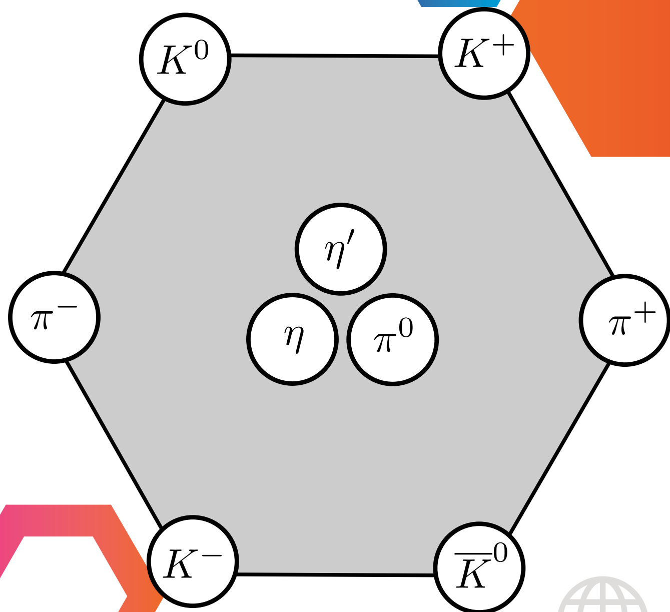
Resonances in the DSE/BSE approach to QCD.



Richard Williams
University of Gießen

Bundesministerium für Bildung und Forschung
HIC for FAIR
Helmholtz International Center

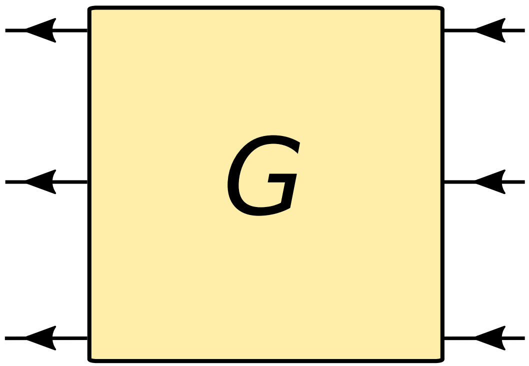
Motivation

Extract properties of hadrons from QCD

- Propagators and vertices
- Formulate description of bound-states in the continuum.

Test truncations against Hadronic Spectrum

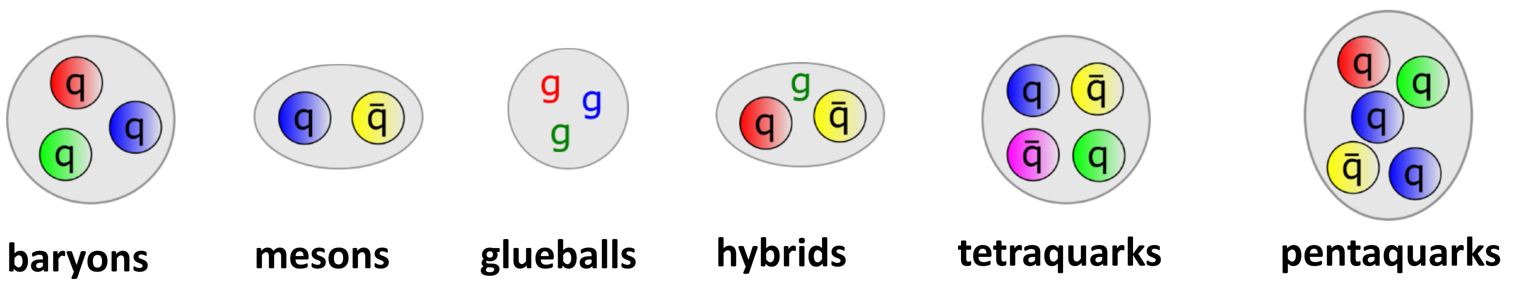
- Include/Exclude interaction terms



Extract from
Green's functions

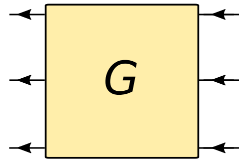
Interaction terms responsible for

- Binding quarks and (anti)quarks
- Unquenching effects
- Decay channels
- Splitting between parity partners …



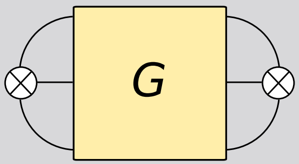
Hadronic states

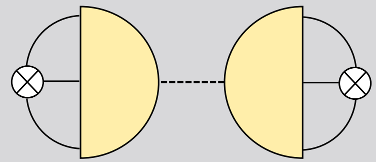
Poles in the Green's function



$$\mathbf{G}_{\alpha\beta\gamma;\alpha'\beta'\gamma'} = \langle 0|\, T\psi_\alpha\psi_\beta\psi_\gamma\bar{\psi}_{\alpha'}\bar{\psi}_{\beta'}\bar{\psi}_{\gamma'}\, |0\rangle$$

Lattice: gauge-invariant current correlators

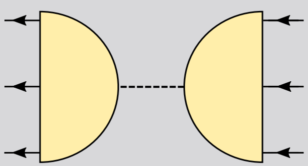




$$e^{-mt} \iff \frac{1}{p^2+m^2}$$

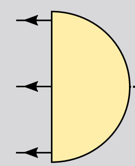
Exponential time-decay.

BSE: gauge-invariant poles from Green's function



$$\mathbf{G} \sim \sum_\lambda \frac{\Psi^\lambda\bar{\Psi}^\lambda}{p^2+m_\lambda^2}$$

Spectral decomposition.



$$\Psi^\lambda_{\alpha\beta\gamma} = \langle 0|\, T\psi_\alpha\psi_\beta\psi_\gamma\, |\lambda\rangle$$

BS wavefunction

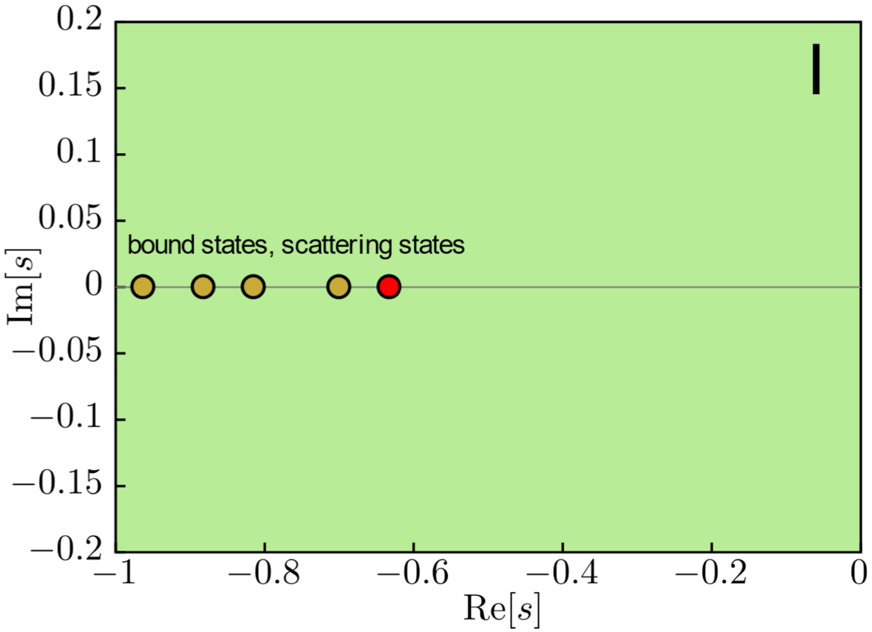
Considerations

See Eichmann

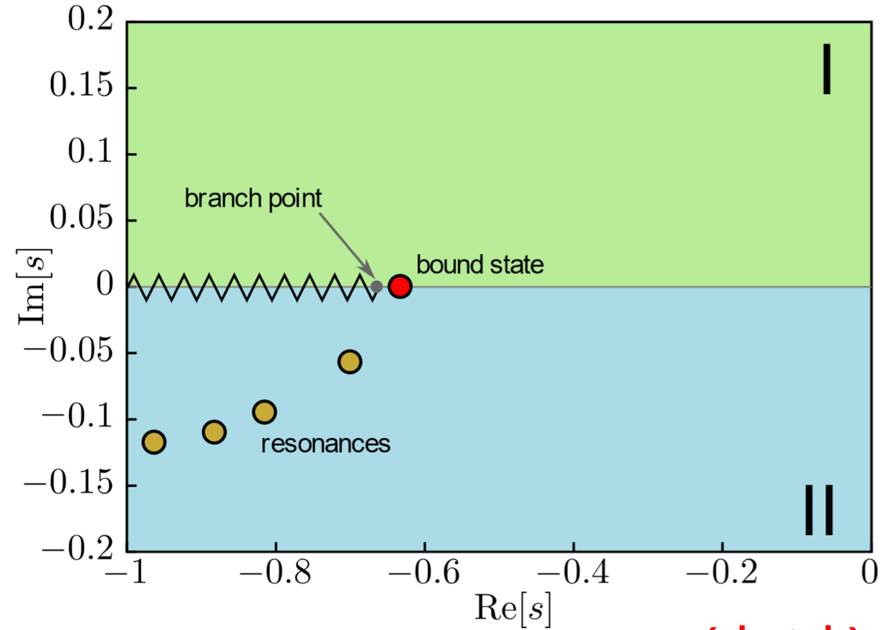
- Bound states **below** strong decay threshold: π, K, D, B
- Most hadrons lie **above** strong decay threshold

(in)finite volume

Lattice: finite volume. No cuts.
Bound states, scattering states



Continuum: infinite volume.
Branch cuts. Bound states, resonances



(sketch)

Resonances

- Appear as poles on the "unphysical sheet" (labelled II).
- Information reconstructed on the Lattice via Lüscher formalism.

Expectations

Consider: function $V(s)$ that exposes "pole" of correlation function e.g. two-point correlator on the lattice, vertex function etc.

