QCD 2- and 3- point Green's functions:

From lattice results to phenomenology

In collaboration with:

A. Athenodorou, D. Binosi, Ph. Boucaud, C. Mezrag, F. De Soto, J. Papavassiliou, C.D. Roberts, S. Zafeiropoulos.

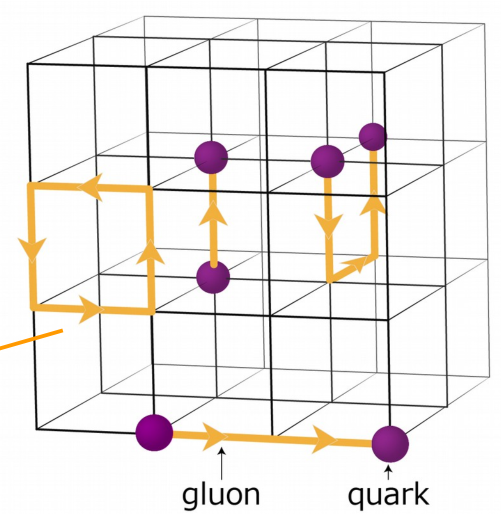
José Rodríguez-Quintero
Universidad de Huelva
Spain

Bad Honnef, 3-6 April, 2018

Lattice two- and three-point Green's function

$$\mathcal{G}^{abc}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle = f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p),$$

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$



$$\tilde{A}^a_\mu(q) = \frac{1}{2}\,\mathrm{Tr}\sum_x A_\mu(x+\hat{\mu}/2)\exp[iq\cdot(x+\hat{\mu}/2)]\lambda^a$$

$$A_\mu(x+\hat{\mu}/2) = \frac{U_\mu(x) - U^\dagger_\mu(x)}{2iag_0} - \frac{1}{3}\,\mathrm{Tr}\,\frac{U_\mu(x) - U^\dagger_\mu(x)}{2iag_0}$$

Tree-level Symanzik gauge action

$$S_g = \frac{\beta}{3}\sum_x \left\{ b_0 \sum_{\substack{\mu,\nu=1 \\ 1\le\mu<\nu}}^{4} \left[1 - \mathrm{Re}\,\mathrm{Tr}\,(U^{1\times1}_{x,\mu,\nu})\right] + b_1 \sum_{\substack{\mu,\nu=1 \\ \mu\neq\nu}}^{4} \left[1 - \mathrm{Re}\,\mathrm{Tr}(U^{1\times2}_{x,\mu,\nu})\right] \right\}$$

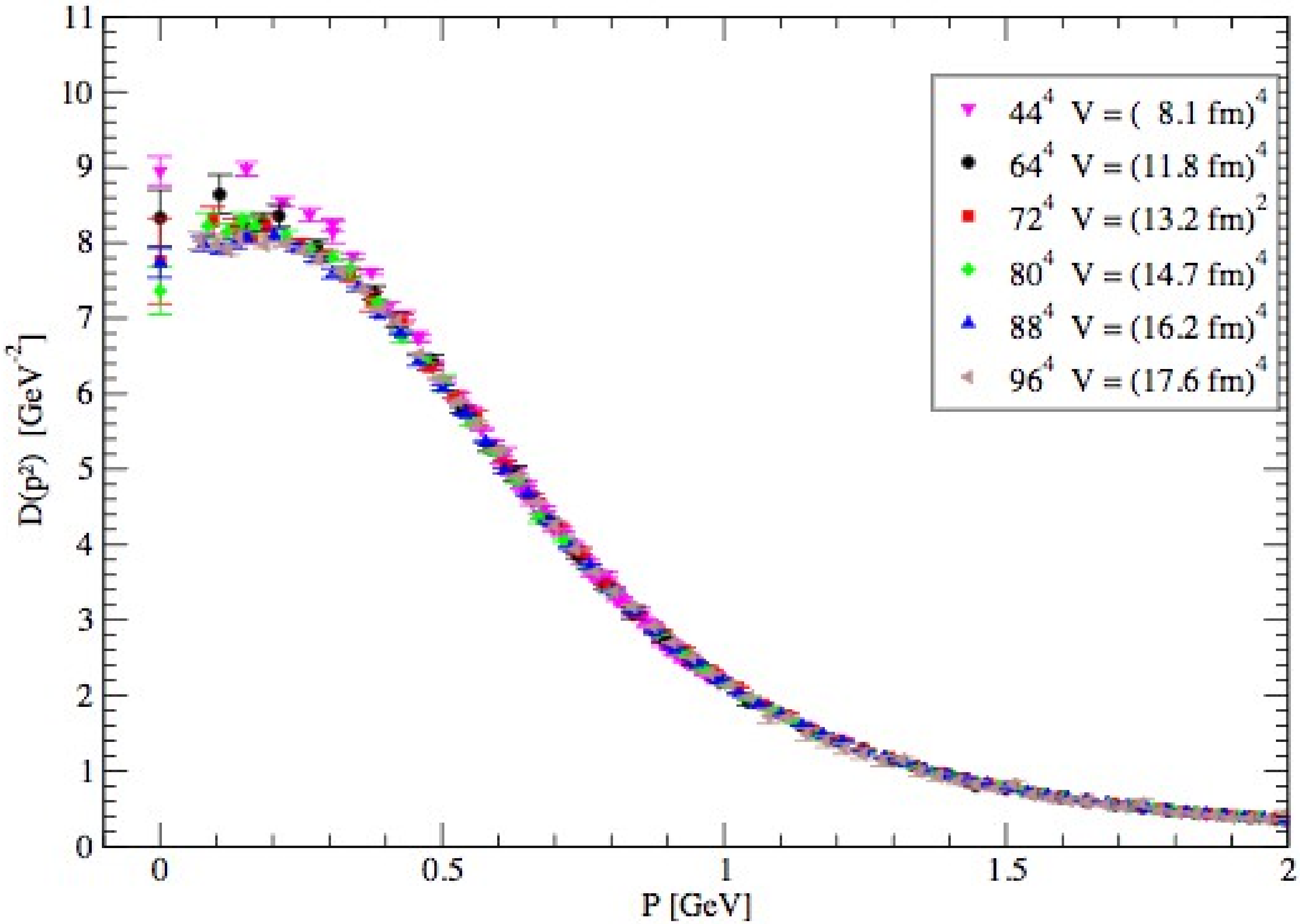
The gauge fields are to be nonperturbatively obtained from lattice QCD simulations and applied then to get the gluon Green's functions

The gluon propagator

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu/q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

Duarte, Oliveira, Silva
PRD94(2016)014502



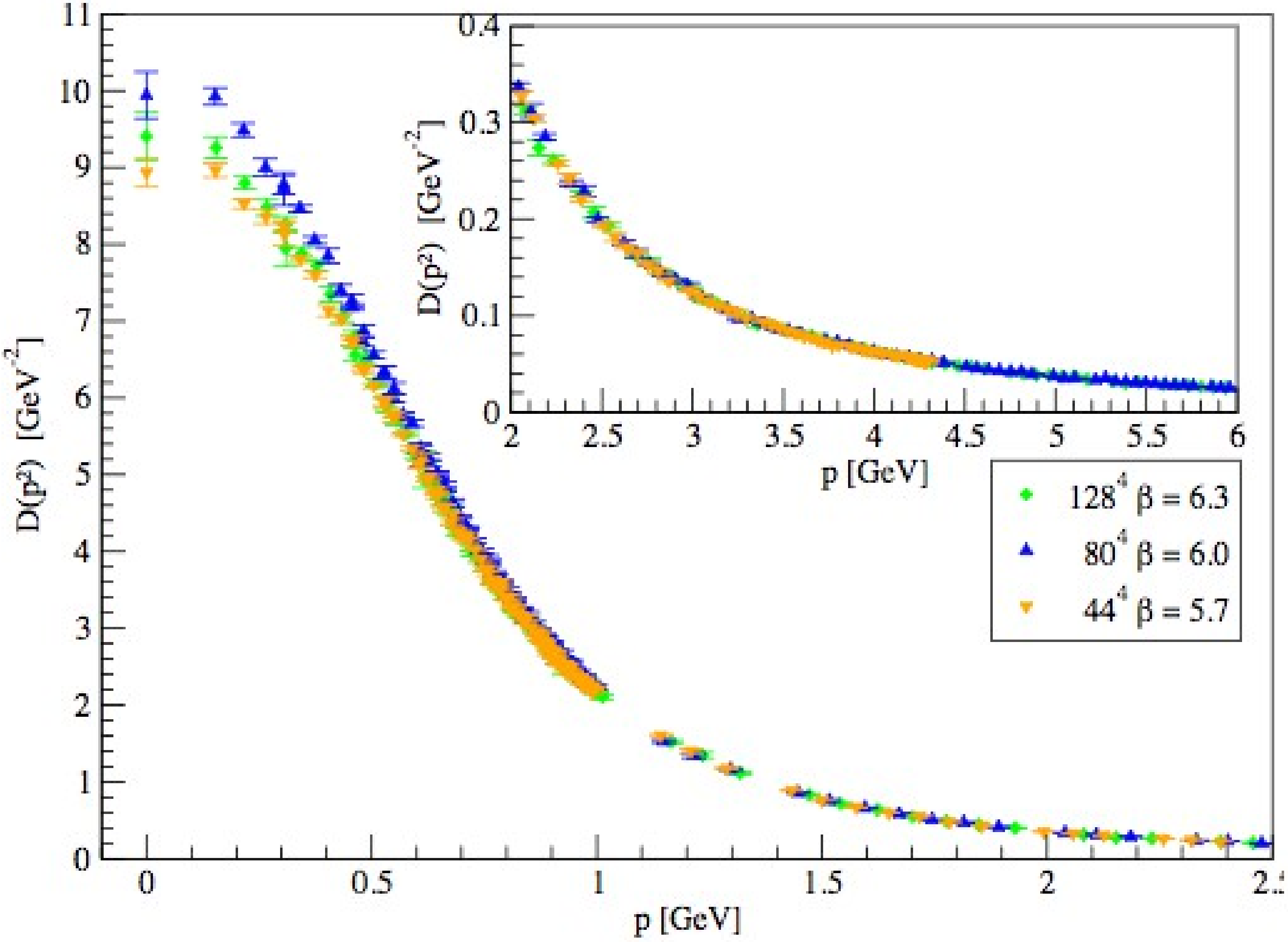
Quenched lattice gluon propagators for different large volumes!

The gluon propagator

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu/q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

Duarte, Oliveira, Silva
PRD94(2016)014502



Quenched lattice gluon propagators for different beta and similar volume!

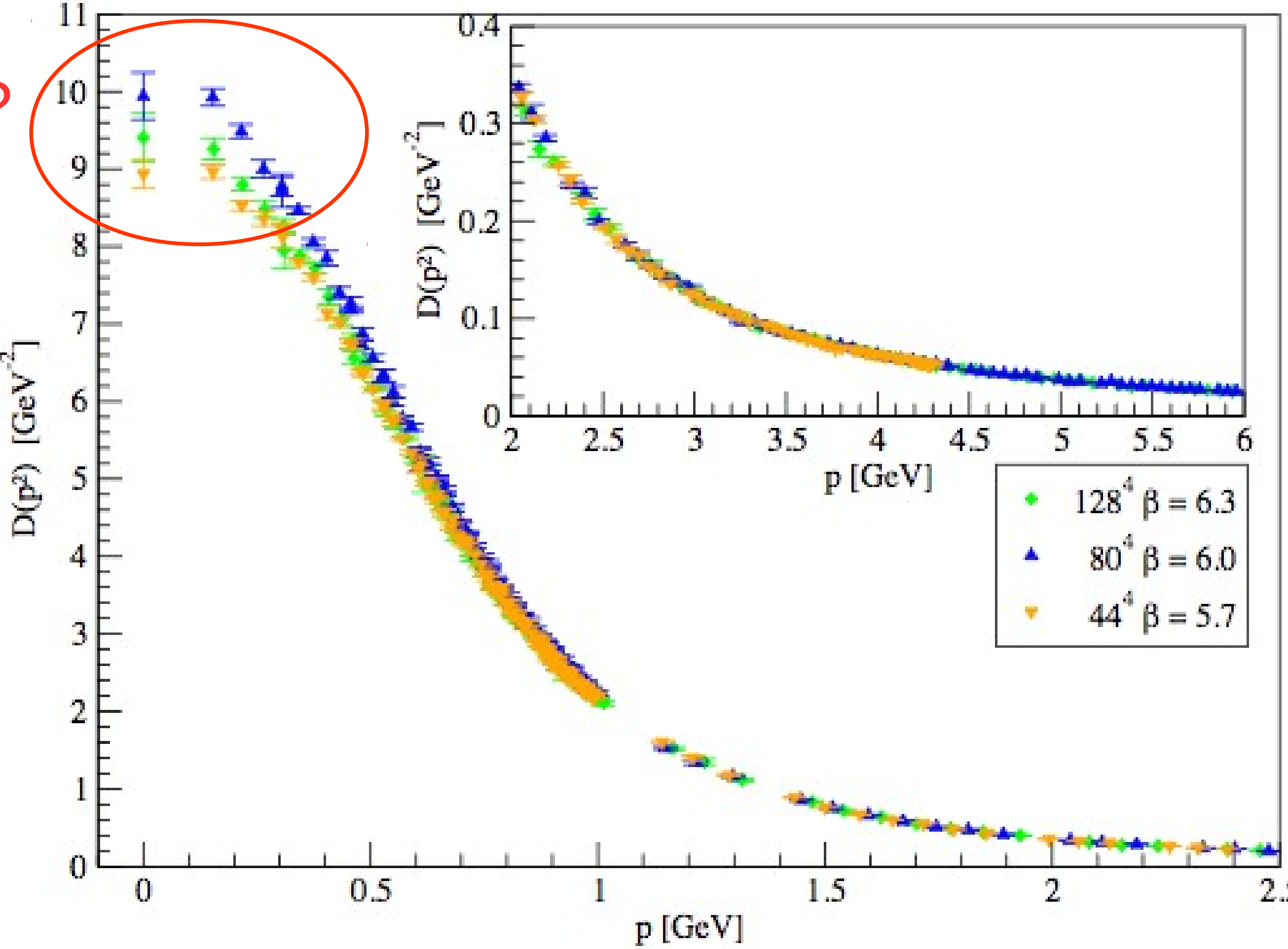
The gluon propagator

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu/q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

Duarte, Oliveira, Silva
PRD94(2016)014502

?



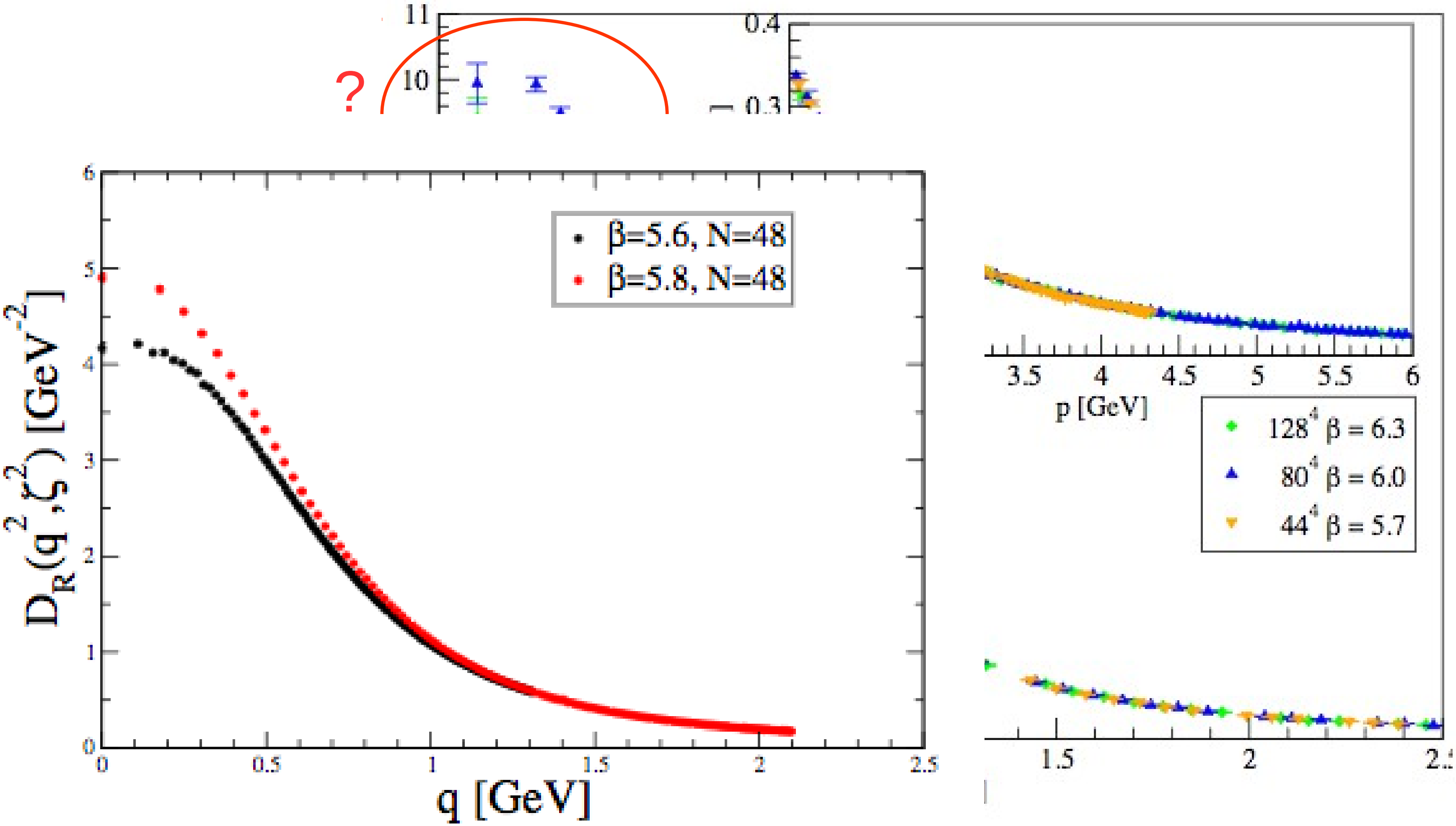
Quenched lattice gluon propagators for different beta and similar volume!

