Spectral representation of lattice gluon and ghost propagators

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   - Tikhonov Regularization
2. Toy Model
   - Breit-Wigner model
   - $\int_0^\infty \rho(\omega)\omega \, \mathrm{d}\omega = 0$ model
3. Lattice Data
   - Gluon Propagator
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Introduction

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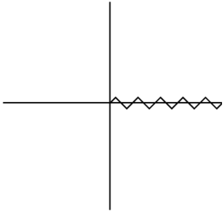

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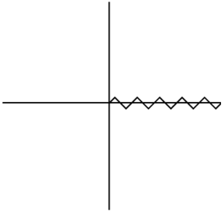

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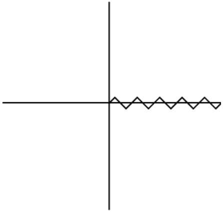

- Lattice simulations give the Euclidean correlator $G(p_4)$, defined for $p^2 > 0$.
- Analytically this has a unique analytical continuation to Minkowski $p^2 < 0$.
- Numerically this is very sensitive to noise: *ill-defined*

Källén–Lehmann representation

Euclidean correlator $G(p_4)$

$$G(p_4) = \int_0^\infty \frac{2\omega\rho(\omega)}{p_4^2 + \omega^2}\,\mathrm{d}\omega = \int_0^\infty \frac{\rho(\sqrt{\mu})}{p_4^2 + \mu}\,\mathrm{d}\mu$$

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Where $\rho(-\omega) = -\rho(\omega)$.

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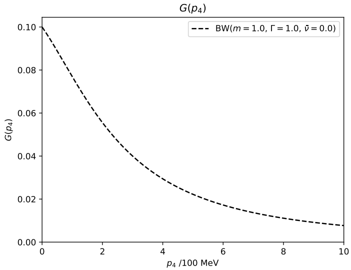
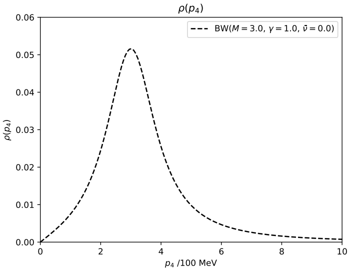
Where $\rho(-\omega) = -\rho(\omega)$. The challenge: find $\rho(\omega)$ for given lattice data p_4 vs $G(p_4)$.





Naive approach

$$G(p_4) = \int_{-\infty}^{\infty} \frac{\rho(\omega)}{\omega - ip_4}\,\mathrm{d}\omega = \int_{-\infty}^{\infty} K(p_4,\omega)\rho(\omega)\,\mathrm{d}\omega\,, \quad K(p_4,\omega) := \frac{1}{\omega - ip_4}$$

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Problem

$\mathbf{K}$ has small singular values, making it ill-conditioned on the numerical level $\rightarrow$ ill-defined inversion problem

Sophisticated Approaches

More sophisticated methods have been proposed.

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- *Tikhonov Regularization*

Result

No noise

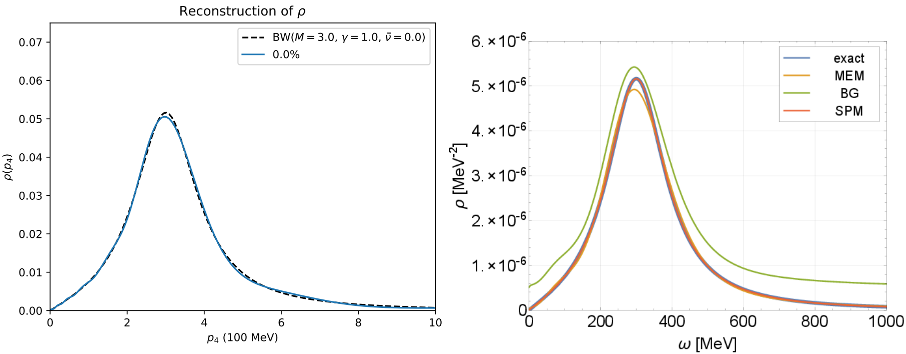

Figure: Left: Tikhonov. Right: Other methods as investigated by Tripolt *et al*[1]

[1] Ralf-Arno Tripolt et al. (2018). "Numerical analytic continuation of Euclidean data". In: arXiv: 1801.10348 [hep-ph].

