The Refined Gribov-Zwanziger scenario beyond the Landau gauge

Antônio D. Pereira
Institute for Theoretical Physics - University of Heidelberg

April 4, 2018

In collaboration with:

M. A. L. Capri, D. Dudal, D. Fiorentini, M. S. Guimaraes, I. F. Justo, B. W. Mintz, L. F. Palhares, R. F. Sobreiro and S. P. Sorella.

Eur.Phys.J. C75 (2015) no.10, 479
Phys.Rev. D92 (2015) no.4, 045039
Phys.Rev. D93 (2016) no.6, 065019
Phys.Rev. D94 (2016) no.2, 025035
Eur.Phys.J. C76 (2016) no.10, 528
Annals Phys. 376 (2017) 40-62
Phys.Rev. D95 (2017) no.4, 045011
Eur.Phys.J. C77 (2017) no.8, 546
Phys.Rev. D96 (2017) no.5, 054022
1802.04582 (to appear in PLB)

Outline

1. Overview of the Gribov problem
2. The Gribov-Zwanziger solution
3. BRST symmetry
4. Extension to linear covariant gauges and more
5. Conclusions

Overview of the Gribov problem

- Yang-Mills theories are invariant under *gauge* transformations.
- It implies that for a given field configuration A^a_μ, there are infinitely many equivalent configurations connected through gauge transformations.
- Possible treatment of such redundancy: gauge-fixing.
- In the path integral formulation we employ the Faddeev-Popov procedure, which seems to work quite well at the perturbative level.

The Faddeev-Popov procedure

Choosing the gauge condition $F[A] = 0$, we rewrite the path integral in flat Euclidean space-time (and $SU(N)$ gauge group) as

$$\mathcal{Z} = \int \mathcal{D}g\mathcal{D}A\delta\left[F[A]\right]\Delta_{\rm FP}\ \mathrm{e}^{-S_{\rm YM}}\,, \tag{1}$$

where $\Delta_{\rm FP}$ is the Faddeev-Popov determinant. To do so, we assumed:

- $F[A] = 0$ has a unique solution *i.e.*, for each gauge orbit, the gauge condition picks just one representative.
- $\Delta_{\rm FP}$ does not contain zero-modes and is positive, so that we can eliminate the absolute value.

To check if the previous assumptions are reasonable for a specific gauge choice, we start with Landau gauge, namely, $\partial_\mu A^a_\mu = 0$.

Uniqueness

- Let us consider a gauge field configuration A^a_μ which satisfies the Landau gauge condition $\partial_\mu A^a_\mu = 0$.
- Now, we perform a gauge transformation on $A^a_\mu \to A'^a_\mu$.
- If the gauge condition is ideal, then $\partial_\mu A'^a_\mu \neq 0$.
- However, if we restrict ourselves to infinitesimal gauge transformations, $A'^a_\mu = A^a_\mu - D^{ab}_\mu \xi^b$, with $D^{ab}_\mu = \delta^{ab}\partial_\mu - gf^{abc}A^c_\mu$ being the covariant derivative in the adjoint representation of the gauge group and ξ^b, the infinitesimal parameter of the transformation, we see that

$$\partial_\mu A'^a_\mu = 0 \;\;\Rightarrow\;\; \underbrace{-\partial_\mu D^{ab}_\mu}_{\textit{FP operator}}\,\xi^b = -(\delta^{ab}\partial^2 - gf^{abc}A^c_\mu\partial_\mu)\xi^b = 0\,. \tag{2}$$

- *Therefore, at the infinitesimal level, the gauge condition selects one representative per orbit if the FP operator does not develop zero-modes.*

WARNING!

- In his seminal paper, V. Gribov proved $-\partial_\mu D^{ab}_\mu$ has zero-modes. Gribov '78
- The presence of zero-modes tells us that Landau gauge is not ideal. So, we have a residual gauge freedom.
- Such configurations, present after the imposition of the gauge condition, are known as Gribov copies and their appearance define the so-called Gribov problem.
- To implement the Faddeev-Popov procedure consistently, we should be able to remove these copies.

