In-Medium Spectral Functions of hadrons with the Functional Renormalization Group

Ralf-Arno Tripolt, ECT*, Trento, Italy

In collaboration with:
Chris Jung, Fabian Rennecke, Naoto Tanji,
Lorenz von Smekal, Jochen Wambach

666. WE-Heraeus-Seminar "From correlation functions to QCD phenomenology"
Bad Honnef, Germany, April 3-6, 2018

ECT*
EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS

Outline

I) Theoretical setup

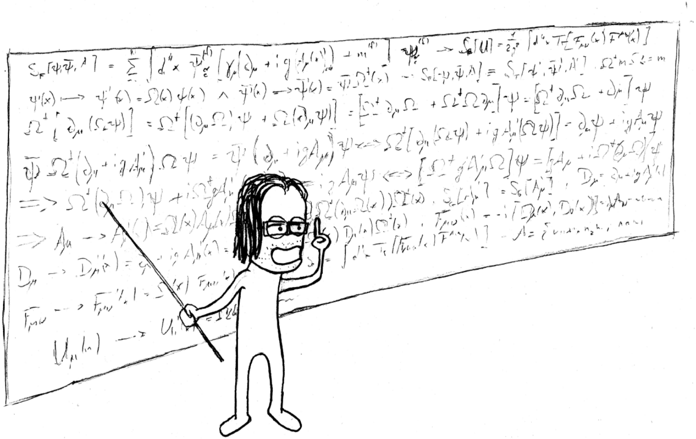
- Functional Renormalization Group (FRG)
- QCD effective model
- Analytic continuation procedure

II) Results

- (Pseudo-)scalar meson spectral functions
- (Axial-)vector meson spectral functions
- Electromagnetic spectral function
- Dilepton rates

III) Summary and outlook

I) Theoretical setup



[courtesy L. Holicki]

Functional Renormalization Group

Euclidean partition function for a scalar field:

$$Z[J] = \int \mathcal{D}\varphi \ \exp\left(-S[\varphi] + \int d^4x \ J(x)\varphi(x)\right)$$

Wilson's coarse-graining: split φ into low- and high-frequency modes

$$\varphi(x) = \varphi_{q\leq k}(x) + \varphi_{q>k}(x)$$

only include fluctuations with $q > k$

$$Z[J] = \int \mathcal{D}\varphi_{q\leq k} \underbrace{\int \mathcal{D}\varphi_{q>k} \ \exp\left(-S[\varphi] + \int d^4x J(x)\varphi(x)\right)}_{Z_k[J]}$$

Functional Renormalization Group

Scale-dependent partition function can be defined as

$$Z_k[J] = \int \mathcal{D}\varphi \; \exp\left(-S[\varphi] - \Delta S_k[\varphi] + \int d^4x J(x)\varphi(x)\right)$$

by introducing a regulator term that suppresses IR modes

$$\Delta S_k[\varphi] = \frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\, \varphi(-q)R_k(q)\varphi(q)$$

Switch to scale-dependent effective action ($\phi(x) = \langle\varphi(x)\rangle$):

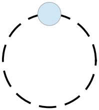
$$\Gamma_k[\phi] = \sup_J\left(\int d^4x\; J(x)\phi(x) - \log Z_k[J]\right) - \Delta S_k[\phi]$$

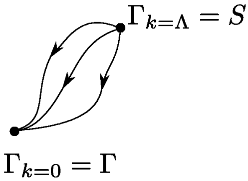
Functional Renormalization Group

Flow equation for the effective average action Γ_k:

$$\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left(\partial_k R_k \left[\Gamma_k^{(2)} + R_k \right]^{-1} \right)$$

[C. Wetterich, Phys. Lett. **B 301** (1993) 90]

$$\partial_k \Gamma_k \;=\; \frac{1}{2}$$ 



[wikipedia.org/wiki/Functional_renormalization_group]

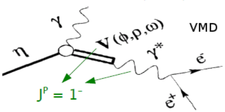
- Γ_k interpolates between bare action S at $k = \Lambda$ and effective action Γ at $k = 0$
- regulator R_k acts as a mass term and suppresses fluctuations with momenta smaller than k
- the use of 3D regulators allows for a simple analytic continuation procedure