Tikhonov Regularization

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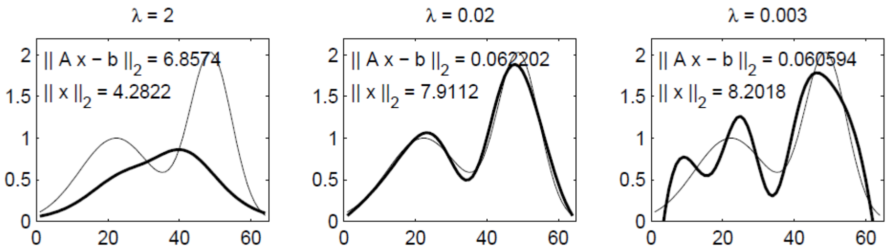
+ Easy to implement
– No unique criterion to choose λ.

How to select the optimal λ?



Too small lambda values mean no regularization at all: over-fitting. Too large results in over-smoothing.

Our preferred solution: Morozov's discrepancy principle

We have data G_n with standard deviation σ_n

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We therefore minimize

$$J_\lambda = \left\| \mathbf{K}\vec{\rho} - \vec{G} \right\|^2 + \lambda \|\vec{\rho}\|^2 \tag{2}$$

subject to $\left\| \mathbf{K}\vec{\rho} - \vec{G} \right\|^2 = \|\vec{\sigma}\|^2$. This has a unique solution.

Tikhonov Regularization: approach

p^2 method, preliminary study published[2]

$$J_\lambda = \sum_{n=1}^{N} \left| \int_{\mu_0}^{\infty} \frac{\rho(\sqrt{\mu})}{\mu + p_n^2} \, \mathrm{d}\mu - G_n \right|^2 + \lambda \int_{\mu_0}^{\infty} \rho(\sqrt{\mu})^2 \, \mathrm{d}\mu$$

[2] David Dudal, Orlando Oliveira, and Paulo J. Silva (2014). "Källén-Lehmann spectroscopy for (un)physical degrees of freedom". In: *Phys. Rev.* D89.1, p. 014010. DOI: 10.1103/PhysRevD.89.014010. arXiv: 1310.4069 [hep-lat]

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$$\vec{c} = -\frac{1}{\lambda}\mathbf{M}\vec{c} - \vec{G} \implies \vec{c} = -(\mathbb{I} + \frac{1}{\lambda}\mathbf{M})^{-1}\vec{G} \quad (7)$$

(8)

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Comparison

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$$c_n = \int_{-\infty}^{\infty} \frac{\rho(\omega)}{\omega - ip_n}\,\mathrm{d}\omega - G_n, \qquad M_{ij} = \begin{cases} \frac{2\pi}{|p_n|+|p_m|} & p_n \cdot p_m \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

Just one parameter, λ. Pole at $p_i = p_j = 0 \rightarrow$ exclude this point.

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p^2 method

$$c_n = \int_{\mu_0}^{\infty} \frac{\rho(\sqrt{\mu})}{\mu + p_n^2}\,\mathrm{d}\mu - G_n, \qquad M_{ij} = \frac{1}{p_j^2 - p_i^2} \ln\left(\frac{p_j^2 + \mu_0}{p_i^2 + \mu_0}\right)$$

No problems when $p_i = p_j = 0$. Two fit parameters: λ, μ_0.

Weighted version

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where $\boldsymbol{\Sigma}$ is the covariance matrix.