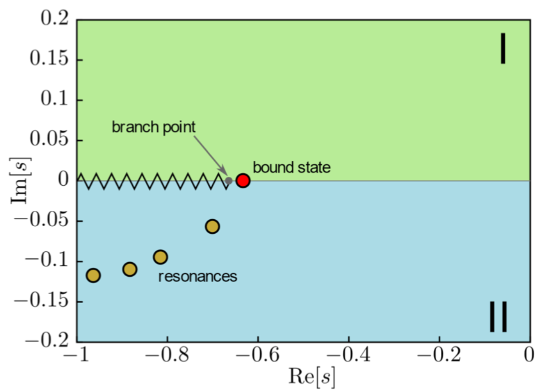
Below decay threshold

- Expect poles on the real-axis
- *Bound state*

$$V(s) \sim \frac{1}{s + M^2}$$

Above decay threshold

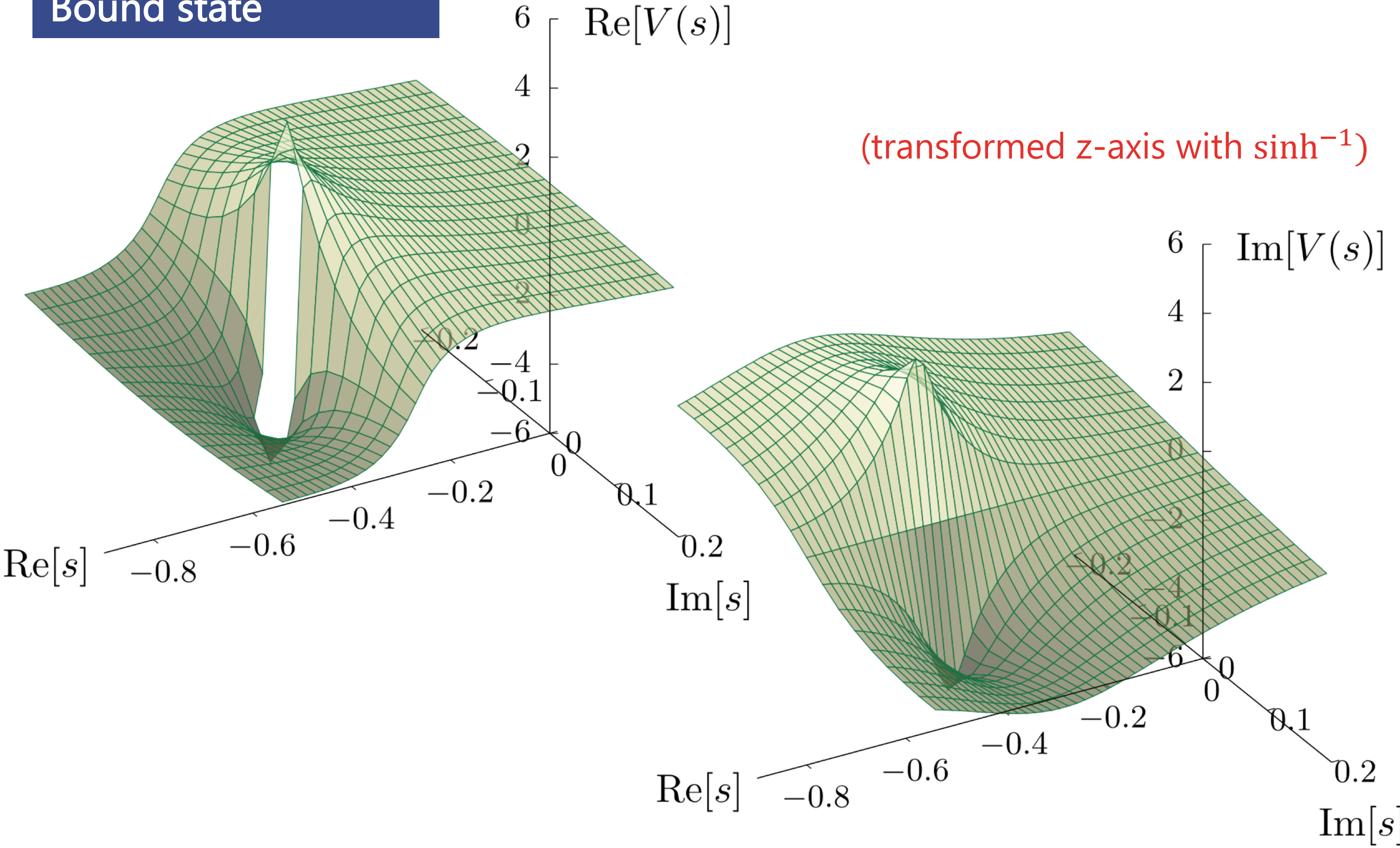
- Expect poles shifted from real-axis, in "unphysical sheet"
- *Resonance*



Let's visualize this:

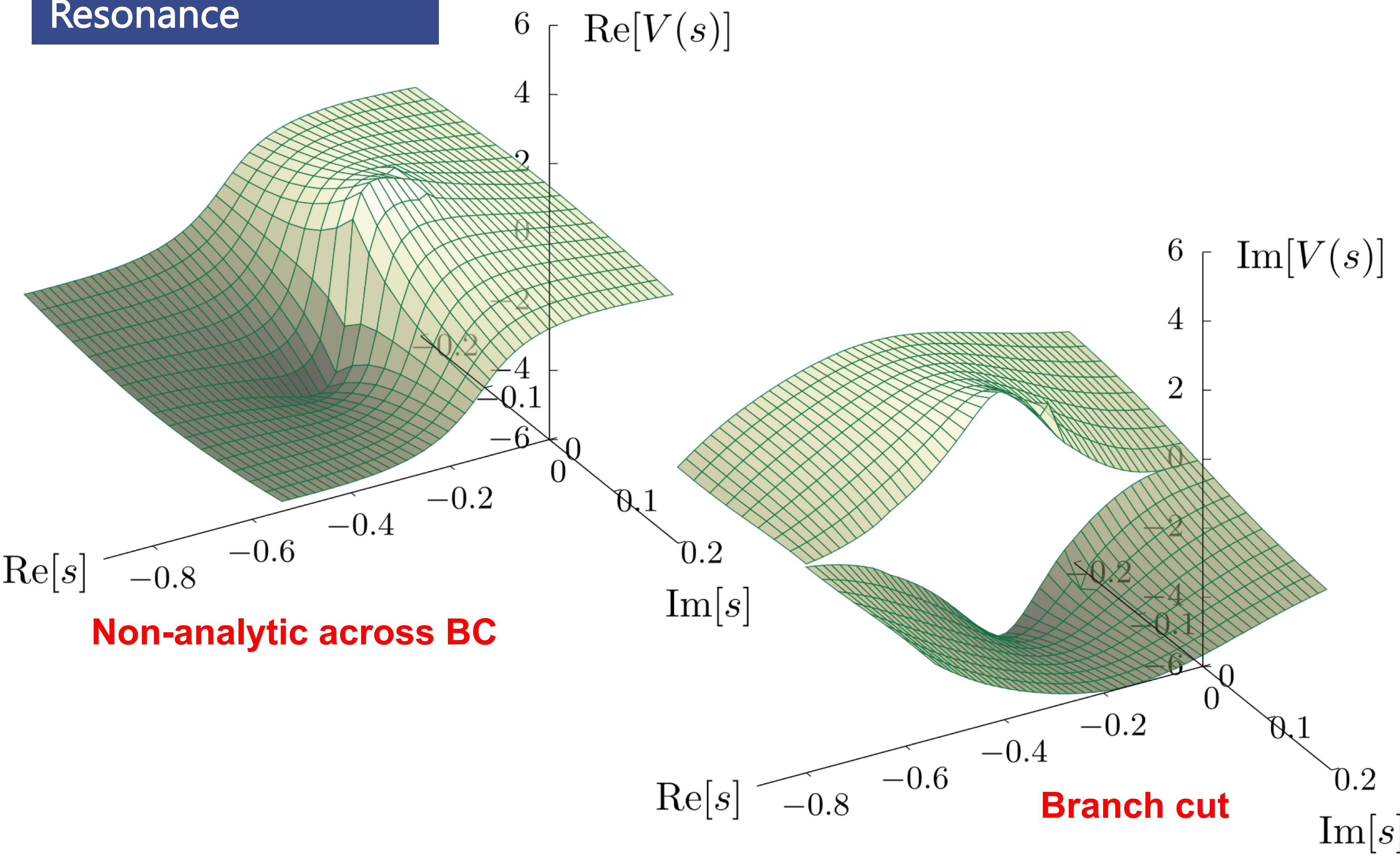
$$V(s) \sim \frac{1}{s + \left(M - \frac{i\Gamma}{2}\right)^2}$$