Modeling vector mesons

- Sakurai (1960): vector mesons as gauge bosons of local gauge symmetry $SU(2)$
  - $\rightarrow$ Electromagnetic-hadronic interaction via exchange of vector mesons
  - $\rightarrow$ Current Field Identity (CFI):

$$j^\mu_{\text{em}} = \frac{m_\rho^2}{g_\rho}\rho^\mu + \frac{m_\omega^2}{g_\omega}\omega^\mu + \frac{m_\phi^2}{g_\phi}\phi^\mu$$

$\Rightarrow$ **Vector Meson Dominance (VMD)**



[Berghaeuser, www.staff.uni-giessen.de (2016)]

- Lee and Nieh (1960s): Gauged linear sigma model, local gauge symmetry $SU(2)_L \times SU(2)_R$

$\Rightarrow$ $\boldsymbol{\rho}$ **meson** and chiral partner $\boldsymbol{a_1}$ **meson** as gauge bosons

Gauged linear-sigma model with quarks

- Local gauge symmetry $SU(2)_L \times SU(2)_R$: Low-energy model of two-flavor QCD
- Additional gauge symmetry $U(1)$ to include photon field

Ansatz for the effective average action $\Gamma_k \equiv \Gamma_k[\sigma, \pi, \rho, a_1, \psi, \bar{\psi}, A_\mu]$:

$$\Gamma_k = \int d^4x \Big\{ \bar{\psi}\left(\not{D} - \mu\gamma_0 + h_S\left(\sigma + \mathrm{i}\vec{\tau}\vec{\pi}\gamma_5\right) + \mathrm{i}h_V\left(\gamma_\mu \vec{\tau}\vec{\rho}^{\mu} + \gamma_\mu\gamma_5\vec{\tau}\vec{a}_1^{\mu}\right)\right)\psi + U_k(\phi^2) \\ - c\sigma + \frac{1}{2}\left|(D_\mu - \mathrm{i}gV_\mu)\Phi\right|^2 + \frac{1}{8}\mathrm{Tr}\left(V_{\mu\nu}V^{\mu\nu}\right) + \frac{1}{4}m_{V,k}^2 \mathrm{Tr}\left(V_\mu V^\mu\right) \Big\}$$

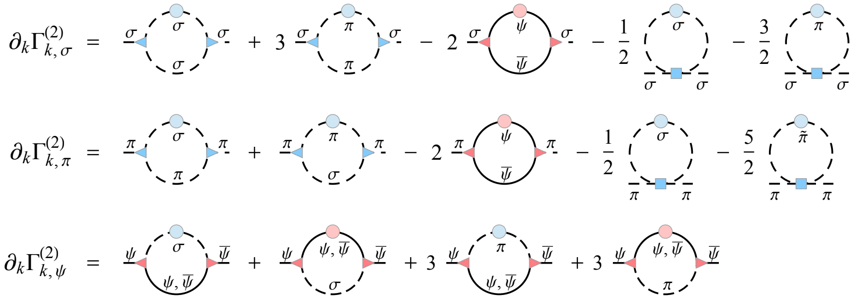
with

$$V_{\mu\nu} = D_\mu V_\nu - D_\nu V_\mu - \mathrm{i}g\left[V_\mu, V_\nu\right], \quad D_\mu\psi = \left(\partial_\mu - \mathrm{i}eA_\mu Q\right)\psi,$$
$$D_\mu V_\mu = \partial_\mu V_\nu - ieA_\mu[T_3, V_\nu], \quad \phi \equiv (\vec{\pi}, \sigma), \quad V_\mu \equiv \vec{\rho}_\mu \vec{T} + \vec{a}_{1,\mu}\vec{T}^5$$

Flow of the effective potential at $\mu = 0$ and $T = 0$

(Loading movie...)