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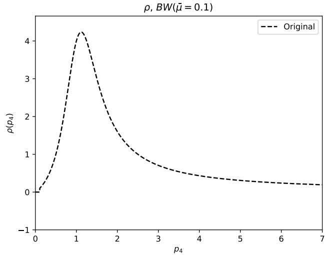
Toy Model

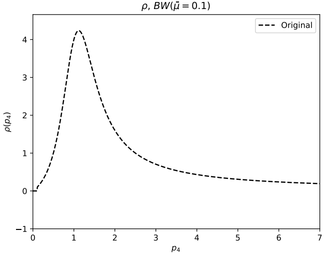
Breit-Wigner model



$$\rho(\omega) = \begin{cases} \frac{\omega}{(\omega^2 - m^2)^2 + \Gamma^2/4} & |\omega| > \bar{\nu} \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

With $\bar{\nu} = 0.1$GeV, $m = 1$GeV and $\Gamma = 1$GeV.

Breit-Wigner model



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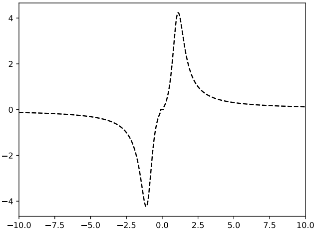
$$\rho(\omega) = \frac{1}{\pi} \frac{2\omega\gamma}{(\omega^2 - \gamma^2 - M^2)^2 + 4\omega^2\gamma^2} \tag{10}$$

With $M = 300$MeV and $\gamma = 100$GeV.

Reconstruction Process

$\rho^{\text{orig}} \rightarrow G_n^{\text{orig}} \rightarrow G_n \in \mathcal{N}(G_n^{\text{orig}}, (\epsilon G_n^{\text{orig}})^2)$



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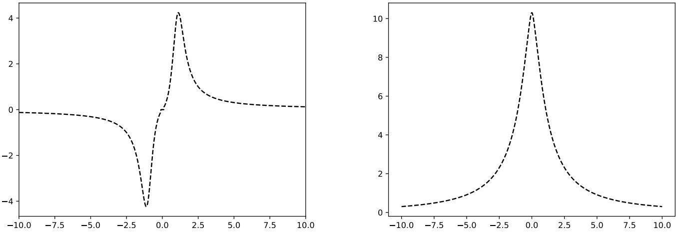

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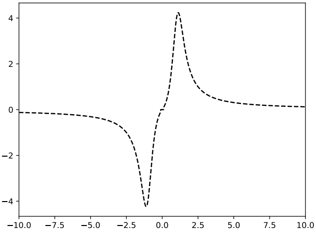

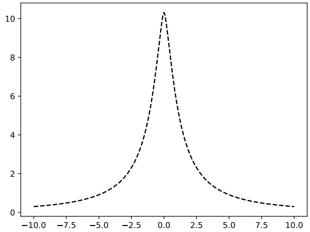

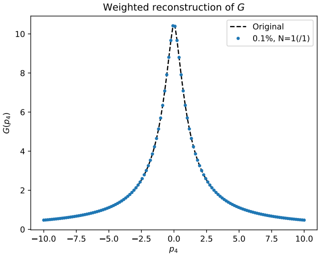

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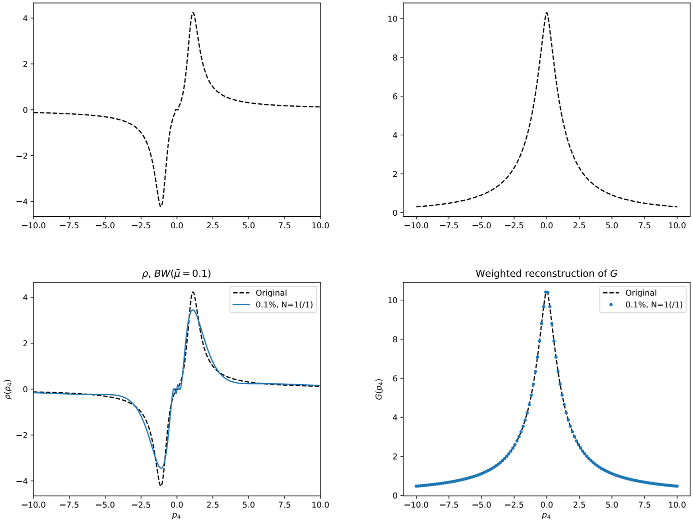