Bound state



Pole readily apparent on the real-axis

Resonance

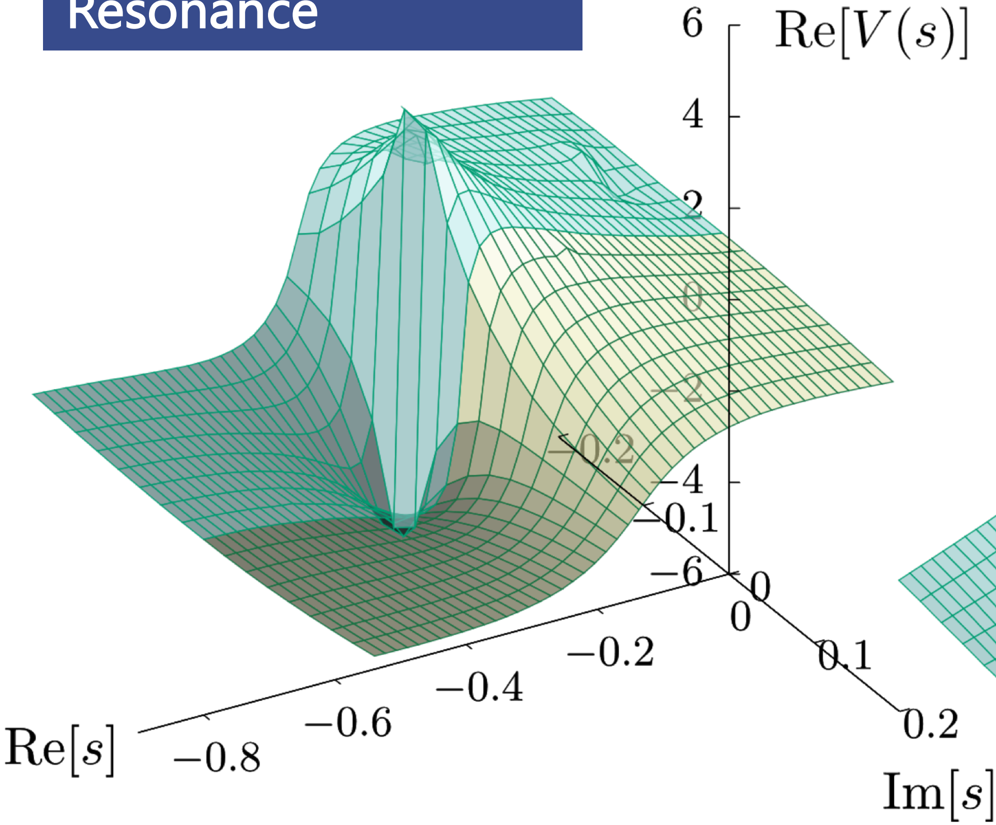


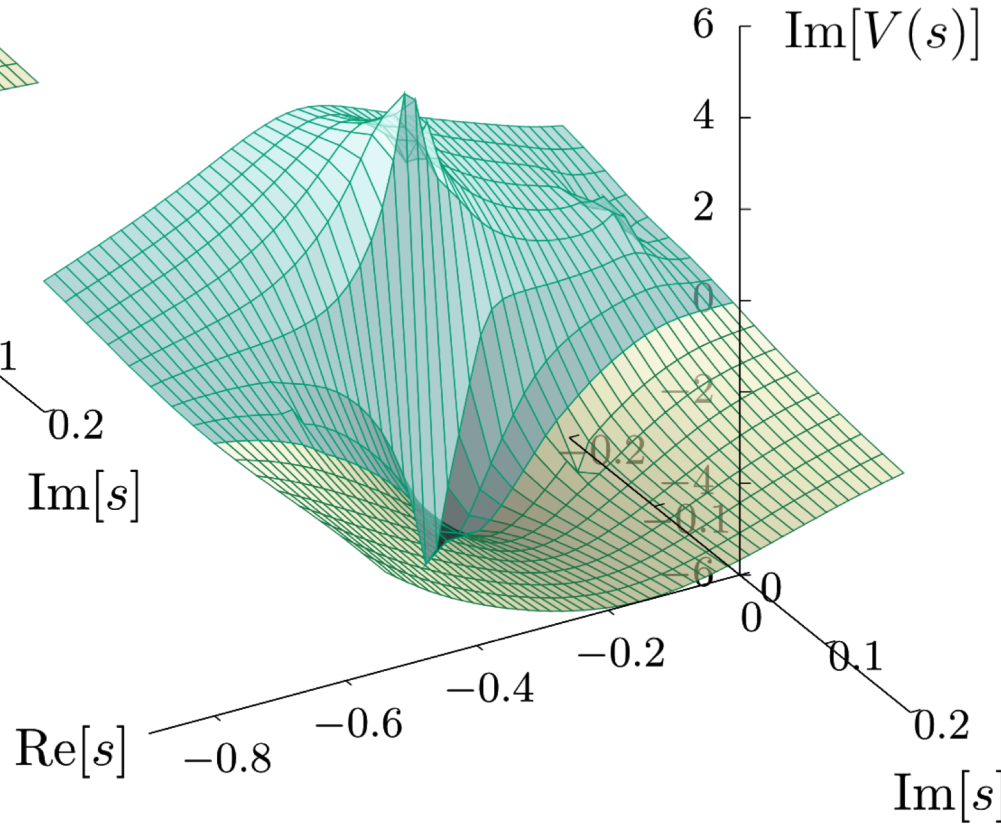
Non-analytic across BC

Branch cut

No poles on the “physical” sheet

Resonance

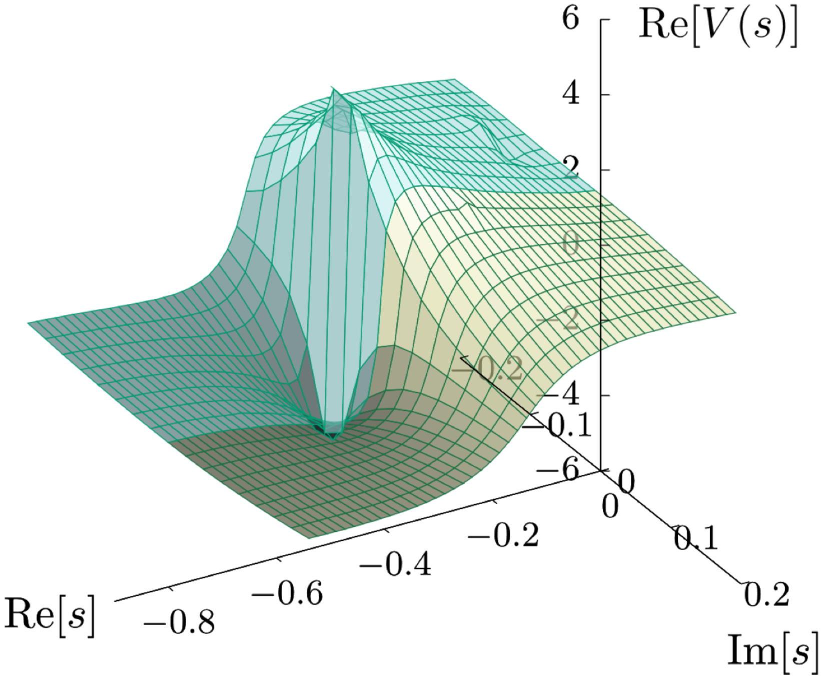


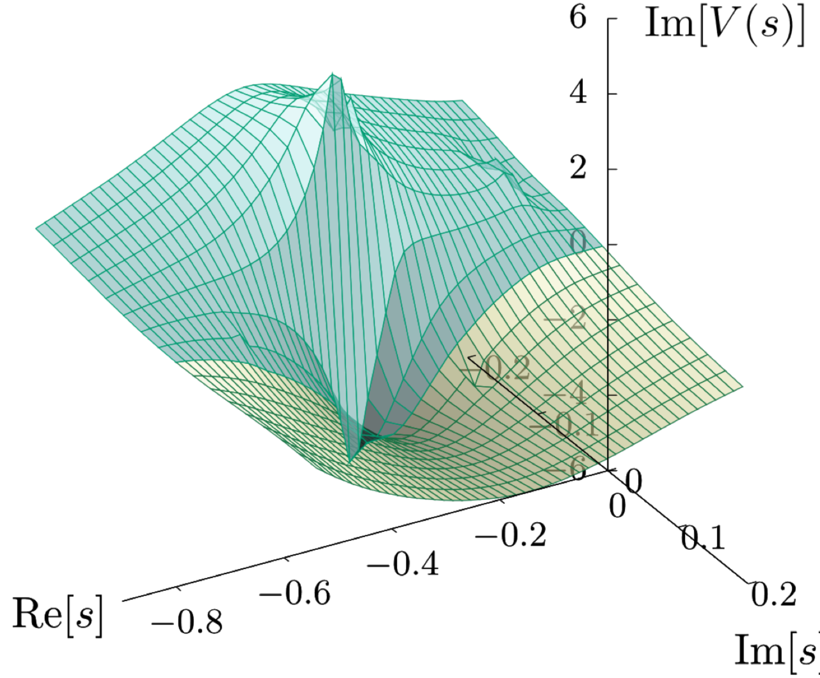


Analytically continue across branch cut to the second Riemann sheet.

Poles on the “unphysical” sheet

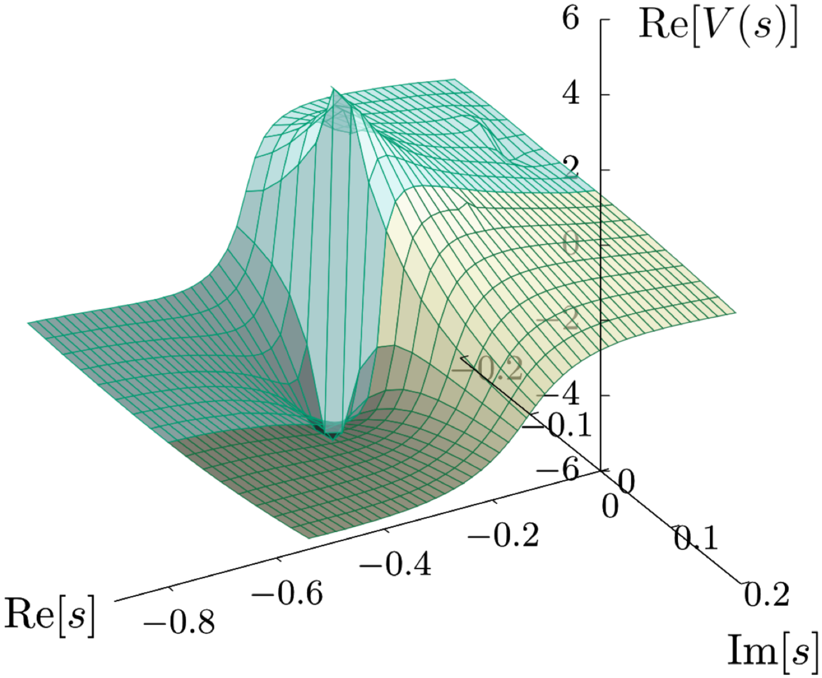
Resonance

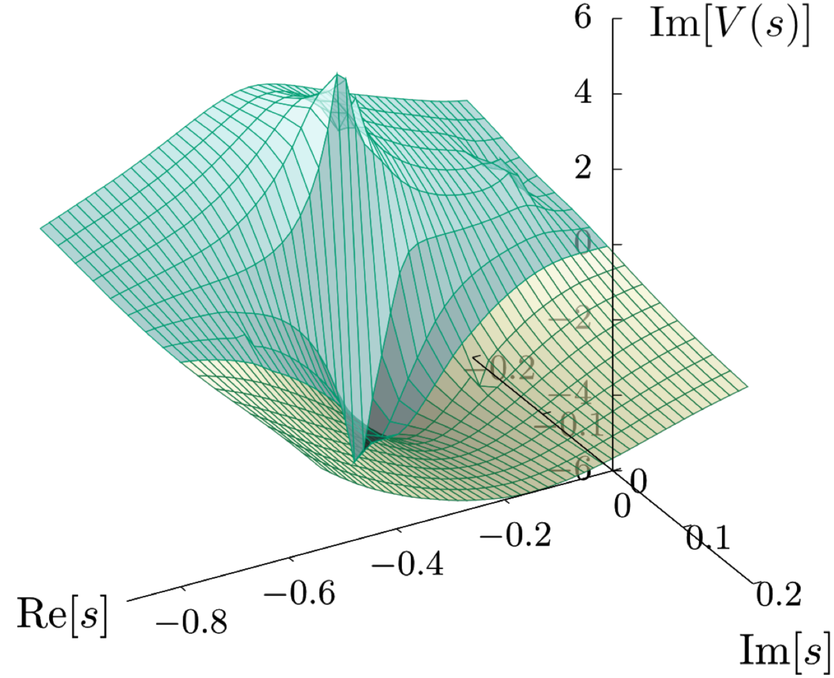




What would we expect to see in the BSE approach?