The Gribov region

Gribov's proposal

- Gribov pointed out the existence of copies, but *also* introduced a way to eliminate them!
- The idea is simple: We should define a suitable region in field space where the Faddeev-Popov operator is positive and, thus, does not develop zero-modes and contains all physical configurations ($\Rightarrow$ All gauge orbits must cross this region) and restrict the path integral domain to this region.
- A first proposal is known as the Gribov region and is defined by

$$\Omega = \left\{ A^a_\mu, \; \partial_\mu A^a_\mu = 0 \middle| \; -\partial_\mu D^{ab}_\mu > 0 \right\} . \tag{3}$$

Properties of Ω

- **Remark 1:** The operator $-\partial_\mu D^{ab}_\mu$, in the Landau gauge, is Hermitian. This makes the definition of a region where it is positive a meaningful task. Dell'Antonio and Zwanziger '91
- The Gribov region Ω has very nice geometrical features: *(i) It is bounded in every direction, (ii) it is convex and (iii) All gauge orbits cross it.*
- Therefore, Ω is a suitable candidate to implement Gribov's idea.
- **Remark 2:** This region is NOT free of all Gribov copies, but at least of all infinitesimal ones. van Baal '92

The action free of (infinitesimal) copies

- Imposing the restriction to Ω, it is possible to lift the modification in the path integral measure to the action.
- The new action is known as the Gribov-Zwanziger action and is given by Zwanziger '89, Vandersickel and Zwanziger '12

$$S_{\mathrm{GZ}} = S + \gamma^4 H(A) - dV\gamma^4(N^2-1)\,, \tag{4}$$

where γ is the so-called Gribov parameter and H, the horizon function,

$$H(A) = g^2 \int d^dx d^dy \; f^{abc} A^b_\mu(x) \left[\mathcal{M}^{-1}(x,y)\right]^{ad} f^{dec} A^e_\mu(y)\,, \tag{5}$$

where $\mathcal{M}^{ab} \equiv -\partial_\mu D^{ab}_\mu$ is the FP operator.

- The Gribov parameter is not free and is fixed by a gap equation,

$$\langle H(A)\rangle = dV(N^2-1)\,. \tag{6}$$

- The horizon function is non-local $\Rightarrow$ the Gribov-Zwanziger action is non-local!

Local Gribov-Zwanziger action

- Remarkably, the Gribov-Zwanziger action can be cast in local form by the introduction of auxiliary fields. Zwanziger '89
- One introduces a pair of bosonic $(\varphi^{ab}_\mu, \bar{\varphi}^{ab}_\mu)$ and fermionic fields $(\omega^{ab}_\mu, \bar{\omega}^{ab}_\mu)$.
- The local Gribov-Zwanziger action is

$$S_{\rm GZ} = S + \int d^dx \, \left(\bar{\varphi}^{ac}_\mu \mathcal{M}^{ab} \varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu \mathcal{M}^{ab} \omega^{bc}_\mu + g\gamma^2 f^{abc} A^a_\mu (\varphi + \bar{\varphi})^{bc}_\mu \right) - dV\gamma^4 (N^2 - 1)\,. \tag{7}$$

- This action is local and renormalizable to all orders in perturbation theory. Zwanziger '89

- On top of the Gribov-Zwanziger framework, one can take into account further non-perturbative effects as the formation of condensates. Dudal, Gracey, Sorella, Vandersickel and Verschelde '08
- Taking into account those effects leads to the *Refined Gribov-Zwanziger* action, which reproduces a *decoupling like* gluon propagator.
- The Refined Gribov-Zwanziger action expressed as

$$S_{\mathrm{RGZ}} = S_{\mathrm{GZ}} + \frac{m^2}{2}\int d^dx\, A^a_\mu A^a_\mu - M^2 \int d^dx \left(\bar{\varphi}^{ab}_\mu \varphi^{ab}_\mu - \bar{\omega}^{ab}_\mu \omega^{ab}_\mu\right) .$$

BRST symmetry

FP action

- A very important concept in the FP quantization is the invariance of the gauge fixed Yang-Mills action under BRST transformations.
- This symmetry plays a very important role in the proof of perturbative renormalizability of the FP action and perturbative unitarity.

