Gribov copies, avalanches and the spontaneous generation of a gluon mass

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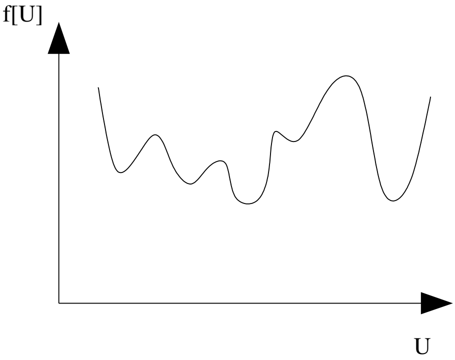
Introduction

- Gribov copies are known to exists in Yang-Mills theory but their influence on physical observables is still debated.
- Strong similarities with the field Stat. Mech. in presence of quenched disorder.
- Recent progresses in this field indicate new paths to attack the problem of Gribov copies.

Gribov copies in a nutshell

- For a given gauge configuration A, consider $f_A[U] = \int \mathrm{Tr}(A^U A^U)$, where A^U is the gauge transform of A.
- The extrema U_i of f_A fulfill $\partial_\mu A_\mu^{U_i} = 0$ (ie are in the Landau gauge).
- Typically, there are many extrema. What/how do we choose?



A toy model for Gribov ambiguity I

- 1975: Imry and Ma introduce the Random Field Ising Model (RFIM)

$$H = -J \sum_{\langle ij \rangle} s_i s_j - \sum_i h_i s_i$$

  with s_i Ising variables, h_i a set of independent, quenched, random variables $\mathcal{P}(h) \propto \exp(-h^2/h_0^2)$.

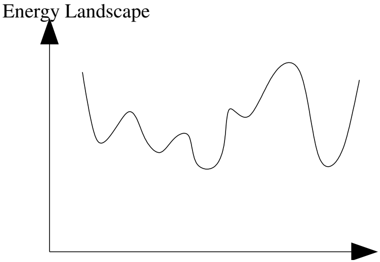
- 1976: dimensional reduction (to all orders!) [Aharony, Imry, Ma, Young]:

$$\nu_{\mathrm{RFIM}}(d) = \nu_{\mathrm{Ising}}(d-2)$$

- 1979, Parisi and Sourlas reinterpret the model by using Faddeev-Popov construction. Dimensional reduction is a consequence of a BRST (super)symmetry!

A toy model for Gribov ambiguity II

- 1980-87, the property of dimensinal reduction is not valid in lower dimensions ($\nu_{\mathsf{RFIM}}(3) \neq \nu_{\mathsf{Ising}}(1)$) [Bray, Moore, Imbrie, Bricmond, Kupiainen].
- 1982, Parisi argues that the RFIM has many local minima in the action. In principle, this invalidates Parisi-Sourlas construction.



- 1986, Fisher realizes that the many minima can lead to singularities in the renormalization-group flow.

Avalanches

- 2007-10, we reformulate the problem to take into account the many minima. Dimensional reduction is realized in high dimensions ($d \gtrsim 5.1$) and broken in low dimensions [MT, Tarjus].
- What counts for dimensional reduction the size of avalanches, not the existence of many minima.
- Compare the ground states for two slightly different external fields (say δJ and $-\delta J$). A large fraction of the spins may have flipped, with a fractal structure.
- What counts for dimensional reduction is the fractal dimension of these spanning avalanches.

Avalanches in Yang-Mills theory

- Objective: test the influence of Gribov copies by using the same methodology.
- Gauge fixing obtained by extremizing

$$f_{A,\eta}[U] = \int \mathrm{Tr}\ (A^U A^U + \eta^\dagger U + U^\dagger \eta)$$

where $A_\mu = A_\mu^a \sigma^a/2$ [for SU(2)] and η is a quenched random variable:

$$\mathcal{P}[\eta] \propto \exp\left[-\frac{g_0^2}{\xi_0}\int_x \mathrm{Tr}\ (\eta^\dagger \eta)\right]$$

- If we omit the Gribov ambiguity (ie if we build the naive Faddeev-Popov action), this leads to the Curci-Ferrari-Delbourgo-Jarvis action [Serreau, Tresmontant, Tissier].
- $\xi_0 = 0$ gives back the Landau gauge.

Gauge fixing I

- To take into account the Gribov ambiguity, we fix the gauge by summing over all extrema (all Gribov copies) [Serreau, Tissier] of $f_{A,\eta}$. For some operator $\mathcal{O}$:

$$\langle \mathcal{O}[A] \rangle = \frac{\sum_i s(i) \mathcal{O}[A^{U_i}] e^{-\rho_0 f_{A,\eta}[U_i]}}{\sum_i s(i) e^{-\rho_0 f_{A,\eta}[U_i]}}$$

  $s(i)$ is a sign which depends on the number of unstable direction of the extremum U_i. ρ_0 is a dimension-2 gauge parameter.
- The denominator ensures that we have a benevolent gauge fixing.

$$\langle \mathcal{O}_{\text{inv}}[A] \rangle = \mathcal{O}_{\text{inv}}[A]$$

- Once we fixed the gauge, we can integrate over A and η.