Resonance

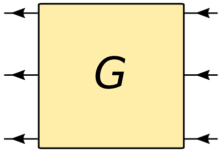




What would we expect to see in the BSE approach?

This is the Bethe-Salpeter approach! ☺

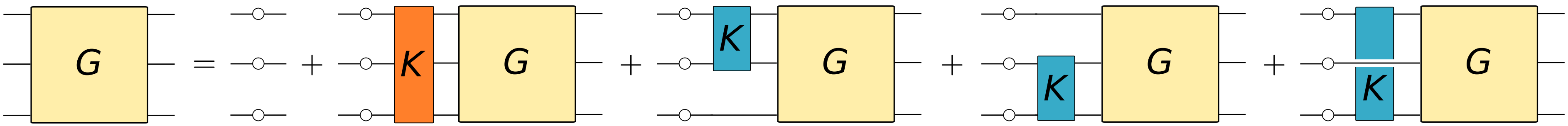
DSE and BSE



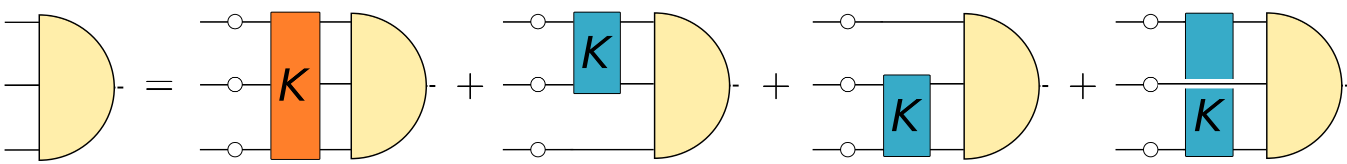
Back to the beginning: **Green's functions**

DSE and BSE

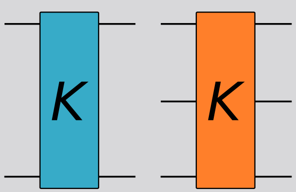
Trade one unknown G, for another unknown K



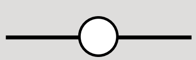
Solution (on-shell) yields Bethe-Salpeter wavefunction



See Eichmann



Irreducible 2-, 3-, 4-body kernels **define** equation

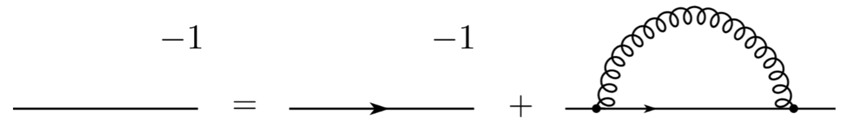


Dressed particle constituents: Green's functions

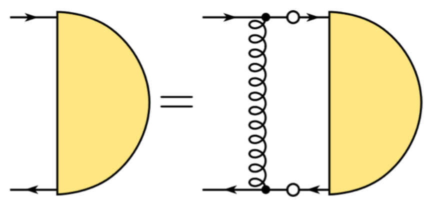
Truncation?

2PI 2-loop (rainbow-ladder)

Quark DSE



Meson BSE



Routinely solved by standard methods

- Quark for complex momenta (Cauchy, shell-method, path deformation)
- One-loop BSE kernel independent of total momentum P

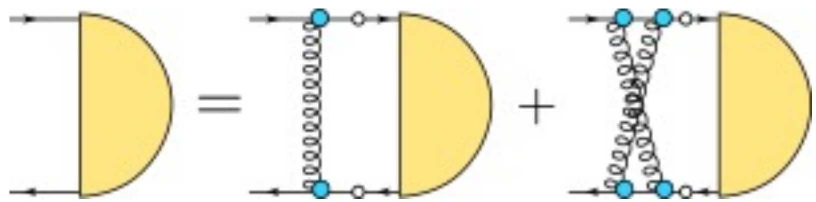
e.g. [Sanchis-Alepuz, RW, arXiv:1710.04903]

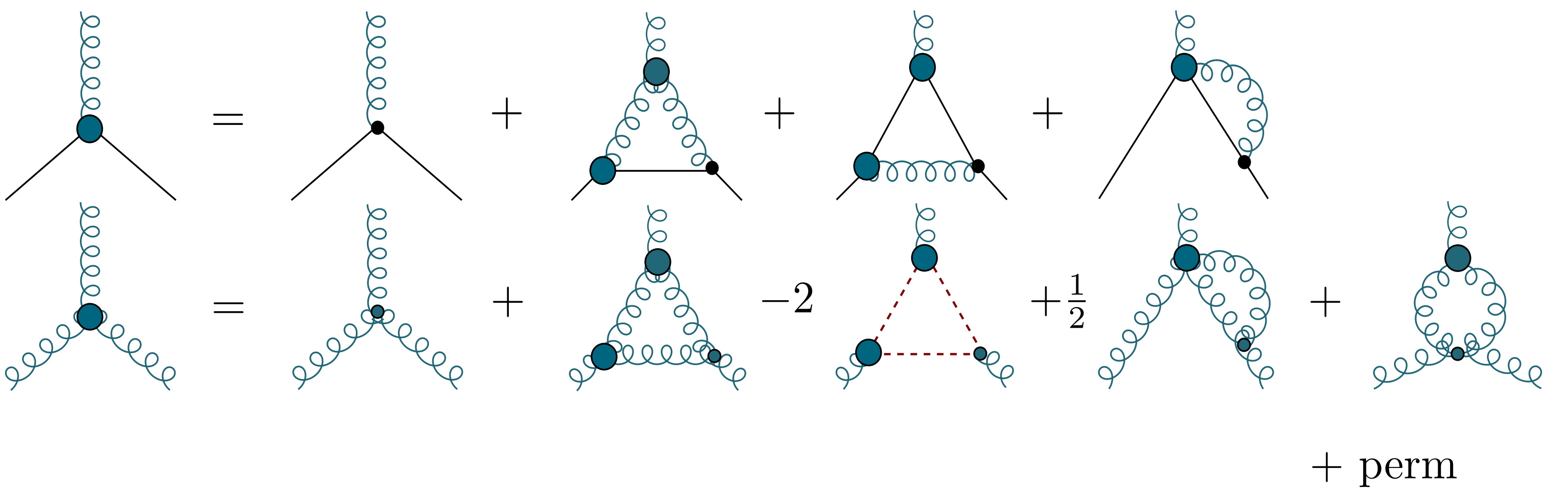
Truncation?

3PI 3-loop

Quark DSE

Meson BSE





See Huber

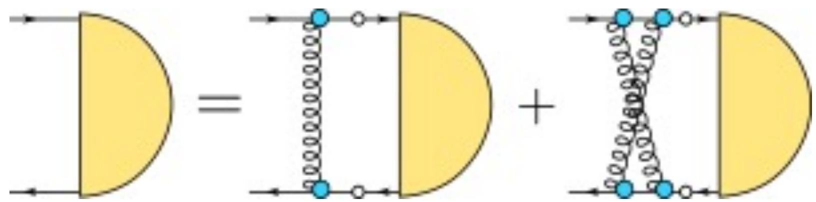
[RW, Fischer, Heupel, PRD93 (2016)]

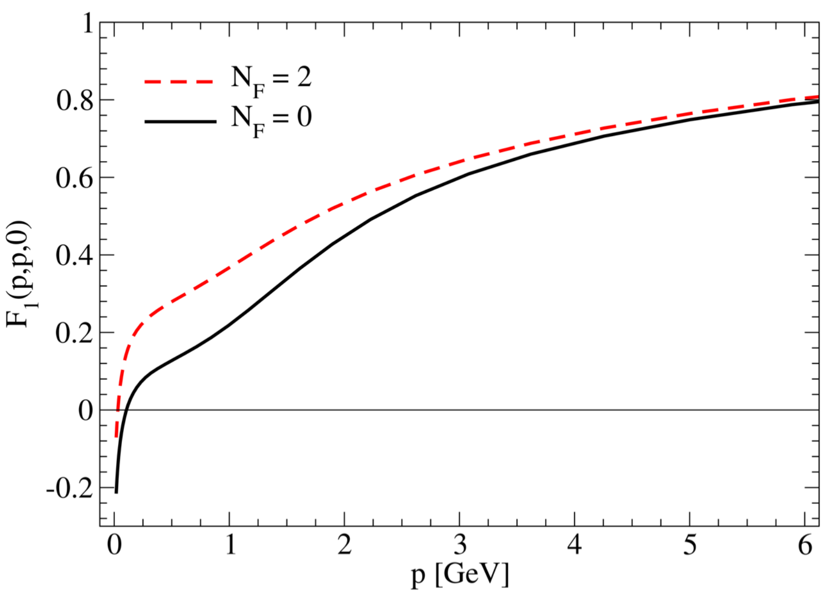
Truncation?