- The BRST transformations are given by

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b\,, & sc^a &= \frac{g}{2}f^{abc}c^b c^c\,, \\
s\bar{c}^a &= b^a\,, & sb^a &= 0\,,
\end{aligned} \tag{8}
$$

with $s^2 = 0$ and

$$
sS = s(S_{\mathrm{YM}} + S_{\mathrm{gf}} + S_{\mathrm{ghosts}}) = 0\,. \tag{9}
$$

Considering the local form of the Gribov-Zwanziger action, we have the following transformations,

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b\,, & sc^a &= \frac{g}{2} f^{abc} c^b c^c\,, \\
s\bar{c}^a &= b^a\,, & sb^a &= 0\,, \\
s\varphi^{ab}_\mu &= \omega^{ab}_\mu\,, & s\omega^{ab}_\mu &= 0\,, \\
s\bar{\omega}^{ab}_\mu &= \bar{\varphi}^{ab}_\mu\,, & s\bar{\varphi}^{ab}_\mu &= 0\,.
\end{aligned}
\tag{10}
$$

and

$$
sS_{\mathrm{GZ}} = \gamma^2 \int d^d x \, \left(g f^{abc} D^{ae}_\mu c^e (\bar{\varphi}^{bc}_\mu + \varphi^{bc}_\mu) + g f^{abc} A^a_\mu \omega^{bc}_\mu \right) \,. \tag{11}
$$

- *The Gribov-Zwanziger action breaks BRST symmetry explicitly!*
- Although explicit, the breaking is *soft*. In the UV, $\gamma \to 0$ and we recover BRST invariance.
- However, when γ is not negligible, *i.e.*, when we are far from the UV regime, standard BRST symmetry seems to be broken within the Gribov-Zwanziger scenario. Cucchieri, Dudal, Lavrov, Lechtenfeld, Kondo, Maggiore, Mendes, Reshetnyak, Serreau, Schaden, Sorella, Tissier, Tresmontant, Vandersickel, Zwanziger,...

Gauge invariant A^h field

- Let us consider the transverse field A^h, $\partial_\mu A^h_\mu = 0$, obtained from the minimization of

$$\mathrm{Tr}\int d^dx\, A^U_\mu A^U_\mu\,. \tag{12}$$

Zwanziger '90, M. Lavelle and D. McMullan '97

- This field is gauge invariant order by order in g and can be formally written as

$$A^h_\mu = \left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)\left(A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu\frac{1}{\partial^2}\partial A\right]\right) + O(A^3)\,. \tag{13}$$

- Its gauge invariance implies $sA^h = 0$.
- The form of the horizon function $H(A)$ and of A^h allow us to write the following expression Phys.Rev. D92 (2015) no.4, 045039

$$H(A) = H(A^h) - R(A)(\partial A)\,. \tag{14}$$

The "new" Gribov-Zwanziger action

The non-local Gribov-Zwanziger action is rewritten as

$$\tilde{S}_{\mathrm{GZ}} = S_{\mathrm{YM}} + \int d^dx \left(b^{h,a}\partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right) + \gamma^4 H(A^h)\,, \tag{15}$$

with

$$b^{h,a} = b^a - \gamma^4 R^a(A) \tag{16}$$

and

$$H(A^h) = g^2 \int d^dx d^dy\; f^{abc} A^{h,b}_\mu(x) \left[\mathcal{M}^{-1}(A^h)\right]^{ad} f^{dec} A^{h,e}_\mu(y)\,. \tag{17}$$