Comparison with MEM[3]

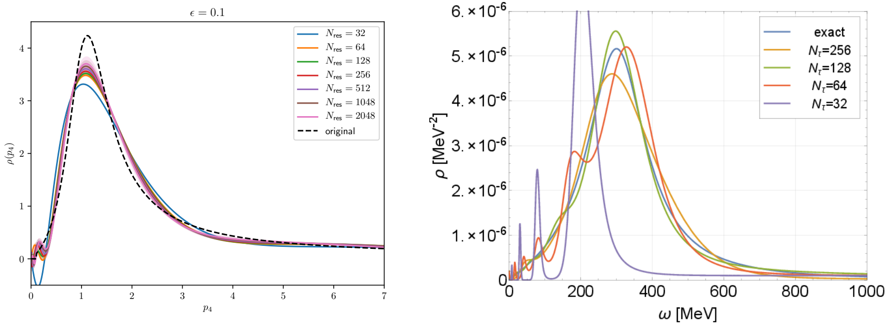
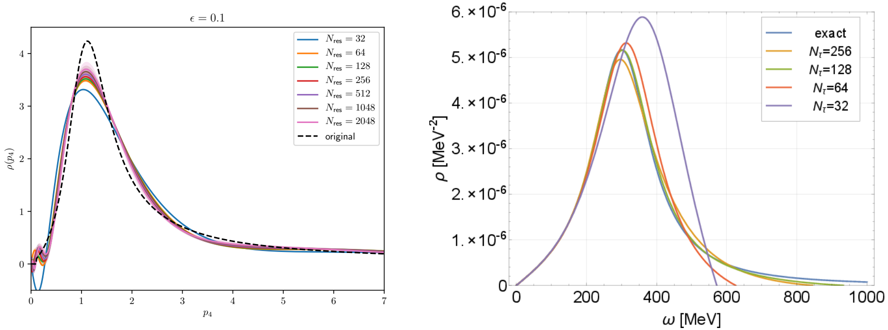


Figure: Tikhonov vs MEM for $\epsilon = 0.1$. Left $BW(m = 1\text{GeV}, \Gamma = 1\text{GeV}, \bar{\nu} = 0.1\text{GeV})$, right $BW(M = 300\text{MeV}, \gamma = 100\text{MeV}, \bar{\nu} = 0)$

Be careful about a direct comparison, the BW models used are not identical.

[3] Ralf-Arno Tripolt et al. (2018). "Numerical analytic continuation of Euclidean data". In: arXiv: 1801.10348 [hep-ph].

Comparison with SPM



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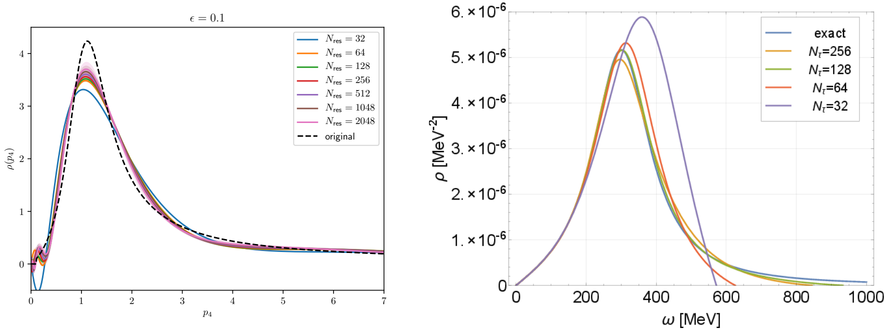

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Integrals are preserved within statistical error with Tikhonov