Flow equations for two-point functions

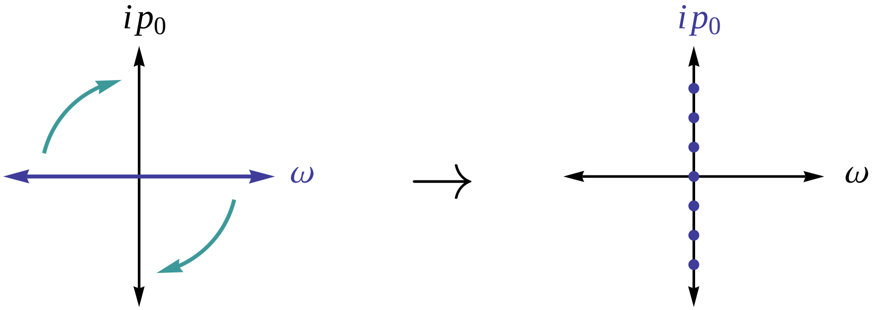


- quark-meson vertices are given by $\Gamma^{(3)}_{\bar{\psi}\psi\sigma} = h, \ \Gamma^{(3)}_{\bar{\psi}\psi\vec{\pi}} = ih\gamma^5\vec{\tau}$
- mesonic vertices from scale-dependent effective potential: $U^{(3)}_{k,\phi_i\phi_j\phi_m}$, $U^{(4)}_{k,\phi_i\phi_j\phi_m\phi_n}$
- one-loop structure and 3D regulators allow for a simple analytic continuation!

[R.-A. Tripolt, L. von Smekal, and J. Wambach, Phys. Rev. **D 90**, 074031 (2014)]

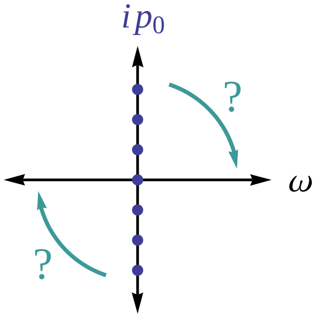
The analytic continuation problem

Calculations at finite temperature are often performed using imaginary energies:



The analytic continuation problem

Analytic continuation problem: How to get back to real energies?



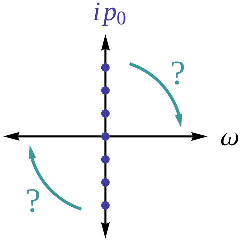
Two-step analytic continuation procedure

1) Use periodicity w.r.t. imaginary energy $ip_0 = i2n\pi T$:

$$n_{B,F}(E + ip_0) \to n_{B,F}(E)$$

2) Substitute p_0 by continuous real frequency ω:

$$\Gamma^{(2),R}(\omega, \vec{p}) = -\lim_{\epsilon \to 0} \Gamma^{(2),E}(ip_0 \to -\omega - i\epsilon, \vec{p})$$

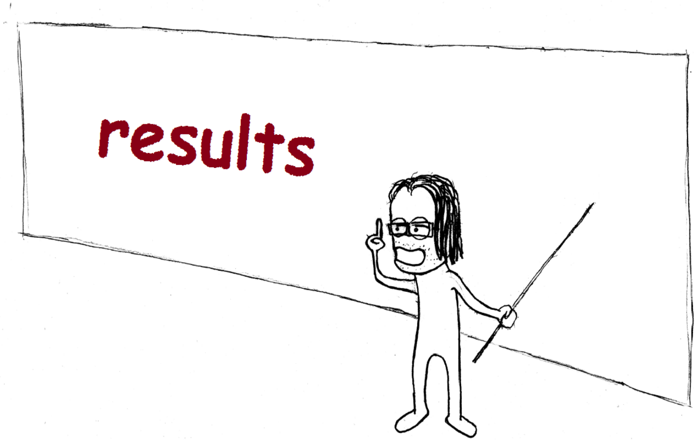


Spectral function is then given by

$$\rho(\omega, \vec{p}) = -\frac{1}{\pi} \mathrm{Im} \frac{1}{\Gamma^{(2),R}(\omega, \vec{p})}$$

[R.-A. T., N. Strodthoff, L. v. Smekal, and J. Wambach, Phys. Rev. **D 89**, 034010 (2014)]
[J. M. Pawlowski, N. Strodthoff, Phys. Rev. **D 92**, 094009 (2015)]
[N. Landsman and C. v. Weert, Physics Reports 145, 3&4 (1987) 141]