As before, we introduce the localizing auxiliary fields. The resulting action is

$$\begin{aligned} S_{\mathrm{GZ}} &= S_{\mathrm{YM}} + \int d^dx \left(b^{h,a}\partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right) \\ &+ \int d^dx \left(\bar{\varphi}^{ac}_\mu \left[\mathcal{M}(A^h)\right]^{ab} \varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu \left[\mathcal{M}(A^h)\right]^{ab} \omega^{bc}_\mu + g\gamma^2 f^{abc} A^{h,a}_\mu (\varphi + \bar{\varphi})^{bc}_\mu \right)\,. \end{aligned} \tag{18}$$

BRST invariance

- We define the BRST transformations, as follows

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b\,, & sc^a &= \frac{g}{2} f^{abc} c^b c^c\,,\\
s\bar{c}^a &= b^{h,a}\,, & sb^{h,a} &= 0\,,\\
s\varphi^{ab}_\mu &= 0\,, & s\omega^{ab}_\mu &= 0\,,\\
s\bar{\omega}^{ab}_\mu &= 0\,, & s\bar{\varphi}^{ab}_\mu &= 0\,.
\end{aligned}
\tag{19}
$$

- These transformations define a symmetry of the Gribov-Zwanziger action, namely,

$$
sS_{\mathrm{GZ}} = 0\,. \tag{20}
$$

- However, due to the presence of A^h, S_{GZ} is still a non-local action.

An important check is that

$$\frac{\partial S_{GZ}}{\partial \gamma^2} \neq s(\text{something})\,, \tag{21}$$

i.e., the Gribov parameter is not akin to a gauge parameter. Therefore, it will enter in correlation functions of physical quantities. Also, written in terms of A^h, the equation that fixes γ, the gap equation, is

$$\langle H(A^h)\rangle = dV(N^2-1)\,, \tag{22}$$

which is *gauge invariant!*

Local action

It is possible to cast the BRST-invariant action in local form through the introduction of a Stueckelberg-like field ξ. PRD 94 (2016) no.2, 025035, PRD 95 (2017) no.4, 045011, PRD 96 (2017) no.5, 054022

$$
\begin{aligned}
S_{\mathrm{GZ}} &= S_{\mathrm{YM}} + S_{\mathrm{FP}} - \int d^4x \left(\bar{\varphi}^{ac}_{\mu}\mathcal{M}^{ab}(A^h)\varphi^{bc}_{\mu} - \bar{\omega}^{ac}_{\mu}\mathcal{M}^{ab}(A^h)\omega^{bc}_{\mu}\right) \\
&- \gamma^2 \int d^4x\, gf^{abc}(A^h)^a_{\mu}(\varphi+\bar{\varphi})^{bc}_{\mu} + \int d^4x\, \tau^a \partial_{\mu}(A^h)^a_{\mu} - \int d^4x\, \bar{\eta}^a \mathcal{M}^{ab}(A^h)\eta^b
\end{aligned}
\tag{23}
$$

The field A^h is now written as

$$
A^h_{\mu} = h^{\dagger} A_{\mu} h + \frac{i}{g} h^{\dagger} \partial_{\mu} h\,, \tag{24}
$$

with

$$
h = \mathrm{e}^{ig\xi^a T^a} \equiv \mathrm{e}^{ig\xi}\,. \tag{25}
$$

Note: The GZ action is written in local form, but it is not polynomial on the local fields.