The gluon propagator

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

Duarte, Oliveira, Silva
PRD94(2016)014502

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu / q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

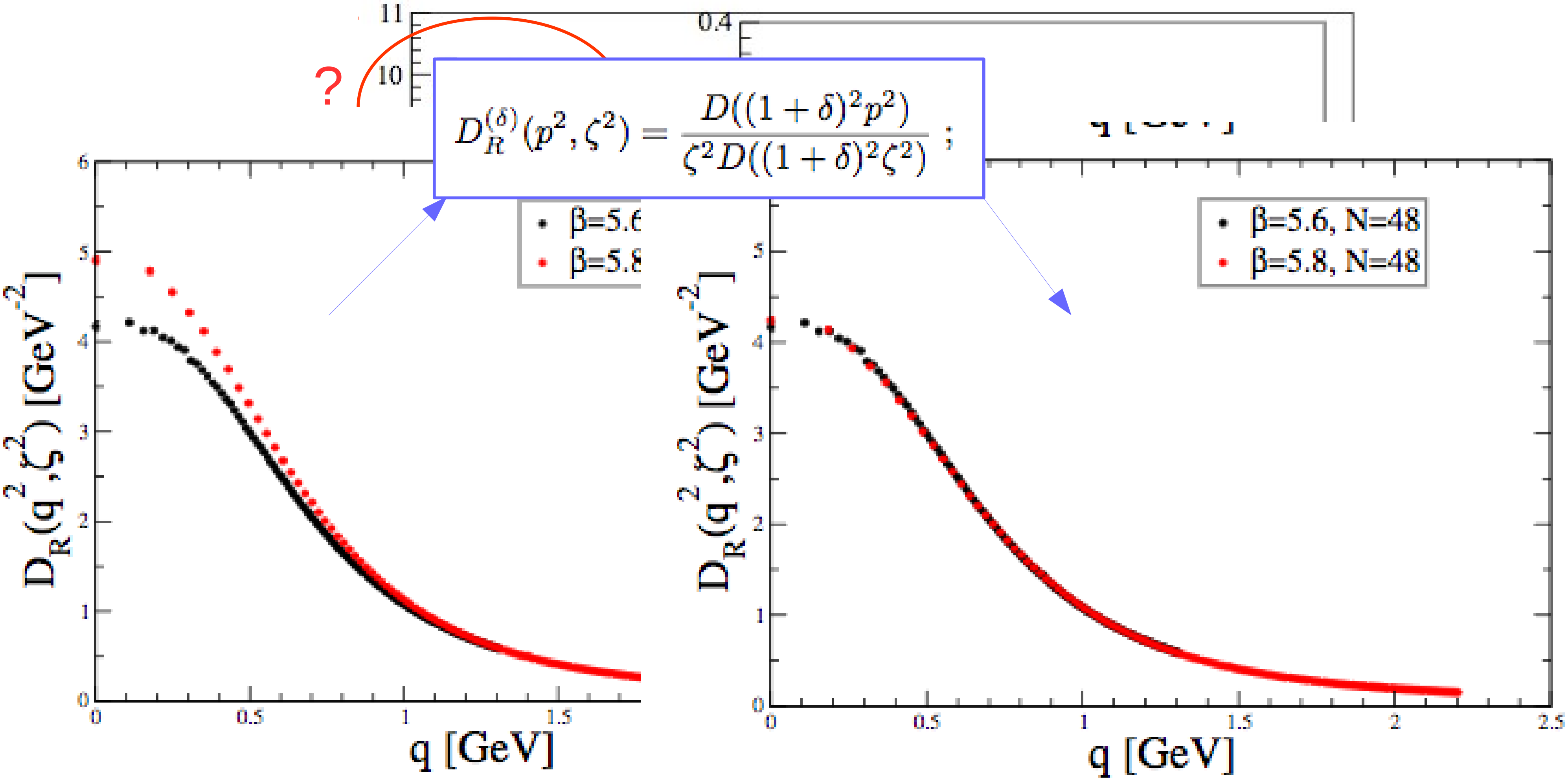


The gluon propagator

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu/q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

Duarte, Oliveira, Silva
PRD94(2016)014502



ArXiv:1704.02053 (PRD): Essentially, a scale setting problem!!

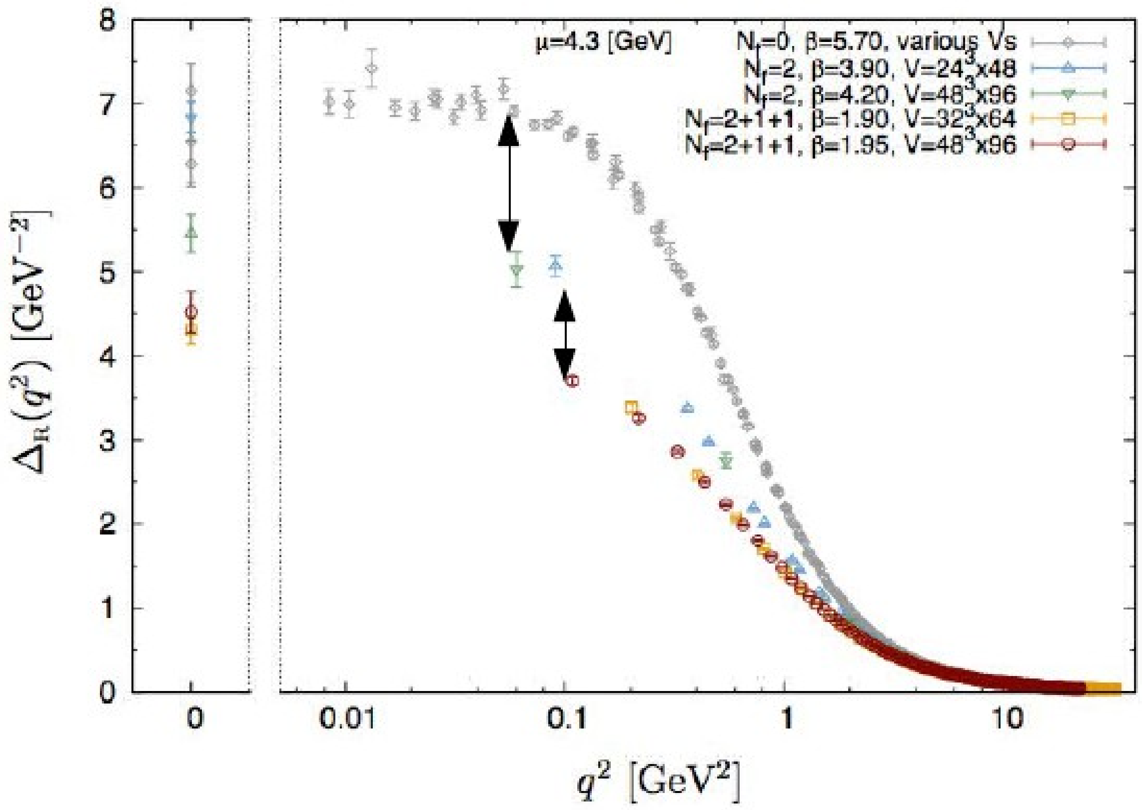
The gluon propagator

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q) \rangle = \delta^{ab} \Delta(p^2) P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu / q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q \cdot \mathcal{G} = r \cdot \mathcal{G} = p \cdot \mathcal{G} = 0$.

Ayala et al.
PRD86(2012)074512

- Effective gluon mass increases with the number of flavours



Unquenched lattice gluon propagators!

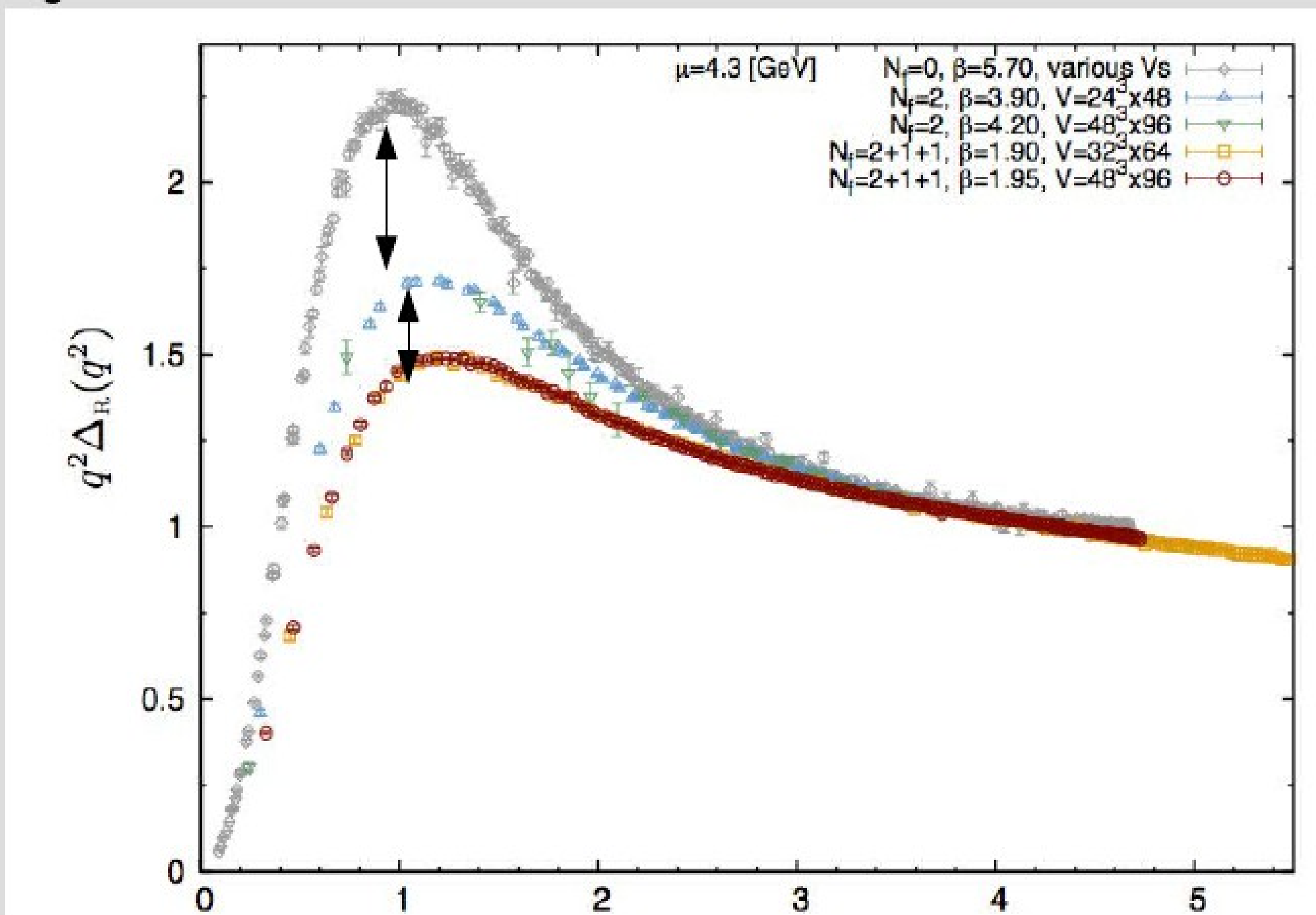
The gluon propagator

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu/q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

Ayala et al.
PRD86(2012)074512

- Effective gluon mass increases with the number of flavours



Unquenched lattice gluon propagators!

The gluon propagator

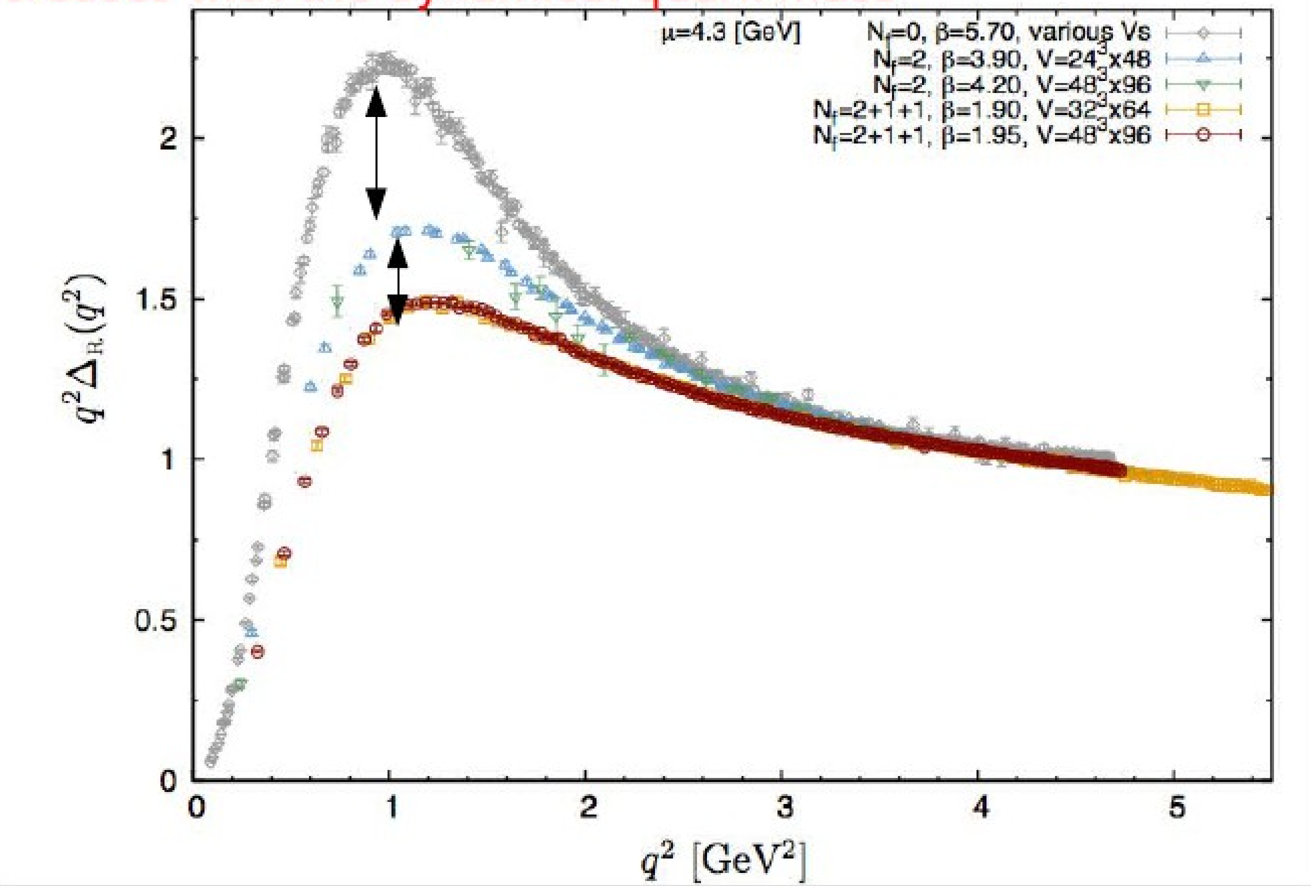
$$\Delta_{\mu\nu}^{ab}(q) = \langle A_\mu^a(q) A_\nu^b(-q) \rangle = \delta^{ab} \Delta(p^2) P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu / q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q \cdot \mathcal{G} = r \cdot \mathcal{G} = p \cdot \mathcal{G} = 0$.

Ayala et al.
PRD86(2012)074512

- Effective gluon mass increases with the number of flavours

… and decreases with the dynamical quark mass



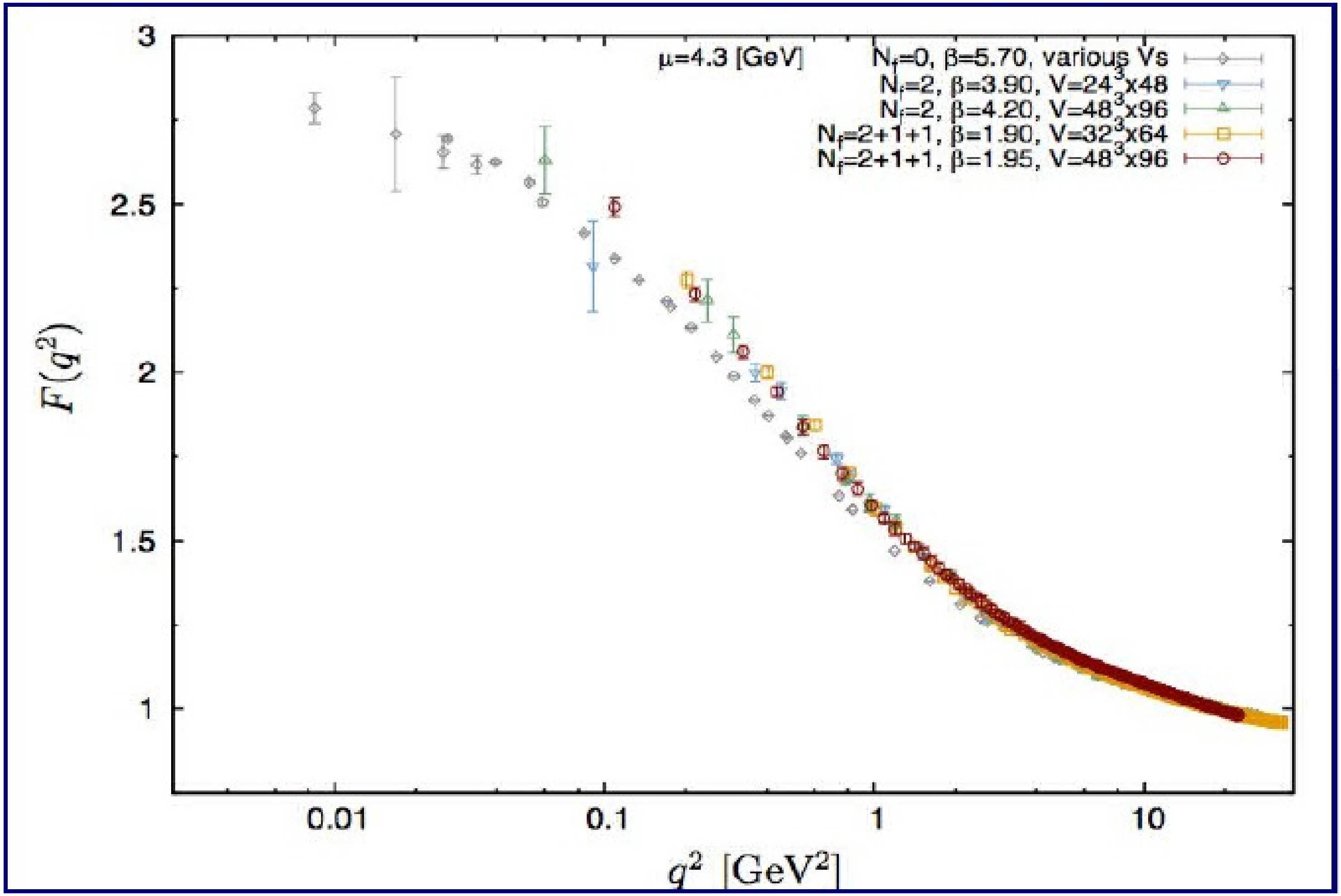
Unquenched lattice gluon propagators!

The ghost propagator

$$\left(F^{(2)}\right)^{ab}(x-y) \equiv \left\langle \left(M^{-1}\right)^{ab}_{xy} \right\rangle, \quad M(U) = -\frac{1}{N}\nabla \cdot \widetilde{D}(U)$$

$$\widetilde{D}(U)\eta(x) = \frac{1}{2}\left(U_\mu(x)\eta(x+\mu) - \eta(x)U_\mu(x) + \eta(x+\mu)U_\mu^\dagger - U_\mu^\dagger(x)\eta(x)\right)$$

Ayala et al.
PRD86(2012)074512



Unquenched lattice ghost propagators!