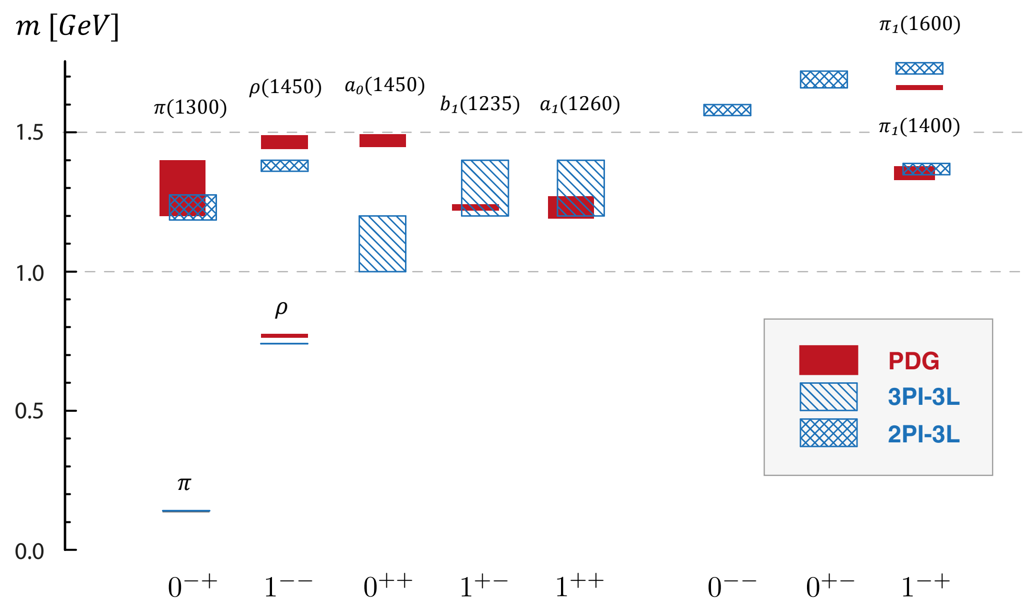
3PI 3-loop

Quark DSE

Meson BSE







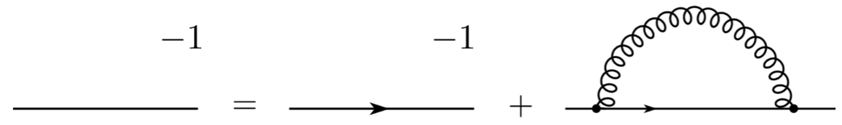
See Huber

[RW, Fischer, Heupel, PRD93 (2016)]

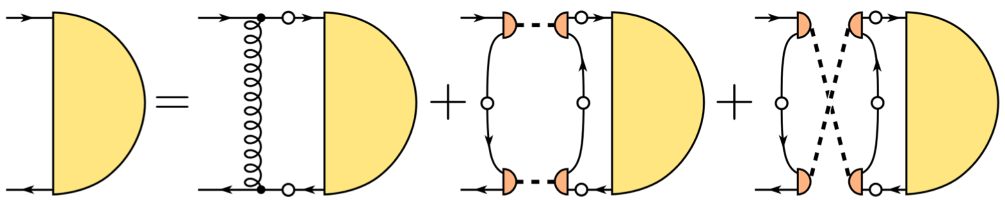
Truncation

Hadronic Unquenching

Quark DSE



incl. decay



[Watson, Cassing, FBS 35 (2004)]
[Fischer, Nickel, Wambach, PRD 76 (2007)]
[Fischer, RW, PRD 78 (2008)]

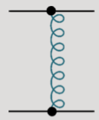
Specifically

- Two-pion decay kernel
- **Couples** to *e.g.* vector and scalar mesons.
- Does **not couple** to pseudoscalar (CP and P): *maintains chiral symmetry*

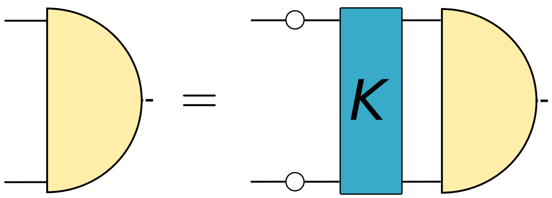
Truncation

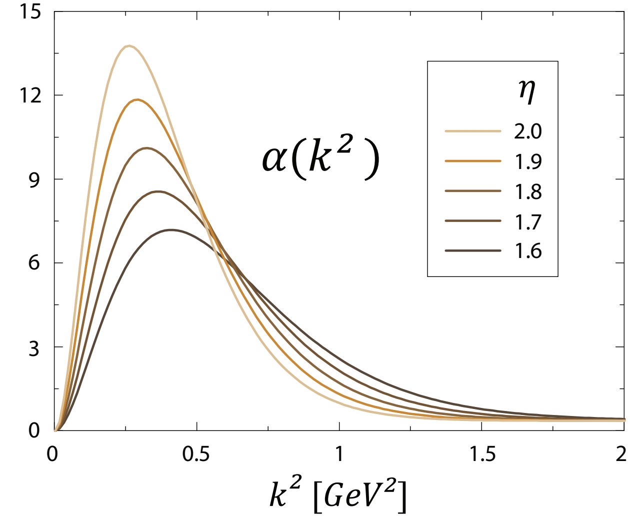
Rainbow-Ladder:

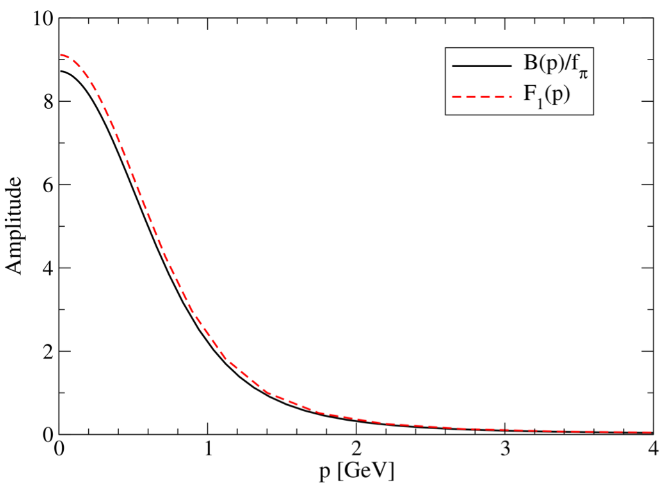
One-gluon exchange



[Maris, Tandy PRC 60 (1999) 055214]

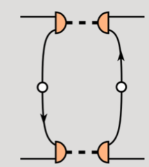






Decay/Unquenching:

Two-pion exchange



$$\Gamma_\pi(k,P) = \gamma_5 \frac{B(k^2)}{f_\pi}, \qquad D_\pi(q^2) = \left(q^2 + m_\pi^2\right)^{-1}$$

See Roberts

[Watson, Cassing, FBS 35 (2004)]
[Fischer, Nickel, Wambach, PRD 76 (2007)]
[Fischer, RW, PRD 78 (2008)]

Decomposition

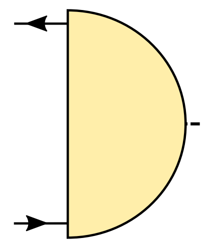
Covariant basis for bound-state: $\Gamma^{(\rho)} = \sum_i g_i \tau_i^{(\rho)}, \quad \chi^{(\rho)} = \sum_i h_i \tau_i^{(\rho)}$

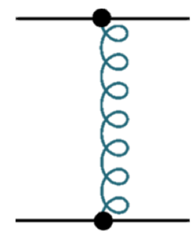
pseudoscalar	
$\tau_1 = \gamma^5$	$\tau_3 = \hat{\not{P}}\gamma^5$
$\tau_2 = \widehat{\not{p}_T}\gamma^5$	$\tau_4 = i\widehat{\not{p}_T}\hat{\not{P}}\gamma^5$

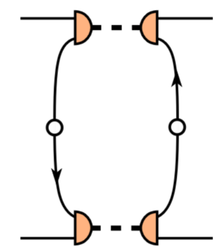
vector			
$\tau_1^\rho = \gamma_T^\rho$	$\tau_3^\rho = i\widehat{p_T}^\rho$	$\tau_5^\rho = 3\widehat{p_T}^\rho\widehat{\not{p}_T} - \gamma_T^\rho$	$\tau_7^\rho = \gamma_T^\rho\widehat{\not{p}_T} - \widehat{p_T}^\rho$
$\tau_2^\rho = \gamma_T^\rho\hat{\not{P}}$	$\tau_4^\rho = \widehat{p_T}^\rho\hat{\not{P}}$	$\tau_6^\rho = (3\widehat{p_T}^\rho\widehat{\not{p}_T} - \gamma_T^\rho)\hat{\not{P}}$	$\tau_8^\rho = i(\gamma_T^\rho\widehat{\not{p}_T} - \widehat{p_T}^\rho)\hat{\not{P}}$

Quark rotation matrix:

$$Y_{ij} = \mathrm{Tr}\left[\overline{\tau}_i^{(\rho)} S(p_+)\tau_j^{(\rho)}(p,P)S(p_-)\right],$$







Kernel trace:

$$L_{ij}^{\mathrm{RL}} = \int_k \mathrm{Tr}\left[\overline{\tau}_i^\rho(p,P)\gamma^\mu\tau_j^\rho(k,P)\gamma^\nu\right] D^{\mu\nu}(q),$$

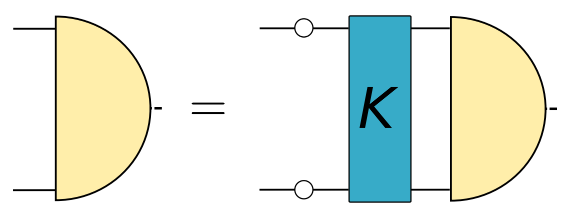
$$L_{ij}^{\pi\pi,\mathrm{s}} = \int_k\int_l \overline{J}_i^\rho(p,l,P) J_j^\rho(k,l,P) D_+^\pi D_-^\pi,$$

$$J_j^\rho(k,l,P) = \mathrm{Tr}\left[\overline{\Gamma}_\pi\tau_j^\rho(k,P)\overline{\Gamma}_\pi S(k-l)\right],$$

$$\overline{J}_i^\rho(p,l,P) = -\left[C^T J_i^\rho(-p,-l,-P)C\right]^T.$$

BSE:

$$g_i = \sum_A L_{ij}^A h_j = \sum_A L_{ij}^A Y_{jk} g_k = M_{ik} g_k,$$



Integrating over Poles

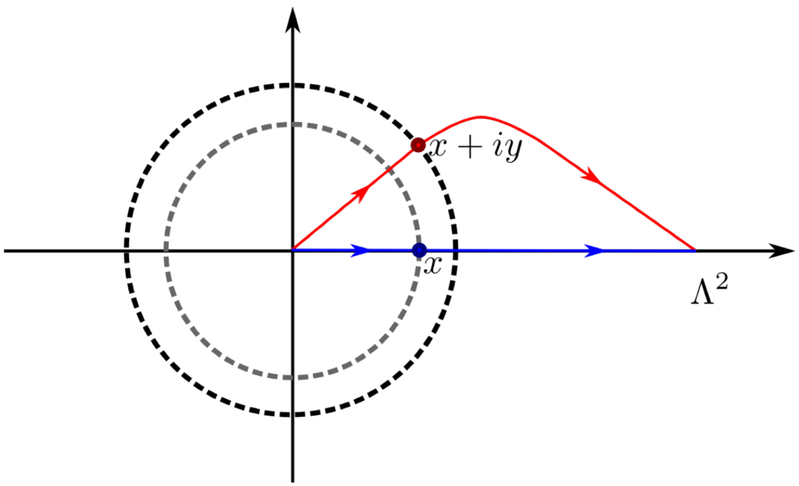
Consider: general integral of the form

$$A(p^2) = \int d^4k\, f(k^2, p^2, z) \qquad z = k \cdot p$$

With $f(k^2, p^2, z)$ containing a pole, dependent upon the angle z

- Angular integral "sweeps" out the pole.
- Radial integral should be deformed to avoid cut structure.