The complete BRST transformations are given by

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b\,, & sc^a &= \frac{g}{2}f^{abc}c^bc^c\,,\\
s\bar{c}^a &= b^a\,, & sb^a &= 0\,,\\
s\varphi^{ab}_\mu &= 0\,, & s\omega^{ab}_\mu &= 0\,,\\
s\bar{\omega}^{ab}_\mu &= 0\,, & s\bar{\varphi}^{ab}_\mu &= 0\,,\\
sh^{ij} &= -igc^a(T^a)^{ik}h^{kj}\,, & sA^{h,a}_\mu &= 0\,,\\
s\tau^a &= 0\,, & s\bar{\eta}^a &= 0\,,\\
s\eta^a &= 0\,, & s^2 &= 0\,.
\end{aligned}
\tag{26}
$$

The Gribov-Zwanziger expressed in the new variables in local, renormalizable to all orders in perturbation theory and BRST-symmetric. Also, for gauge-invariant operators $\mathcal{O}$ one has

$$
\langle\mathcal{O}(x)\mathcal{O}(y)\rangle\Big|^{\text{Landau}}_{\text{Original GZ}} = \langle\mathcal{O}(x)\mathcal{O}(y)\rangle\Big|^{\text{Landau}}_{\text{“New” GZ}}
\tag{27}
$$

Linear covariant gauges

Linear covariant gauges

- This class of gauges contains a gauge parameter α and is defined by

$$\partial_\mu A^a_\mu = -i\alpha b^a \,. \tag{28}$$

- In this case, the construction of a Gribov region is not as clear as in the Landau gauge.
- For general α (different from zero), the Faddeev-Popov operator is not Hermitian.

Proposal

- We propose to restrict the path integral to Ω^h,

$$\Omega^h = \left\{ A^a_\mu \;\middle|\; \partial_\mu A^a_\mu = -i\alpha b^a \,, \;\; \mathcal{M}^{ab}(A^h) > 0 \right\} \,. \tag{29}$$

- This region is free of "regular" zero-modes and

$$\lim_{\alpha\to 0} \Omega^h = \Omega^{\mathrm{Landau}} \tag{30}$$

- In view of the results discussed before in the Landau gauge, we extend our results to linear covariant gauges. PRD 92 (2015) no.4, 045039, PRD 93 (2016) no.6, 065019,

Gribov-Zwanziger action in linear covariant gauges

The BRST symmetric Gribov-Zwanziger action in linear covariant gauges is constructed as PRD 96 (2017) no.5, 054022

$$\begin{aligned} S^{\mathrm{LCG}}_{\mathrm{GZ}} &= S^{\mathrm{FP}}_{\mathrm{LCG}} - \int d^4x \left(\bar{\varphi}^{ac}_{\mu}\mathcal{M}^{ab}(A^h)\varphi^{bc}_{\mu} - \bar{\omega}^{ac}_{\mu}\mathcal{M}^{ab}(A^h)\omega^{bc}_{\mu}\right) \\ &- \gamma^2 \int d^4x\, gf^{abc}(A^h)^a_{\mu}(\varphi+\bar{\varphi})^{bc}_{\mu} + \int d^4x\, \tau^a\partial_{\mu}(A^h)^a_{\mu} - \int d^4x\, \bar{\eta}^a\mathcal{M}^{ab}(A^h)\eta^b\,. \end{aligned} \tag{31}$$

- This action is local, BRST-invariant and renormalizable to all orders in perturbation theory. PRD 96 (2017) no.5, 054022
- It effectively restricts the path integral domain to a region which removes a class of copies in linear covariant gauges.

- As in the Landau gauge, further non-perturbative effects can be taken into account in this case. PRD 93 (2016) no.6, 065019, PRD 96 (2017) no.5, 054022
- In $d=3,4$ the Gribov-Zwanziger action in linear covariant gauges can be "refined" by the inclusion of the following operators.

$$\frac{m^2}{2}\int d^dx\, A^{h,a}_{\mu}A^{h,a}_{\mu} \qquad \text{and} \qquad -M^2\int d^dx\left(\bar{\varphi}^{ab}_{\mu}\varphi^{ab}_{\mu}-\bar{\omega}^{ab}_{\mu}\omega^{ab}_{\mu}\right). \tag{32}$$