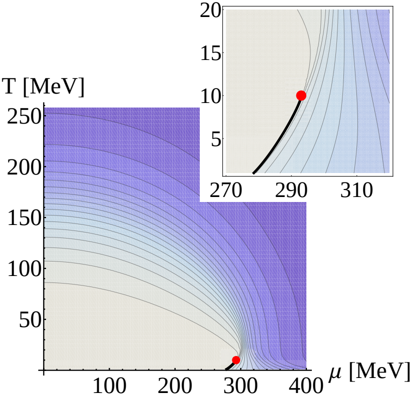
III) Results



[courtesy L. Holicki]

Phase diagram of the quark-meson model

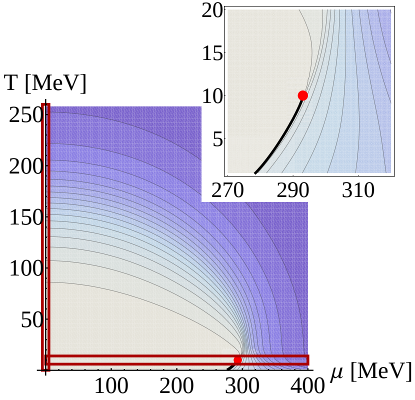
- chiral order parameter σ_0 decreases towards higher T and μ
- a crossover is observed at $T \approx 175$ MeV and $\mu = 0$
- critical endpoint (CEP) at $\mu \approx 292$ MeV and $T \approx 10$ MeV
- we will study spectral functions along $\mu = 0$ and $T \approx 10$ MeV



[R.-A. T., N. Strodthoff, L. v. Smekal, and J. Wambach, Phys. Rev. **D 89**, 034010 (2014)]

Phase diagram of the quark-meson model

- chiral order parameter σ_0 decreases towards higher T and μ
- a crossover is observed at $T \approx 175$ MeV and $\mu = 0$
- critical endpoint (CEP) at $\mu \approx 292$ MeV and $T \approx 10$ MeV
- we will study spectral functions along $\mu = 0$ and $T \approx 10$ MeV



[R.-A. T., N. Strodthoff, L. v. Smekal, and J. Wambach, Phys. Rev. **D 89**, 034010 (2014)]

Flow of σ and π spectral function at $\mu = T = 0$

(Loading movie...)

σ and π spectral function for $T > 0$ at $\mu = 0$

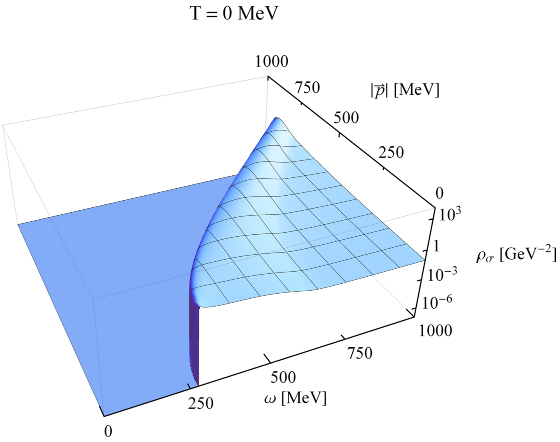
(Loading movie...)

σ and π spectral function for $\mu > 0$ at T_c

(Loading movie...)

σ spectral function vs. ω and $\vec{p}$ at $\mu = T = 0$

- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes



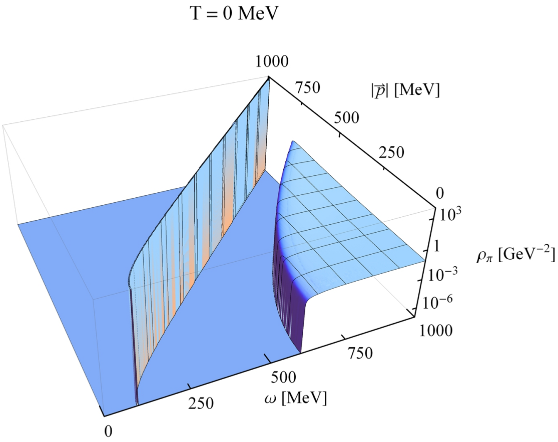
σ spectral function vs. ω and $\vec{p}$ for $T > 0$, $\mu = 0$

- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes

(Loading movie...)