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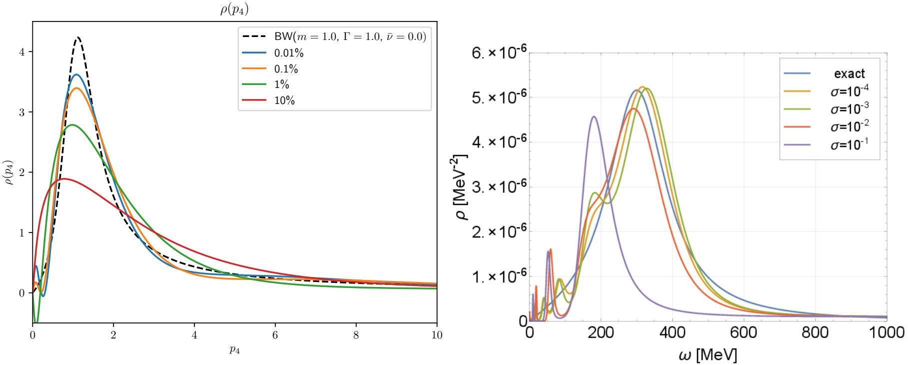

Figure: Tikhonov vs MEM for $N_{\mathrm{res}} = 64$. Left $BW(m = 1\text{GeV}, \Gamma = 1\text{GeV}, \bar{\nu} = 0.1\text{GeV})$, right $BW(M = 300\text{MeV}, \gamma = 100\text{MeV}, \bar{\nu} = 0)$

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Comparison with SPM

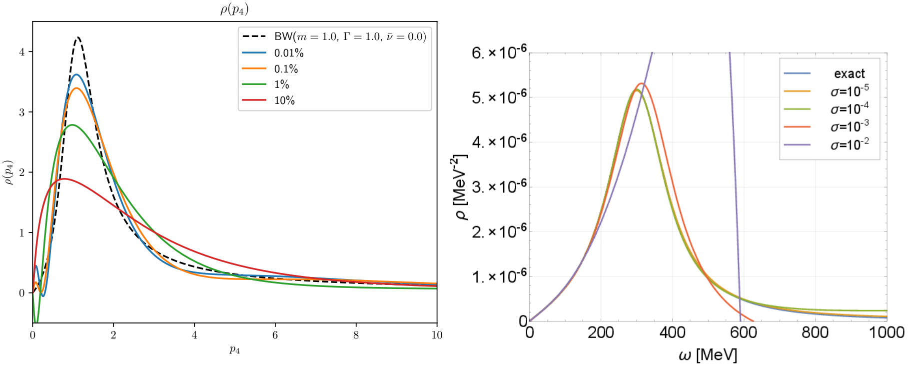

Figure: Tikhonov vs SPM for $N_{\text{res}} = 64$. Left $BW(m = 1\text{GeV}, \Gamma = 1\text{GeV}, \bar{\nu} = 0.1\text{GeV})$, right $BW(M = 300\text{MeV}, \gamma = 100\text{MeV}, \bar{\nu} = 0)$

Be careful about a direct comparison, the BW models used are not identical.

ip-Formalism vs p^2 Formalism

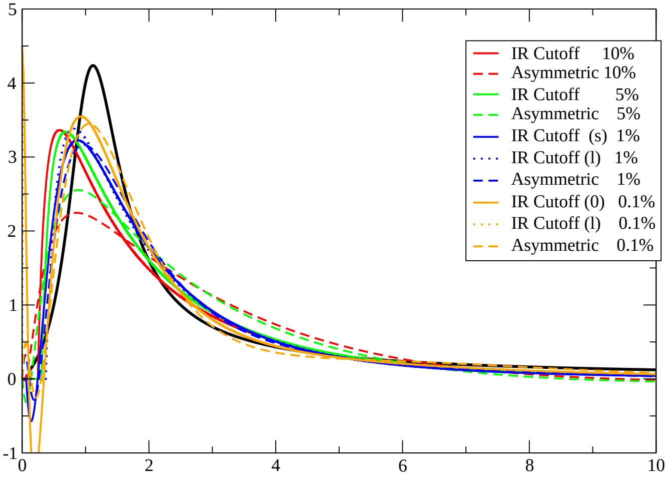

Figure: Dashed ip, solid p^2.

Reproducibility/Stability

Does every sample $G_n \in \mathcal{N}(G_n, (\epsilon G_n)^2)$ give approximately the same λ?

