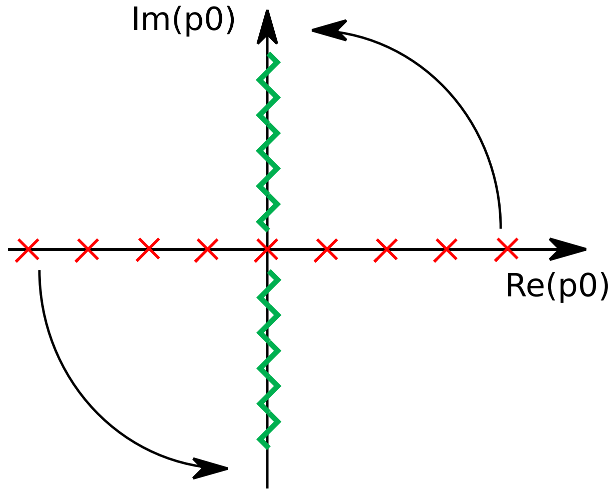
Spectral functions from the FRG

666. WE-Heraeus-Seminar in Bad Honnef

Nicolas Wink



Reconstructing the gluon

Anton K. Cyrol,[1] Jan M. Pawlowski,[1,2] Alexander Rothkopf,[1,3] and Nicolas Wink[1]

[1] *Institute for Theoretical Physics, Universität Heidelberg, Philosophenweg 12, D-69120 Germany*
[2] *ExtreMe Matter Institute EMMI, GSI, Planckstr. 1, D-64291 Darmstadt, Germany*
[3] *Faculty of Science and Technology, University of Stavanger, NO-4036 Stavanger, Norway*

We reconstruct the gluon spectral function in Landau gauge QCD from numerical data for the gluon propagator. The reconstruction relies on two novel ingredients: Firstly we derive analytically the low frequency asymptotics of the spectral function. Secondly we construct a functional basis from a careful consideration of the analytic properties of the gluon propagator in Landau gauge. This allows us to reliably capture the non-perturbative regime of the gluon spectrum. We also compare different reconstruction methods and discuss the respective systematic errors.

Uploaded today!

Base on:

Finite temperature calculation

Pawlowski, Strodthoff, NW arxiv:1711.07444

Gluon reconstruction

Cyrol, Rothkopf, Pawlowski, NW arxiv:1804.today

Spectral functions in QCD

What is a spectral function?

- Defined via the retarded propagator

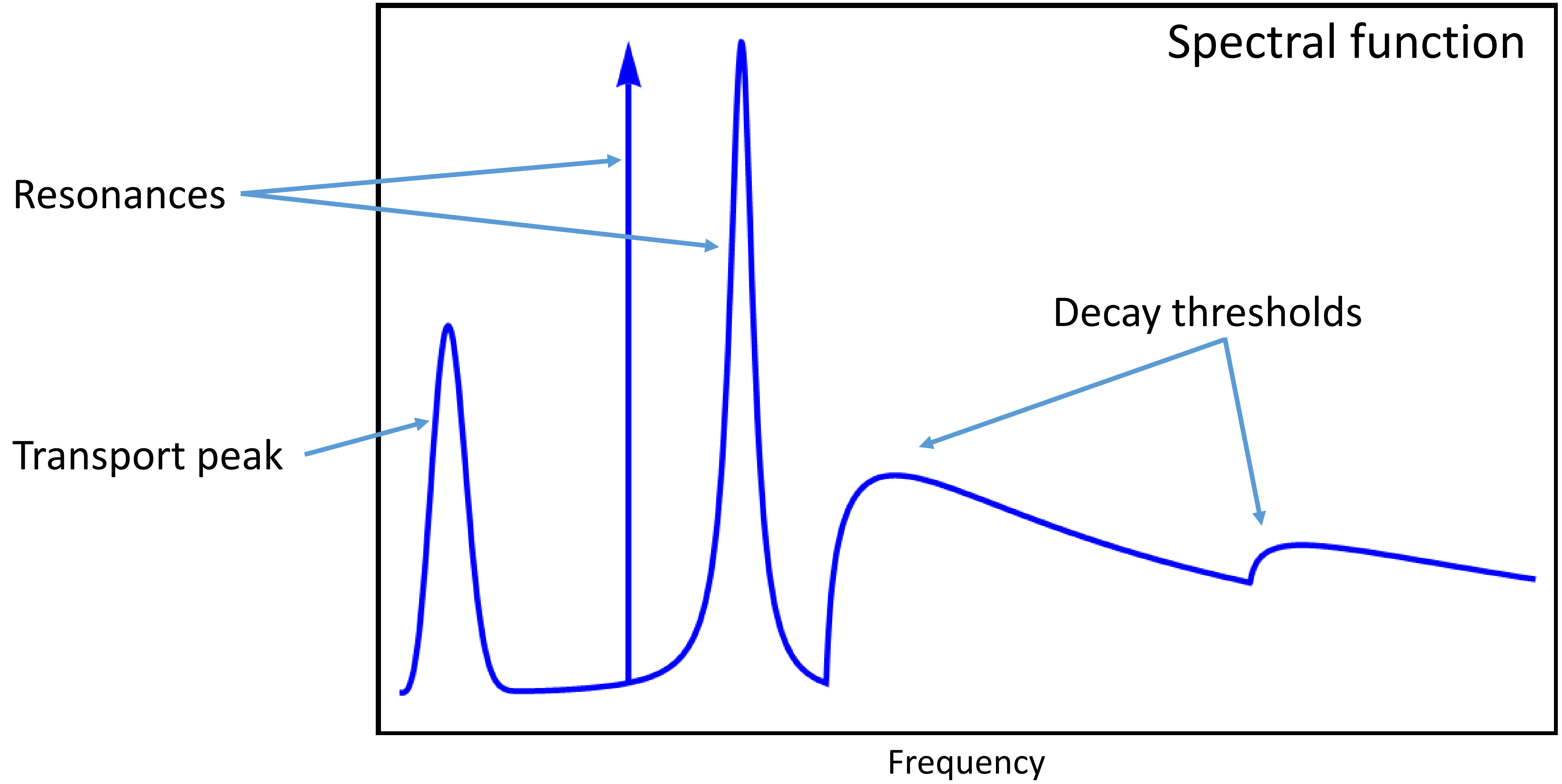
  $$\rho(\omega) = 2\,\mathrm{Im}\,G_{\mathrm{R}}(\omega)$$

- Encodes the spectrum of the theory

Why calculate spectral functions?

- Encodes the spectrum of the theory
- “Easy” to extract other observables

Spectral functions in QCD



Spectral functions in QCD

➡ Requires non-perturbative correlation functions in Minkowski space-time

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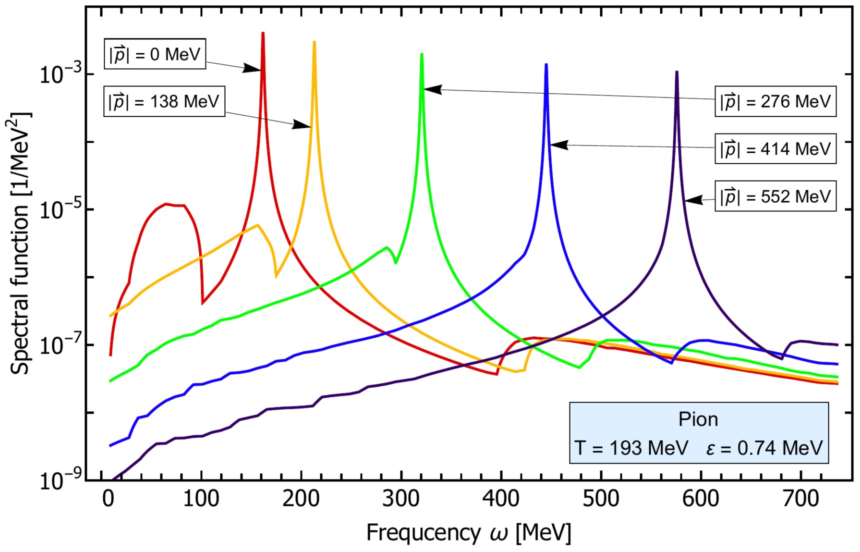
Direct calculation



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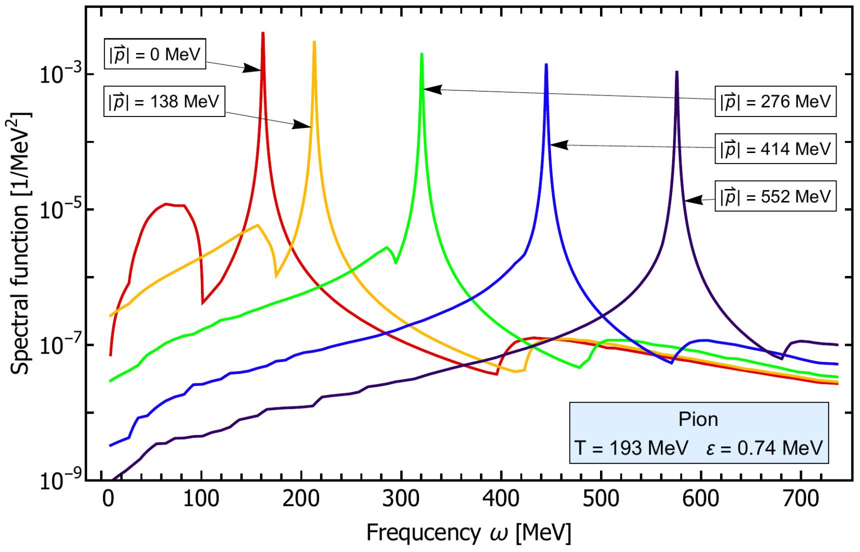

