiral symmetry restoration.
Electromagnetic fields as trigger of deconfinemen.