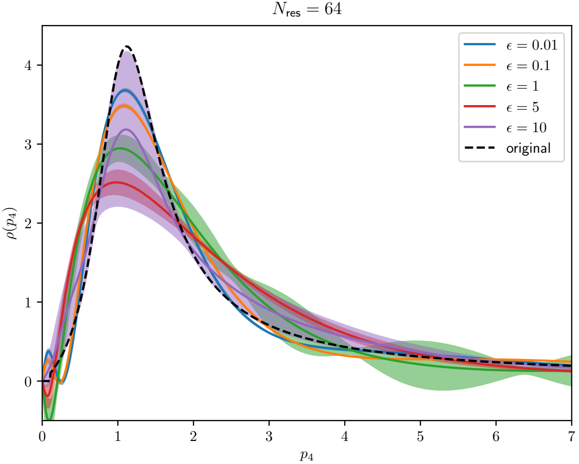

Figure: Bootstrap, $N = 1000$. Fixed $N_{\text{res}} = 64$

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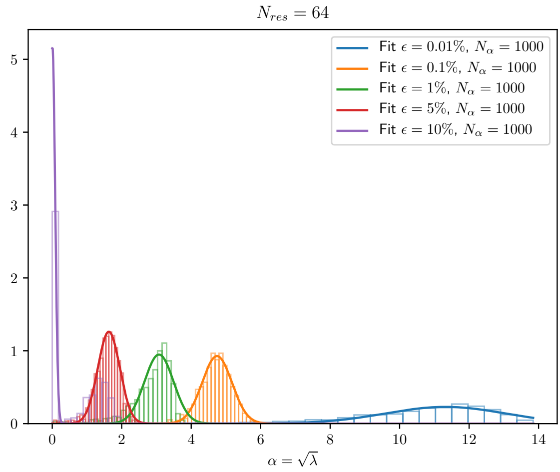

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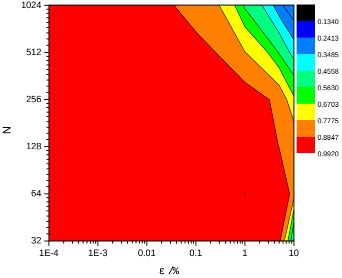

Figure: R^2 plot, shows where we have good reproducibility

Observations

- At any noise level, the reconstruction doesn't become better than $N_p = 64$.
- At realistic noise levels α is Gaussian distributed.

$\int_0^\infty \rho(\omega)\omega \, \mathrm{d}\omega = 0$ model

For gluons we expect the following sum rule:

$$\int_0^\infty \rho(\omega)\omega \, \mathrm{d}\omega = 0 \tag{11}$$

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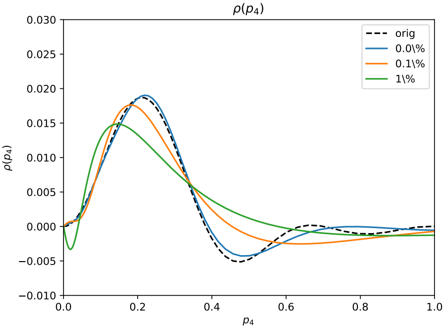
Toy model