Applied to **quark propagator**



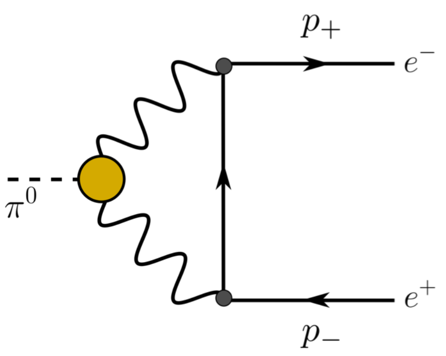
$$I(p^2) = \int dq^2 q^2 \sigma(q^2) K(q^2, p^2)$$

$$K(q^2, p^2) = \int dz \sqrt{1-z^2} \frac{f((q-p)^2)}{(q-p)^2} K_\theta(q, p)$$

e.g. [Alkofer, Detmold, Fischer, Maris, PRD 70 (2004)]

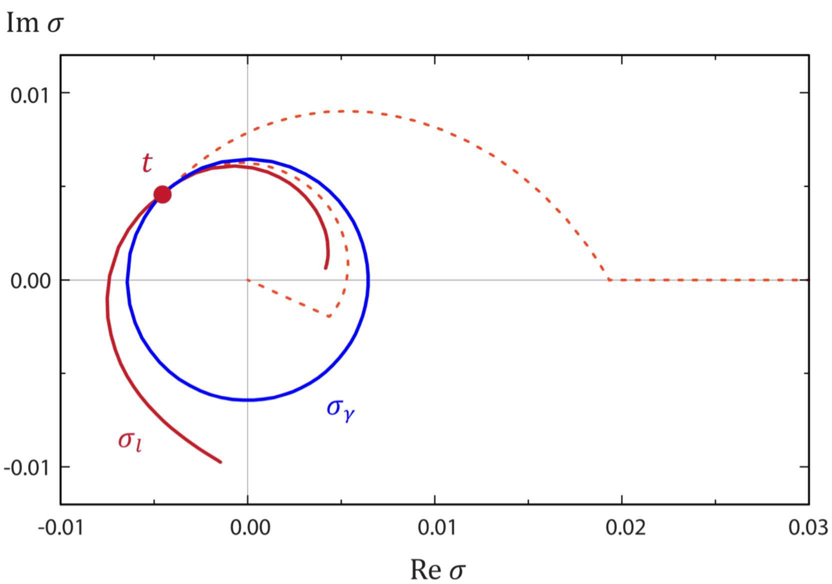
Integrating over Poles

Applied to **rare pion decay** $\pi^0 \to e^+ e^-$ to avoid cut structure during integration

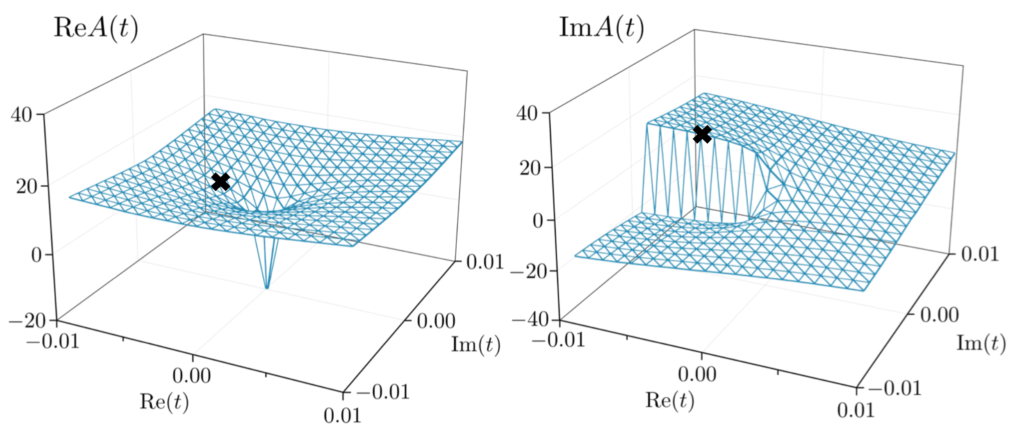


$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2 \, Q'^2}$$

- Results in agreement with dispersion relations
- Technique has further applications



See Weil (Poster)

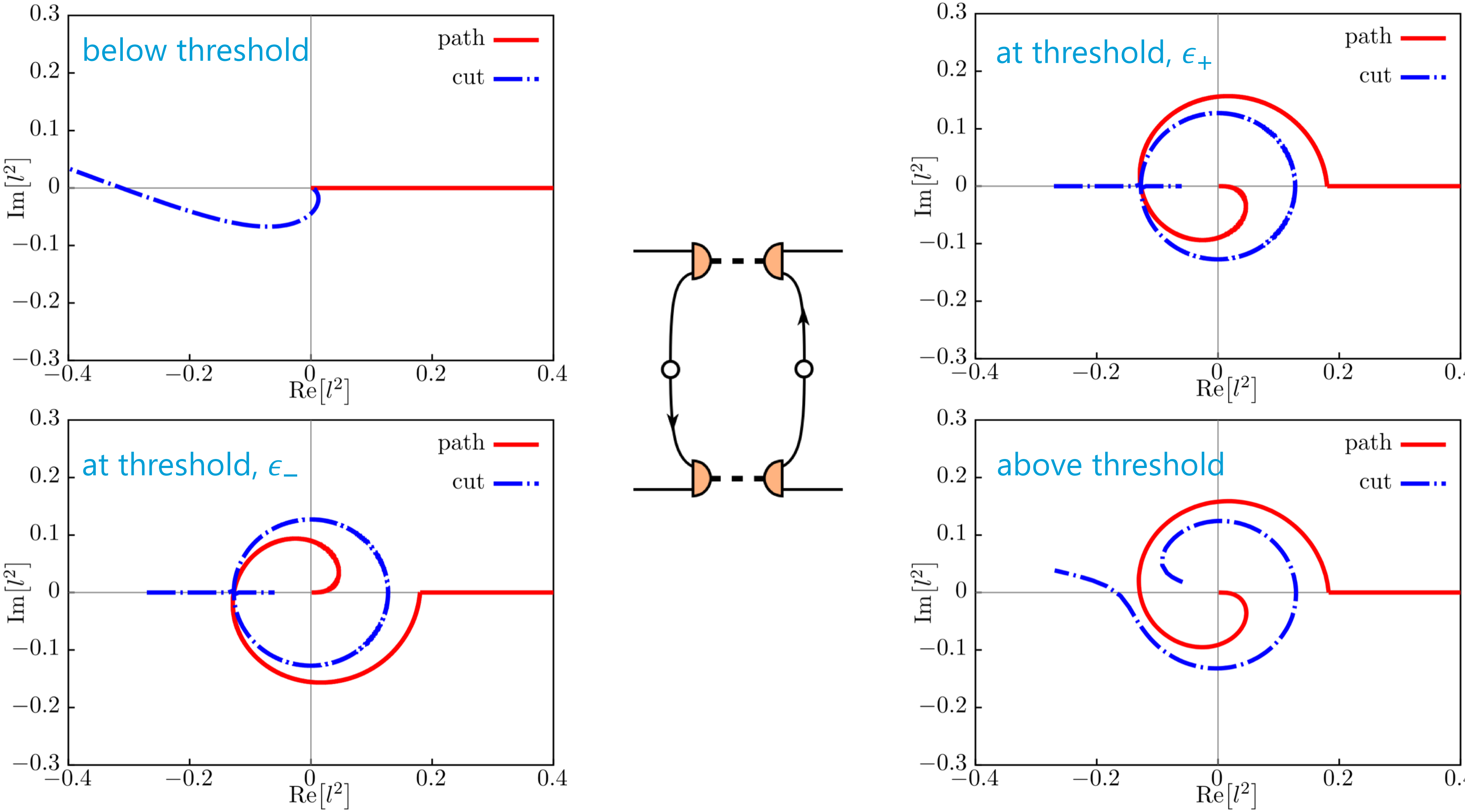


[Weil, Eichmann, Fischer, RW, PRD 96 (2017)]

Integrating over Poles

Two-pion cuts

$$l^2_{\text{cut}} = -z\sqrt{t} + \sqrt{t(z^2-1) - m_\pi^2}, \qquad t = P^2/4$$

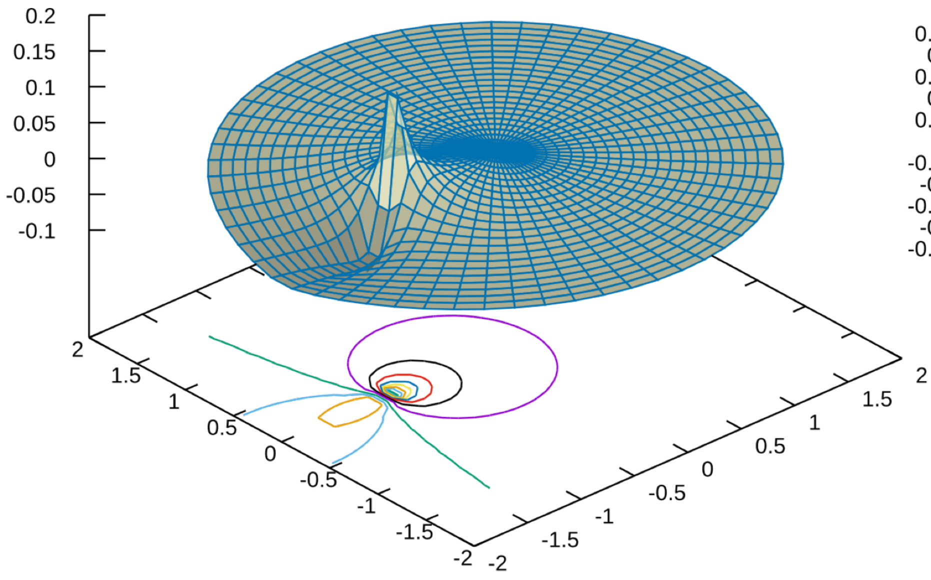


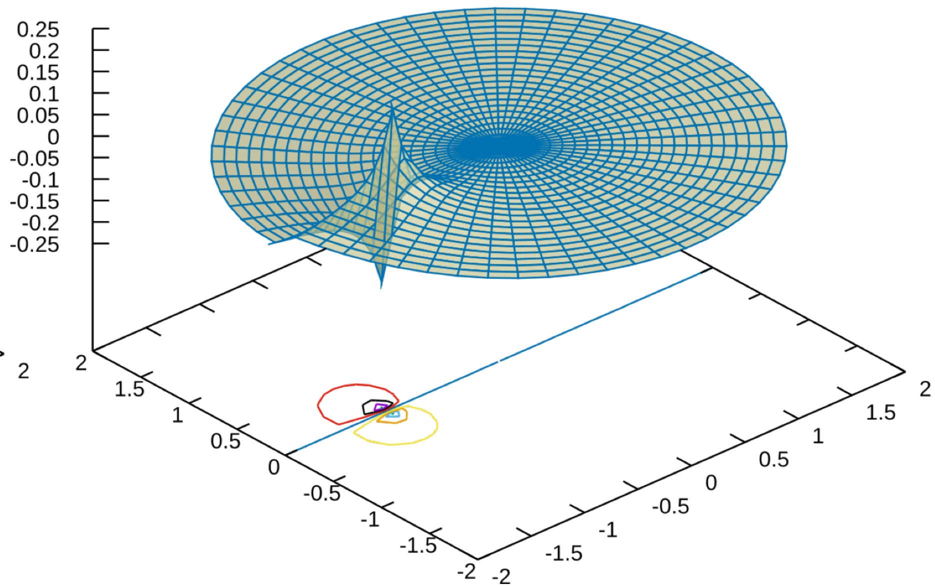
See also [Windisch, Huber, Alkofer, APPS 6 (2013)]

