STRANGENESS NEUTRALITY AND THE QCD PHASE STRUCTURE

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[FR, Fu, Pawlowski, work in progress]

FROM CORRELATION FUNCTIONS TO QCD PHENOMENOLOGY
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QCD PHASE DIAGRAM

- Early universe
- Critical point?
- Deconfinement and chiral transition
- Hadrons
- Net Baryon Density
- Quarks and Gluons
- Neutron stars
- Color Superconductor?
- RHIC, LHC
- FAIR SIS 300
- Nuclei
PROBING THE PHASE DIAGRAM IN HICS

• increasing baryon chemical potential with decreasing beam energy

• net baryon content determined by incident nuclei
  (quark numbers are conserved under strong interactions)

• net strangeness has to be zero  \(\rightarrow\) strangeness neutrality
NOMENCLATURE

- chemical potentials:

\[
\begin{bmatrix}
u \\
d \\
s
\end{bmatrix}
\rightarrow
\begin{bmatrix}
\mu_u \\
\mu_d \\
\mu_s
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{3} \mu_B + \frac{1}{2} \mu_I \\
\frac{1}{3} \mu_B - \frac{1}{2} \mu_I \\
\frac{1}{3} \mu_B - \mu_S
\end{bmatrix}
\]

- here: \( \mu_I = 0 \)

- cumulants of conserved charges

\[
\chi^{BS}_{ij} = T^{i+j-4} \frac{\partial^{i+j} p(T, \mu_B, \mu_S)}{\partial \mu_B^i \partial \mu_S^j}
\]

net baryon number: \( B = \langle N_B - N_B \rangle = \chi^{BS}_{10} V T^3 \)

net strangeness: \( S = \langle N_S - N_S \rangle = \chi^{BS}_{01} V T^3 \)
BARYON-STRANGENESS CORRELATION

• B-S correlations:

\[ C_{BS} \equiv -3 \frac{\chi_{11}^{BS}}{\chi_{02}^{BS}} = -3 \frac{\langle BS \rangle - \langle B \rangle \langle S \rangle}{\langle S^2 \rangle - \langle S \rangle^2} = -3 \frac{\langle BS \rangle}{\langle S^2 \rangle} \]

[Koch, Majumder, Randrupp, nucl-th/0505052]

• diagnostic tool for deconfinement:

**QGP**
- all strangeness is carried by \( s, \bar{s} \)
- strict relation between B and S: \( B_s = -S_s / 3 \)
- if all flavors are independent: \( \chi_{11}^{BS} = -\chi_{02}^{BS} / 3 \)

\[ C_{BS} = 1 \]

**hadronic phase**
- mesons can carry only strangeness, baryons both
- \( \chi_{11}^{BS} \): only strange baryons
- \( \chi_{02}^{BS} \): strange baryons & mesons

\[ C_{BS} \neq 1 \]
STRANGENESS NEUTRALITY

• HIC: colliding nuclei have zero strangeness  \( S = 0 \forall T, \mu_B \)

• strangeness neutrality implicitly defines \( \mu_S(\mu_B) \)

\[
\chi^{BS}_{01}(\mu_B, \mu_S(\mu_B)) = 0 \Rightarrow \frac{d\chi^{BS}_{01}}{d\mu_B} = 0 \Leftrightarrow \frac{\partial \mu_S}{\partial \mu_B} = \frac{1}{3} C_{BS}
\]

→ access B-S correlation through strangeness neutrality

→ effect of strangeness neutrality on thermodynamics / phase structure?
LOW-ENERGY MODELING OF QCD
PQM MODEL

• Polyakov-loop enhanced Quark-Meson model (in the local potential approximation LPA):

\[
\Gamma_k = \int_x \left\{ \bar{q} \left( \gamma_\nu D_\nu + \gamma_\nu C_\nu \right) q + \bar{q} h \cdot \Sigma_5 q + \text{tr} \left( \bar{D}_\nu \Sigma \cdot \bar{D}_\nu \Sigma^\dagger \right) + \tilde{U}_k (\Sigma) + U_{\text{glue}} (\langle L \rangle, \langle \bar{L} \rangle) \right\}
\]
PQM MODEL