$$\rho(\omega) = \frac{J_1(\omega)J_3(\omega)}{\omega^2}$$



Lattice Data

Gluon Propagator

$T = 0$ Gluon propagator data, 80^4 lattice, $\beta = 6.0$, normalised at 3GeV

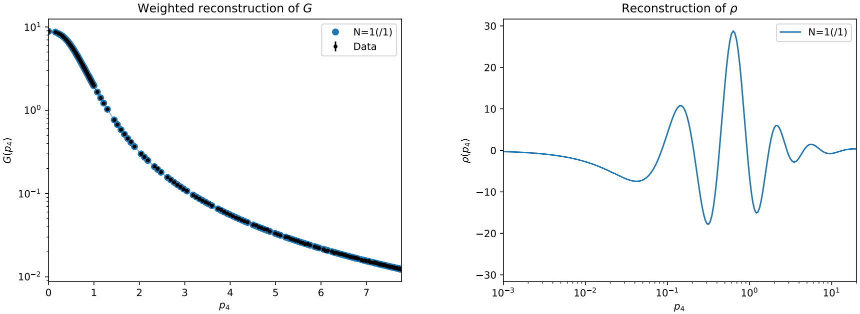
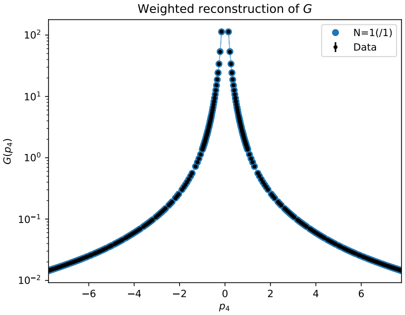
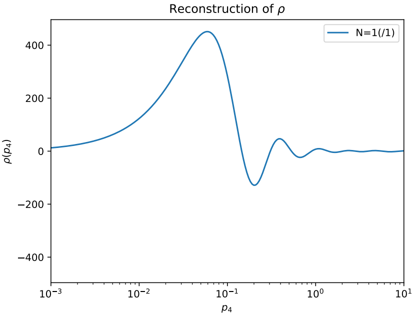


Figure: p_4 (GeV)

Ghost Propagator

$T = 0$ Ghost propagator data, 80^4 lattice, $\beta = 6.0$, normalised at 3GeV





Conclusion/Outlook

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+ Tikhonov Regularisation is competitive with the other tools on the market today

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+ Easy to implement
+ In realistic noise ranges, it is robust
- When including the covariance matrix, the UV tail is approximated better

Outlook

- Study $\int_0^\infty \rho(\omega)\omega \, \mathrm{d}\omega = 0$ toy-models

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- Study $\int_0^\infty \rho(\omega)\omega \, \mathrm{d}\omega = 0$ toy-models
- Apply to various sets of Gluon and Ghost propagator data, $T \geq 0$

Dudal, David, Orlando Oliveira, and Paulo J. Silva (2014). "Källén-Lehmann spectroscopy for (un)physical degrees of freedom". In: *Phys. Rev.* D89.1, p. 014010. DOI: 10.1103/PhysRevD.89.014010. arXiv: 1310.4069 [hep-lat].

Tripolt, Ralf-Arno et al. (2018). "Numerical analytic continuation of Euclidean data". In: arXiv: 1801.10348 [hep-ph].

Roelfs et al. 2018?