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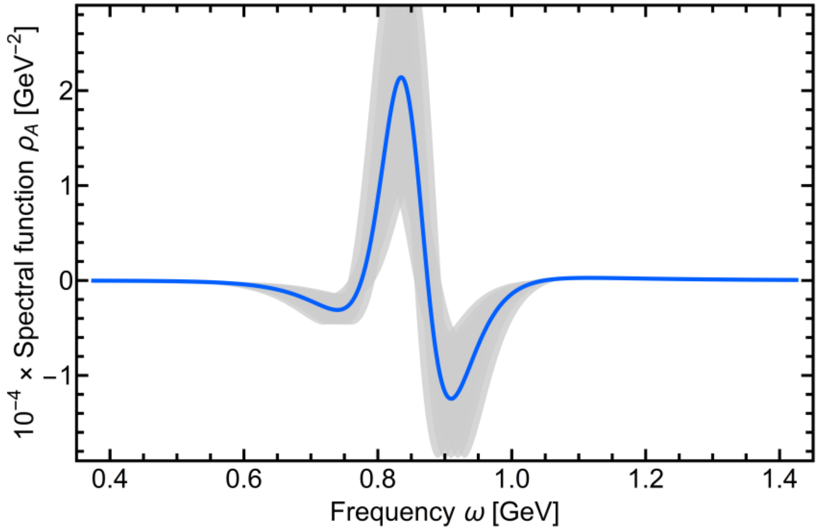
Reconstruction



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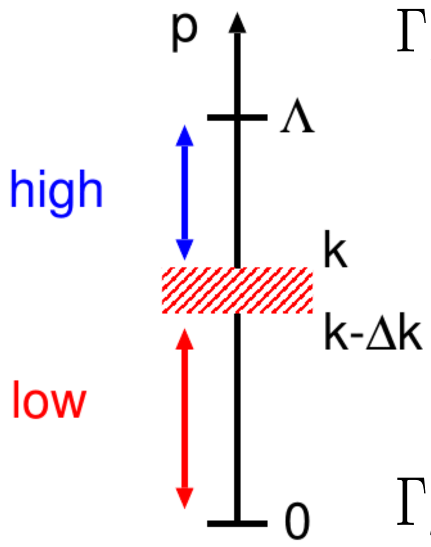
QCD from the FRG

Functional Renormalization Group



$$\Gamma_{k=\Lambda} = S$$

- Non-perturbative first principle method
- Access to physical mechanisms
- No sign problem
  - Chemical potential
  - Real time

$$\Gamma_{k=0} = \Gamma$$

cf. poster by Anton Cyrol

cf. talk by Mario Mitter

cf. talk by Fabian Rennecke

……

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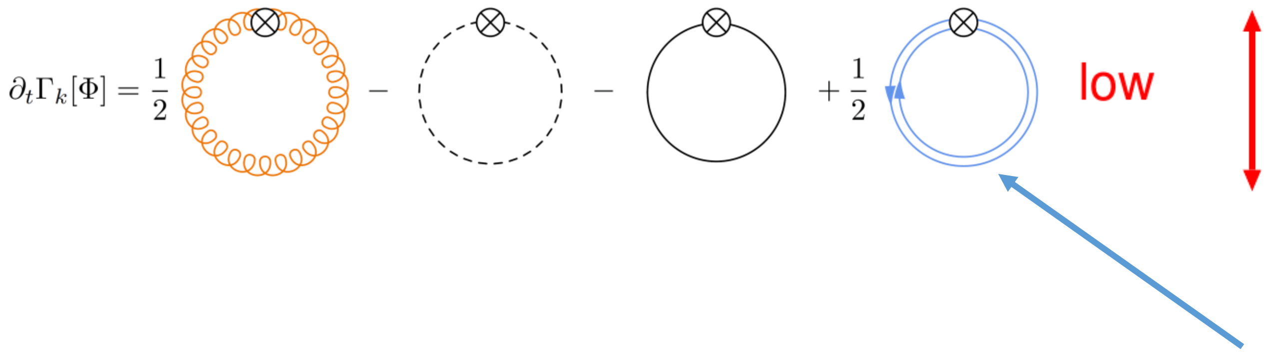
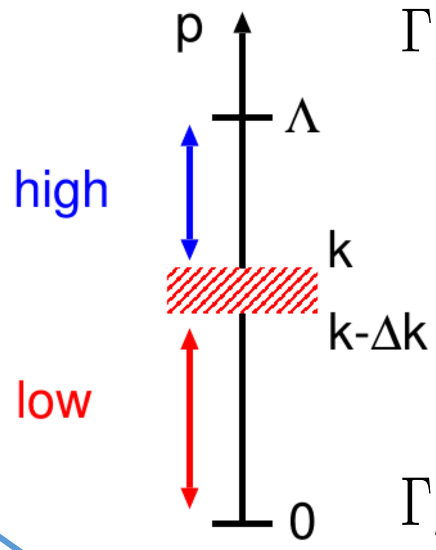
Flow equation for QCD





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Bound states efficiently taken into account via Dynamical Hadronization

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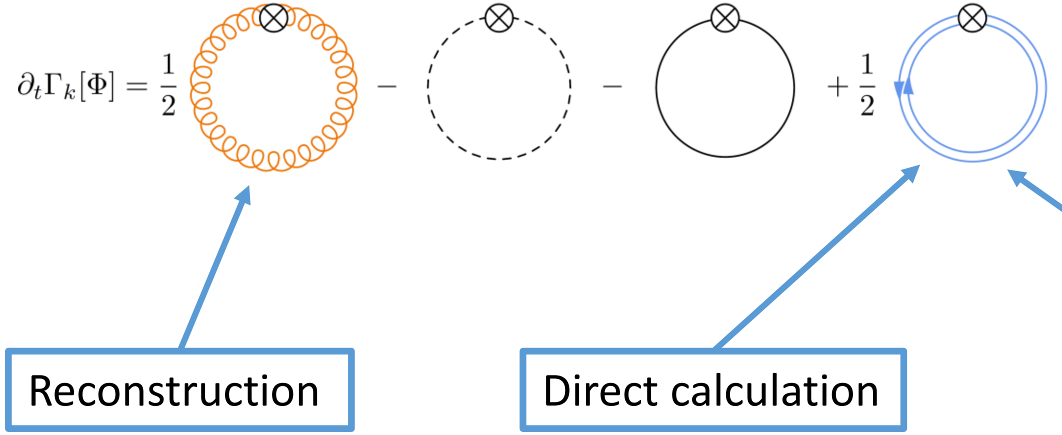

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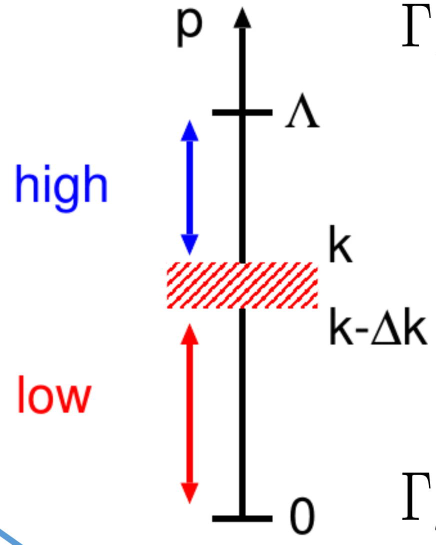

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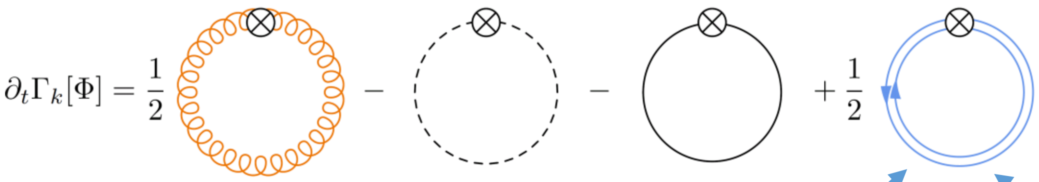

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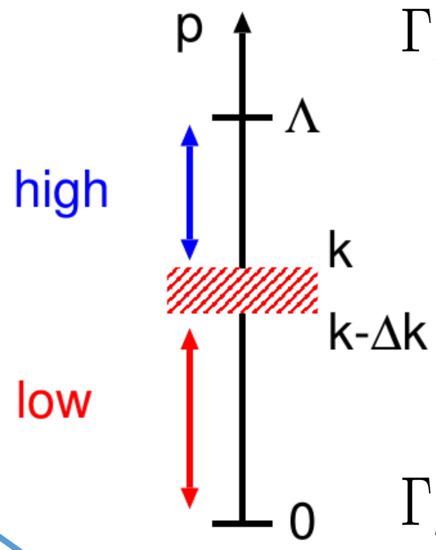

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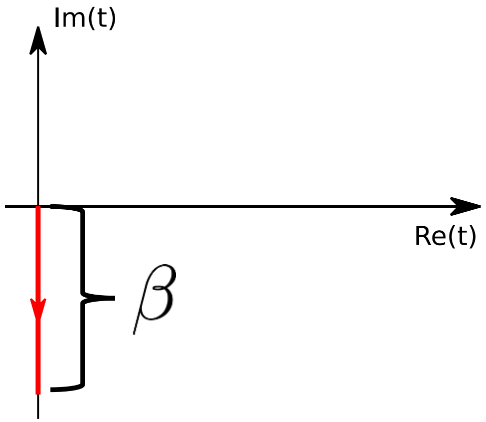
Collaborative effort fQCD collaboration:

J. Braun, A. Cyrol, W.-j. Fu, M. Leonhardt, M. Mitter, J.M. Pawlowski, M. Pospiech, F. Rennecke, C. Schneider, NW

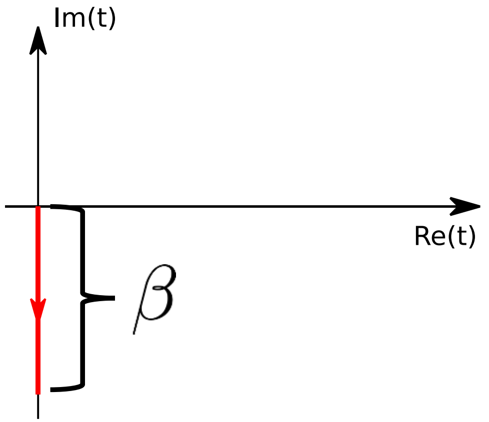
Introduction

From imaginary to real times



Matsubara contour

From imaginary to real times



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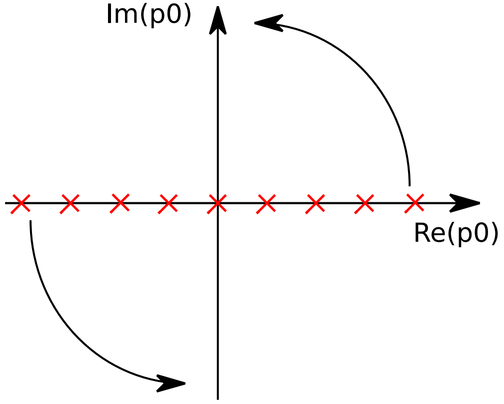