The vertex and the three-gluon Green's function

$$\mathcal{G}^{abc}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle = f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p),$$

Symmetric configuration:

$q^2 = r^2 = p^2$ and $q\cdot r = q\cdot p = r\cdot p = -q^2/2$;

$$\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\Gamma_{\alpha'\mu'\nu'}(q,r,p)\Delta_{\alpha'\alpha}(q)\Delta_{\mu'\mu}(r)\Delta_{\nu'\nu}(p),$$

$$G_{\alpha\mu\nu}(q,r,p) = T^{sym}(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + S^{sym}(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)$$

$$\Gamma_{\alpha\mu\nu}(q,r,p) = \Gamma^{sym}_T(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + \Gamma^{sym}_S(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)$$

$$\lambda^{\text{tree}}_{\alpha\mu\nu}(q,r,p) = \Gamma^{(0)}_{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p).$$

$$\lambda^{S}_{\alpha\mu\nu}(q,r,p) = (r-p)_\alpha(p-q)_\mu(q-r)_\nu/r^2.$$

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q)A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu/q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

In Landau gauge and for particular kinematical configurations, transversality and Bose symmetry make possible a simple tensorial decomposition of the gluon Green's function

The vertex and the three-gluon Green's function

$$\mathcal{G}^{abc}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle = f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p),$$

Symmetric configuration:

$q^2 = r^2 = p^2$ and $q\cdot r = q\cdot p = r\cdot p = -q^2/2$;

$$\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\Gamma_{\alpha'\mu'\nu'}(q,r,p)\Delta_{\alpha'\alpha}(q)\Delta_{\mu'\mu}(r)\Delta_{\nu'\nu}(p),$$

$$G_{\alpha\mu\nu}(q,r,p) = T^{sym}(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + S^{sym}(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)$$

$$\Gamma_{\alpha\mu\nu}(q,r,p) = \Gamma^{sym}_T(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + \Gamma^{sym}_S(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)$$

$$T^{\rm sym}(q^2) = g\,\Gamma^{\rm sym}_T(q^2)\,\Delta^3(q^2),$$
$$S^{\rm sym}(q^2) = g\,\Gamma^{\rm sym}_S(q^2)\,\Delta^3(q^2).$$

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu/q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

$$\lambda^{\rm tree}_{\alpha\mu\nu}(q,r,p) = \Gamma^{(0)}_{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p).$$

$$\lambda^{S}_{\alpha\mu\nu}(q,r,p) = (r-p)_\alpha(p-q)_\mu(q-r)_\nu/r^2.$$

In Landau gauge and for particular kinematical configurations, transversality and Bose symmetry make possible a simple tensorial decomposition of the gluon Green's function

The vertex and the three-gluon Green's function

$$\mathcal{G}^{abc}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle = f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p),$$

Symmetric configuration:

$q^2 = r^2 = p^2$ and $q\cdot r = q\cdot p = r\cdot p = -q^2/2;$

$$\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\Gamma_{\alpha'\mu'\nu'}(q,r,p)\Delta_{\alpha'\alpha}(q)\Delta_{\mu'\mu}(r)\Delta_{\nu'\nu}(p),$$

$$G_{\alpha\mu\nu}(q,r,p) = T^{sym}(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + S^{sym}(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)$$

$$W_{\alpha\mu\nu} = \lambda^{tree}_{\alpha\mu\nu} + \lambda^{S}_{\alpha\mu\nu}/2$$

$$T^{\rm sym}(q^2) = g\,\Gamma^{\rm sym}_T(q^2)\,\Delta^3(q^2),$$
$$S^{\rm sym}(q^2) = g\,\Gamma^{\rm sym}_S(q^2)\,\Delta^3(q^2).$$

$$T^{\rm sym}(q^2) = \left.\frac{W_{\alpha\mu\nu}(q,r,p)\,\mathcal{G}_{\alpha\mu\nu}(q,r,p)}{W_{\alpha\mu\nu}(q,r,p)W_{\alpha\mu\nu}(q,r,p)}\right|_{\rm sym},$$

$$\Gamma_{\alpha\mu\nu}(q,r,p) = \Gamma^{sym}_T(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + \Gamma^{sym}_S(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)$$

$$\lambda^{\rm tree}_{\alpha\mu\nu}(q,r,p) = \Gamma^{(0)}_{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p).$$

$$\lambda^{S}_{\alpha\mu\nu}(q,r,p) = (r-p)_\alpha(p-q)_\mu(q-r)_\nu/r^2.$$

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q)A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu/q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

In Landau gauge and for particular kinematical configurations, transversality and Bose symmetry make possible a simple tensorial decomposition of the gluon Green's function

The vertex and the three-gluon Green's function

$$\mathcal{G}^{abc}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle = f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p),$$

Asymmetric configuration:

$$q \rightarrow 0\,;\; r^2 = p^2 = -p\cdot r$$

$$\boxed{\mathcal{G}_{\alpha\mu\nu}(q,r,p) = g\Gamma_{\alpha'\mu'\nu'}(q,r,p)\Delta_{\alpha'\alpha}(q)\Delta_{\mu'\mu}(r)\Delta_{\nu'\nu}(p),}$$

$$W_{\alpha\mu\nu} = \lambda^{tree}_{\alpha\mu\nu} + \cancel{\lambda^{S}_{\alpha\mu\nu}/2}$$

$$\boxed{G_{\alpha\mu\nu}(q,r,p) = T^{sym}(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + \cancel{S^{sym}(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)}}$$

$$\boxed{T^{asym}(r^2) = \left.\frac{W_{\alpha\mu\nu}(q,r,p)\,\mathcal{G}_{\alpha\mu\nu}(q,r,p)}{W_{\alpha\mu\nu}(q,r,p)W_{\alpha\mu\nu}(q,r,p)}\right|_{asym}}$$

$$\boxed{T^{asym}(r^2) = g\,\Gamma^{asym}_T(r^2)\,\Delta(0)\,\Delta^2(r^2),}$$

$$\boxed{\Gamma_{\alpha\mu\nu}(q,r,p) = \Gamma^{sym}_T(q^2)\,\lambda^{tree}_{\alpha\mu\nu}(q,r,p) + \cancel{\Gamma^{sym}_S(q^2)\,\lambda^{S}_{\alpha\mu\nu}(q,r,p)}}$$

$$\lambda^{tree}_{\alpha\mu\nu}(q,r,p) = \Gamma^{(0)}_{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p).$$

$$\cancel{\lambda^{S}_{\alpha\mu\nu}(q,r,p) = (r-p)_\alpha(p-q)_\mu(q-r)_\nu/r^2.}$$

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q)A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2)P_{\mu\nu}(q),$$

where $P_{\mu\nu}(q) = \delta_{\mu\nu} - q_\mu q_\nu/q^2$, implies directly that $\mathcal{G}$ is totally transverse: $q\cdot\mathcal{G} = r\cdot\mathcal{G} = p\cdot\mathcal{G} = 0$.

In Landau gauge and for particular kinematical configurations, transversality and Bose symmetry make possible a simple tensorial decomposition of the gluon Green's function

The vertex and the three-gluon Green's function

$$\mathcal{G}^{abc}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle = f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p),$$

Symmetric configuration:

$q^2 = r^2 = p^2$ and $q\cdot r = q\cdot p = r\cdot p = -q^2/2$;

$$\Delta_R(q^2;\mu^2) = Z_A^{-1}(\mu^2)\,\Delta(q^2),$$
$$T_R^{\rm sym}(q^2;\mu^2) = Z_A^{-3/2}(\mu^2) T^{\rm sym}(q^2),$$

$$g^{\rm sym}(q^2) = q^3 \frac{T^{\rm sym}(q^2)}{[\Delta(q^2)]^{3/2}} = q^3 \frac{T_R^{\rm sym}(q^2;\mu^2)}{[\Delta_R(q^2;\mu^2)]^{3/2}}.$$

MOM renormalization prescription:

$$\Delta_R(q^2;q^2) = Z_A^{-1}(q^2)\,\Delta(q^2) = 1/q^2,$$
$$T_R^{\rm sym}(q^2;q^2) = Z_A^{-3/2}(q^2)\,T^{\rm sym}(q^2) = g_R^{\rm sym}(q^2)/q^6.$$

$$T^{\rm sym}(q^2) = g\,\Gamma_T^{\rm sym}(q^2)\,\Delta^3(q^2),$$

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2) P_{\mu\nu}(q),$$

$$T^{\rm sym}(q^2) = \left.\frac{W_{\alpha\mu\nu}(q,r,p)\,\mathcal{G}_{\alpha\mu\nu}(q,r,p)}{W_{\alpha\mu\nu}(q,r,p) W_{\alpha\mu\nu}(q,r,p)}\right|_{\rm sym},$$

$$g^{sym}(\mu^2)\,\Gamma^{sym}_{T,R}(q^2;\mu^2) = \frac{g^{sym}(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

After the required projection and the appropriate renormalization, one can define a QCD coupling from the Green's functions, and relate it to the 1PI vertex form factor, in both symmetric...

The vertex and the three-gluon Green's function

$$\mathcal{G}^{abc}_{\alpha\mu\nu}(q,r,p) = \langle A^a_\alpha(q) A^b_\mu(r) A^c_\nu(p)\rangle = f^{abc}\mathcal{G}_{\alpha\mu\nu}(q,r,p),$$

Asymmetric configuration:

$$q \to 0\,;\; r^2 = p^2 = -\,p\cdot r$$

$$\Delta_R(q^2;\mu^2) = Z_A^{-1}(\mu^2)\,\Delta(q^2),$$
$$T_R^{\rm sym}(q^2;\mu^2) = Z_A^{-3/2}(\mu^2) T^{\rm sym}(q^2),$$

$$g^{\rm asym}(r^2) = r^3 \frac{T^{\rm asym}(r^2)}{[\Delta(r^2)]^{1/2}\Delta(0)} = r^3 \frac{T_R^{\rm asym}(r^2;\mu^2)}{[\Delta_R(r^2;\mu^2)]^{1/2}\Delta_R(0;\mu^2)}$$

MOM renormalization prescription:

$$T^{\rm asym}(r^2) = g\,\Gamma_T^{\rm asym}(r^2)\,\Delta(0)\,\Delta^2(r^2),$$

$$\Delta_R(q^2;q^2) = Z_A^{-1}(q^2)\,\Delta(q^2) = 1/q^2,$$

$$T_R^{\rm asym}(r^2;r^2) = Z_A^{-3/2}(r^2)\,T^{\rm asym}(r^2) = \Delta_R(0;q^2)\,g_R^{\rm asym}(r^2)/r^4,$$

$$g^{asym}(\mu^2)\,\Gamma^{asym}_{T,R}(q^2;\mu^2) = \frac{g^{asym}(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$$\Delta^{ab}_{\mu\nu}(q) = \langle A^a_\mu(q) A^b_\nu(-q)\rangle = \delta^{ab}\Delta(p^2) P_{\mu\nu}(q),$$

$$T^{\rm asym}(r^2) = \left.\frac{W_{\alpha\mu\nu}(q,r,p)\,\mathcal{G}_{\alpha\mu\nu}(q,r,p)}{W_{\alpha\mu\nu}(q,r,p) W_{\alpha\mu\nu}(q,r,p)}\right|_{\rm asym}$$

After the required projection and the appropriate renormalization, one can define a QCD coupling from the Green's functions, and relate it to the 1PI vertex form factor, in both symmetric and asymmetric kinematical configurations.

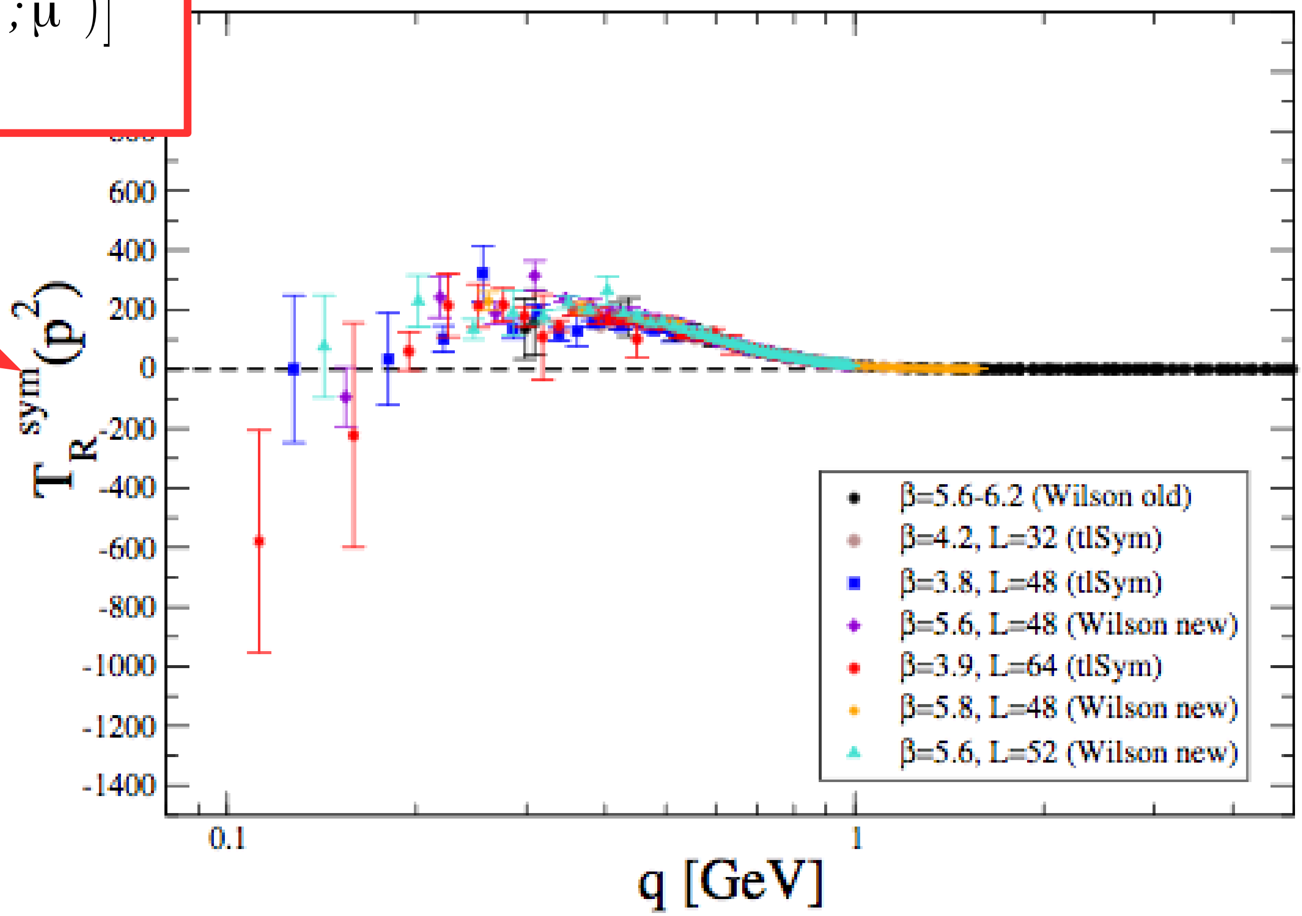
The zero-crossing of the three-gluon vertex

$$g^i(\mu^2)\,\Gamma^i_{T,R}(q^2;\mu^2) = \frac{g^i(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$i = sym\,,\, asym\,.$

$$g^{sym}(q^2) = q^3\,\frac{T^{sym}(q^2)}{[\Delta(q^2)]^{3/2}}$$

$$g^{asym}(q^2) = q^3\,\frac{T^{asym}(q^2)}{\Delta(0)[\Delta(q^2)]^{1/2}}$$



Let's then focus (again) on the symmetric case: the form factor appears to change its sign at very deep IR momenta and show then a zero-crossing. This appears to happen below ~0.2 GeV.

The zero-crossing of the three-gluon vertex

M. Tissier, N. Wschebor, PRD84(2011)045018
A.C Aguilar et al.; PRD89(2014)05008
A. Blum et al.; PRD89(2014)061703
G. Eichmann et al.; PRD89(2014)105014
A.K. Cyrol et al.; arXiv:1605.01856[hep-ph]
A. Cucchieri, A. Maas, T. Mendes; PRD74(2006)014503;PRD77(2008)094510

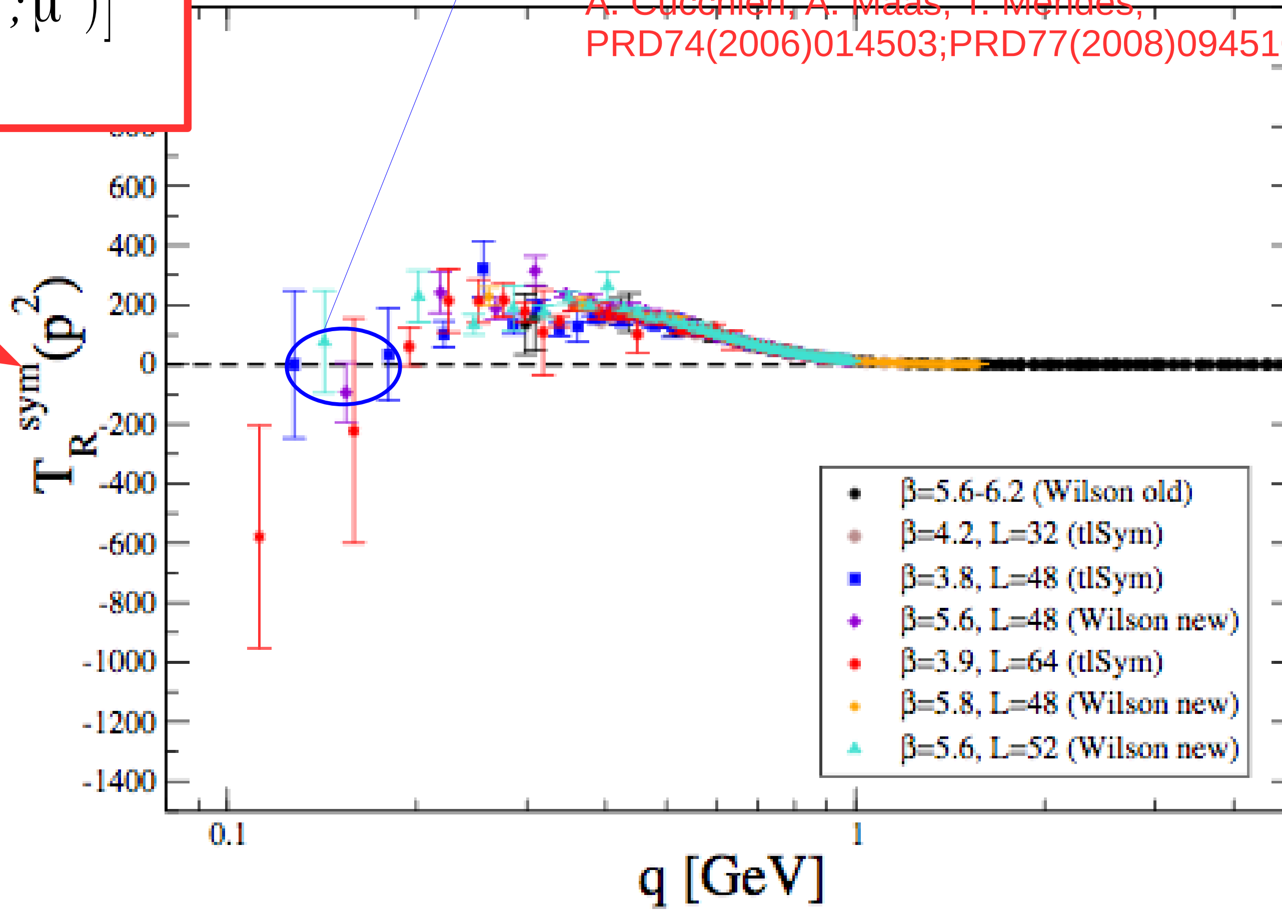
$$g^i(\mu^2)\,\Gamma^i_{T,R}(q^2;\mu^2) = \frac{g^i(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$i = sym\,, asym.$

$$g^{sym}(q^2) = q^3\frac{T^{sym}(q^2)}{[\Delta(q^2)]^{3/2}}$$

$$g^{asym}(q^2) = q^3\frac{T^{asym}(q^2)}{\Delta(0)[\Delta(q^2)]^{1/2}}$$

zero-crossing



Let's then focus (again) on the symmetric case: the form factor appears to change its sign at very deep IR momenta and show then a zero-crossing. This appears to happen below ~0.2 GeV.