π spectral function vs. ω and $\vec{p}$ at $\mu = T = 0$

- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- capture process $\pi^* + \pi \to \sigma$ is suppressed at large $\vec{p}$
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes



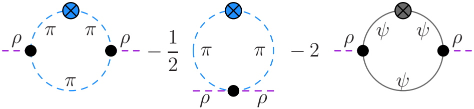
π spectral function vs. ω and $\vec{p}$ for $T > 0$, $\mu = 0$

- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- capture process $\pi^* + \pi \to \sigma$ is suppressed at large $\vec{p}$
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes

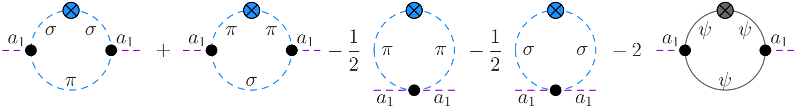
(Loading movie...)

Flow equations for ρ and a_1 2-point functions

$\partial_k \Gamma^{(2)}_{\rho,k} =$

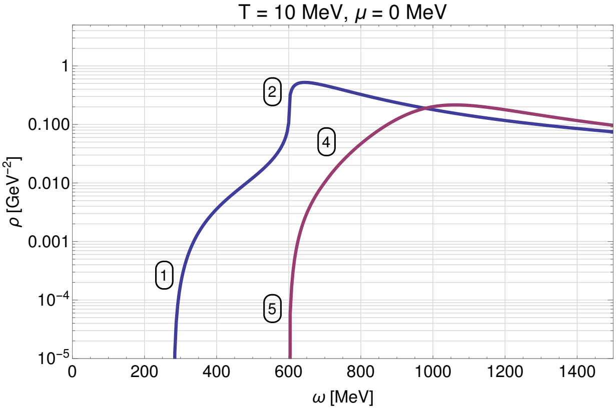


$\partial_k \Gamma^{(2)}_{a_1,k} =$



- neglect vector mesons inside the loops
- vertices extracted from ansatz for the effective average action Γ_k
- tadpole diagrams give ω-independent contributions