Continuation from
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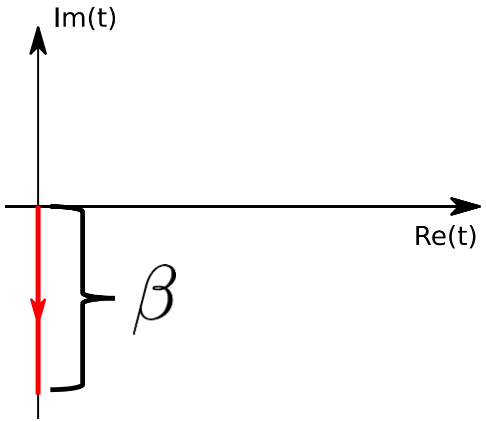

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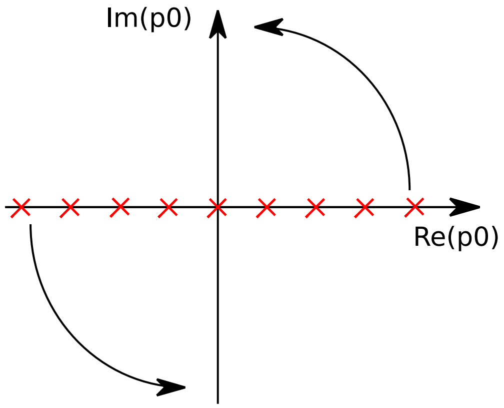

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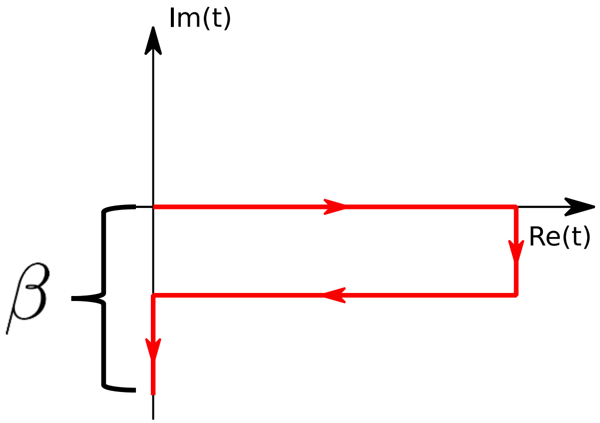

Schwinger-Keldysh contour

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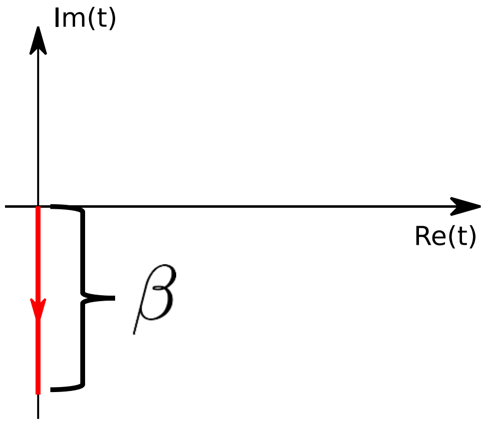

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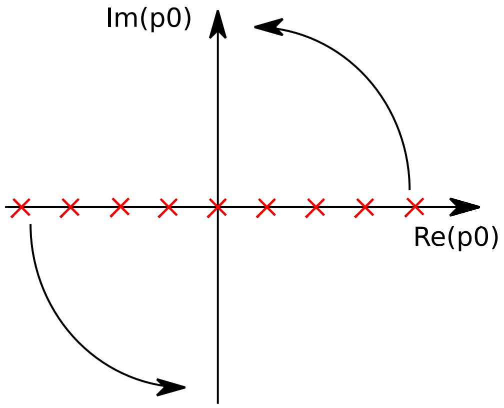

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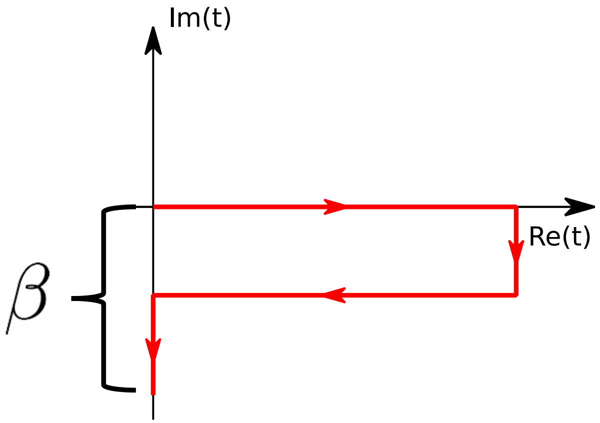

Schwinger-Keldysh contour

Use analyticity constrains and KMS condition to obtain real time correlation functions form Matsubara formalism

From imaginary to real times

Prerequisites :

Assume the existence of a spectral representation

$$G(p_0,\vec{p}) = \int_{\eta>0} 2\eta \frac{\rho(\eta,\vec{p})}{p_0^2+\eta^2} + \sum_{j\in\{\text{poles}\}} \frac{R_j}{p_0^2+M_j^2}$$

Possible to allow for additional complex conjugate poles

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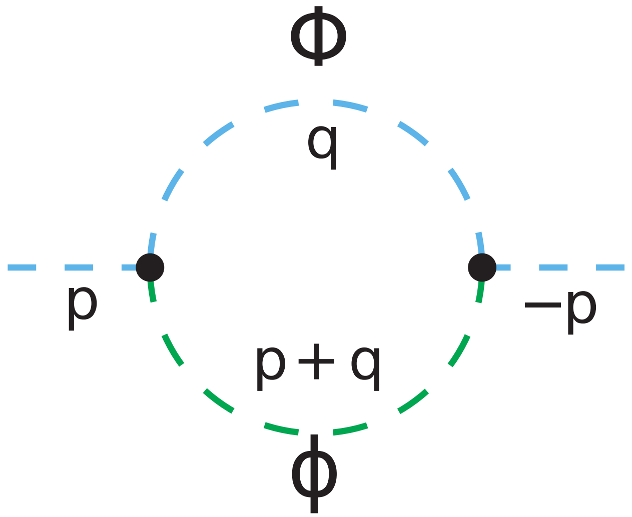
Example :

One-loop perturbation theory

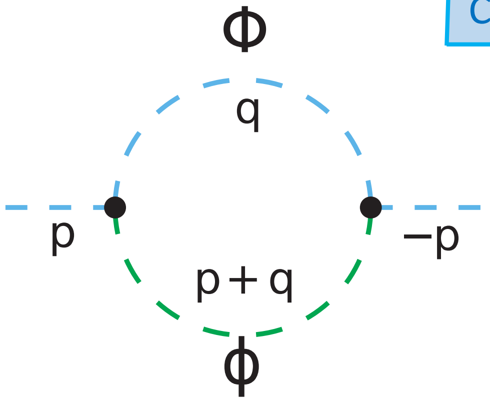
Two bosonic fields with interaction $\sim \Phi\Phi\varphi$

Calculate $\Gamma^{(2)}(p)$ for $p^0 \in \mathbb{C}$

Calculate Matsubara sum $\sum\!\!\!\!\!\!\int_{T,q} G_1(q+p)G_2(q)$



Illustrative example



$$\sum_T \frac{1}{(q_0+p_0)^2+(\epsilon^1_{q+p})^2}\frac{1}{(q_0)^2+(\epsilon^2_q)^2}$$

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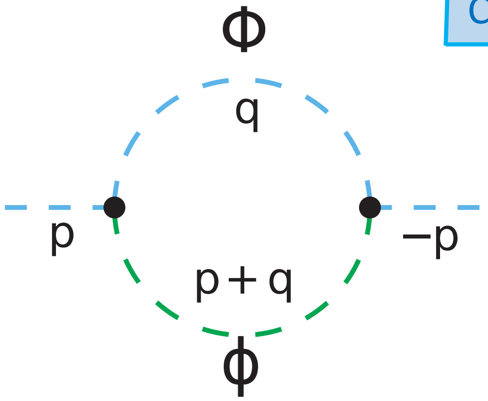
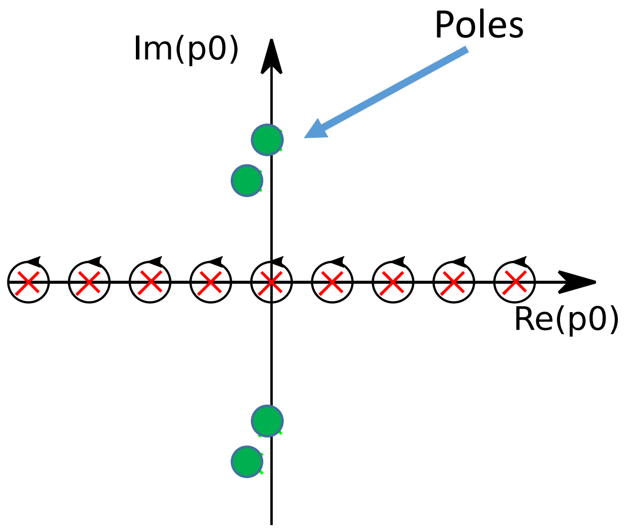


$$\sum_T \frac{1}{(q_0+p_0)^2 + (\epsilon^1_{q+p})^2} \frac{1}{(q_0)^2 + (\epsilon^2_q)^2}$$

Replace sum by contour integral:

Bosonic occupation number

$$T\sum_n f(2\pi n T) = -\frac{1}{2}\int_C \mathrm{d}z \;\; f(z)[1 + 2n_B(iz)]$$



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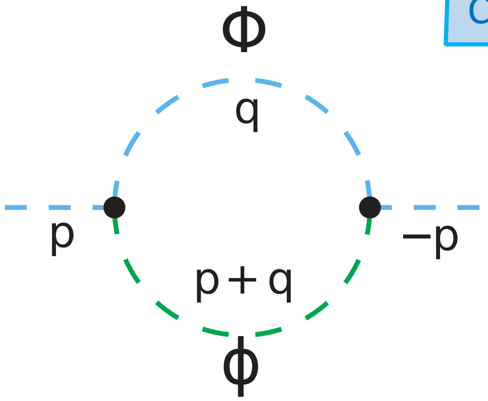