The zero-crossing of the three-gluon vertex

M. Tissier, N. Wschebor, PRD84(2011)045018
A.C Aguilar et al.; PRD89(2014)05008
A. Blum et al.; PRD89(2014)061703
G. Eichmann et al.; PRD89(2014)105014
A.K. Cyrol et al.; arXiv:1605.01856[hep-ph]
A. Cucchieri, A. Maas, T. Mendes; PRD74(2006)014503;PRD77(2008)094510

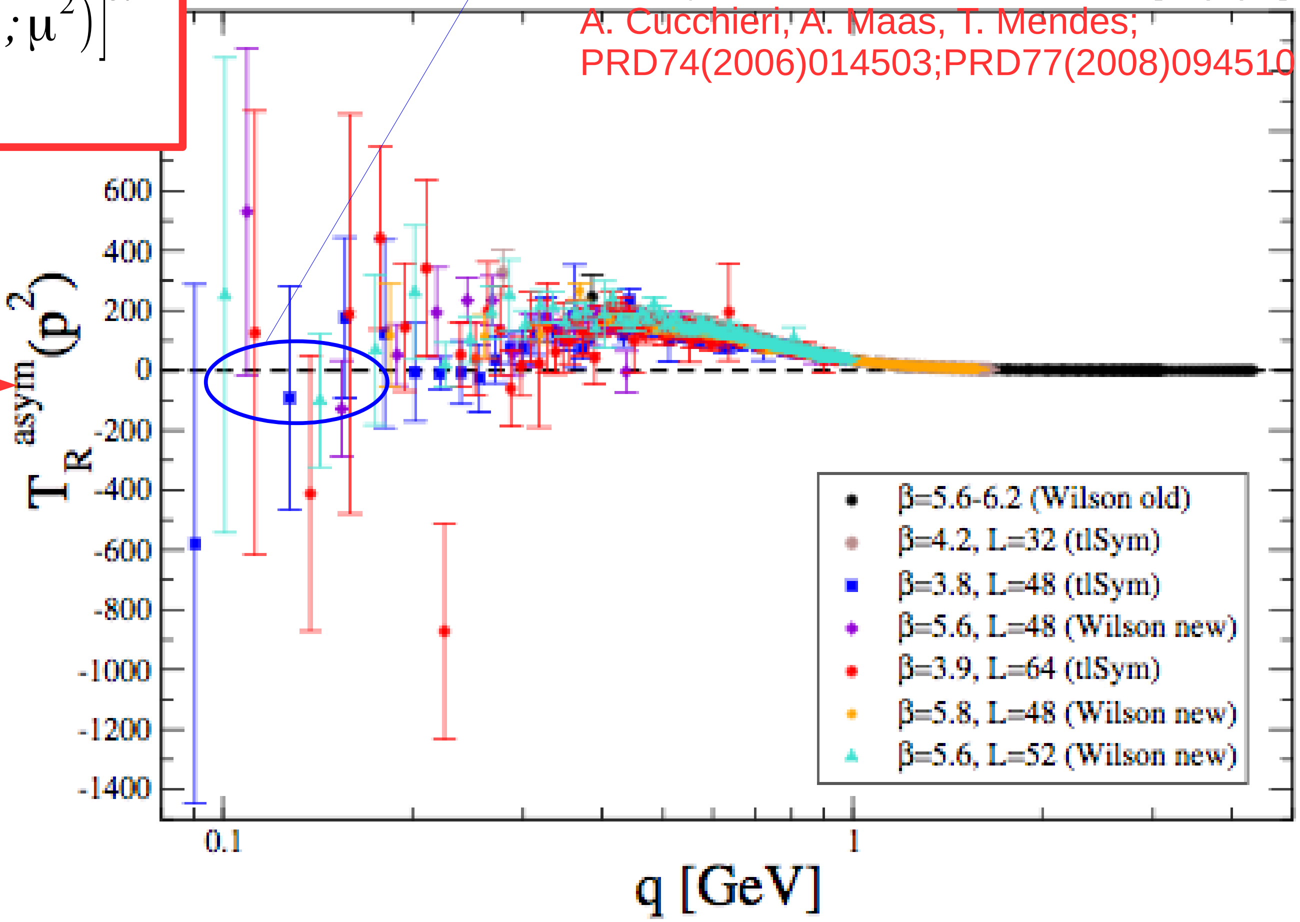
$$g^i(\mu^2)\,\Gamma^i_{T,R}(q^2;\mu^2) = \frac{g^i(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$i = sym, asym.$

$$g^{sym}(q^2) = q^3\frac{T^{sym}(q^2)}{[\Delta(q^2)]^{3/2}}$$

$$g^{asym}(q^2) = q^3\frac{T^{asym}(q^2)}{\Delta(0)[\Delta(q^2)]^{1/2}}$$

Zero-crossing?

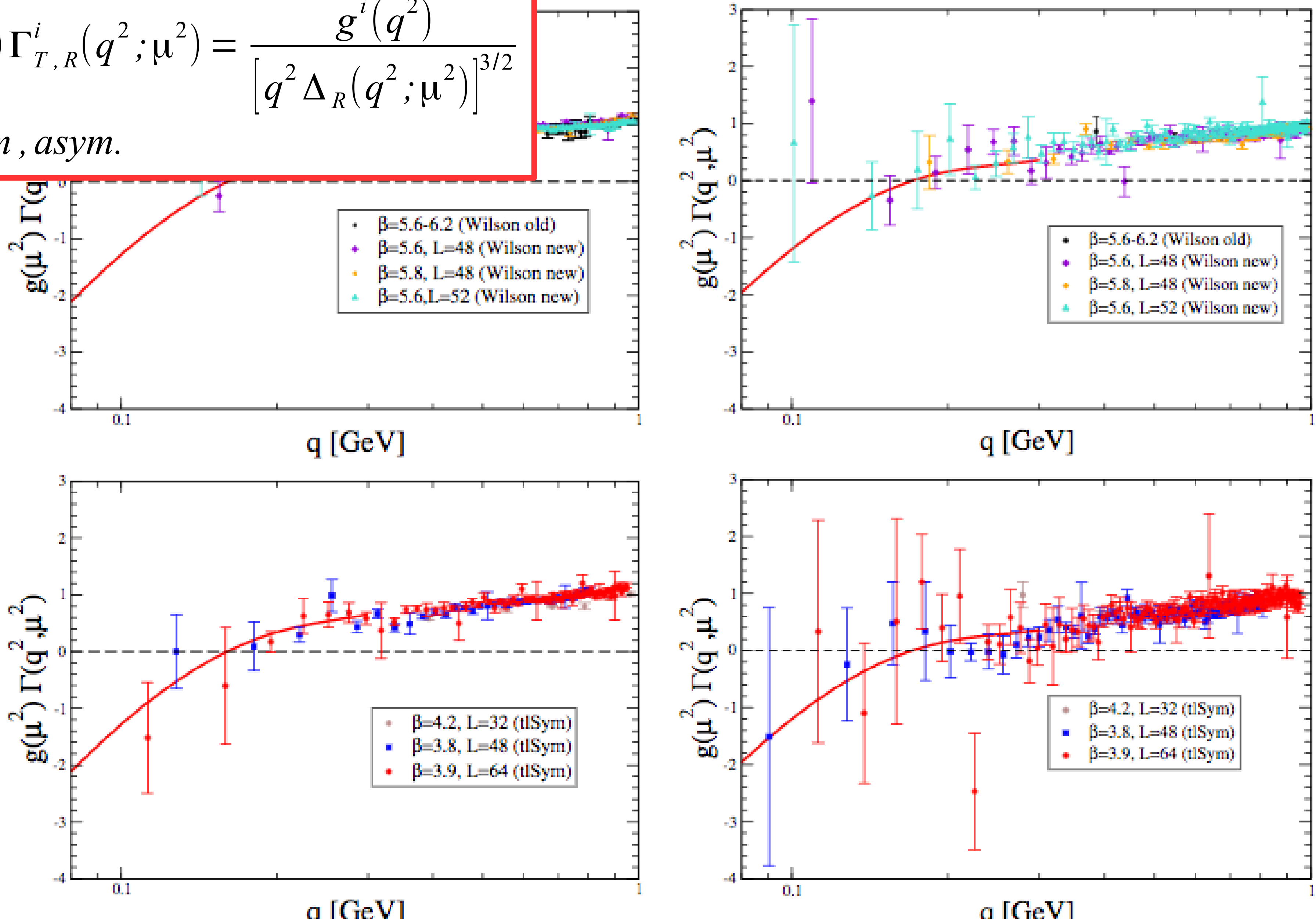


Let's consider now the asymmetric case: the results are much noisier (surely because of the zero-momentum gluon field in the correlation function), although there appear to be strong indications for the happening of the zero-crossing.

The zero-crossing of the three-gluon vertex

$$g^{i}(\mu^2)\,\Gamma^{i}_{T,R}(q^2;\mu^2) = \frac{g^{i}(q^2)}{\left[q^2\Delta_R(q^2;\mu^2)\right]^{3/2}}$$

$i = sym\,,\, asym.$

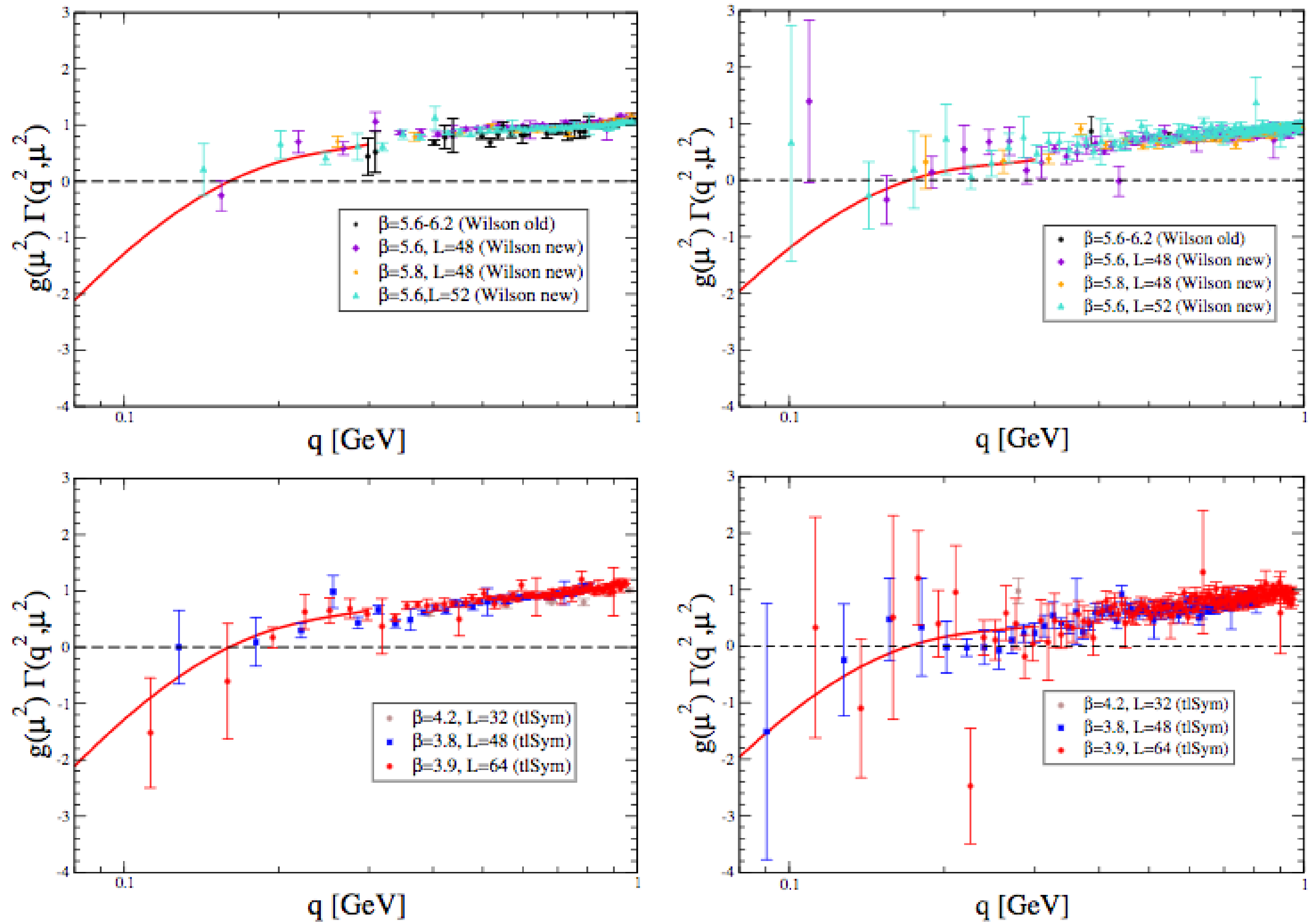


After le
crossin
asymm

The zero-crossing of the three-gluon vertex

$g^i(\mu^2$

$i = sym$

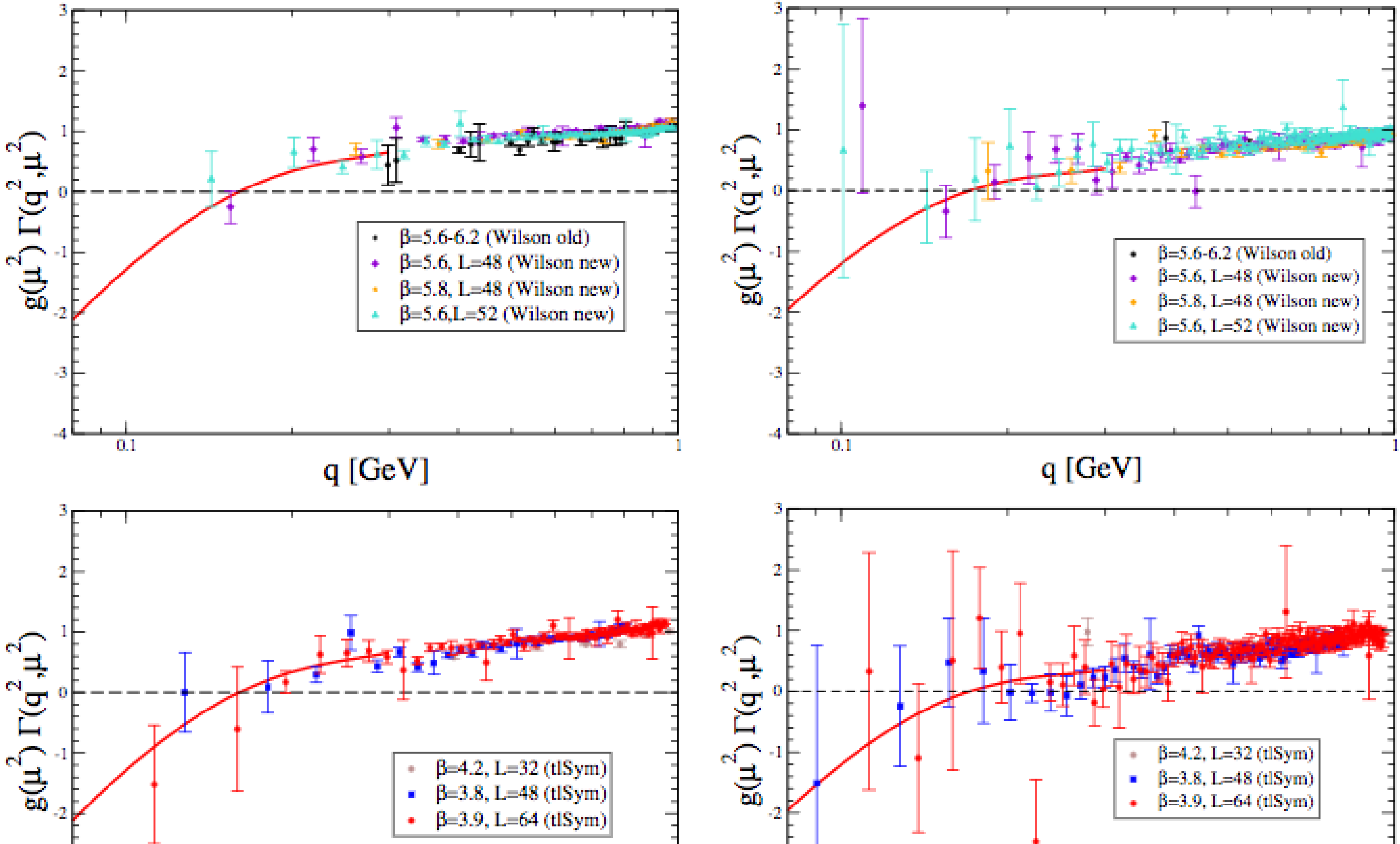


After le
crossin
asymm

The zero-crossing of the three-gluon vertex

$g^i(\mu^2$

$i = syn$



After leg amputation, the 1PI form factor for the tree-level tensor shows clearly the zero-crossing. The trend is the same for both Wilson and tlSym actions and symmetric and asymmetric configurations.

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008

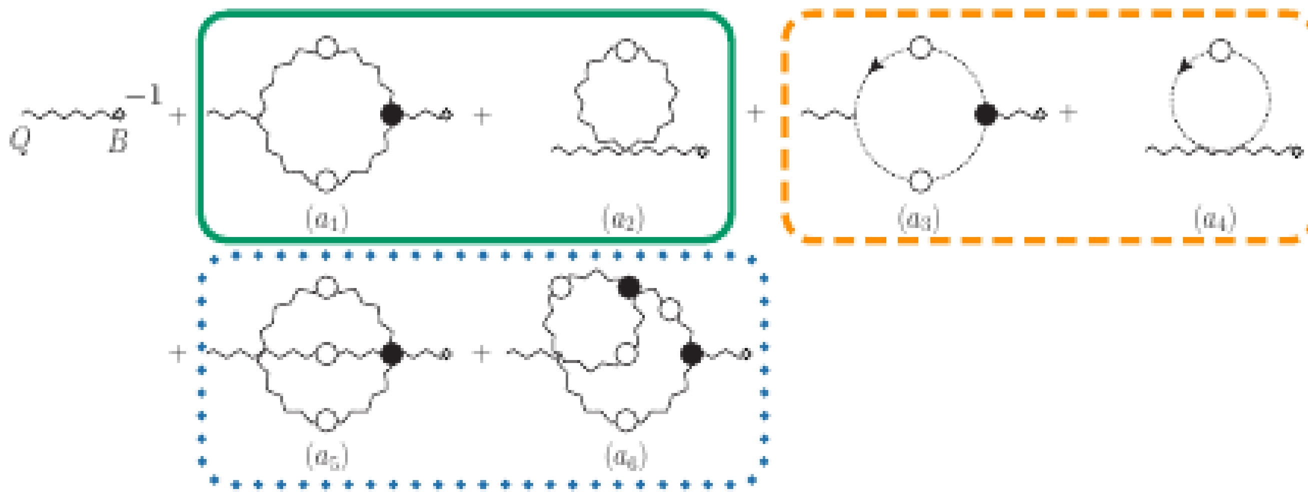
DSE-based explanation:

In PT-BFM truncation

$$\Gamma^{i,(B)}_{T,R}(p^2;\mu^2) \underset{p^2/\mu^2 \ll 1}{\simeq} F_R(0;\mu^2)\frac{\partial}{\partial p^2}\Delta_R^{-1}(p^2;\mu^2) + \ldots$$

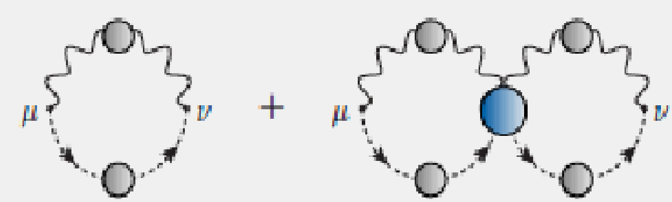
cf. D. Binosi's talk!!!