Two-pion integral

$$F(l,P) \propto \frac{l_T^\rho}{l_T^2}\int_k J_j^\rho(k,l,P)h_j(k,P)\ .$$

$$I(P^2) = \int_l \frac{1}{l^2\left(l_+^2+m_\pi^2\right)\left(l_-^2+m_\pi^2\right)}.$$





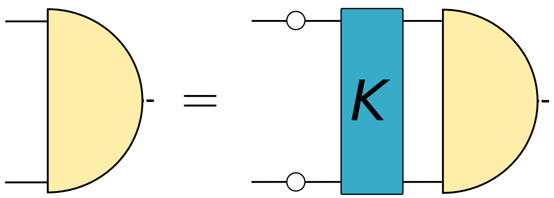
$$I(P^2) = \frac{1}{4\pi^2P^2}\left[\ln\left(\frac{a^2-1}{a^2}\right)+\frac{1}{a}\ln\left(\frac{a+1}{a-1}\right)\right],$$

$$a=\sqrt{1+4m_\pi^2/P^2}$$

Calculation

Put together:

- Solve quark for complex momenta
- Calculate one-loop RL kernel
- Calculate two-loop pi-pi kernel
- Choose appropriate path deformation

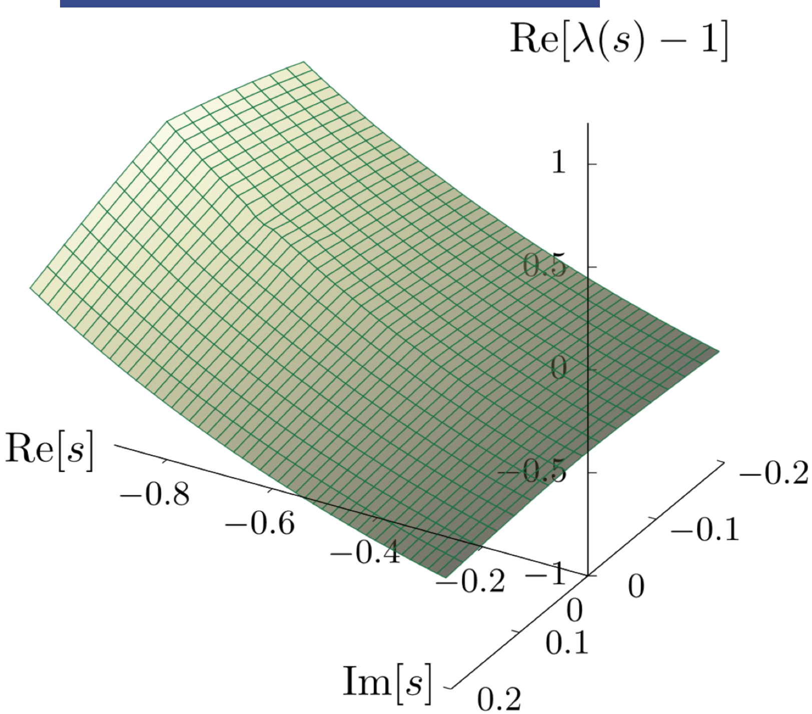


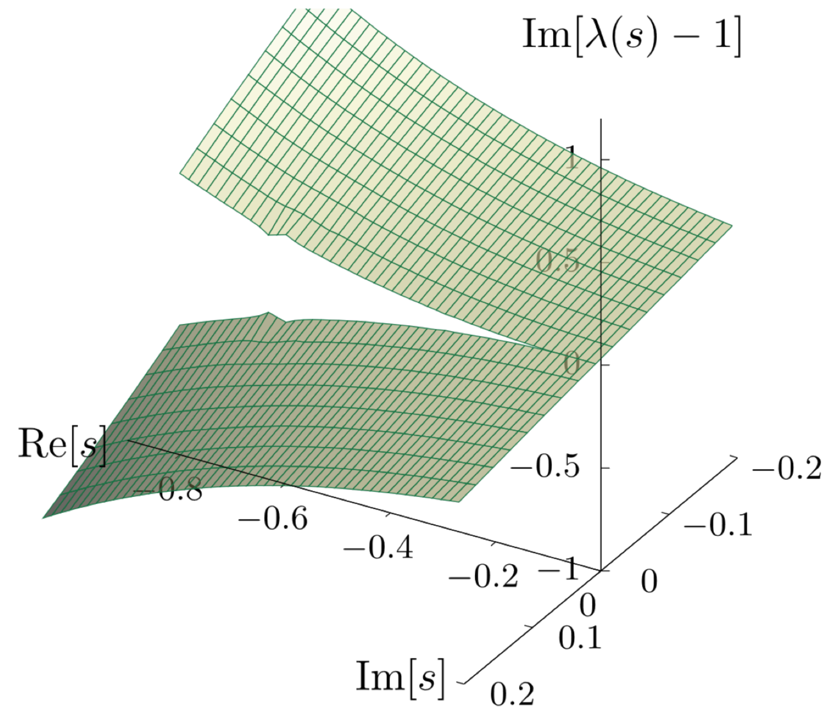
Solve BSE as eigenvalue equation in $\lambda(P^2)$ *complex*