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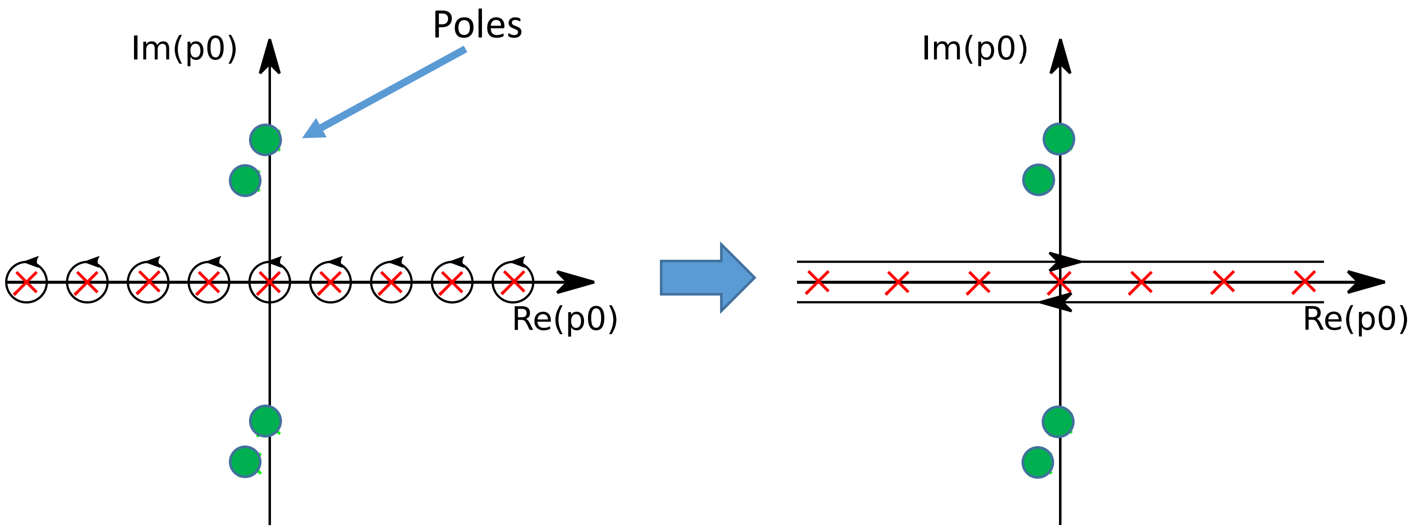

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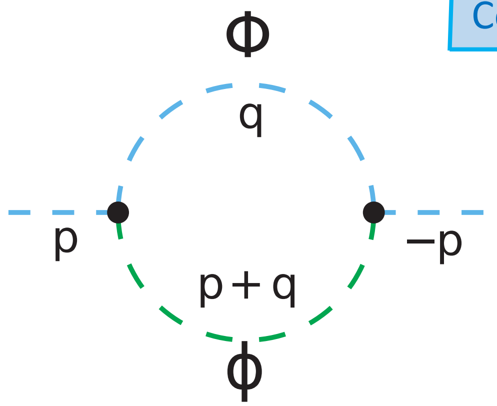

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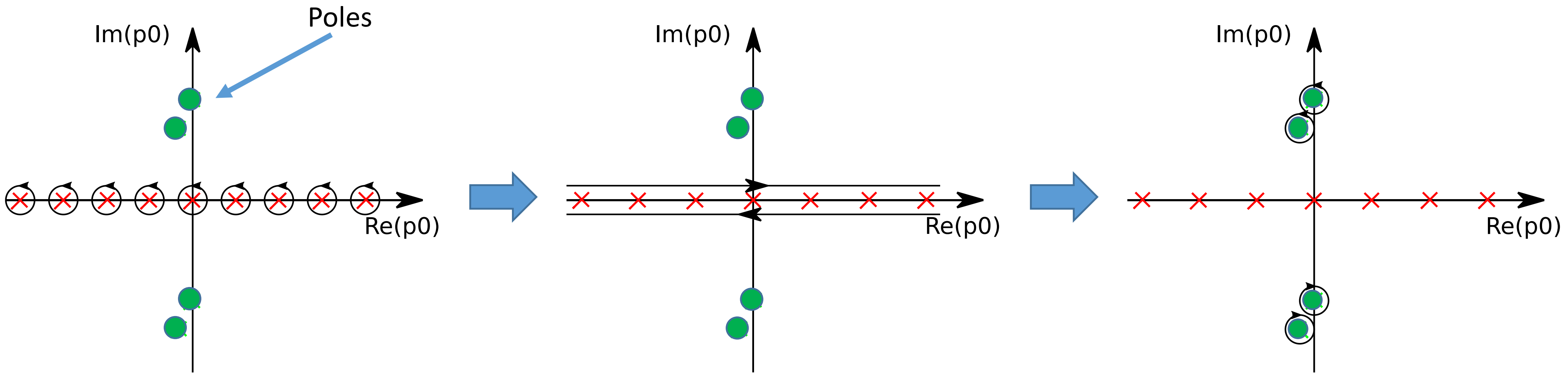
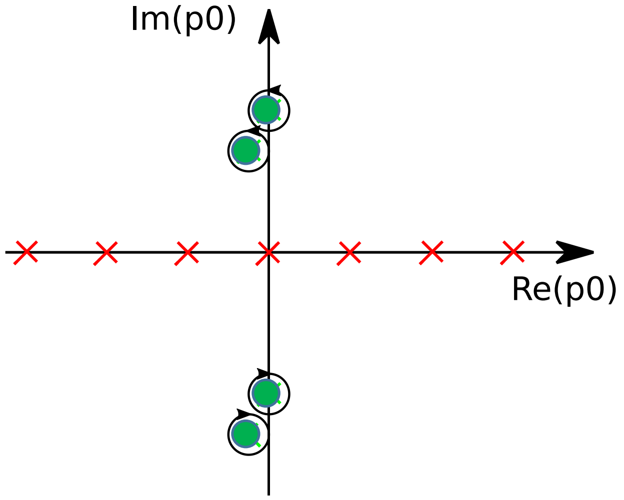


Illustrative example

$$\frac{1}{i}\sum_{\pm}\left(\mathrm{Res}_1^{\pm}\cdot[1+2n_B(-ip_0+\epsilon^1_{q+p})]+\mathrm{Res}_2^{\pm}\cdot[1+2n_B(\epsilon^2_q)]\right)$$



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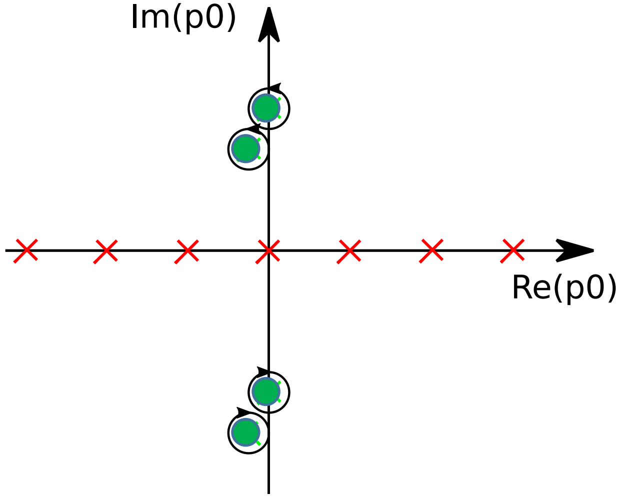

$$p_0 = 2m\pi T \quad m\in\mathbb{Z}$$

Identify ambiguity of the analytic continuation

Illustrative example

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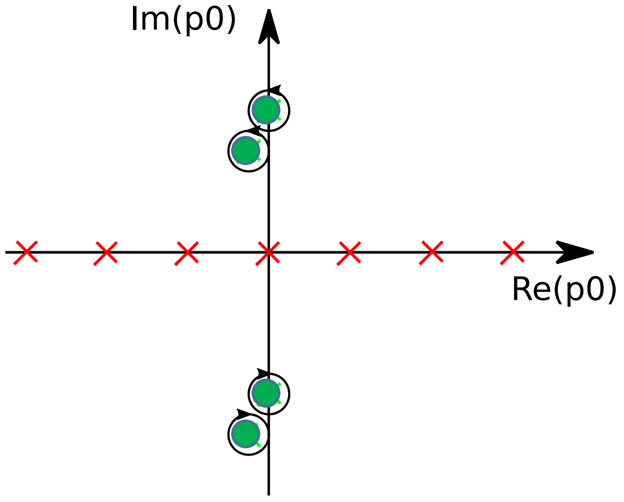

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$$e^{\mathrm{i}p_0} = 1$$

Illustrative example

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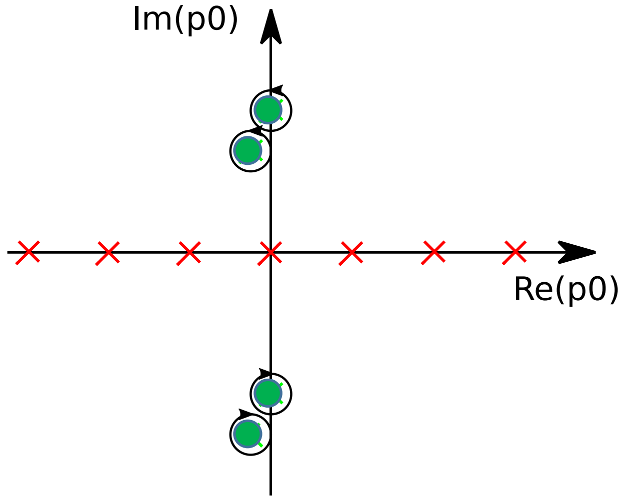

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Mathematically rigorous

Baym and Mermin, Journal of Mathematical Physics 2, 232 (1961)

Evans, Nucl.Phys. B374 (1992)

Analyticity off the imaginary axis

Correct decay behaviour at infinity

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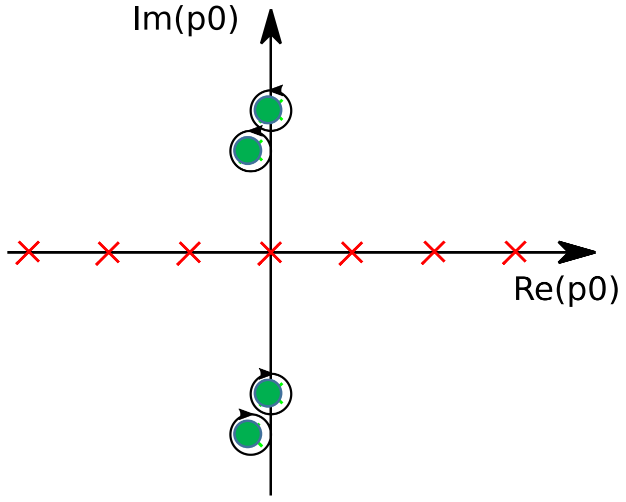