$\widetilde{\Delta}^{-1}(q^2) =$



$$[1+G(q^2)]^2\Delta^{-1}(q^2) = \widehat{\Delta}^{-1}(q^2).$$

$\Lambda_{\mu\nu}(q) =$



$$= G(q^2)g_{\mu\nu} + L(q^2)\frac{q_\mu q_\nu}{q^2}$$

The zero-crossing of the three-gluon vertex

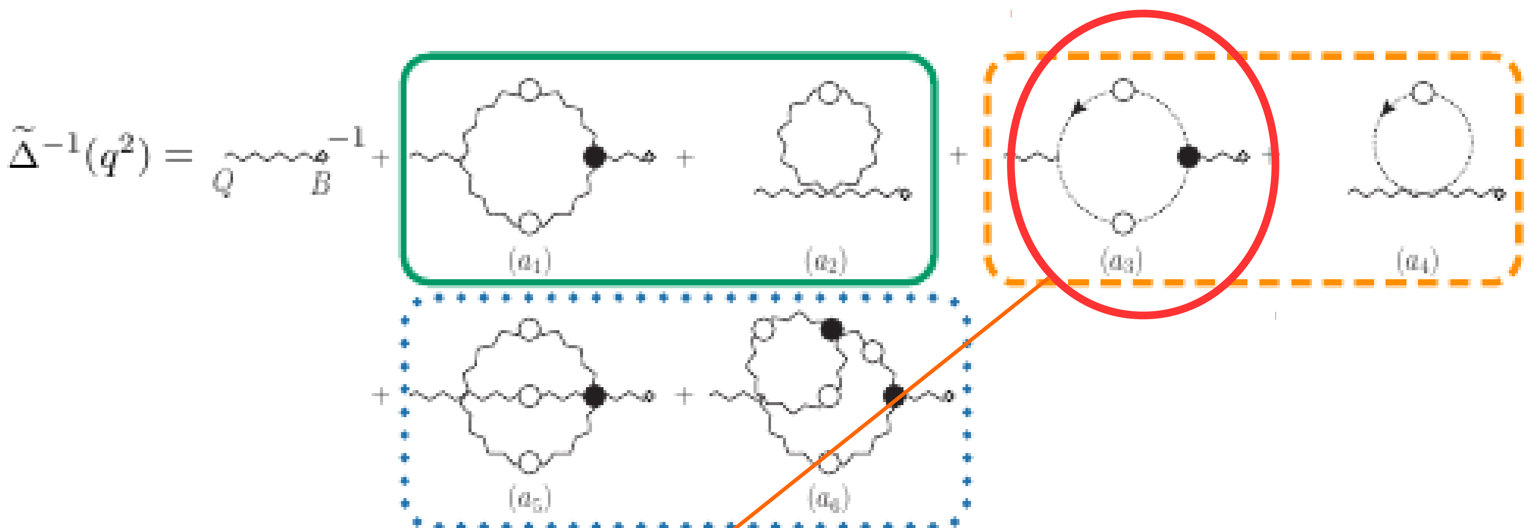
A.C Aguilar et al.; PRD89(2014)05008

DSE-based explanation:

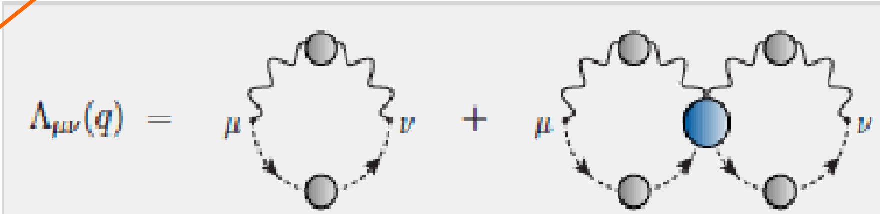
In PT-BFM truncation

$$\Gamma^{i,(B)}_{T,R}(p^2;\mu^2) \underset{p^2/\mu^2 \ll 1}{\simeq} F_R(0;\mu^2)\frac{\partial}{\partial p^2}\Delta_R^{-1}(p^2;\mu^2) + \ldots$$

cf. D. Binosi's talk!!!



$$[1+G(q^2)]^2\Delta^{-1}(q^2) = \widehat{\Delta}^{-1}(q^2).$$



$$= G(q^2)g_{\mu\nu} + L(q^2)\frac{q_\mu q_\nu}{q^2}$$

$$\Pi_c(q^2) = \frac{g^2C_A}{6}q^2F(q^2)\int_k \frac{F(k^2)}{k^2(k+q)^2},$$

The zero-crossing of the three-gluon vertex

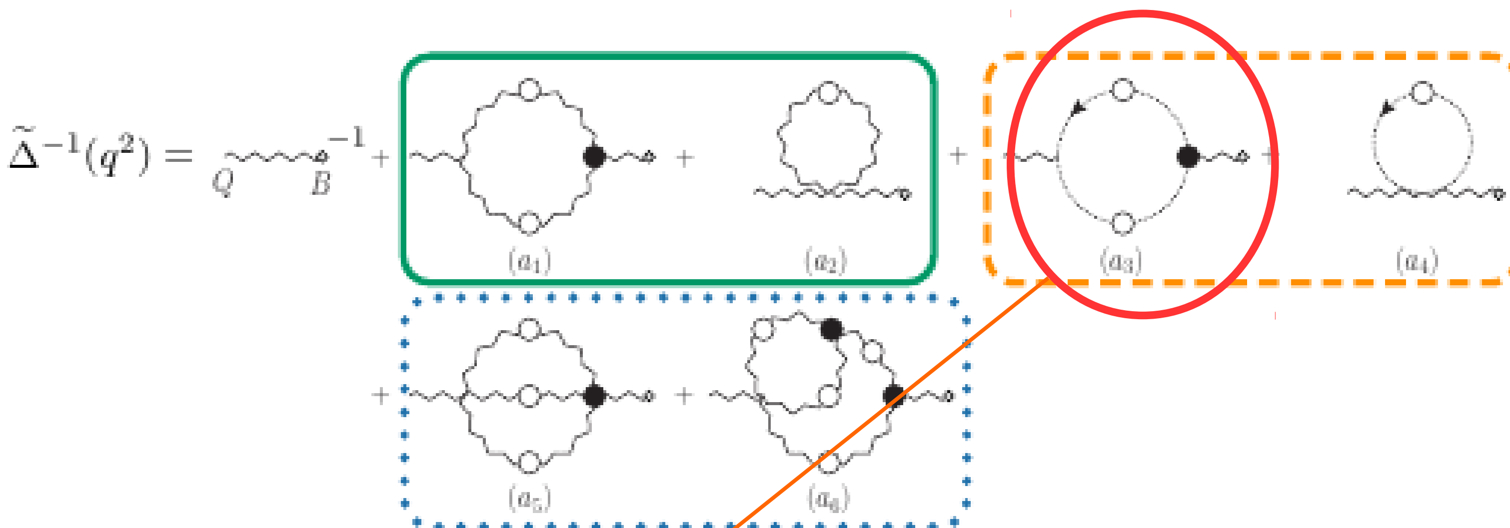
A.C Aguilar et al.; PRD89(2014)05008

DSE-based explanation:

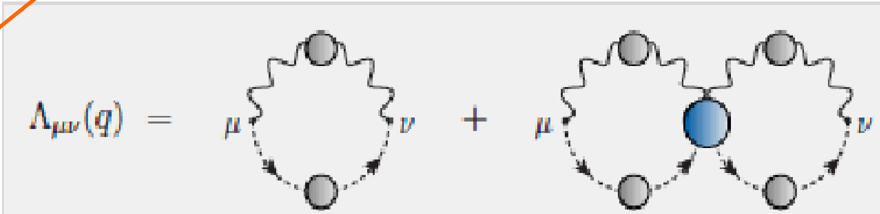
$$\boxed{\Gamma^{i,(B)}_{T,R}(p^2;\mu^2) \underset{p^2/\mu^2 \ll 1}{\simeq} F_R(0;\mu^2)\frac{\partial}{\partial p^2}\Delta_R^{-1}(p^2;\mu^2) \; + \; \ldots}$$

In PT-BFM truncation

cf. D. Binosi's talk!!!



$$[1+G(q^2)]^2\Delta^{-1}(q^2) = \widehat{\Delta}^{-1}(q^2).$$



$$= G(q^2)g_{\mu\nu} + L(q^2)\frac{q_\mu q_\nu}{q^2}$$

$$\boxed{\Pi_c(q^2) = \frac{g^2 C_A}{6} q^2 F(q^2) \int_k \frac{F(k^2)}{k^2(k+q)^2},}$$

d=4

$$\boxed{\Delta_R^{-1}(q^2;\mu^2) \underset{q^2\to 0}{=} q^2\left[a + b\log\frac{q^2+m^2}{\mu^2} + c\log\frac{q^2}{\mu^2}\right] + m^2,}$$

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008

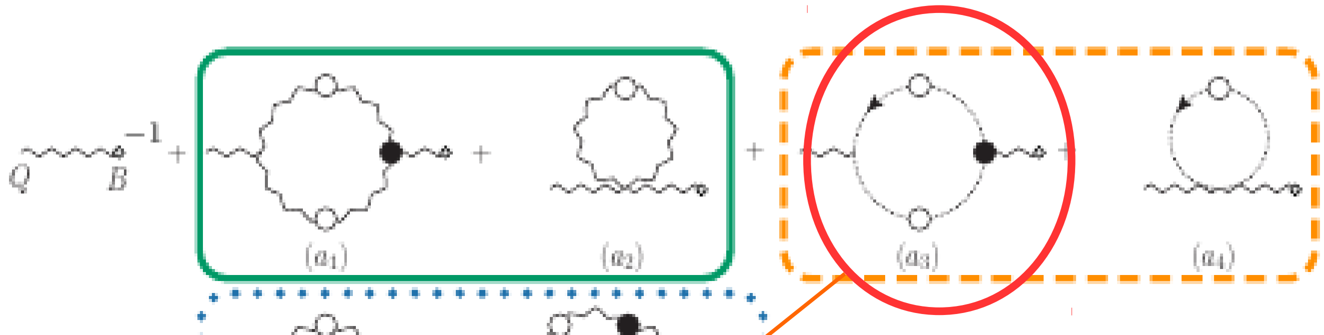
DSE-based explanation:

In PT-BFM truncation

$$\Gamma^{i,(B)}_{T,R}(p^2;\mu^2) \underset{p^2/\mu^2 \ll 1}{\simeq} F_R(0;\mu^2)\left(a + b\ln\frac{m^2}{\mu^2} + c\right) + c\, F_R(0;\mu^2)\ln\frac{p^2}{\mu^2} + \ldots$$

. Binosi's talk!!!

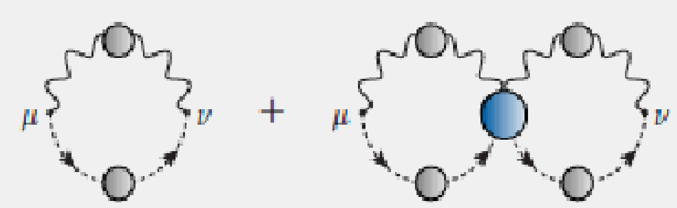
$$\widetilde{\Delta}^{-1}(q^2) =$$



A logarithmic divergent contribution at vanishing momentum, pulling down the 1PI form factor and generating a zero crossing, can be understood within a DSE framework.

$$[1 + G(q^2)]^2 \Delta^{-1}(q^2) = \widehat{\Delta}^{-1}(q^2).$$

$$\Lambda_{\mu\nu}(q) =$$



$$= G(q^2) g_{\mu\nu} + L(q^2)\frac{q_\mu q_\nu}{q^2}$$

$$\Pi_c(q^2) = \frac{g^2 C_A}{6} q^2 F(q^2) \int_k \frac{F(k^2)}{k^2(k+q)^2},$$

d=4

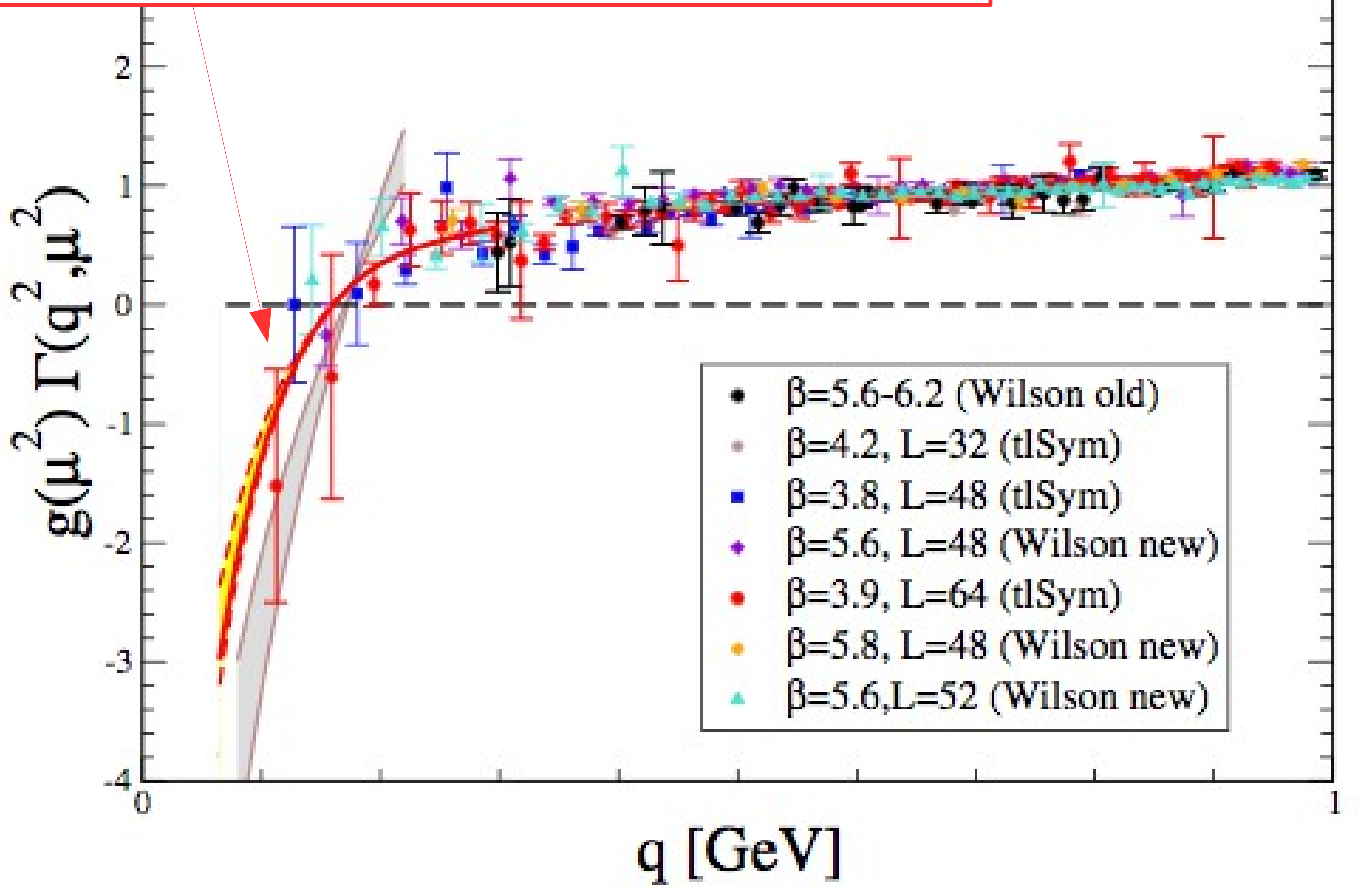
$$\Delta_R^{-1}(q^2;\mu^2) \underset{q^2\to 0}{=} q^2\left[a + b\log\frac{q^2+m^2}{\mu^2} + c\log\frac{q^2}{\mu^2}\right] + m^2,$$

The zero-crossing of the three-gluon vertex

A.C Aguilar et al.; PRD89(2014)05008
Ph. Boucaud et al.; PRD95(2017)114503

$$g_R^i(\mu^2)\Gamma_R^i(p^2;\mu^2) = a_{ln}^i(\mu^2)\ln\frac{p^2}{\mu^2} + a_0^i(\mu^2) + a_2^i(\mu^2)\, p^2 \ln\frac{p^2}{M^2} + o(p^2)$$

i = symmetric



We can thus perform a fit, only over a deep IR domain, of our data to a DSE-grounded formula and describe the behaviour of the 1PI form factor.

The zero-crossing of the three-gluon vertex

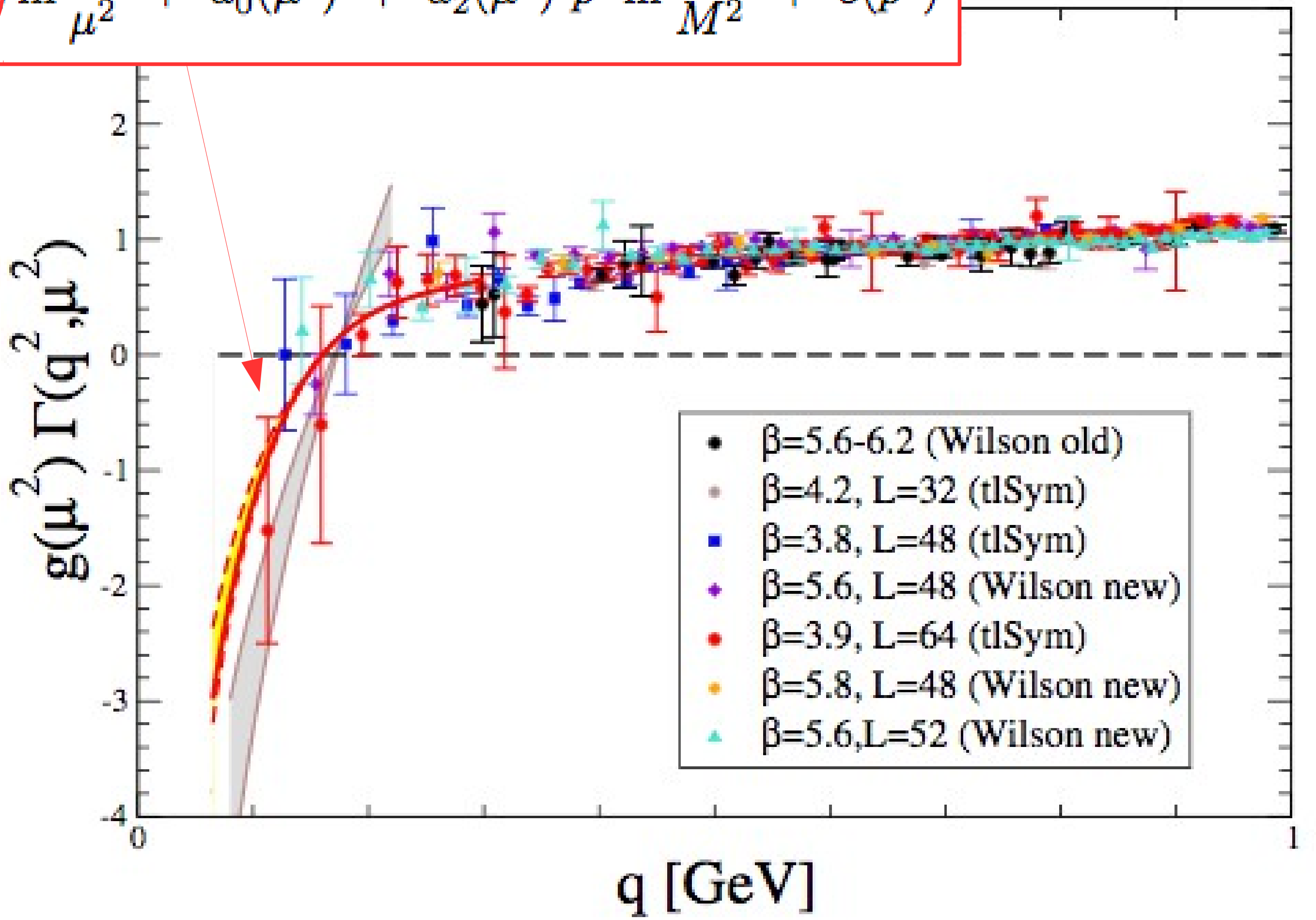
A.C Aguilar et al.; PRD89(2014)05008
Ph. Boucaud et al.; PRD95(2017)114503

$$g_R^i(\mu^2)\Gamma_R^i(p^2;\mu^2) = a_{ln}^i(\mu^2)\ln\frac{p^2}{\mu^2} + a_0^i(\mu^2) + a_2^i(\mu^2)\, p^2 \ln\frac{p^2}{M^2} + o(p^2)$$

i = symmetric

$$g_R^i(\mu^2)\, c\, F_R(0,\mu^2)$$

Consistent with direct large-volume lattice evaluations of the gluon and ghost two-point Green functions.



We can thus perform a fit, only over a deep IR domain, of our data to the DSE-grounded formula and describe the behaviour of the 1PI form factor.

The zero-crossing of the three-gluon vertex

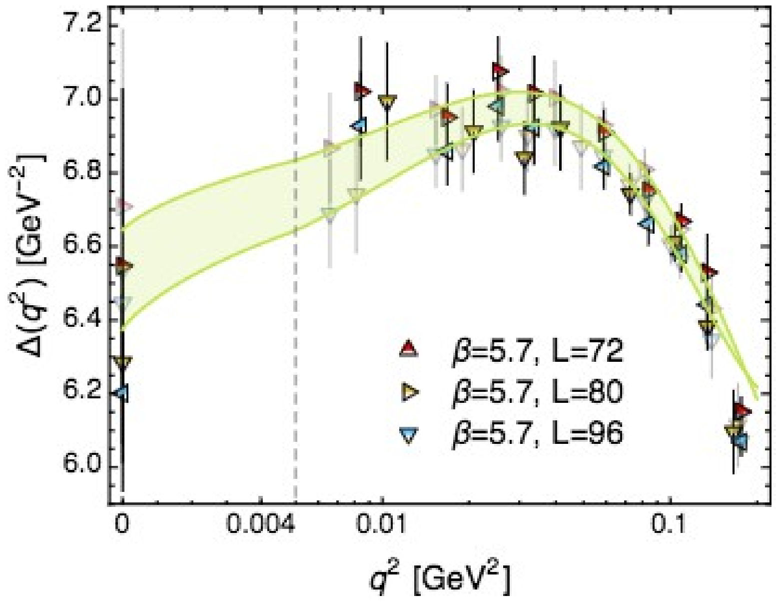
A.C Aguilar et al.; PRD89(2014)05008
Ph. Boucaud et al.; PRD95(2017)114503

$$g_R^i(\mu^2)\Gamma_R^i(p^2;\mu^2) = a_{ln}^i(\mu^2)\ln\frac{p^2}{\mu^2} + a_0^i(\mu^2) + a_2^i(\mu^2)\,p^2\ln\frac{p^2}{M^2} + o(p^2)$$

i = symmetric

$$g_R^i(\mu^2)\, c\, F_R(0,\mu^2)$$

Consistent with direct large-volume lattice evaluations of the gluon and ghost two-point Green functions.