$$\Gamma = \lambda(P^2)\, K\, \Gamma, \qquad P^2 \in \mathbb{C}$$

Use **right tools** for solving the eigensystem

Eigenvalues

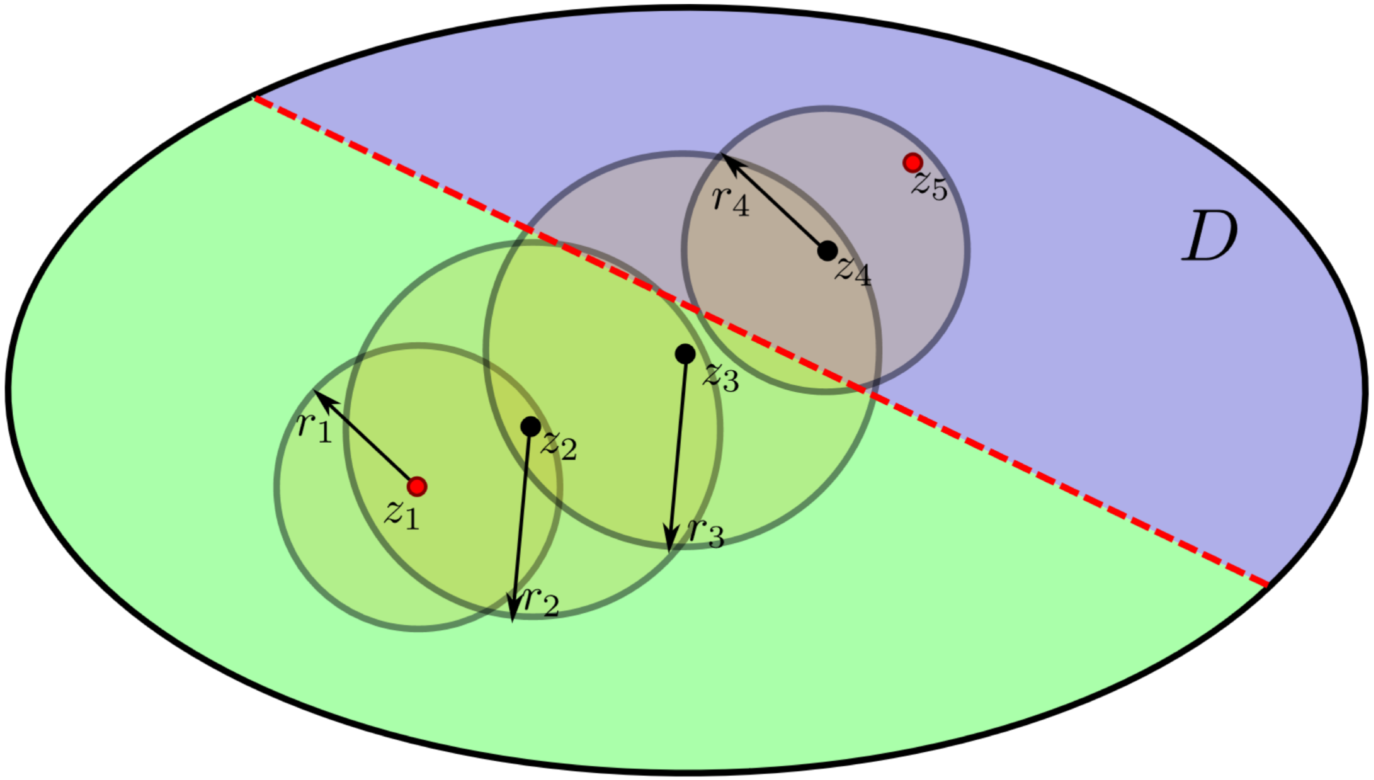




- “tent structure” in real part
- Branch cut in imaginary part

No solution on “physical sheet” where: $\lambda(s) = 1$

Analytic Continuation

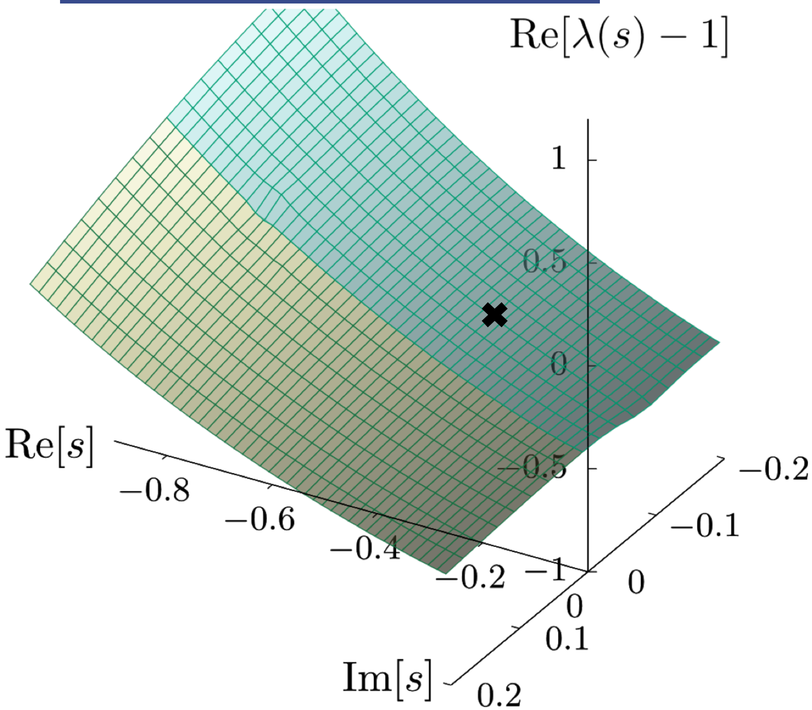


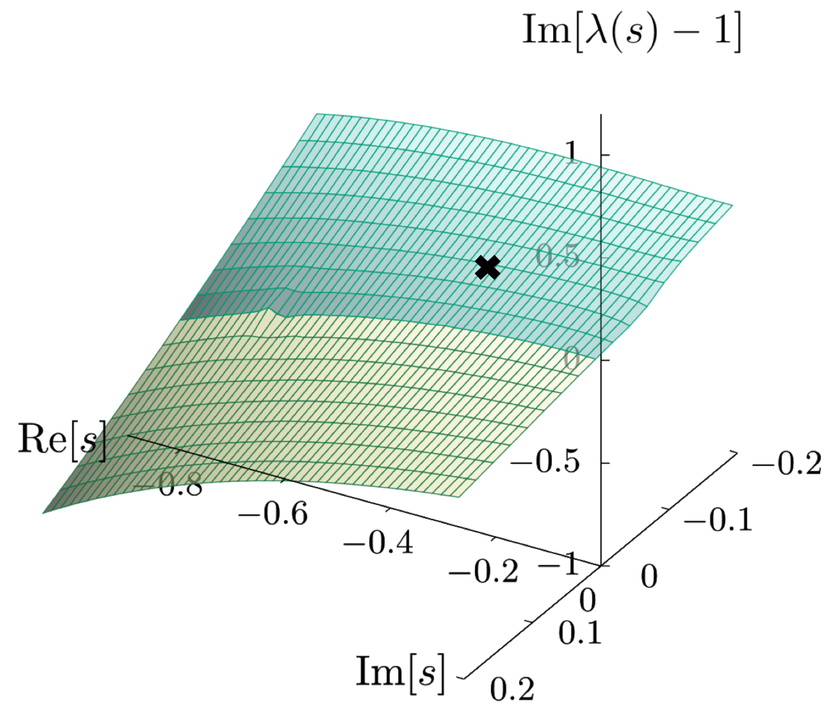
Analytic continuation (from e.g. z_1 to z_5)

- Using power series (i.e. Hadamard method)
- Pade approximants. RVP and Schlessinger point method.

[Tripolt et al, arXiv:1801.10384]

Eigenvalues





Analytically continue to find $\lambda(s) = 1$ on "unphysical sheet"

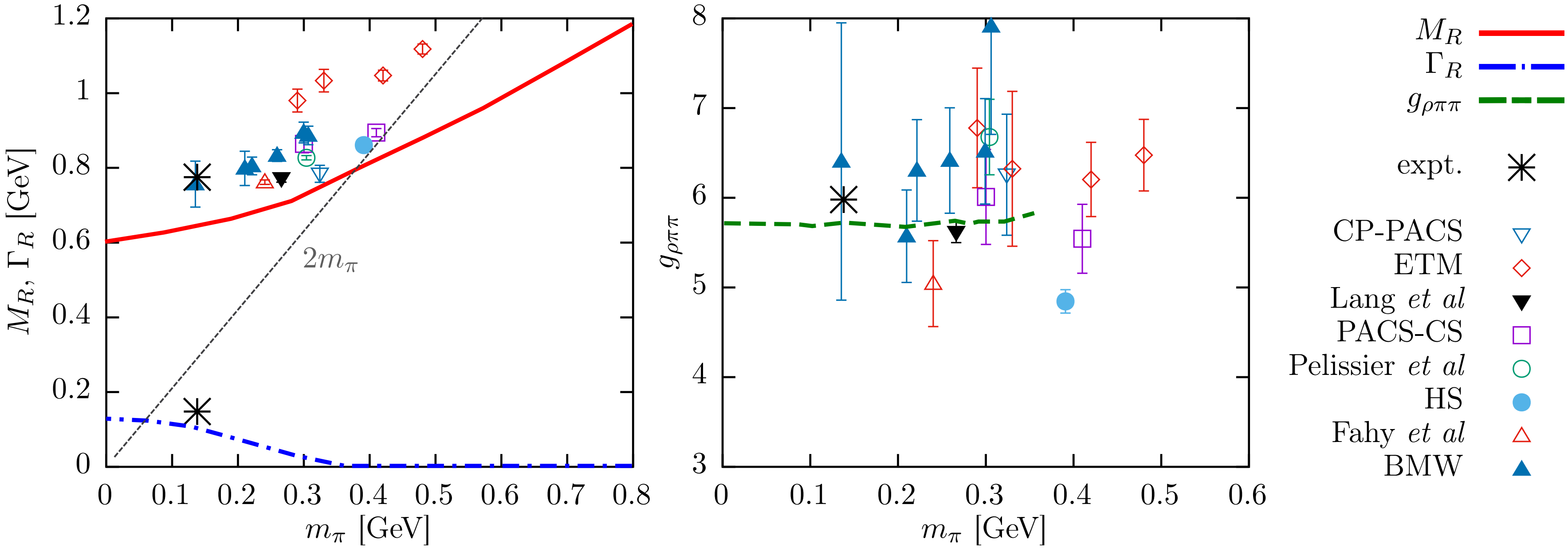
$$s = -0.408 - 0.065i \; [GeV^2]$$

$$M = 0.641 \; [GeV]$$

$$\Gamma = 0.101 \; [GeV]$$

$$\Gamma_R = \frac{p^3}{M_R^2}\frac{g_{\rho\pi\pi}^2}{6\pi}, \quad p = \sqrt{M_R^2/4 - m_\pi^2}\,,$$

Mass dependence



Here: strong coupling constant $g_{\rho\pi\pi} \sim 5.7$ **(experimental value** $g_{\rho\pi\pi} \sim 6.0$**)**

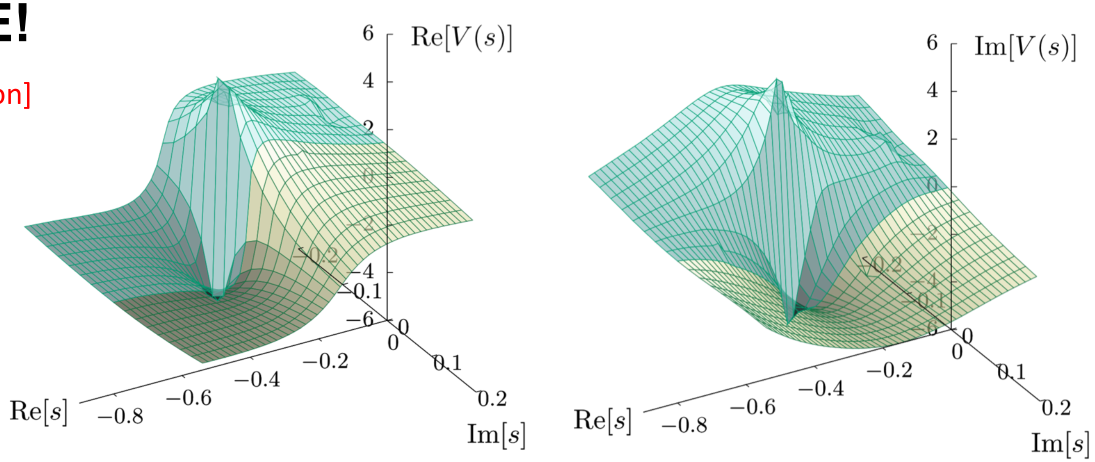
RL: (impulse approximation) $g_{\rho\pi\pi} \sim 5.2$

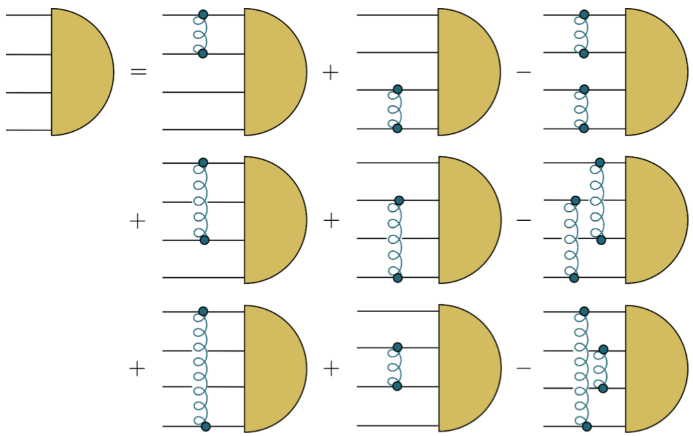
[Jarecke, Maris, Tandy, PRC67 (2003)]
[Mader, Eichmann, Blank, Krassnigg, PRD84 (2011)]

Summary

- **Resonances in BSE!**

[RW, in preparation]





See Wallbott (Poster)

Next Steps

- Extend to other bound-states
  - Baryons
  - Tetraquarks
- Solidify truncation + … *more*

Review

Eichmann, Sanchis-Alepuz, RW, Alkofer, Fischer 1606.9602 Prog. Part. Nucl. Phys. (in press)