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Unique physical analytic continuation identified by setting $e^{ip_0} = \pm 1$ everywhere

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- Corresponds to a contour deformation at vanishing temperature

Strodthoff, PRD 95 (2017) no.7, 076002
Pawlowski, Strodthoff, NW, arxiv:1711.07444

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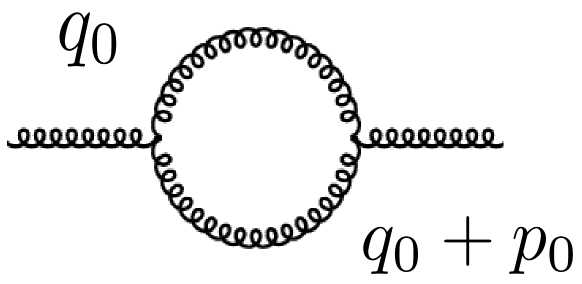

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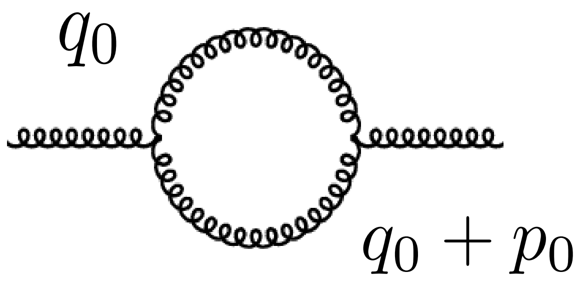

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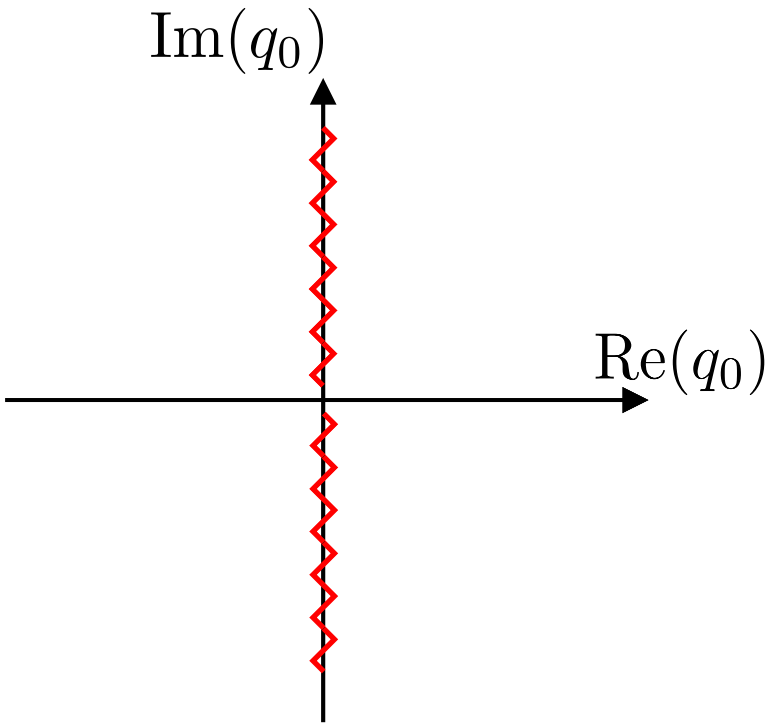

Analytic structure of the gluon

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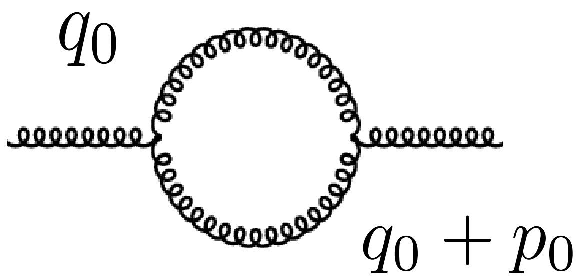

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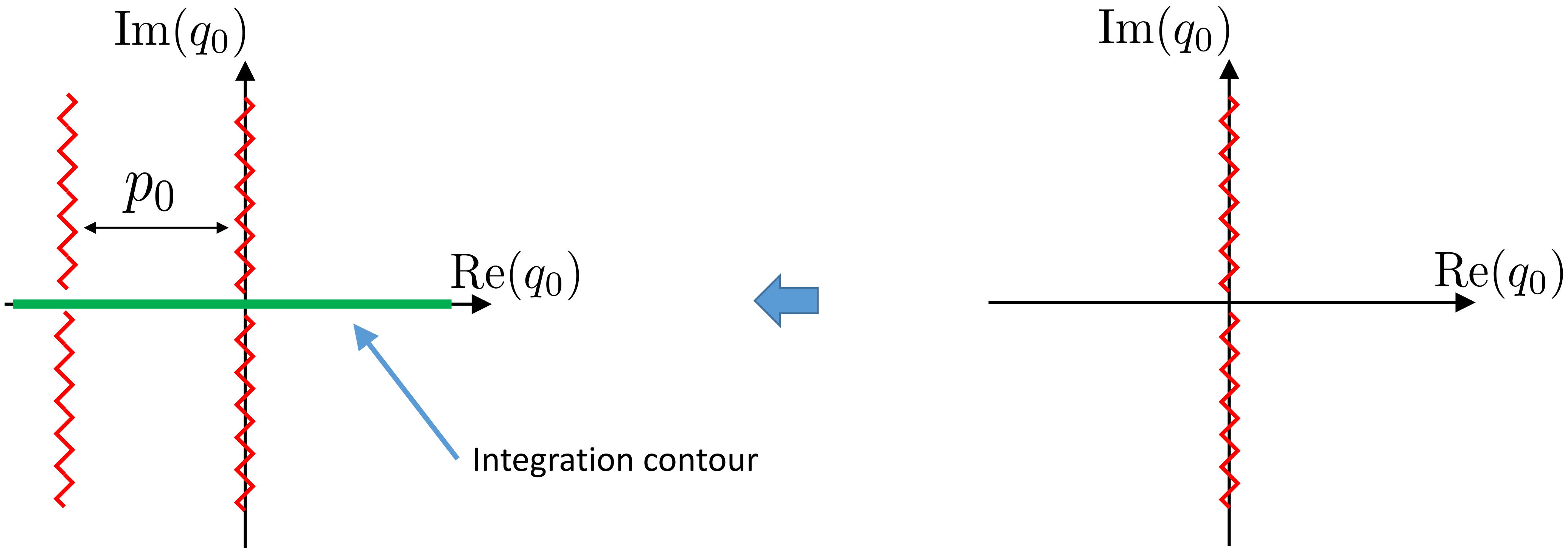

Analytic structure gluon polarization diagram

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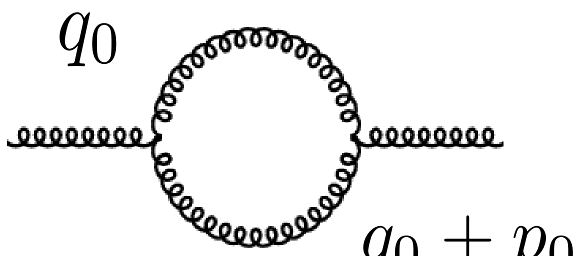

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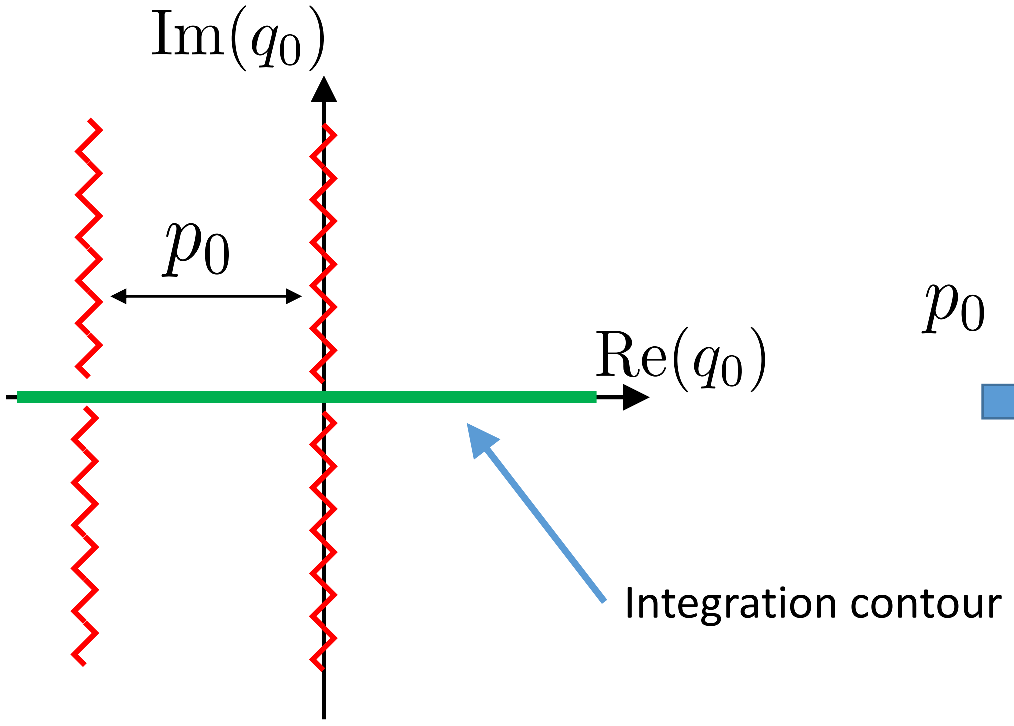
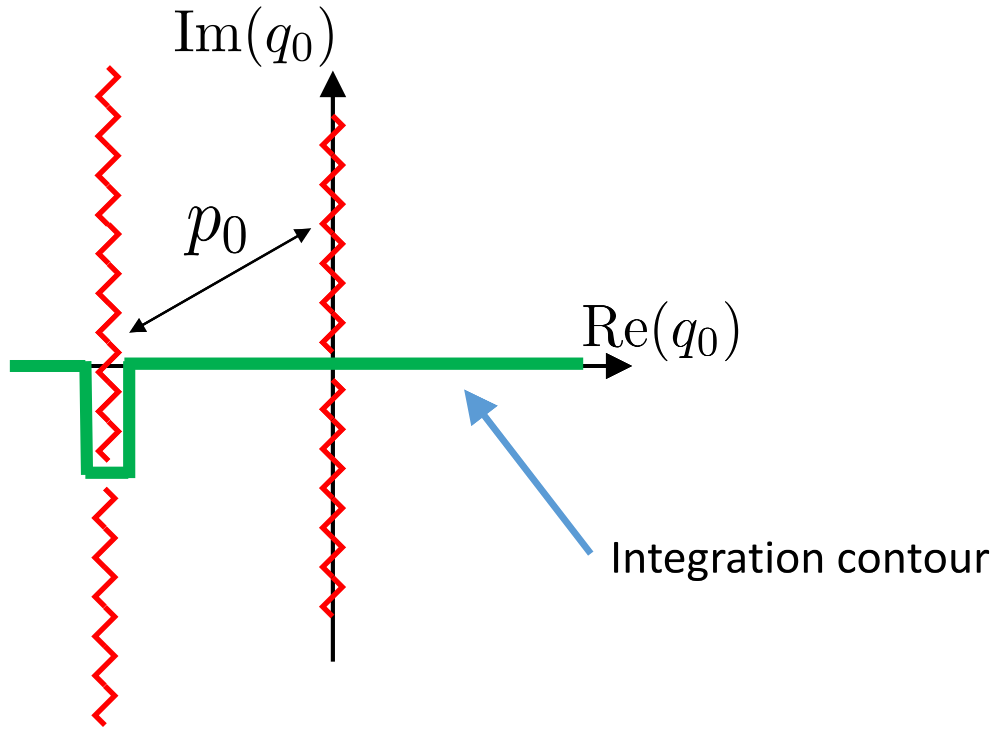