We can thus perform a fit, only over a deep IR domain, of our data to the DSE-grounded formula and describe the behaviour of the 1PI form factor.

The zero-crossing of the three-gluon vertex

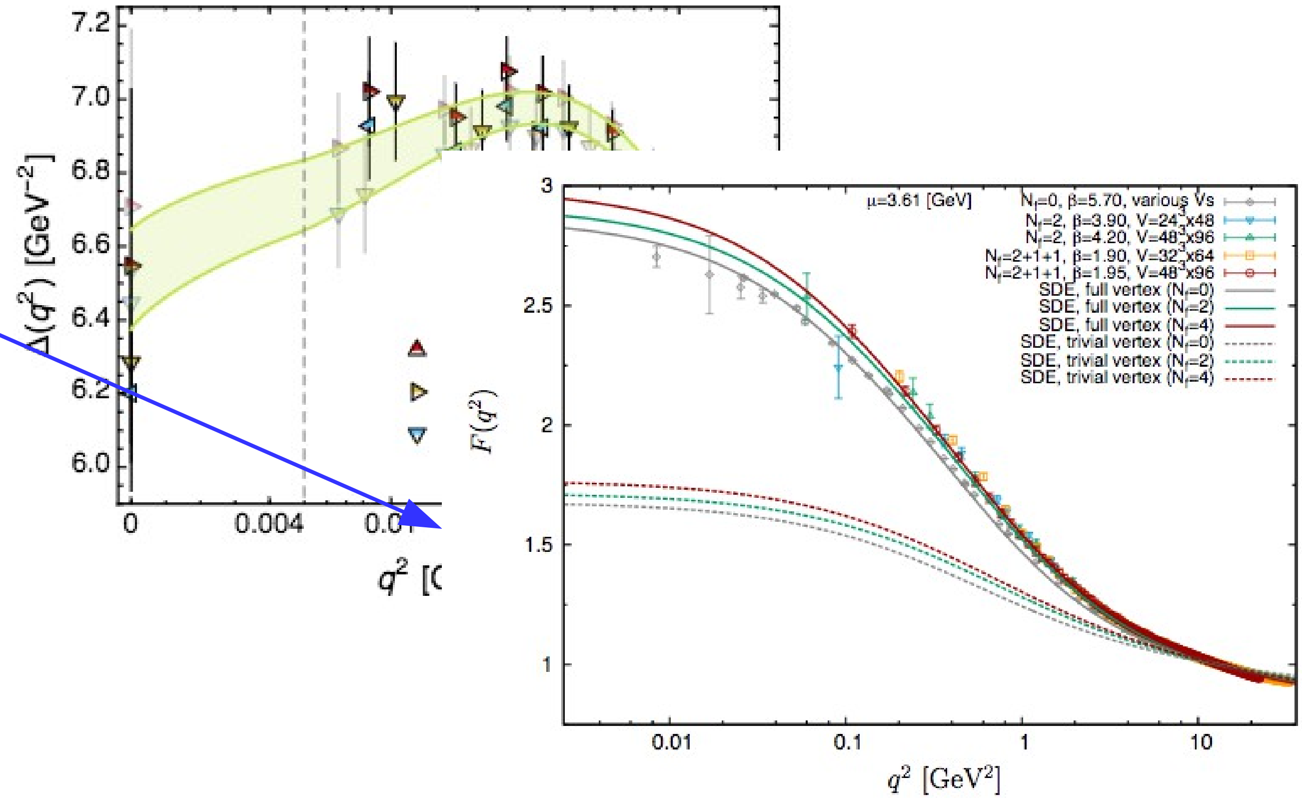
A.C Aguilar et al.; PRD89(2014)05008
Ph. Boucaud et al.; PRD95(2017)114503

$$g^i_R(\mu^2)\Gamma^i_R(p^2;\mu^2) \;=\; a^i_{ln}(\mu^2)\ln\frac{p^2}{\mu^2} \;+\; a^i_0(\mu^2) \;+\; a^i_2(\mu^2)\,p^2\ln\frac{p^2}{M^2} \;+\; o(p^2)$$

i = symmetric

$$g^i_R(\mu^2)\,c\,F_R(0,\mu^2)$$

Consistent with direct large-volume lattice evaluations of the gluon and ghost two-point Green functions.



We can thus perform a fit, only over a deep IR domain, of our data to the DSE-grounded formula and describe the behaviour of the 1PI form factor.

The zero-crossing of the three-gluon vertex

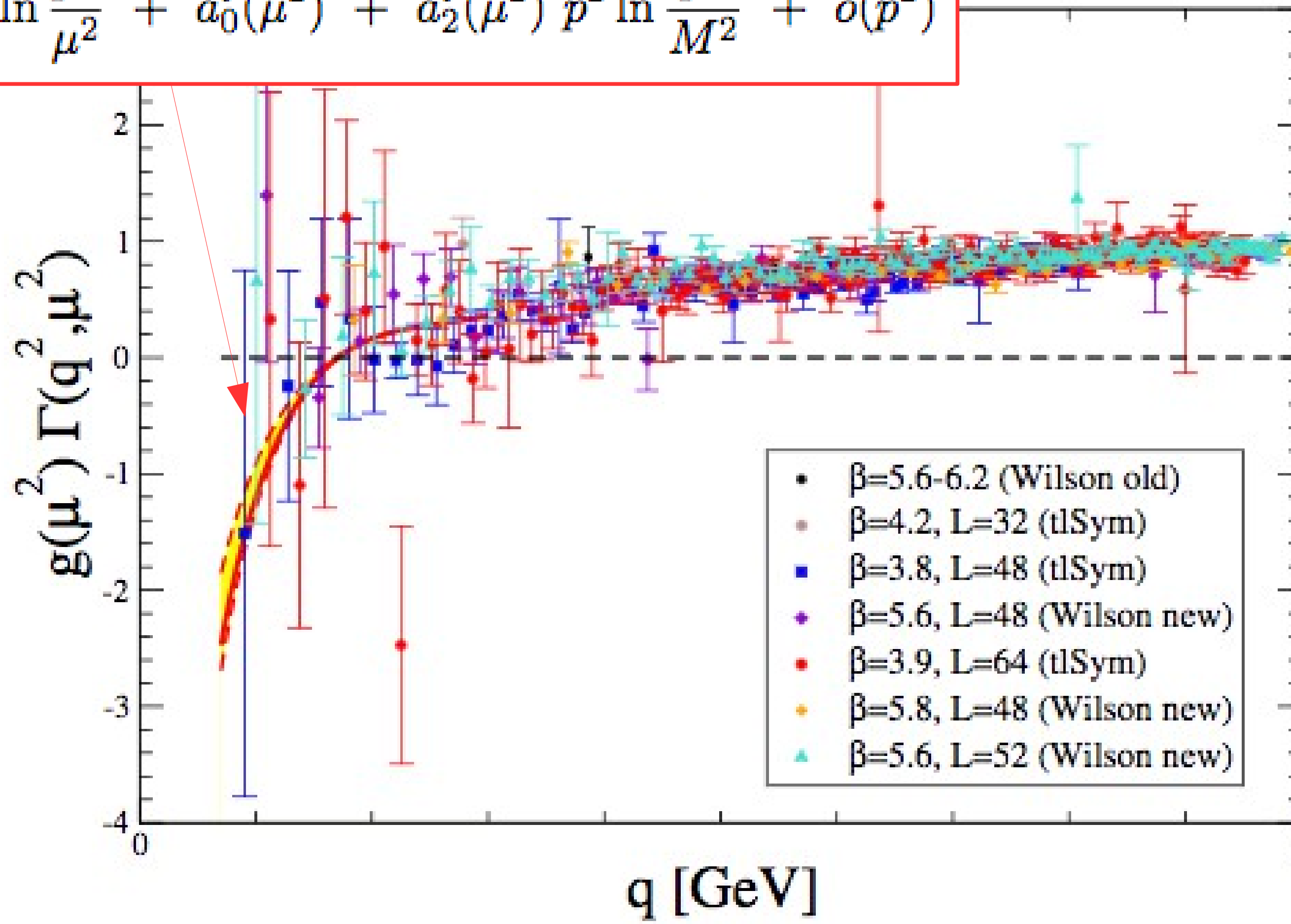
A.C Aguilar et al.; PRD89(2014)05008
Ph. Boucaud et al.; PRD95(2017)114503

i = asymmetric

$$g_R^i(\mu^2)\Gamma_R^i(p^2;\mu^2) = a_{ln}^i(\mu^2)\ln\frac{p^2}{\mu^2} + a_0^i(\mu^2) + a_2^i(\mu^2)\, p^2 \ln\frac{p^2}{M^2} + o(p^2)$$

$$g_R^i(\mu^2)\, c\, F_R(0,\mu^2)$$

Consistent with direct large-volume lattice evaluations of the gluon and ghost two-point Green functions.



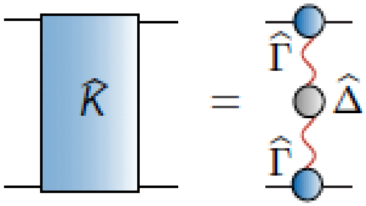
The low-momenta asymptotic 1PI form factor obtained from DSE within the PT-BFM is fully consistent with lattice data for both symmetric and asymmetric kinematic configurations.

Quark's gap equation: RGI interaction

cf. (again) D. Binosi's talk!!!

Convert vertices/propagators into PT-BFM ones
new RG invariant combination appears

$$\widehat{d}(k^2) = \alpha(\mu^2)\widehat{\Delta}(k^2;\mu^2)$$

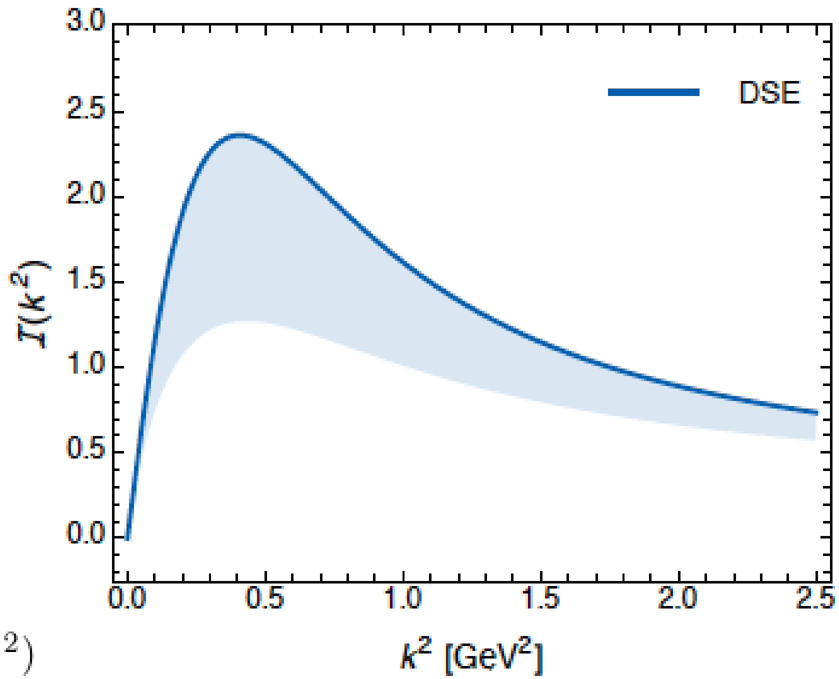


Use symmetry identity
to identify the interaction strength

A.C Aguilar, D. Binosi, J. Papavassiliou, J. R-Q, PRD90(2009)
D. Binosi, L. Chang, J. Papavassiliou, C.D. Roberts, PLb742(2015)

$$\mathcal{I}(k^2) = k^2\widehat{d}(k^2) \longrightarrow = \left[\frac{1}{1-L(q^2)F(q^2)}\right]^2 \alpha_T(q^2)\ .$$

$$\widehat{d}(k^2) = \frac{\alpha(\mu^2)\Delta(k^2;\mu^2)}{[1+G(k^2;\mu^2)]^2}$$



1+*G* and *L* determined by their own SDEs
under simplifying assumptions:

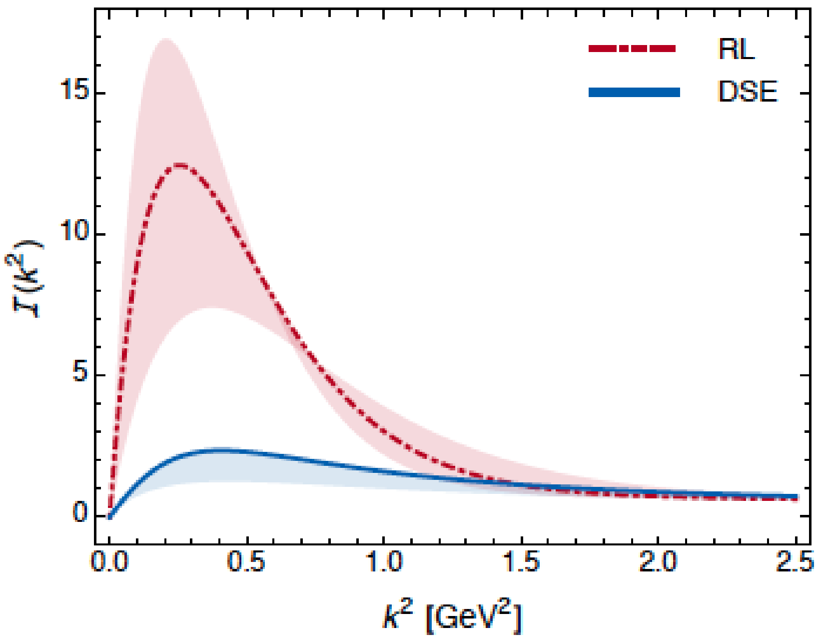
$$1+G(p^2) = Z_c - g^2\int_k \left[2+\frac{(k\cdot p)^2}{k^2p^2}\right] B_1(k)\Delta(k)\frac{F((k+p)^2)}{(k+p)^2},$$

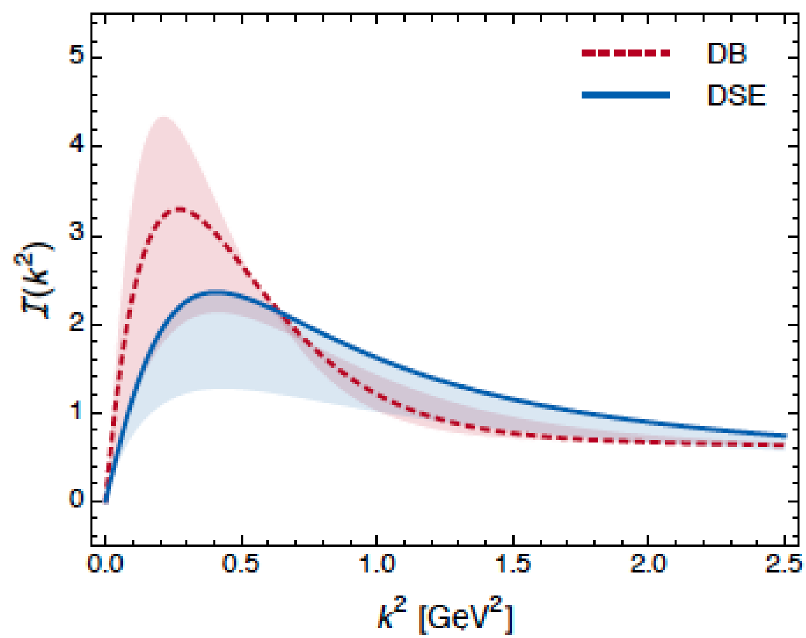
$$L(p^2) = -g^2\int_k \left[1-4\frac{(k\cdot p)^2}{k^2p^2}\right] B_1(k)\Delta(k)\frac{F((k+p)^2)}{(k+p)^2}.$$

$$F^{-1}(q^2) \ = \ Z_c - 3\ g^2\int_k \left[1-\frac{(k\cdot p)^2}{k^2p^2}\right] B_1(k)\Delta(k)\frac{F((k+p)^2)}{(k+p)^2}$$

- **Main source of uncertainties:**
  needs assumptions on ghost vertex behavior
- **Parametrized by δ∈[0,1]**
  lower bound (δ=0): *1/F=1+G*

Top-down vs. Bottom-up approaches





D. Binosi, L. Chang, J. Papavassiliou, C.D. Roberts, PLB 742 (2015)

RGI interaction kernel

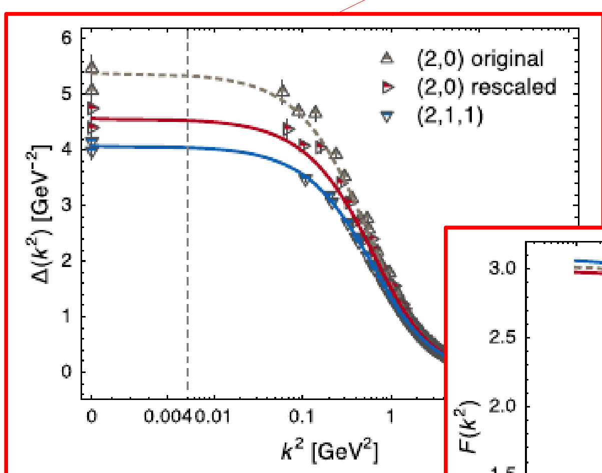
Let us now carefully examine the RGI Interaction:

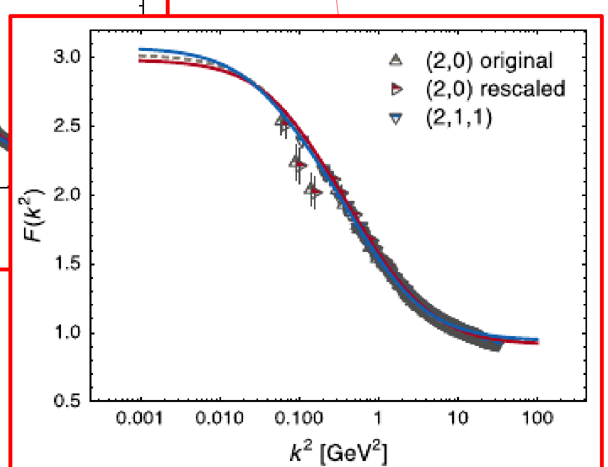
D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

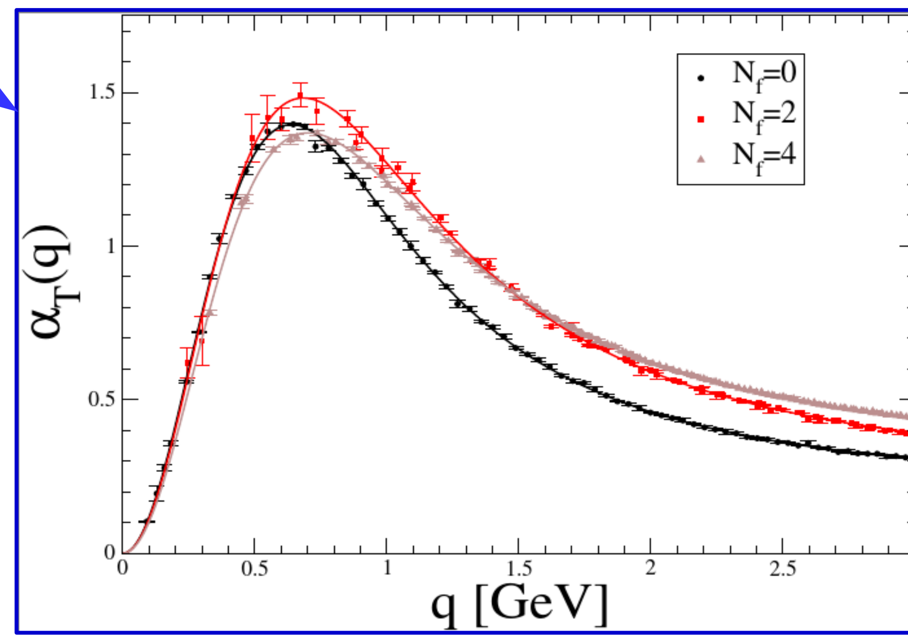
$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\mathrm{T}}(k^2)}{[1 - L(k^2)F(k^2)]^2}$$

$$\boxed{\alpha_T(k^2) = \lim_{a \to 0} g^2(a) k^2 \Delta(k^2;a) F^2(k^2;a)}$$

A running strong coupling in a particular scheme (Taylor), well-known in perturbation and easy-to-handle in Lattice QCD