ip-Formalism, weighted vs unweighted

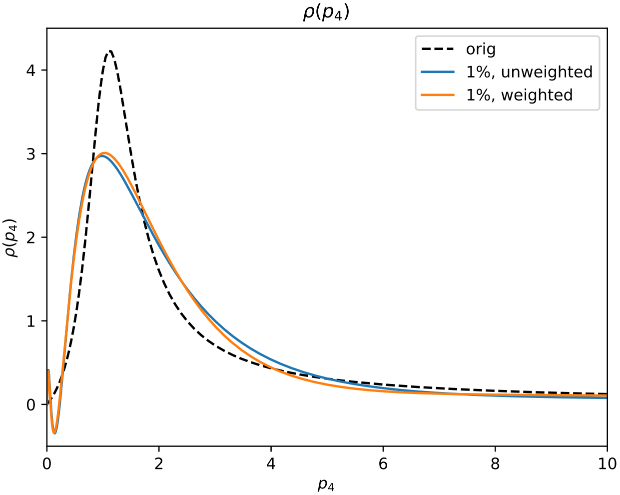

Figure: Weighted vs unweighted, resp. $N_p = 256$

ip-Formalism, weighted vs unweighted

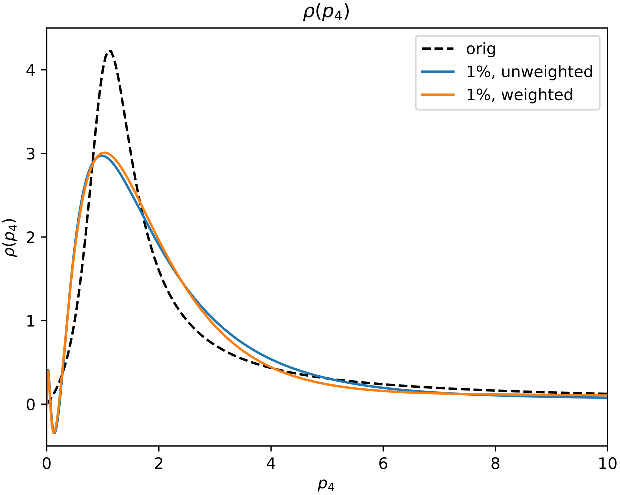

Figure: Weighted vs unweighted, resp. $N_p = 256$

Weighted performs better in the UV, use the weighted algorithm from now

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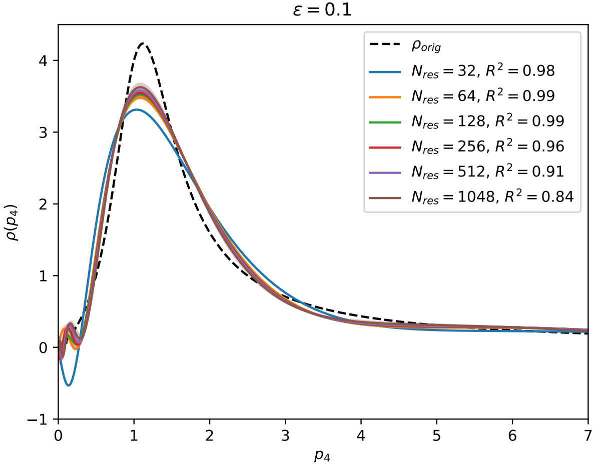

Figure: Bootstrap, $N = 1000$. Fixed $\epsilon = 0.1$

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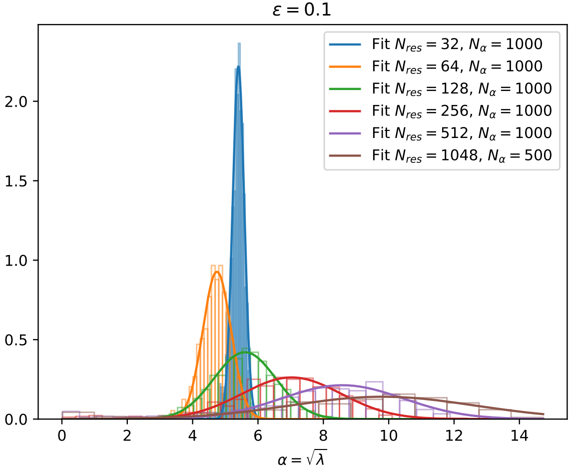

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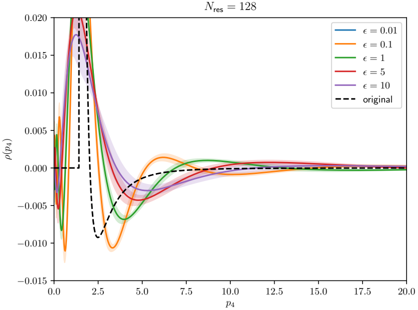
Toy model

$$\rho(\omega) = \begin{cases} \frac{-1}{\omega^4+4} + \frac{A}{\omega^6+2} & \omega > \sqrt{2} \\ \rho(-\omega) = -\rho(\omega) \end{cases}$$



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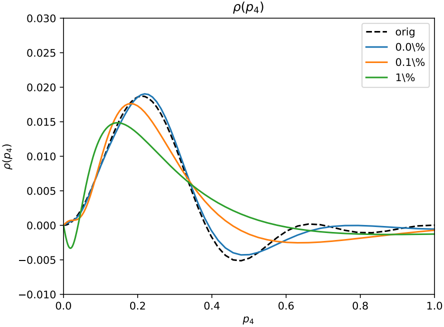

$\int \rho(\omega)\omega \, \mathrm{d}\omega = 0$ model, ip vs p^2