• Polyakov-loop enhanced Quark-Meson model (in the local potential approximation LPA):

\[
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\]

• scalar and pseudoscalar meson nonets:

\[
\Sigma = T^a (\sigma^a + i \pi^a) \ni \{ \sigma, f_0, a_0^0, a_0^+, a_0^-, \kappa^0, \bar{\kappa}^0, \kappa^+, \kappa^- \}
\]

\[
\Sigma_5 = T_a (\sigma + i \gamma_5 \pi_a)
\]

• open strange mesons: \( l\bar{s}, s\bar{l} \)

• quarks (assume light isospin symmetry):

\[
q = \begin{pmatrix} l \\ l \\ s \end{pmatrix} \quad \mu = \begin{pmatrix} \frac{1}{3} \mu_B \\ \frac{1}{3} \mu_B \\ \frac{1}{3} \mu_B - \mu_s \end{pmatrix}
\]
• Polyakov-loop enhanced Quark-Meson model (in the local potential approximation LPA):

\[ D_\nu = \partial_\nu - ig\delta_\nu_0 A_0 \]

\[ \Gamma_k = \int_x \left\{ \bar{q} \left( \gamma_\nu D_\nu + \gamma_\nu C_\nu \right) q + \bar{q} h \cdot \Sigma_5 q + \text{tr} \left( \bar{D}_\nu \Sigma \cdot \bar{D}_\nu \Sigma^\dagger \right) + \bar{U}_k (\Sigma) + U_{\text{glue}} (\langle L \rangle, \langle \bar{L} \rangle) \right\} \]

quarks and mesons:

\[ q = \begin{pmatrix} i \\ i \\ i \\ s \end{pmatrix} \]

\[ \Sigma = T^a (\sigma^a + i\pi^a) \]

\[ \Sigma_5 = T_a (\sigma_a + i\gamma_5 \pi_a) \]
PQM MODEL

- Polyakov-loop enhanced Quark-Meson model (in the local potential approximation LPA):

\[
\begin{align*}
\text{cov. derivative with } A_0 \text{ background} & \quad D_\nu = \partial_\nu - ig\delta_\nu A_0 \\
\text{vector source / cov. derivative for the chemical potential} & \quad C_\nu = \delta_\nu \mu \\
\text{cov. derivative} & \quad \bar{D}_\nu \Sigma = \partial_\nu \Sigma + [C_\nu, \Sigma] \\
\end{align*}
\]

\[
\Gamma_k = \int_x \left\{ \bar{q} (\gamma_\nu D_\nu + \gamma_\nu C_\nu) q + \bar{q} h \cdot \Sigma_5 q + \text{tr}(\bar{D}_\nu \Sigma \cdot \bar{D}_\nu \Sigma^\dagger) + \bar{U}_k (\Sigma) + U_{\text{glue}}(\langle L \rangle, \langle \bar{L} \rangle) \right\}
\]

quarks and mesons:

\[
q = \begin{pmatrix} i_{1} \\ i_{s} \end{pmatrix}, \quad \Sigma = T^a (\sigma^a + i\pi^a), \quad \Sigma_5 = T_a (\sigma_a + i\gamma_5\pi_a)
\]
• Polyakov-loop enhanced Quark-Meson model (in the local potential approximation LPA):

\[ D_\nu = \partial_\nu - ig\delta_{\nu 0}A_0 \]

\[ C_\nu = \delta_{\nu 0} \mu \]

\[ \bar{D}_\nu \Sigma = \partial_\nu \Sigma + [C_\nu, \Sigma] \]

couples \( \mu \)s to mesons!