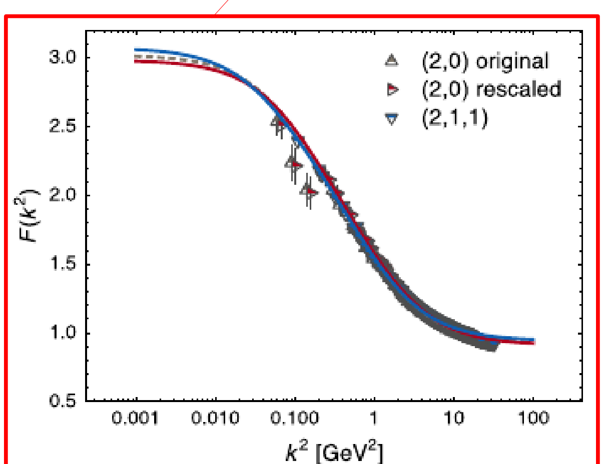


RGI interaction kernel

Let us now carefully examine the RGI Interaction:

D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\mathrm{T}}(k^2)}{[1 - L(k^2)F(k^2)]^2}$$



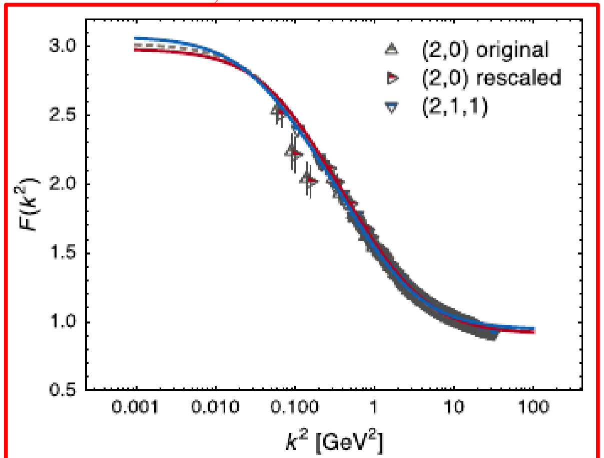
RGI interaction kernel

Let us now carefully examine the RGI Interaction:

D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\mathrm{T}}(k^2)}{[1 - L(k^2)F(k^2)]^2}$$

$$\boxed{L(p^2) = -g^2 \int_k \left[1 - 4\frac{(k\cdot p)^2}{k^2 p^2}\right] B_1(k)\Delta(k)\frac{F((k+p)^2)}{(k+p)^2}.}$$



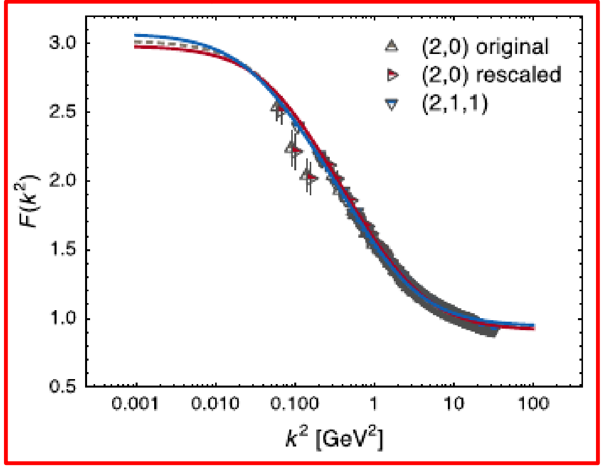
RGI interaction kernel

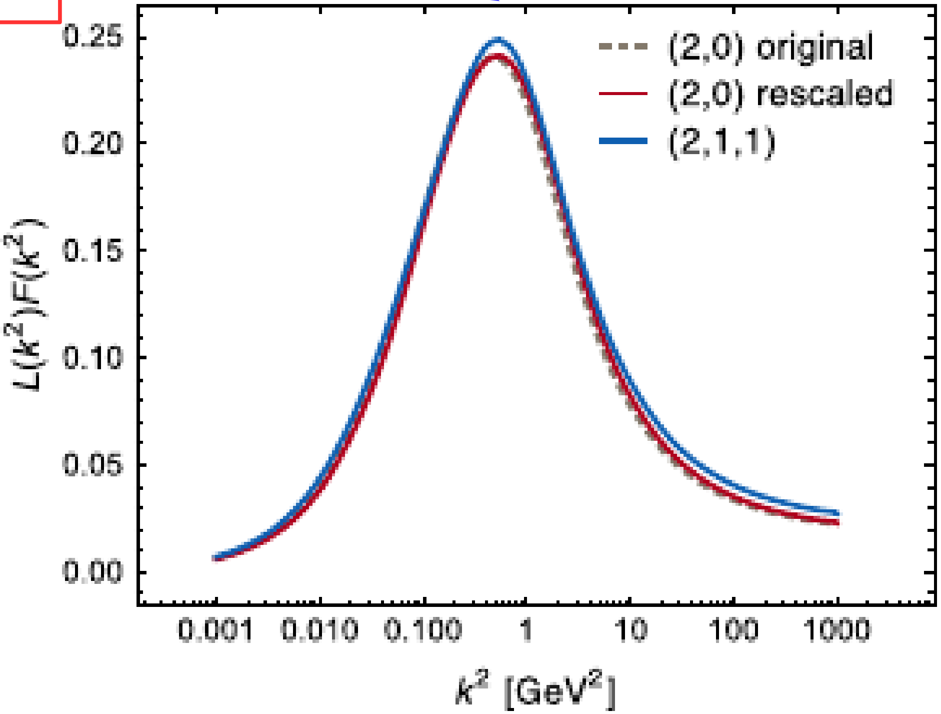
Let us now carefully examine the RGI Interaction:

D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\mathrm{T}}(k^2)}{[1 - L(k^2)F(k^2)]^2}$$

$$L(p^2) = -g^2 \int_k \left[1 - 4\frac{(k\cdot p)^2}{k^2 p^2}\right] B_1(k)\Delta(k)\frac{F((k+p)^2)}{(k+p)^2}.$$





RGI interaction kernel

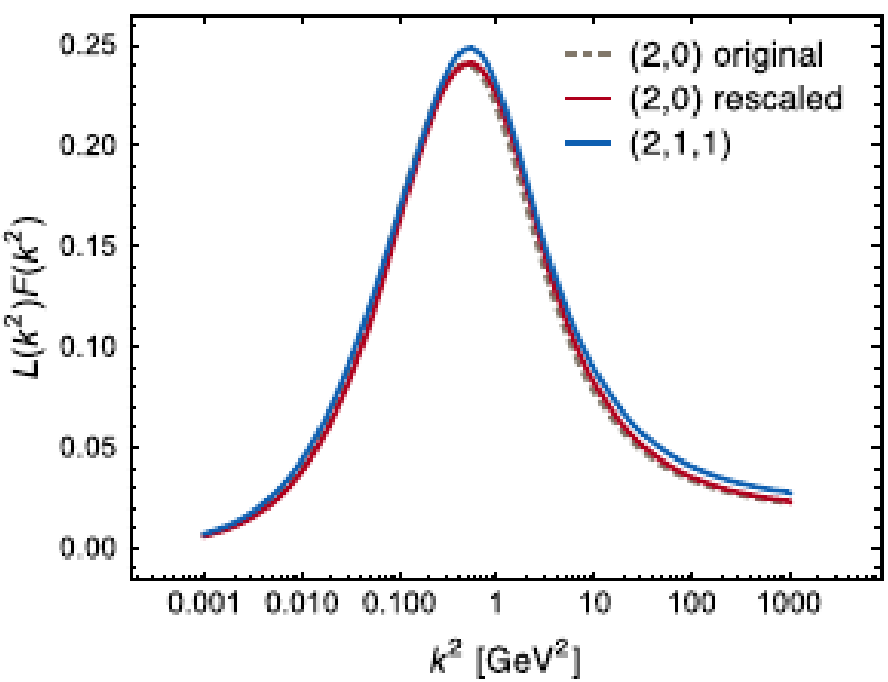
Let us now carefully examine the RGI Interaction:

D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\mathrm{T}}(k^2)}{[1 - L(k^2) F(k^2)]^2}$$

$$F(k^2;\zeta^2) \underset{k^2/\Lambda_{\mathrm{T}}^2 \gg 1}{\approx} \left(\ln \frac{k^2}{\Lambda_{\mathrm{T}}^2} \Big/ \ln \frac{\zeta^2}{\Lambda_{\mathrm{T}}^2} \right)^{-\widetilde{\gamma}_0/\beta_0},$$

$$L(k^2;\zeta^2) \underset{k^2/\Lambda_{\mathrm{T}}^2 \gg 1}{\approx} \frac{3g^2(\zeta^2)}{32\pi^2} \left(\ln \frac{k^2}{\Lambda_{\mathrm{T}}^2} \Big/ \ln \frac{\zeta^2}{\Lambda_{\mathrm{T}}^2} \right)^{-(\widetilde{\gamma}_0+\gamma_0)/\beta_0}$$



RGI interaction kernel

Let us now carefully examine the RGI Interaction:

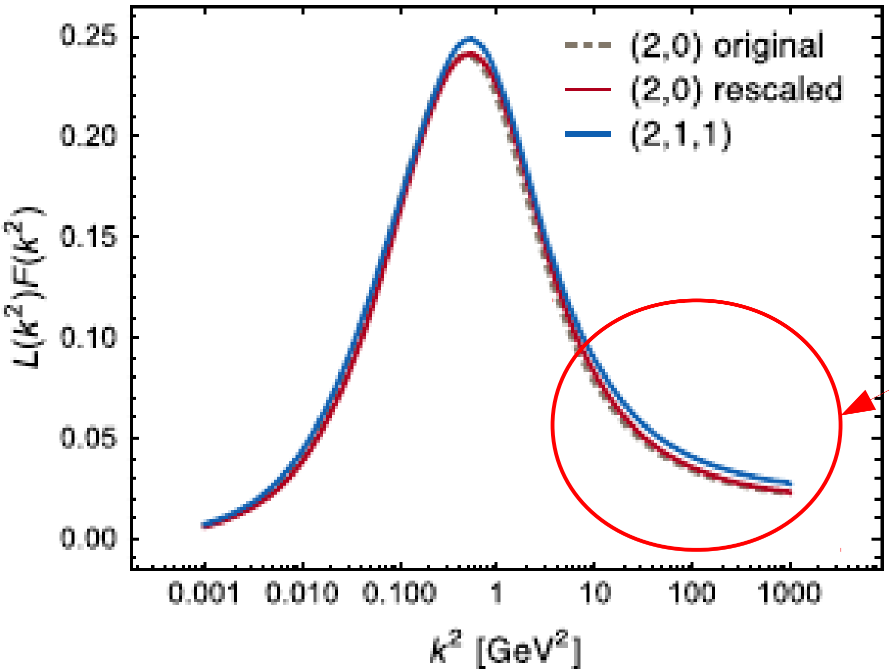
D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\mathrm{T}}(k^2)}{[1 - L(k^2) F(k^2)]^2}$$

$$F(k^2;\zeta^2) \underset{k^2/\Lambda_{\mathrm{T}}^2 \gg 1}{\approx} \left(\ln \frac{k^2}{\Lambda_{\mathrm{T}}^2} \Big/ \ln \frac{\zeta^2}{\Lambda_{\mathrm{T}}^2} \right)^{-\widetilde{\gamma}_0/\beta_0},$$

$$L(k^2;\zeta^2) \underset{k^2/\Lambda_{\mathrm{T}}^2 \gg 1}{\approx} \frac{3g^2(\zeta^2)}{32\pi^2} \left(\ln \frac{k^2}{\Lambda_{\mathrm{T}}^2} \Big/ \ln \frac{\zeta^2}{\Lambda_{\mathrm{T}}^2} \right)^{-(\widetilde{\gamma}_0+\gamma_0)/\beta_0}$$

$$L(k^2)F(k^2) \underset{q^2/\Lambda_{\mathrm{T}}^2 \gg 1}{\approx} \frac{3}{2\beta_0 \ln\left(k^2/\Lambda_{\mathrm{T}}^2\right)},$$



RGI interaction kernel

Let us now carefully examine the RGI Interaction:

D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\rm T}(k^2)}{[1 - L(k^2) F(k^2)]^2}$$

$$F(k^2;\zeta^2) \underset{k^2/\Lambda_{\rm T}^2 \gg 1}{\approx} \left(\ln \frac{k^2}{\Lambda_{\rm T}^2} \Big/ \ln \frac{\zeta^2}{\Lambda_{\rm T}^2} \right)^{-\widetilde{\gamma}_0/\beta_0} ,$$

$$L(k^2;\zeta^2) \underset{k^2/\Lambda_{\rm T}^2 \gg 1}{\approx} \frac{3 g^2(\zeta^2)}{32\pi^2} \left(\ln \frac{k^2}{\Lambda_{\rm T}^2} \Big/ \ln \frac{\zeta^2}{\Lambda_{\rm T}^2} \right)^{-(\widetilde{\gamma}_0+\gamma_0)/\beta_0}$$

$$L(k^2)F(k^2) \underset{q^2/\Lambda_{\rm T}^2 \gg 1}{\approx} \frac{3}{2\beta_0 \ln\left(k^2/\Lambda_{\rm T}^2\right)},$$

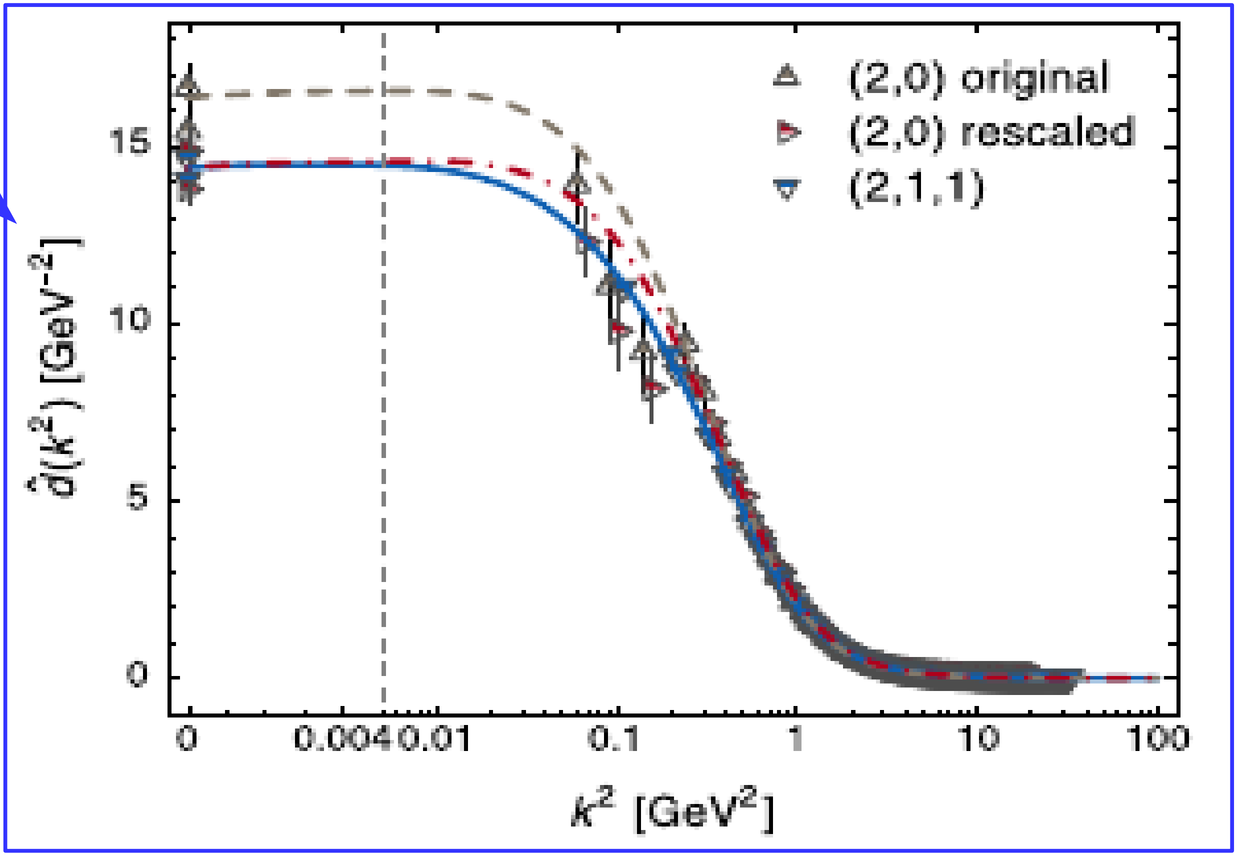
$$\alpha_{\overline{\rm MS}}(k^2)(1 + 1.09\, \alpha_{\overline{\rm MS}}(k^2) + \ldots)$$

RGI interaction kernel

Let us now carefully examine the RGI Interaction:

D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\mathrm{T}}(k^2)}{[1 - L(k^2)F(k^2)]^2}$$



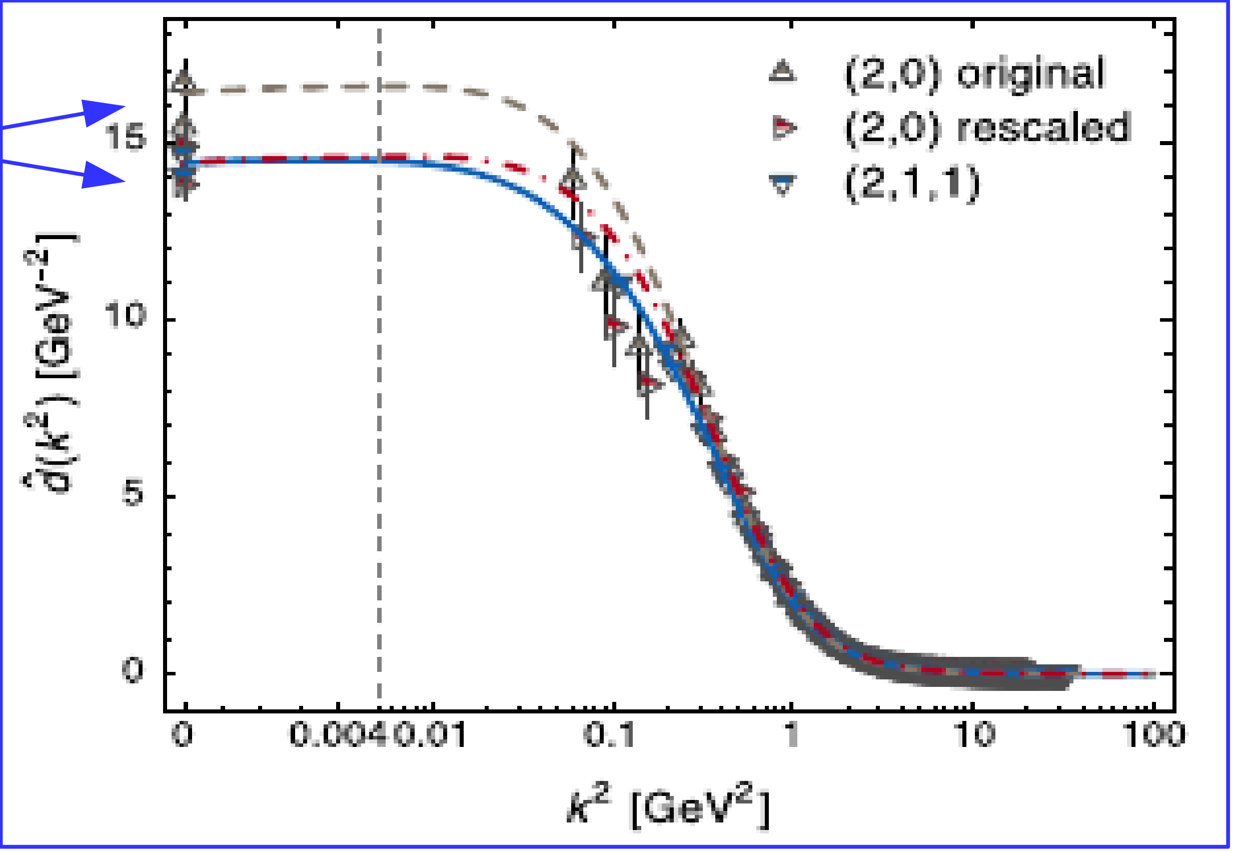
RGI interaction kernel

Let us now carefully examine the RGI Interaction:

D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\mathrm{T}}(k^2)}{[1 - L(k^2)F(k^2)]^2}$$

Zero-momentum freezing!
Flavor-dependent?