$p_0 \in \mathbb{C}$



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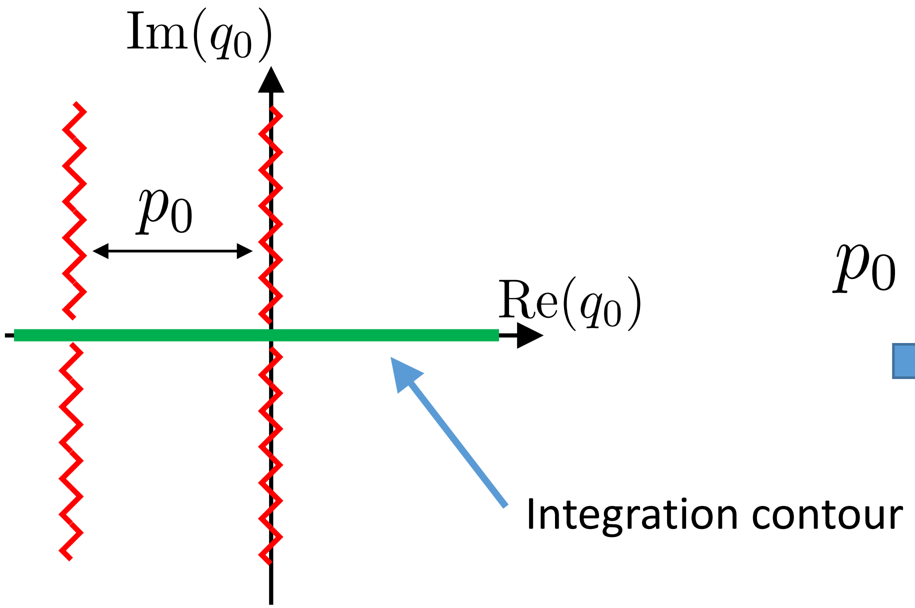

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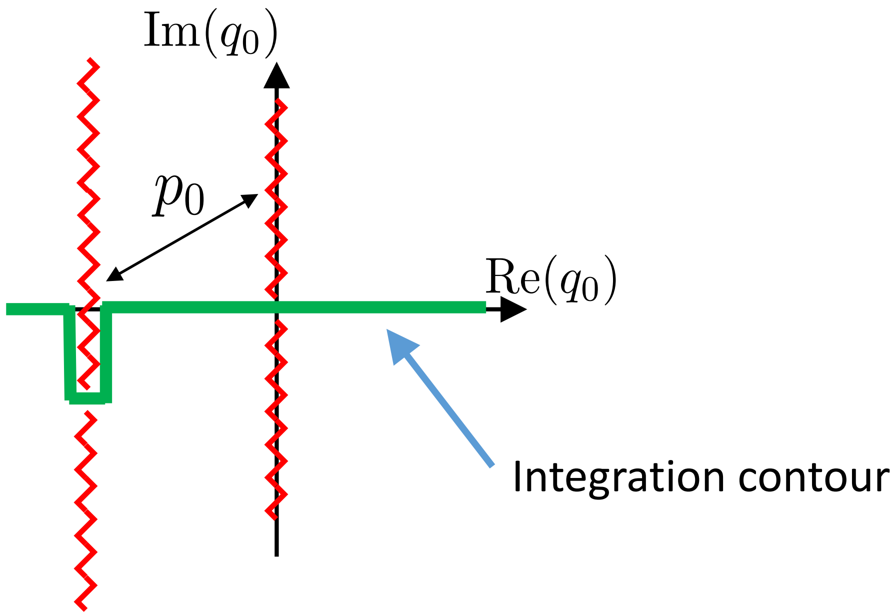

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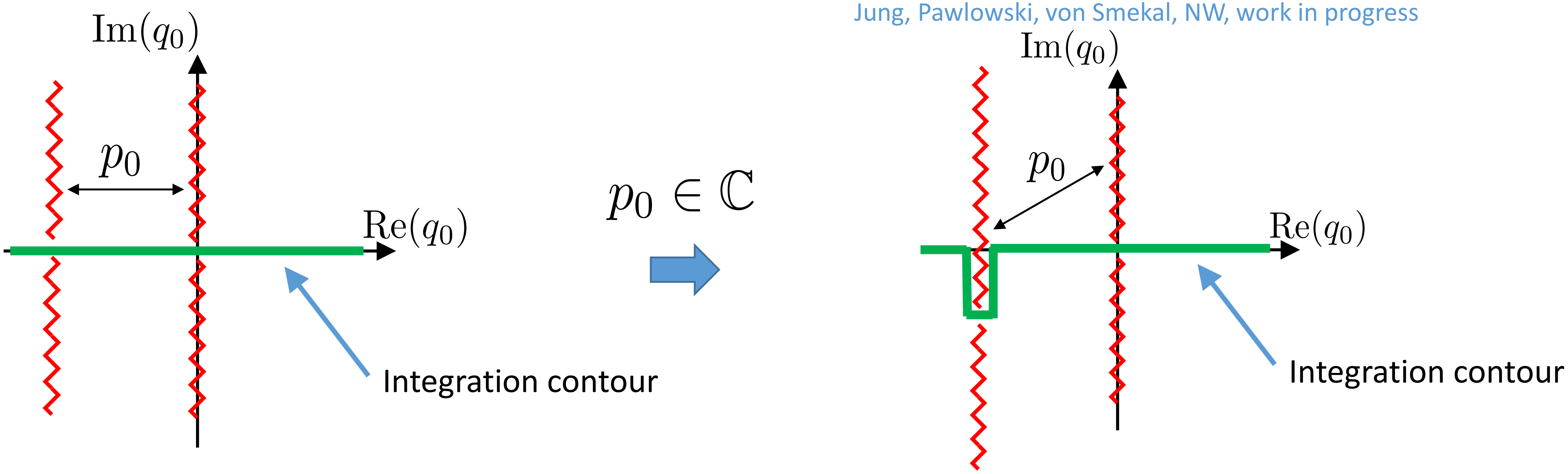
  Strodthoff, PRD 95 (2017) no.7, 076002
  Pawlowski, Strodthoff, NW, arxiv:1711.07444
- Considering poles is sufficient
- Branch cuts can be mapped to poles via spectral/integral representations
  Jung, Pawlowski, von Smekal, NW, work in progress



Remarks

$$G(p_0, \vec{p}) = \int_{\eta>0} 2\eta \frac{\rho(\eta, \vec{p})}{p_0^2 + \eta^2}$$

- Numerically accessible
- Corresponds to a contour deformation at vanishing temperature

  Strodthoff, PRD 95 (2017) no.7, 076002
  Pawlowski, Strodthoff, NW, arxiv:1711.07444
- Considering poles is sufficient
- Branch cuts can be mapped to poles via spectral/integral representations

  Jung, Pawlowski, von Smekal, NW, work in progress
- Generalization to the FRG → no new conceptual problems

Regulator poles

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Regulator poles

$R_k(\vec{q}^{\,2})$ → No changes

Kamikado, Strodthoff, von Smekal, Wambach, Eur.Phys.J. C74, 2806 (2014)
Tripolt, Strodthoff , von Smekal, Wambach, Phys.Rev. D89, 034010 (2014)

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$R_k(q^2)$ → (Lorentz invariant) Additional poles

Foerchinger, JHEP 1205 (2012) 021
Pawlowski, Strodthoff, Phys.Rev. D92 (2015)
Pawlowski, Strodthoff, NW arxiv:1711.07444

Direct calculation

Application to the O(N)-Model

Spectral functions of the O(N) model

Application to the O(N)-Model

Effective description of the lightest mesons

Spectral functions of the O(N) model

$$\rho(\omega, \vec{p}) = -2 \, \mathrm{Im} \, G_{\mathrm{R}}(\omega, \vec{p})$$

Application to the O(N)-Model

Effective description of the lightest mesons

Truncation:

$$\Gamma_k = \sum\!\!\!\!\!\!\!\int_{T,q} \Delta\Gamma_\sigma^{(2)} + \Delta\Gamma_\pi^{(2)} + V(\sigma)$$