\[ \Gamma_k = \int_x \left\{ \bar{q}(\gamma_\nu D_\nu + \gamma_\nu C_\nu)q + \bar{q} h \cdot \Sigma_5 q + \text{tr}(\bar{D}_\nu \Sigma \cdot \bar{D}_\nu \Sigma^\dagger) + \tilde{U}_k(\Sigma) + U_{\text{glue}}(\langle L \rangle, \langle \bar{L} \rangle) \right\} \]

quarks and mesons:

\[ q = \begin{pmatrix} i \\ i \\ s \end{pmatrix} \]

\[ \Sigma = T^a (\sigma^a + i\pi^a) \]

\[ \Sigma_5 = T_a (\sigma_a + i\gamma_5 \pi_a) \]

effective meson potential:

\[ \tilde{U}_k = U_k(\rho_1, \rho_2) - j_l \sigma_l - j_s \sigma_s - c_k \xi \]

U(3) x U(3) symmetric potential (as a function of two chiral invariants)

\[ \rho_i = \text{tr}(\Sigma \cdot \Sigma^\dagger)^i \]

explicit chiral symmetry breaking: finite light & strange current quark masses

anomalous U(1)_A breaking via 't Hooft determinant

\[ \xi = \det(\Sigma + \Sigma^\dagger) \]
PQM MODEL

• Polyakov-loop enhanced Quark-Meson model (in the local potential approximation LPA):

\[
D_\nu = \partial_\nu - ig\delta_{\nu 0} A_0 \\
C_\nu = \delta_{\nu 0} \mu \\
\bar{D}_\nu \Sigma = \partial_\nu \Sigma + [C_\nu, \Sigma]
\]

\[
\Gamma_k = \int \{ \bar{q} (\gamma_\nu D_\nu + \gamma_\nu C_\nu) q + \bar{q} h \cdot \Sigma_5 q + \text{tr} (\bar{D}_\nu \Sigma \cdot \bar{D}_\nu \Sigma^\dagger) + \tilde{U}_k (\Sigma) + U_{\text{glue}}(\langle L \rangle, \langle \bar{L} \rangle) \}
\]

\[
q = \begin{pmatrix} i \\ i \\ i \\ s \end{pmatrix} \\
\Sigma = T^a (\sigma^a + i \pi^a) \\
\Sigma_5 = T_a (\sigma_a + i \gamma_5 \pi_a)
\]

Effective meson potential:

\[
\tilde{U}_k = U_k(\rho_1, \bar{\rho}_2) - j_l \sigma_l - j_s \sigma_s - c_k \xi
\]

\[
\text{Polyakov loop potential:} [\text{Lo et al., hep-lat/1307.5958}]
\]

\[
L = \frac{1}{N_c} \left\langle \text{Tr}_{\mathcal{P}} e^{ig \int_0^\beta d\tau A_0(\tau)} \right\rangle \\
\frac{U_{\text{glue}}(L, \bar{L})}{T^4} = -\frac{1}{2} a(T) \bar{L} L + b(T) \ln [M_H(L, \bar{L})] + \frac{1}{2} c(T) (L^3 + \bar{L}^3) + d(T) (\bar{L} L)^2
\]

\[
M_H(L, \bar{L}) = 1 - 6LL + 4(L^3 + \bar{L}^3) - 3(\bar{L} L)^2
\]

• parameters fitted to reproduce lattice pressure and Polyakov loop susceptibilities

• approximate \( N_f \) and \( \mu \) dependence from QCD and HTL/HDL arguments

[Herbst et al., hep-ph/1008.0081, 1302.1426]  
[Haas et al., hep-ph/1302.1993]

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non-perturbative quantum fluctuations via FRG

- relevant object: scale dependent effective action $\Gamma_k$

successively integrate out fluctuations from UV to IR (Wilson RG)

$\rightarrow \Gamma_k$ is eff. action that incorporates all fluctuations down to scale $k$