ρ and a_1 vacuum spectral functions



① : $\rho^* \rightarrow \pi + \pi$

② : $\rho^* \rightarrow \bar{\psi} + \psi$

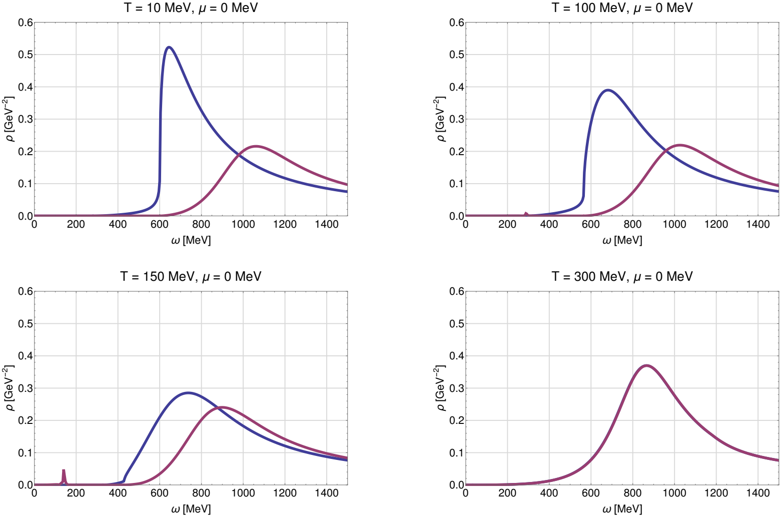
③ : $a_1^* + \pi \rightarrow \sigma$

④ : $a_1^* \rightarrow \pi + \sigma$

⑤ : $a_1^* \rightarrow \bar{\psi} + \psi$

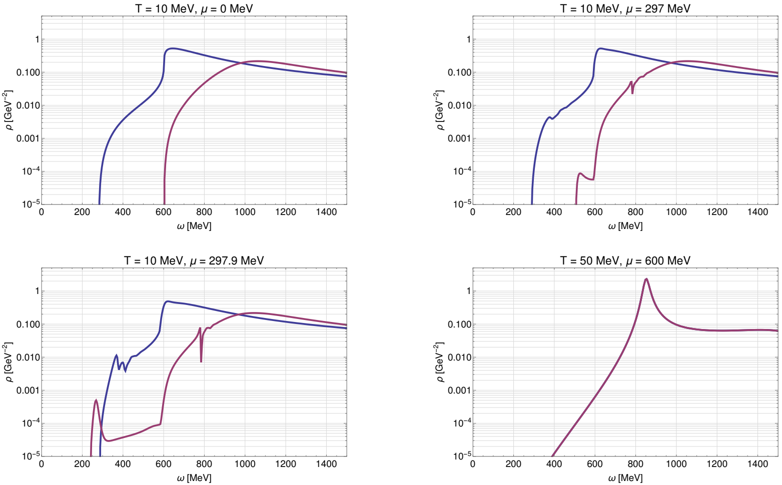
[C. Jung, F. Rennecke, R.-A. T., L. von Smekal, and J. Wambach, Phys. Rev. **D 95**, 036020 (2017)]

T-dependence of ρ and a_1 spectral functions



[C. Jung, F. Rennecke, R.-A. T., L. von Smekal, and J. Wambach, Phys. Rev. **D 95**, 036020 (2017)]

μ-dependence of ρ and a_1 spectral functions



[C. Jung, F. Rennecke, R.-A. T., L. von Smekal, and J. Wambach, Phys. Rev. **D 95**, 036020 (2017)]

T-dependence of ρ and a_1 spectral functions

(Loading movie...)

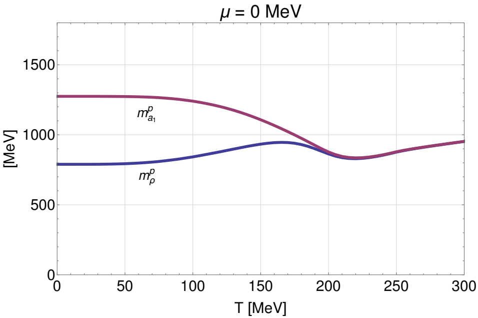
T-dependence of ρ and a_1 pole masses

- pole masses in the vacuum:

  $m_\rho^p = 789$ MeV, $\quad m_{a_1}^p = 1275$ MeV

- degeneration of ρ and a_1 spectral functions in chirally symmetric phase
- broadening of spectral functions with increasing T
- pole masses do not vary much, no dropping ρ mass

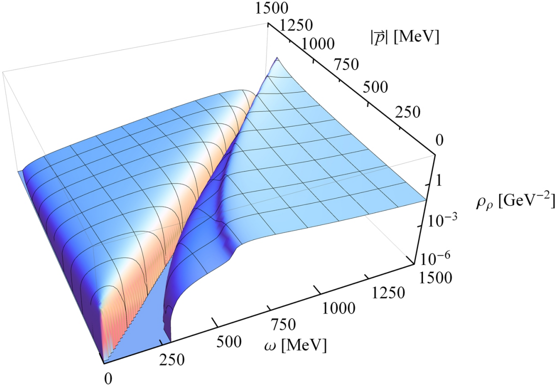
$\Rightarrow$ consistent with broadening/melting-ρ-scenario



[C. Jung, F. Rennecke, R.-A. T., L. von Smekal, and J. Wambach, Phys. Rev. **D 95**, 036020 (2017)]

Momentum-dependence of ρ spectral function

- shown for $\mu = 0$ and $T = 100$ MeV
- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes



[C. Jung, F. Rennecke, R.-A. T., L. von Smekal, and J. Wambach, Phys. Rev. **D 95**, 036020 (2017)]

Temperature-dependence of ρ spectral function

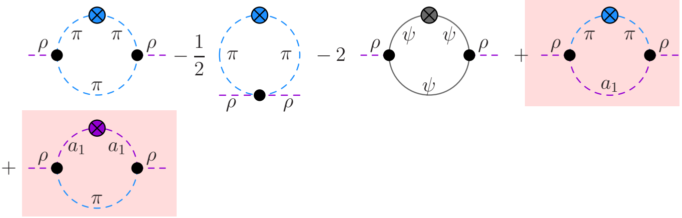
- time-like region ($\omega > \vec{p}$) is Lorentz-boosted to higher energies
- space-like region ($\omega < \vec{p}$) is non-zero at finite T due to space-like processes

(Loading movie...)