Vacuum : $\Delta\Gamma_x^{(2)} = \Gamma_x^{(2)}(q^2) - \Gamma_x^{(2)}(0)$

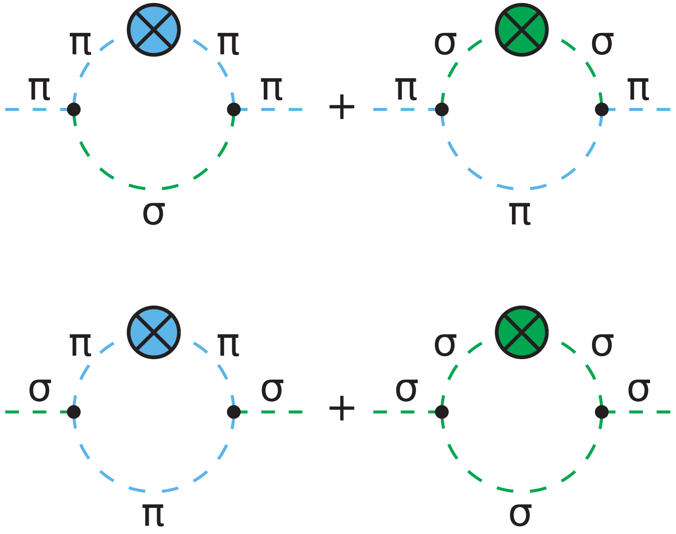
Finite Temperature : $\Delta\Gamma_x^{(2)} = Z_x q^2$

Spectral functions of the O(N) model

$$\rho(\omega, \vec{p}) = -2 \ \mathrm{Im} \ G_\mathrm{R}(\omega, \vec{p})$$

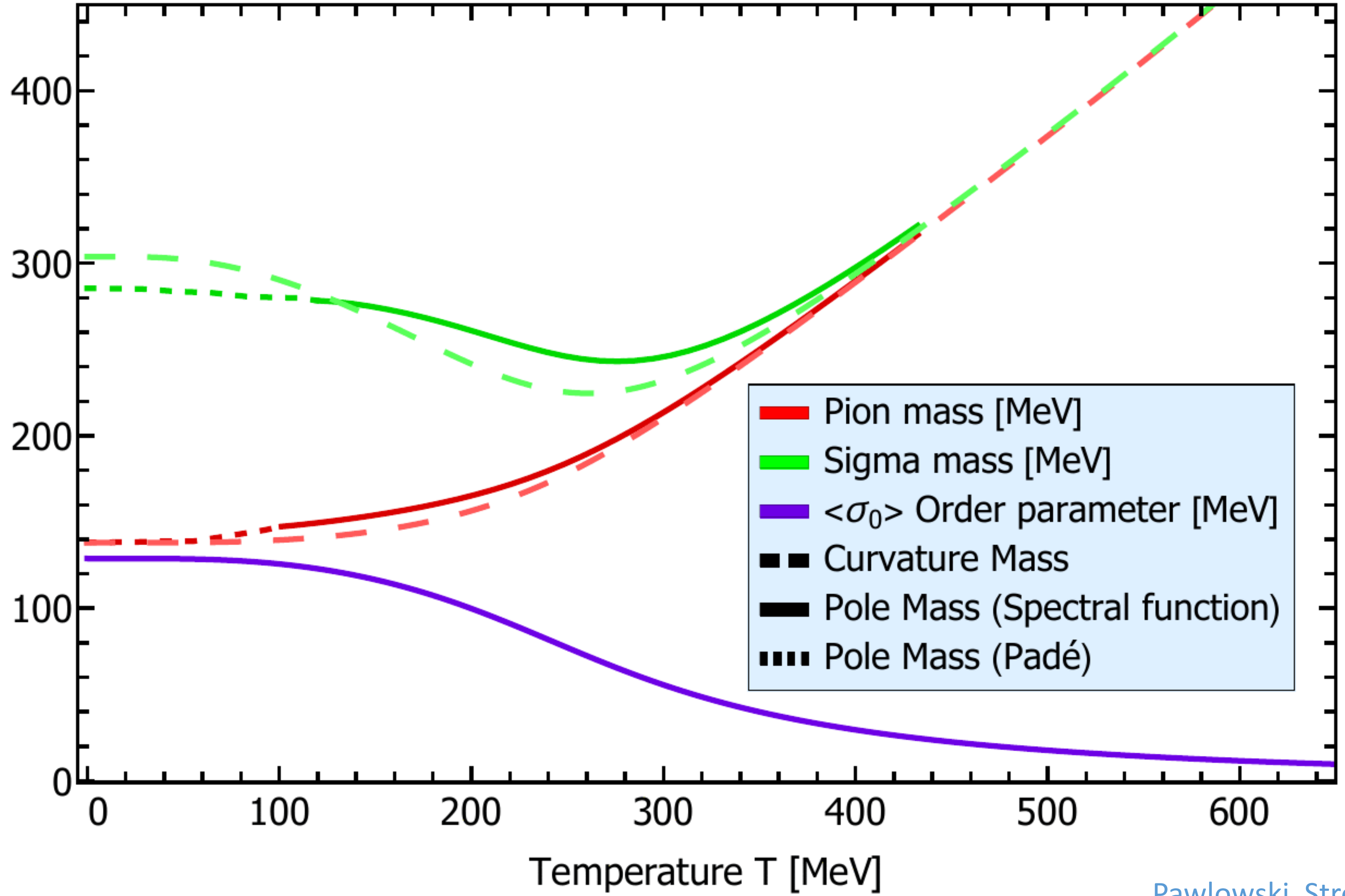
$\partial_k \Delta\Gamma_{\pi,k}^{(2)} =$