The BRST invariance allows for the following proofs: PRD 95 (2017) no.4, 045011

- Correlation functions of gauge-invariant operators are independent of α.
- The mass parameters (γ^2, m^2, M^2) are α-independent and can enter physical correlators.
- The longitudinal part of the gluon propagator is tree-level exact.
- The pole mass of the transverse component of the gluon propagator is α-independent.
- The poles of the gauge-invariant correlation function $\langle A^{h,a}_\mu(-p)A^{h,b}_\nu(p)\rangle$ are the same as those of the transverse part of the gluon propagator.
- Also,

$$\langle A^{h,a}_\mu(-p)A^{h,b}_\nu(p)\rangle = \langle A^{h,a}_\mu(-p)A^{h,b}_\nu(p)\rangle_{\alpha=0} = \langle A^{a}_\mu(-p)A^{b}_\nu(p)\rangle_{\rm Landau}\,. \qquad (33)$$

Universality

One can consider more general covariant, color invariant and renormalizable gauges as 1802.04582 (to appear in PLB)

$$
\begin{aligned}
S_{gf} &= \int d^4x\, s\left(\bar{c}^a(\partial_\mu A^a_\mu - \mu^2\xi^a + \frac{g}{2}\beta f^{abc}\bar{c}^b c^c) - i\frac{\alpha}{2}\bar{c}^a b^a\right) \\
&= \int d^4x \left(ib^a\partial_\mu A^a_\mu + \frac{\alpha}{2}b^a b^a - i\mu^2 b^a\xi^a + ig\beta f^{abc}b^a\bar{c}^b c^c\right. \\
&+ \left.\frac{g^2}{4}\beta f^{abc}f^{cmn}\bar{c}^a\bar{c}^b c^m c^n\right) + \int d^4x \left(\bar{c}^a\partial_\mu D^{ab}_\mu(A)c^b + \mu^2\bar{c}^a g^{ab}(\xi)c^b\right)\,,
\end{aligned}
$$

and the following action

$$
\begin{aligned}
\tilde{S} &= S_{\mathrm{YM}} + S_{gf} - \int d^4x \left(\bar{\varphi}^{ac}_\mu \mathcal{M}^{ab}(A^h)\varphi^{bc}_\mu - \bar{\omega}^{ac}\mathcal{M}^{ab}(A^h)\omega^{bc}_\mu\right) \\
&- \gamma^2 \int d^4x\, gf^{abc}(A^h)^a_\mu(\varphi+\bar{\varphi})^{bc}_\mu + \int d^4x \left(\tau^a\partial_\mu(A^h)^a_\mu - \bar{\eta}^a\mathcal{M}^{ab}(A^h)\eta^b\right)\,.
\end{aligned}
$$

For gauge invariant local operators $\mathcal{O}(x)$, one has

$$\langle\mathcal{O}(x)\mathcal{O}(y)\rangle\Big|_{\tilde{S}} = \left(\langle\mathcal{O}(x)\mathcal{O}(y)\rangle\Big|_{\tilde{S}}\right)_{\sigma_i=0} = \langle\mathcal{O}(x)\mathcal{O}(y)\rangle\Big|^{\text{Landau}}_{\text{“new” GZ}} = \langle\mathcal{O}(x)\mathcal{O}(y)\rangle\Big|^{\text{Landau}}_{\text{GZ}},$$

with $\sigma_i = (\alpha, \beta, \mu^2)$.

In this sense, Zwanziger's horizon function gains universal character.

Conclusions

- A reformulation of the Gribov-Zwanziger action in Landau gauge with gauge invariant variables was proposed.
- The action enjoys a BRST symmetry which is compatible with its refinement.
- The reformulation puts the Gribov parameter and the refining parameters as manifestly gauge invariant quantities.
- It is possible to use the gauge invariant horizon function to eliminate Gribov copies in linear covariant gauges.
- This leads to gauge parameter control, exactness of the longitudinal component of the gluon propagator, gauge parameter independent pole mass,...
- The BRST symmetric (R)GZ action can be expressed in local form. It is renormalizable to all orders in perturbation theory.
- Such a symmetry enables a universal character to the horizon function.

Thank You!