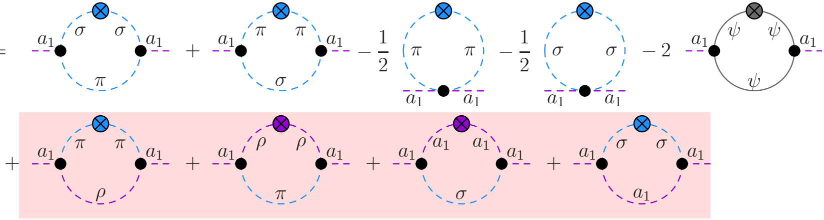
[C. Jung, F. Rennecke, R.-A. T., L. von Smekal, and J. Wambach, Phys. Rev. **D 95**, 036020 (2017)]

Including fluctuating (axial-)vector mesons

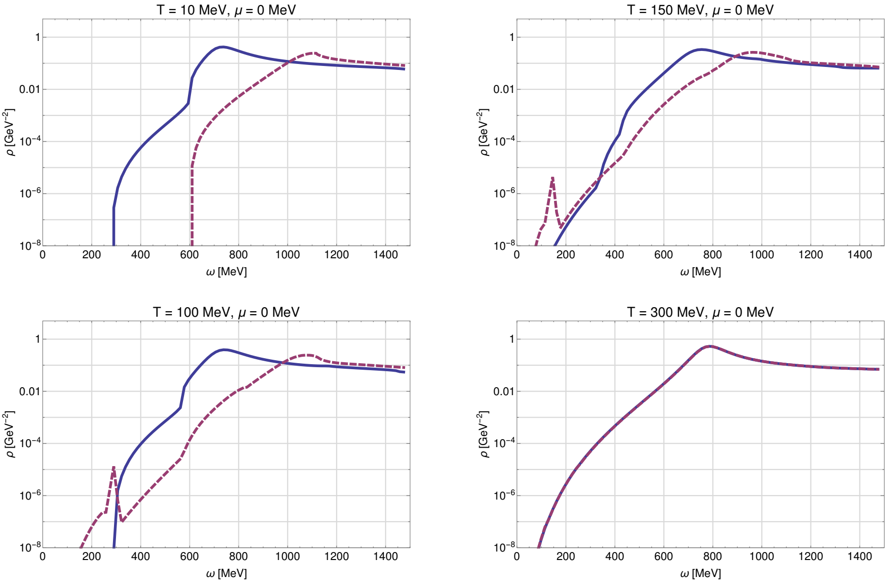
$\partial_k \Gamma^{(2)}_{\rho\rho,k} =$