Gauge fixing II

- The denominator can be computed via the replica trick $1/x = \lim_{n\to 0} x^{n-1}$.
- Both numerator and denominator can be written in terms of a SU(2) nonlinear sigma model (super)field $\mathcal{V} = n^0 1 + i n^a \sigma_a$ (with n a unit 4-vector) which depend x and on 2 grassman variables, θ and $\bar{\theta}$.
- We can factorize the gauge group and obtain a continuous theory for: A, c,$\bar{c}$, h and $n-1$ copies of the superfield $\mathcal{V}_i(x, \theta, \bar{\theta})$ $(i = 2 \cdots n)$.

Gauge fixing III

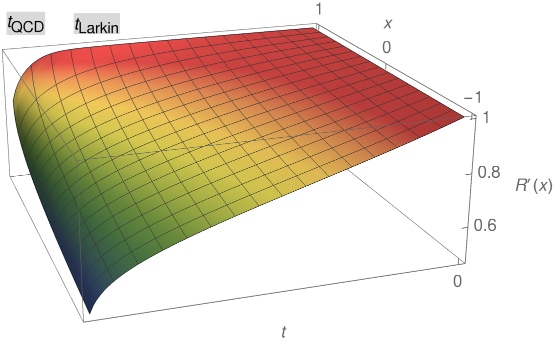
(Skipping technical details) the microscopic action reads:
$S = S_{\mathrm{YM}} + S_0 + S_1 + S_2$ with

$$
\begin{aligned}
S_0 = &\int_x \frac{1}{2}(D_\mu \overline{c}^a \partial_\mu c^a + \partial_\mu \overline{c}^a D_\mu c^a) + h^a \partial_\mu A^a_\mu \\
&- \frac{\xi_0}{2}(h^a)^2 - \frac{g_0^2 \xi_0}{4}(\epsilon^{abc}\overline{c}^b c^c)^2 + \frac{\rho_0}{2}(A^a_\mu)^2 + \rho_0 \xi_0 \overline{c}^a c^a \\
S_1 = &\sum_{i=2}^{n} \int_{x\underline{\theta}_i} \mathrm{Tr}\ (A^{\mathcal{V}_i}_\mu)^2 + \xi_0 (n_i^0 \overline{c}^a c^a - \frac{4}{g_0^2} n_i^0 \rho_0 + \frac{2}{g_0} n_i^a h^a) \\
S_2 = &- \frac{2\xi_0}{g_0^2} \sum_{i,j=2}^{n} \int_{x\underline{\theta}_i\underline{\theta}_j} (n_i^0 n_j^0 + n_i^a n_j^a)
\end{aligned}
$$

Last term permits to test the presence of avalanches.
Both gluons and ghosts are massless.

Renormalization-group flow I

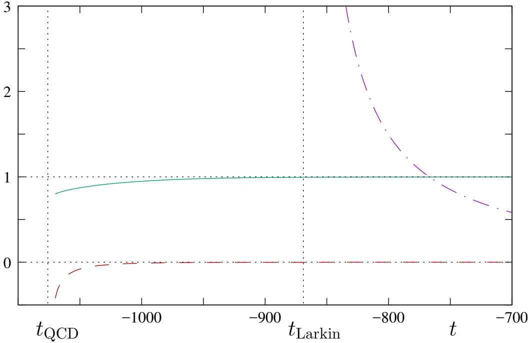
- A standard (but lengthy!) calculation leads to the 1-loop calculation for the theory.
- the 2-replica potential evolves form $R(x) = x$ to some nontrivial function:



Renormalization-group flow II

- In the first part of the flow, the theory is identical to the naive Curci-Ferrari-Delbourgo-Jarvis theory: coupling constant and gauge parameter ξ have the same flows, ghosts and gluons are massless. Can be traced back to the fact that $R'(1) = 1$ is a fixed point. BRST is realized.
- At the Larkin scale, $R''(1)$ diverges. The function generates a cusp $R'(x) \sim cte - a\sqrt{1-x}$.
- Below the Larkin scale, $R'(1) < 1$. Both gluons and ghosts are massive. BRST is broken.

Renormalization-group flow III



- Interplay between the Larkin length and $\Lambda_{\rm QCD}$ (which one is bigger?). At 1-loop, for $26/3 > \xi_0 > \xi_c \simeq 4.35$, the Larkin scale is more ultraviolet than the Landau pole.

$$\Lambda_{\rm Larkin} = \Lambda_{\rm QCD} \exp\left[\frac{6\pi}{22\alpha_{\rm UV}}\left(1 - \frac{\xi_c}{\xi_{\rm UV}}\right)^{22/13}\right].$$

Conclusions

- Gluon mass can be generated perturbatively because of the Gribov copies.
- This works for ξ sufficiently large. Could be tested numerically?
- The gluon mass squared is negative, in lines with the observation of Orsay's group.
- Does it lead to a stabilization of the theory in the IR, as in the Landau gauge?