$\rightarrow$ lowering k: zooming out / coarse graining

- evolution equation for $\Gamma_k$:

$$\partial_t \Gamma_k = \frac{1}{2} \text{STr} \left[ \left( \Gamma_k^{(2)} + R_k \right)^{-1} \partial_t R_k \right]$$

mass-like IR regulator

[Wetterich 1993]
PQM + FUNCTIONAL RG

- dynamical chiral symmetry breaking though $U_k$
- `statistical` confinement through $A_0$ background ($U_{\text{glue}}$)
- thermal quark distributions modified through feedback from $A_0$

\[
\begin{align*}
n_F(E) &= \frac{1}{e^{(E-\mu)/T} + 1} \\
N_F(E; L, \bar{L}) &= \frac{1 + 2Le^{(E-\mu)/T} + Le^{2(E-\mu)/T}}{1 + 3Le^{(E-\mu)/T} + 3Le^{2(E-\mu)/T} + e^{3(E-\mu)/T}} \\
&\rightarrow \begin{cases} 
\frac{1}{e^{3(E-\mu)/T} + 1}, & L \to 0 \ (\text{confinement}) \\
\frac{1}{e^{(E-\mu)/T} + 1}, & L \to 1 \ (\text{deconfinement})
\end{cases}
\end{align*}
\]

- `interpolation` between baryon and quark d.o.f.
- correct `N_c - scaling` of particle number fluctuations

- thermodynamics from Euclidean (off-shell) formulation

\[\text{simple hierarchy of relevant fluctuations: the lighter the particle, the more relevant it is fluctuations of kaons and s-quarks (coupled to $A_0$) already sufficient for qualitative description of the relevant strangeness effects (for moderate $\mu$)}\]
RESULTS
MODEL VS LATTICE EOS

cf. [Herbst et al., hep-ph/1308.3621]

\[ t = \frac{T - T_c}{T_c} \]
STRANGENESS NEUTRALITY

\( \mu_S(\mu_B) \) at strangeness neutrality

- slope directly related to baryon-strangeness correlations:

\[
\frac{\partial \mu_S}{\partial \mu_B} = \frac{1}{3} C_{BS}
\]

\( C_{BS} \) for any \( T \) and \( \mu \)

competition between baryonic and mesonic sources of strangeness!
STRAINFENESS NEUTRALITY

$\mu_s(\mu_B)$ at strangeness neutrality

- slope directly related to baryon-strangeness correlations:

$$\frac{\partial \mu_S}{\partial \mu_B} = \frac{1}{3} C_{BS}$$

$C_{BS}$ for any $T$ and $\mu$

competition between baryonic and mesonic sources of strangeness!

$$3 \frac{\partial \mu_S}{\partial \mu_B} = C_{BS} \sim \frac{\langle \text{strange baryons} \rangle}{\langle \text{(strange baryons & mesons)}^2 \rangle}$$

\[ \begin{cases} < 1 & \text{mesons dominate} \\ = 1 & \text{mesons & baryons (or free flavors)} \\ > 1 & \text{baryons dominate} \end{cases} \]
STRANGENESS NEUTRALITY

EoS at strangeness neutrality

\[ \mu_B = 300 \text{ MeV} \]
\[ S = 0 \]
\[ \mu_S = 0 \]

\[ \mu_B = 450 \text{ MeV} \]
\[ S = 0 \]
\[ \mu_S = 0 \]

\[ \mu_B = 600 \text{ MeV} \]
\[ S = 0 \]
\[ \mu_S = 0 \]
STRANGENESS NEUTRALITY
phase structure at strangeness neutrality

- critical temperature starts increasing at moderate $\mu_B$ due to strangeness neutrality
- smaller curvature of the phase boundary
- CEP at larger $T$ (?)

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SUMMARY & OUTLOOK

- **strangeness neutrality in heavy ion collisions**
  - intimate relation to baryon strangeness correlations
  - sensitive to QCD phase transition

- **relevant for phase structure and thermodynamics at finite $\mu_B$**
  - phase transition at larger $T$ for moderate $\mu_B$
  - likely to affect position of the CEP

In progress:
- study larger $\mu$ and the CEP
- eigenvalue potential vs loop potential: minimum vs saddle point
- going beyond LPA
- including gluon fluctuations: dynamical hadronization
- self-consistent computation of the $A_0$ potential
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