RGI interaction kernel

Let us now carefully examine the RGI Interaction:

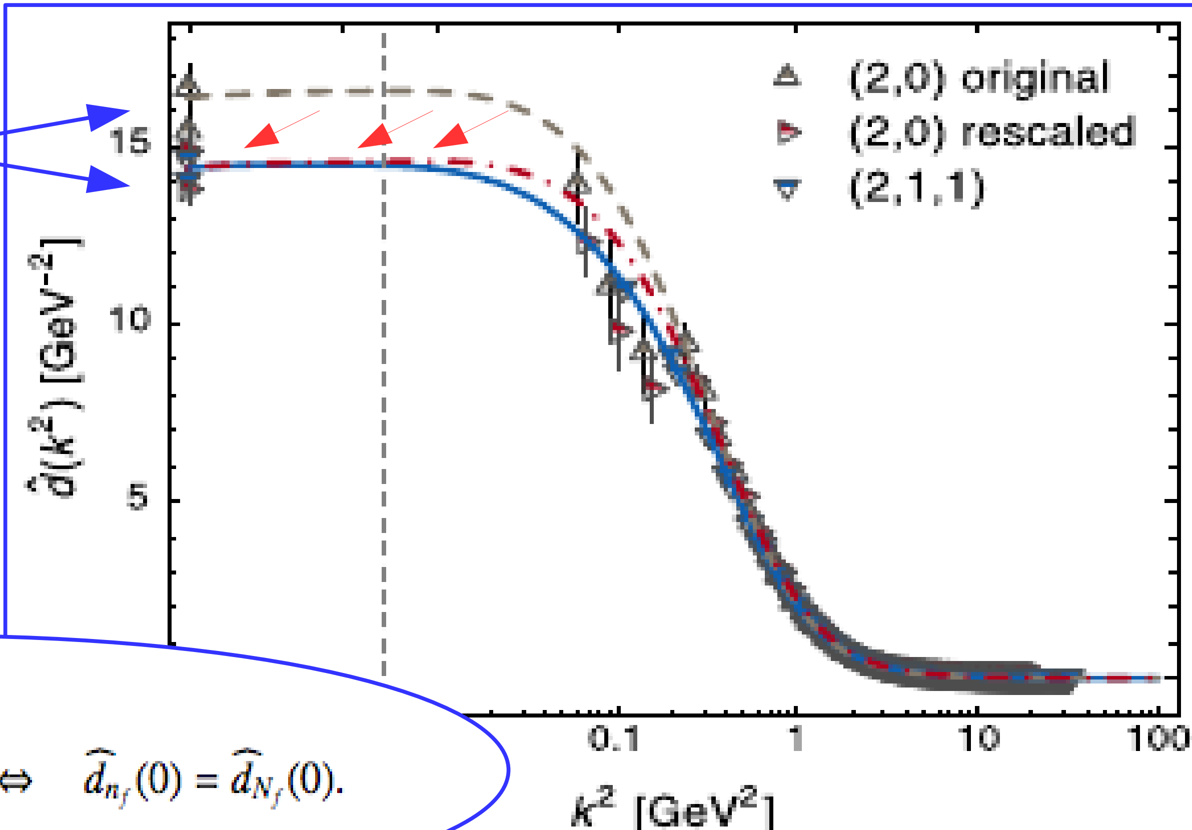
D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\rm T}(k^2)}{[1 - L(k^2)F(k^2)]^2}$$

Zero-momentum freezing!
Flavor-dependent?
Yes, but not when the quark thresholds are far above …

...As happens for

$N_f = n_f + \delta$
$n_f = 2$
$\delta = 1(s) + 1(c)$



$$\lim_{k^2\to 0} \frac{I_{n_f}(k^2)}{k^2} = \lim_{k^2\to 0} \frac{I_{N_f}(k^2)}{k^2} \quad \Leftrightarrow \quad \widehat{d}_{n_f}(0) = \widehat{d}_{N_f}(0).$$

RGI interaction kernel

Let us now carefully examine the RGI Interaction:

D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\mathrm{T}}(k^2)}{[1 - L(k^2)F(k^2)]^2}$$

Low-momentum asymptotic expansion

$$\boxed{I(k^2) \underset{k^2/\Lambda_{\mathrm{T}}^2 \ll 1}{\approx} k^2 \widehat{d}(0) \left[1 - \left(\frac{\widehat{d}(0)}{8\pi} + \frac{\ell_w}{m_g^2}\right) k^2 \ln \frac{k^2}{\Lambda_{\mathrm{T}}^2}\right]}$$

RGI interaction kernel

Let us now carefully examine the RGI Interaction:

D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_T(k^2)}{[1 - L(k^2)F(k^2)]^2}$$

Low-momentum asymptotic expansion

$$I(k^2) \underset{k^2/\Lambda_T^2 \ll 1}{\approx} k^2 \widehat{d}(0) \left[1 - \left(\frac{\widehat{d}(0)}{8\pi} + \frac{\zeta_w}{m_g^2} \right) k^2 \ln \frac{k^2}{\Lambda_T^2} \right]$$

$$\widehat{d}(0) = \frac{\alpha_0}{m_0^2} \approx \frac{0.9\pi}{(m_P/2)^2}$$

Ir mass scale of about one half of the proton mass (cf. C. Roberts' talk!!!)

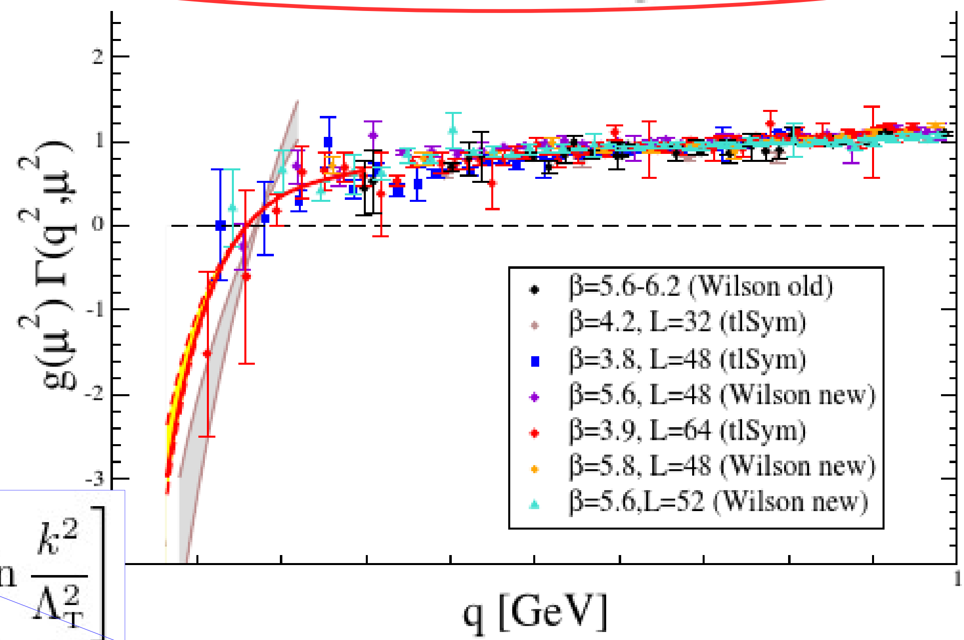
D. Binosi, L. Chang, J. Papavassiliou, C.D. Roberts, PLB 742 (2015)

RGI interaction kernel

Let us now carefully examine the RGI Interaction:

$$\Gamma^{i,(B)}_{T,R}(p^2;\mu^2) \underset{p^2/\mu^2 \ll 1}{\simeq} F_R(0;\mu^2)\frac{\partial}{\partial p^2}\Delta_R^{-1}(p^2;\mu^2) + \dots$$

$$I(k^2) := k^2\widehat{d}(k^2) = \frac{\alpha_{\mathrm{T}}(k^2)}{[1-L(k^2)F(k^2)]^2}$$



Low-momentum asymptotic expansion

$$I(k^2) \underset{k^2/\Lambda_{\mathrm{T}}^2 \ll 1}{\approx} k^2\widehat{d}(0)\left[1-\left(\frac{\widehat{d}(0)}{8\pi}+\frac{l_w}{m_g^2}\right)k^2\ln\frac{k^2}{\Lambda_{\mathrm{T}}^2}\right]$$

$$\boxed{\Delta^{-1}(k^2;\zeta^2) \underset{k^2 \ll \zeta^2}{\approx} k^2\left[a_\Delta + l_g\ln\frac{k^2+m_g^2}{\zeta^2} + l_w\ln\frac{k^2}{\zeta^2}\right] + m_g^2,}$$

A divergent ghost-loop contribution to the gluon vacuum polarization in its DSE

A.C. Aguilar et al,. PRD89(2014)05008
A.K. Cyrol et al. PRD94(2016)054005
Ph. Boucaud et al,. PRD95(2017)114503

QCD effective charge

cf. C. Roberts' talk!!!

Let us first carefully examine the RGI Interaction:

D. Binosi, J. R-Q, C.D. Roberts, PRD95(2017)114009

$$I(k^2) := k^2\widehat{d}(k^2) = \frac{\alpha_{\rm T}(k^2)}{[1 - L(k^2)F(k^2)]^2}$$

Remarkable QCD feature: saturation of the RG key ingredient $\hat{d}(0)$

$$\boxed{\widehat{d}(0) = \frac{\alpha_0}{m_0^2} \approx \frac{0.9\pi}{(m_P/2)^2}}$$

D. Binosi, L. Chang, J. Papavassiliou, C.D. Roberts, PLB 742 (2015)

Define then the RGI invariant function

$$\boxed{\mathcal{D}(k^2) = \frac{\Delta(k^2;\mu^2)}{\Delta(0;\mu^2)m_0^2} = \frac{\Delta\left(k^2,\zeta_0^2\right)}{z\left(\zeta_0^2\right)} = \begin{cases} 1/m_0^2 & k^2 \ll m_0^2 \\ 1/k^2 & k^2 \gg m_0^2 \end{cases}}$$

Extract the (process-independent) coupling

Using the quark gap equation

$$\Sigma(p) = Z_2 \int_{dq}^{\Lambda} 4\pi\widehat{\alpha}_{\rm PI}(k^2)\mathcal{D}_{\mu\nu}(k^2)\gamma_\mu S(q)\hat{\Gamma}^a_\nu(q,p)$$

$$\boxed{\widehat{\alpha}(k^2) = \frac{\widehat{d}(k^2)}{\mathcal{D}(k^2)} \xrightarrow[k^2\gg m_0^2]{} \mathcal{I}(k^2)}$$

D. Binosi, C. Mezrag, J. Papavassiliou, J.R-Q, C.D. Roberts, arXiv:1612.04835

Conclusions

- Lattice contemporary results for the three-gluon Green's functions provide, as a main feature, a zero-crossing at very low-momenta...

Conclusions

• Lattice contemporary results for the three-gluon Green's functions provide, as a main feature, a zero-crossing at very low-momenta...

• … that can be understood as being driven by a negative logarithmic singularity for the 3-gluon 1-PI vertex.

Conclusions

• Lattice contemporary results for the three-gluon Green's functions provide, as a main feature, a zero-crossing at very low-momenta...

• … that can be understood as being driven by a negative logarithmic singularity for the 3-gluon 1-PI vertex.

• Combining 2-point Green's functions, within the PT-BFM approach, a RGI interaction kernel can be built...

Conclusions

• Lattice contemporary results for the three-gluon Green's functions provide, as a main feature, a zero-crossing at very low-momenta...

• … that can be understood as being driven by a negative logarithmic singularity for the 3-gluon 1-PI vertex.

• Combining 2-point Green's functions, within the PT-BFM approach, a RGI interaction kernel can be built...

• … and applied to define a process-independent effective charge.

Conclusions

• Lattice contemporary results for the three-gluon Green's functions provide, as a main feature, a zero-crossing at very low-momenta

• … th … ive
logar …

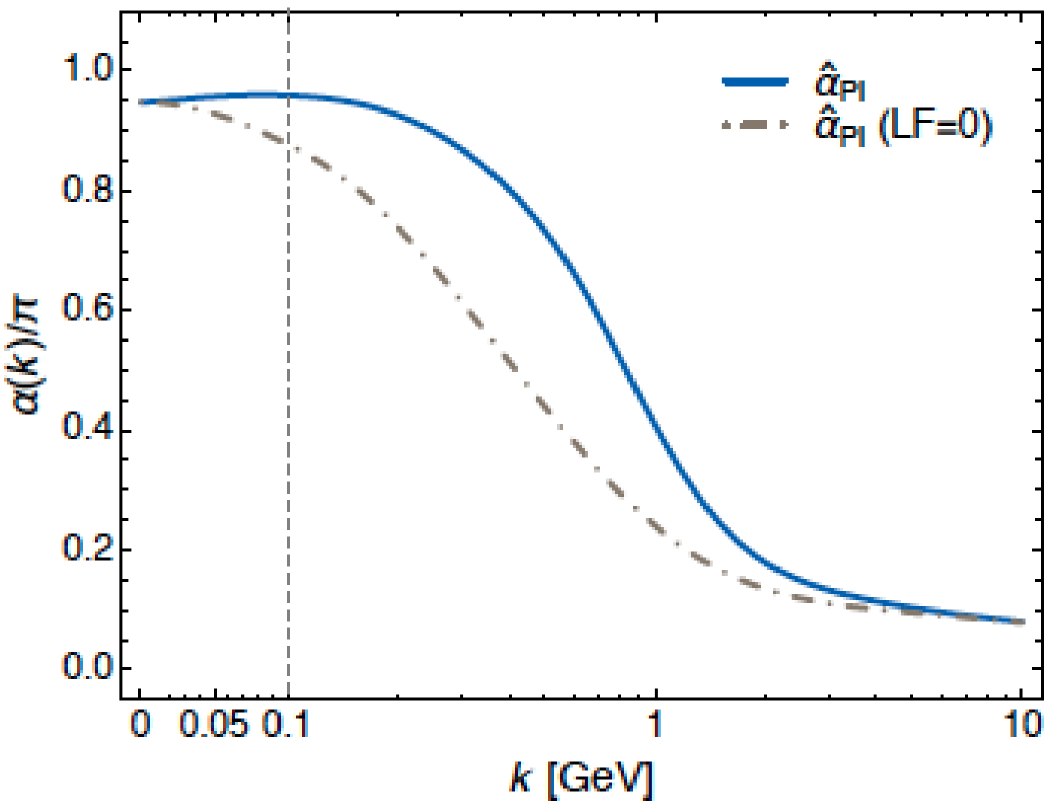
Thank you!!!

• Combi … FM
approach, a …

• … and applied to define a process-independent effective charge.

QCD effective charge

$$\widehat{\alpha}(k^2) = \frac{\widehat{d}(k^2)}{\mathcal{D}(k^2)} \underset{k^2 \gg m_0^2}{\longrightarrow} \mathcal{I}(k^2)$$



•Parameter free
completely determined from 2-points sector

•No Landau pole
physical coupling showing an IR fixed point

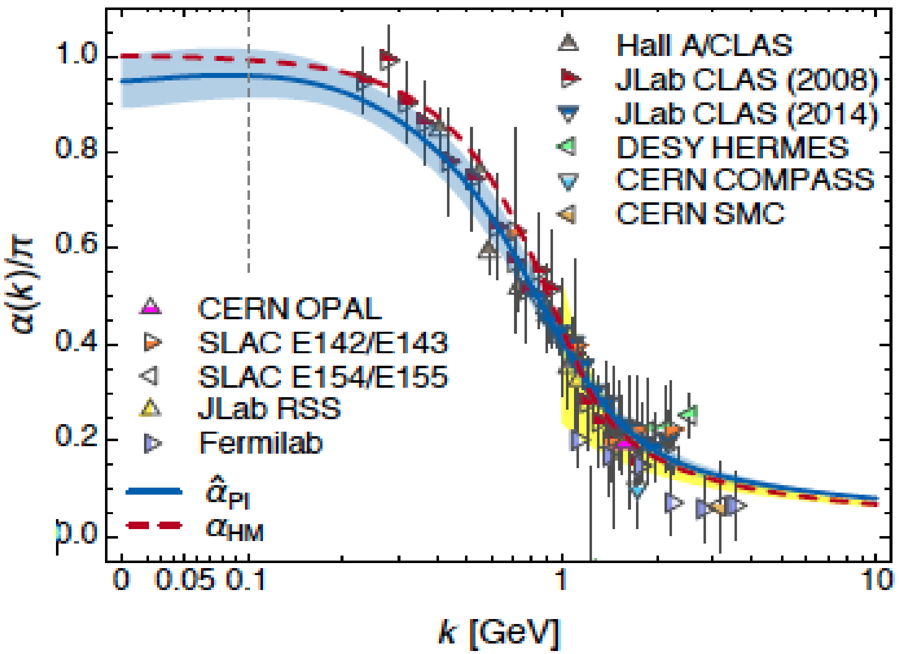
•Smoothly connects IR and UV domains
no explicit matching procedure

•Essentially non-perturbative result
continuum/lattice results plus setting of single mass scale (from the gluon)

•Ghost gluon dynamics critical
enhancement at intermediate momenta

D. Binosi, C. Mezrag, J. Papavassiliou, J.R-Q, C.D. Roberts, arXiv:1612.04835

QCD effective charge: comparison



- **Equivalence in the perturbative domain**
  reasonable definitions of the charge

$$\alpha_{g_1}(k^2) = \alpha_{\overline{\rm MS}}(k^2)[1 + 1.14\alpha_{\overline{\rm MS}}(k^2) + \cdots]$$
$$\widehat{\alpha}_{PI}(k^2) = \alpha_{\overline{\rm MS}}(k^2)[1 + 1.09\alpha_{\overline{\rm MS}}(k^2) + \cdots]$$

- **Equivalence in the non-perturbative domain**
  highly non-trivial (ghost-gluon interactions)

- **Agreement with light-front holography**
  model for α_{g_1}
  Deur, Brodsky, de Teramond, PPNP 90 (2016)

- **Process dependent effective charges**
  fixed by the leading-order term in the expansion of a given observable
  Grunberg, PRD 29 (1984)

- **Bjorken sum rule**
  defines such a charge
  Bjorken, PR 148 (1966); PRD 1 (1970)

$$\int_0^1 {\rm d}x\,[g_1^p(x,k^2) - g_1^n(x,k^2)] = \frac{g_A}{6}[1 - \alpha_{g_1}(k^2)/\pi]$$

  - $g_1^{p,n}$ **spin dependent p/n structure functions**
    extracted from measurements using unpolarized targets
  - g^A **nucleon flavour-singlet axial charge**

- **Many merits**
  - **Existence of data**
    for a wide momentum range
  - **Tight sum rules constraints on the integral**
    at IR and UV extremes
  - **Isospin non-singlet**
    suppress contributions from hard-to-compute processes

D. Binosi, C. Mezrag, J. Papavassiliou, J.R-Q, C.D. Roberts, arXiv:1612.04835