$\partial_k \Gamma^{(2)}_{a_1 a_1,k} =$

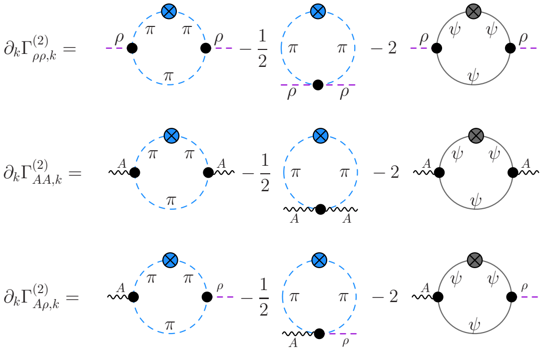


ρ and a_1 spectral functions at $\mu = 0$ - preliminary

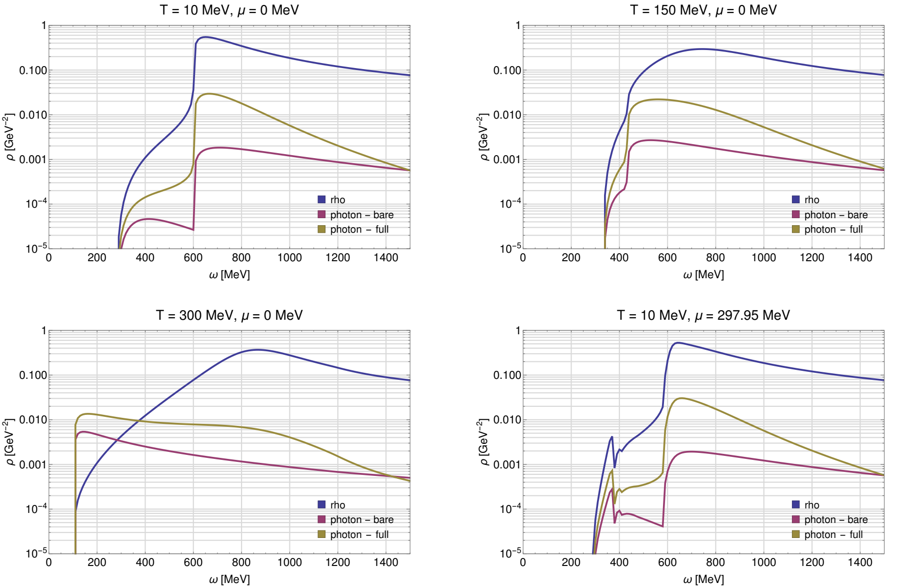


Rho-photon mixing

$$\begin{pmatrix} \Gamma^{(2)}_{AA} & \Gamma^{(2)}_{A\rho} \\ \Gamma^{(2)}_{\rho A} & \Gamma^{(2)}_{\rho\rho} \end{pmatrix} \xrightarrow{\text{diagonalize}} \begin{pmatrix} \tilde{\Gamma}^{(2)}_{AA} & 0 \\ 0 & \tilde{\Gamma}^{(2)}_{\rho\rho} \end{pmatrix}, \quad \tilde{\Gamma}^{(2)}_{AA} = \overbrace{\Gamma^{(2)}_{AA} - \frac{\Gamma^{(2)}_{A\rho}\Gamma^{(2)}_{\rho A}}{\Gamma^{(2)}_{\rho\rho}}}^{\mathcal{O}(e^2)} + \mathcal{O}(e^4)$$



EM spectral functions - preliminary



Calculation of dilepton rates

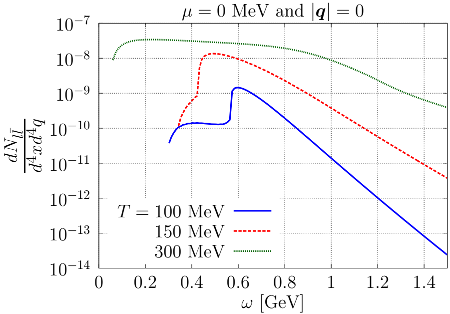
- We use the Weldon formula for the thermal dilepton rate:

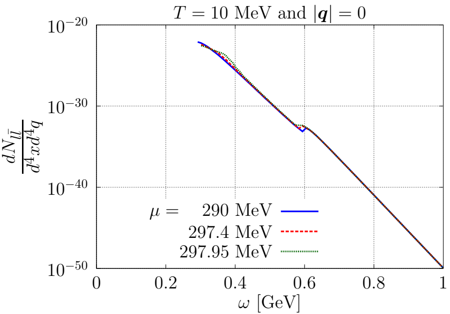
$$\frac{d^8 N_{l\bar{l}}}{d^4x d^4q} = \frac{\alpha}{12\pi^3}\left(1+\frac{2m^2}{q^2}\right)\left(1-\frac{4m^2}{q^2}\right)^{1/2} q^2(2\rho_T+\rho_L)n_B(q_0)$$

- in the following we assume $m = 0$ and set the external spatial momentum to zero, such that $\rho_T = \rho_L = \rho_{\tilde{A}\tilde{A}}$

[H. A. Weldon, Phys. Rev. D42, 2384 (1990)]

Dilepton rates - preliminary





Summary and Outlook

- Analytically continued flow equations for vector meson two-point functions based on a gauged linear sigma model within the FRG
- Chiral order parameter and in-medium spectral functions obtained by the same theoretical framework
  - $\Rightarrow$ Spectral functions of vector mesons with fluctuating vector mesons
  - $\Rightarrow$ Calculation of EM spectral functions and dilepton rates

Work in progress:

- Improve truncation (e.g. self-consistent solution of flow equations)
- Improve phenomenology (e.g. include baryons)