$\partial_k \Delta\Gamma_{\sigma,k}^{(2)} =$



Application to the O(N)-Model

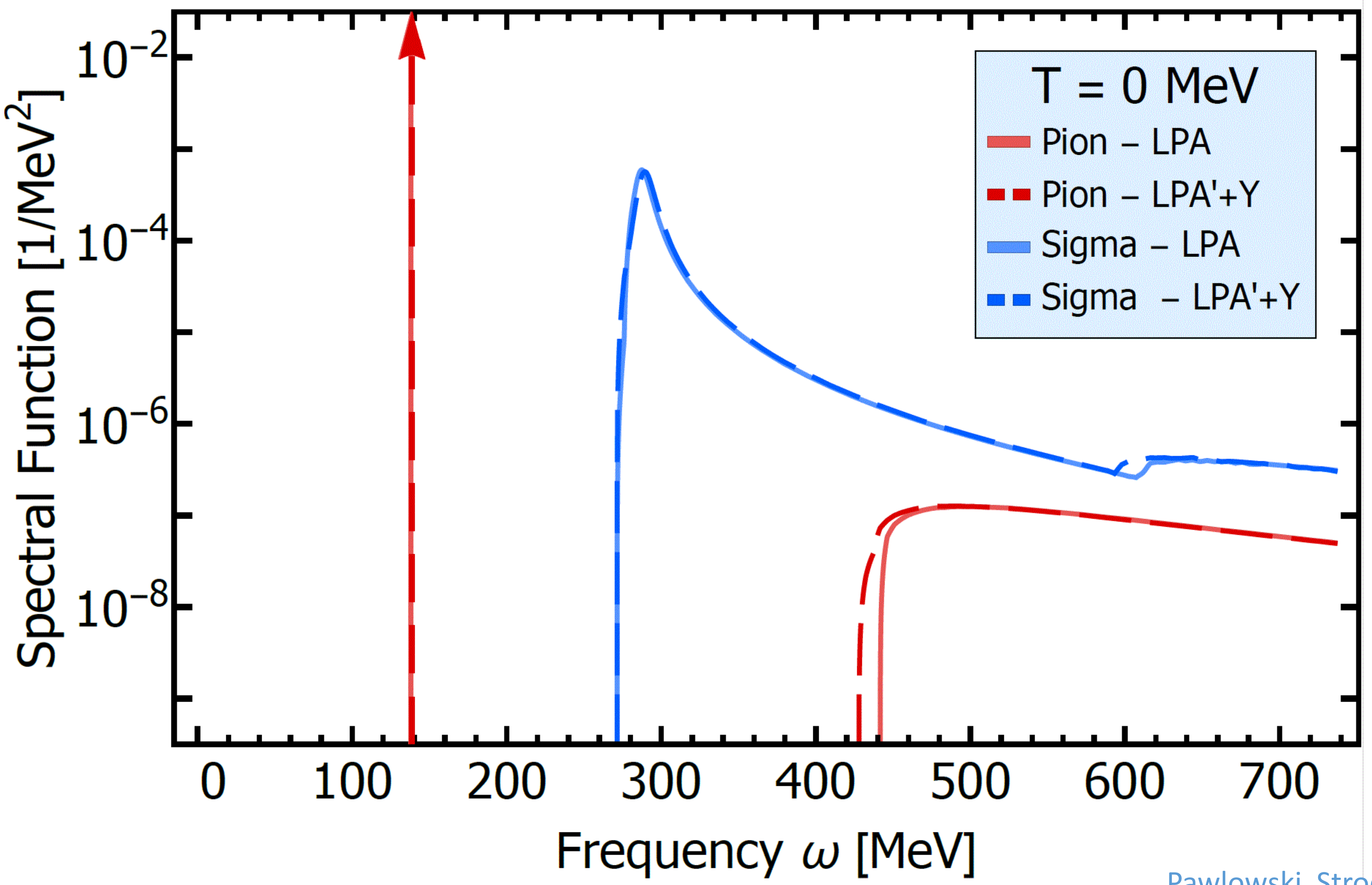
Phase structure



Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model

Finite temperature spectral functions



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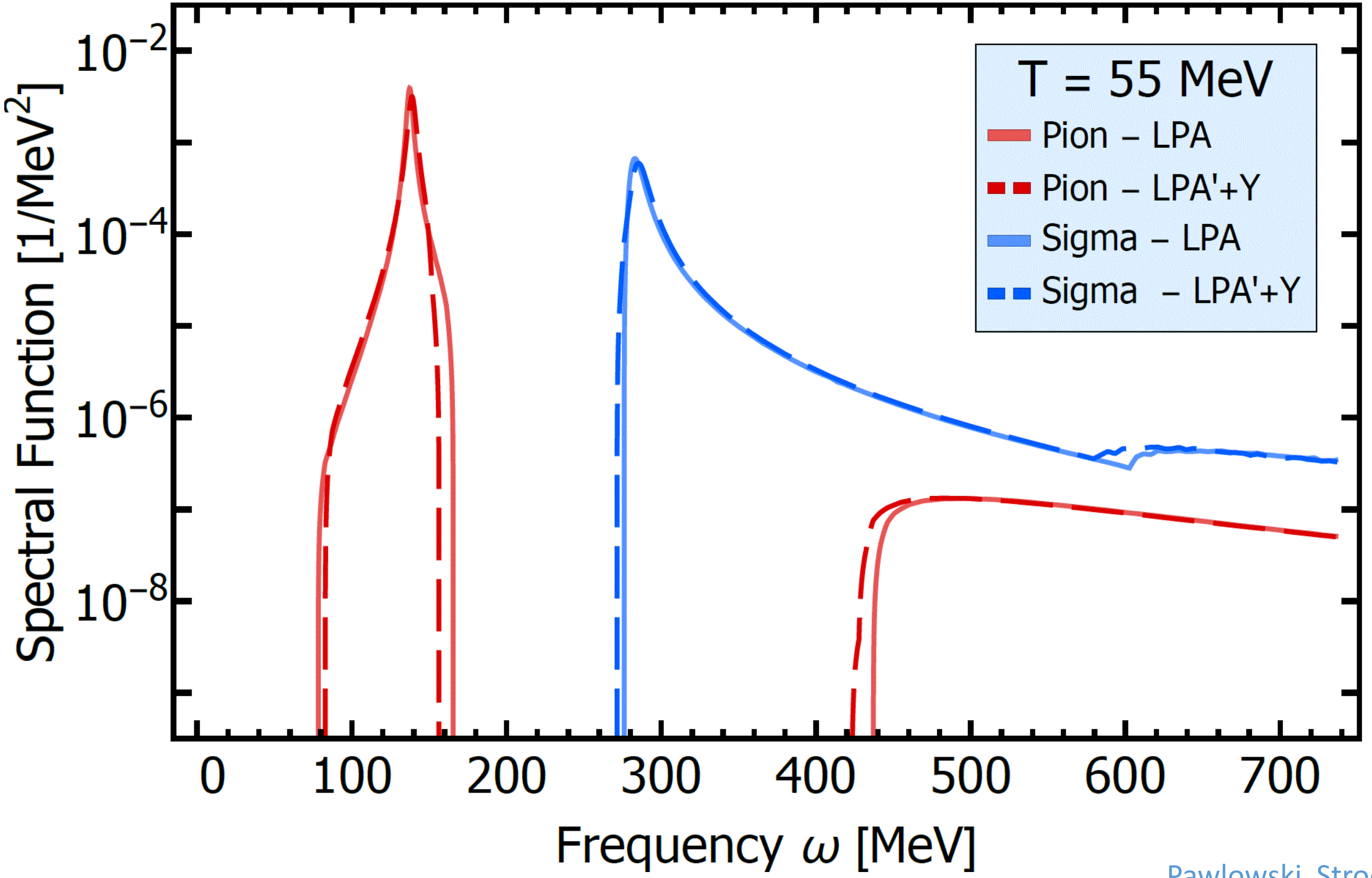

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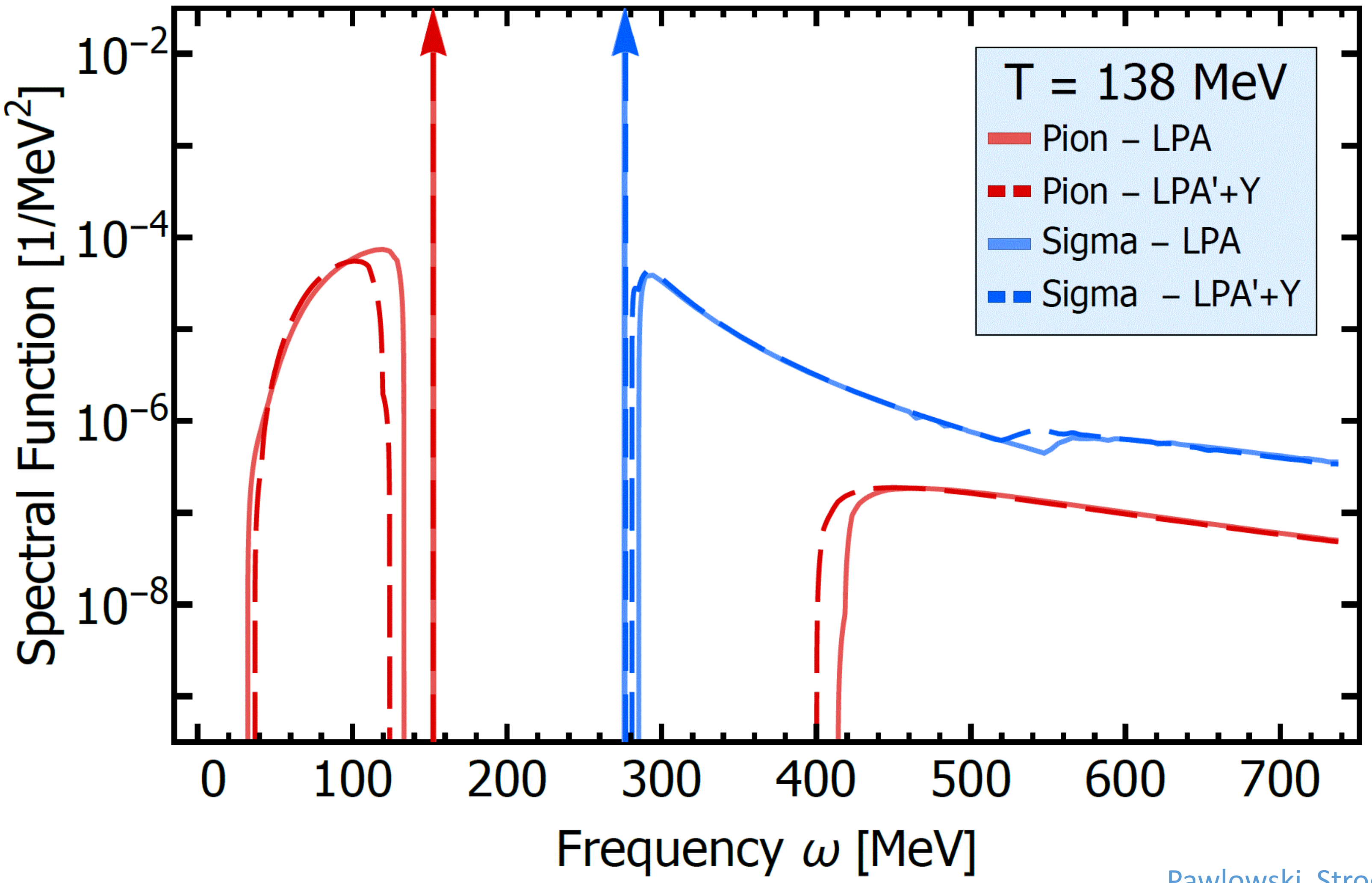

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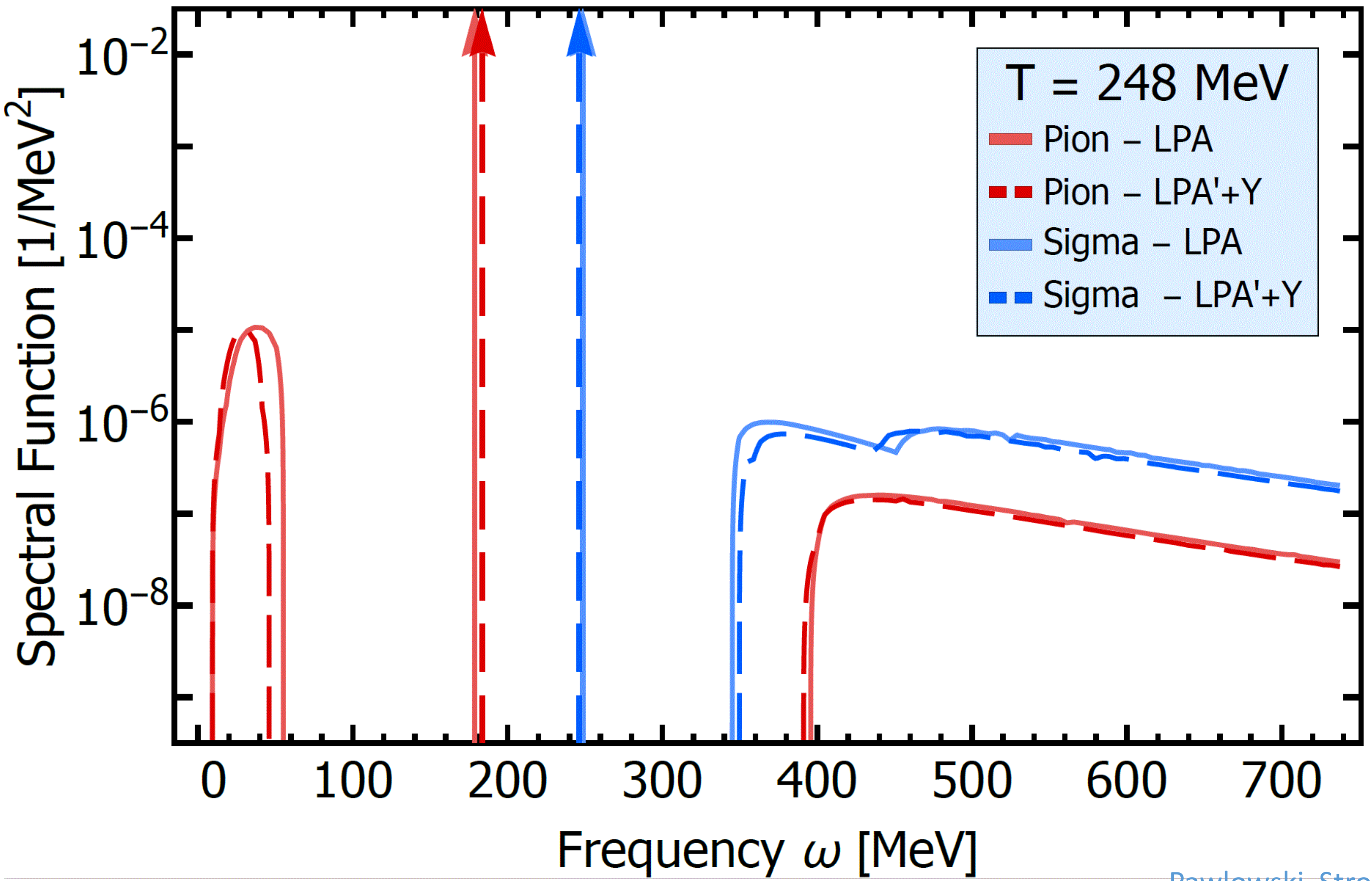

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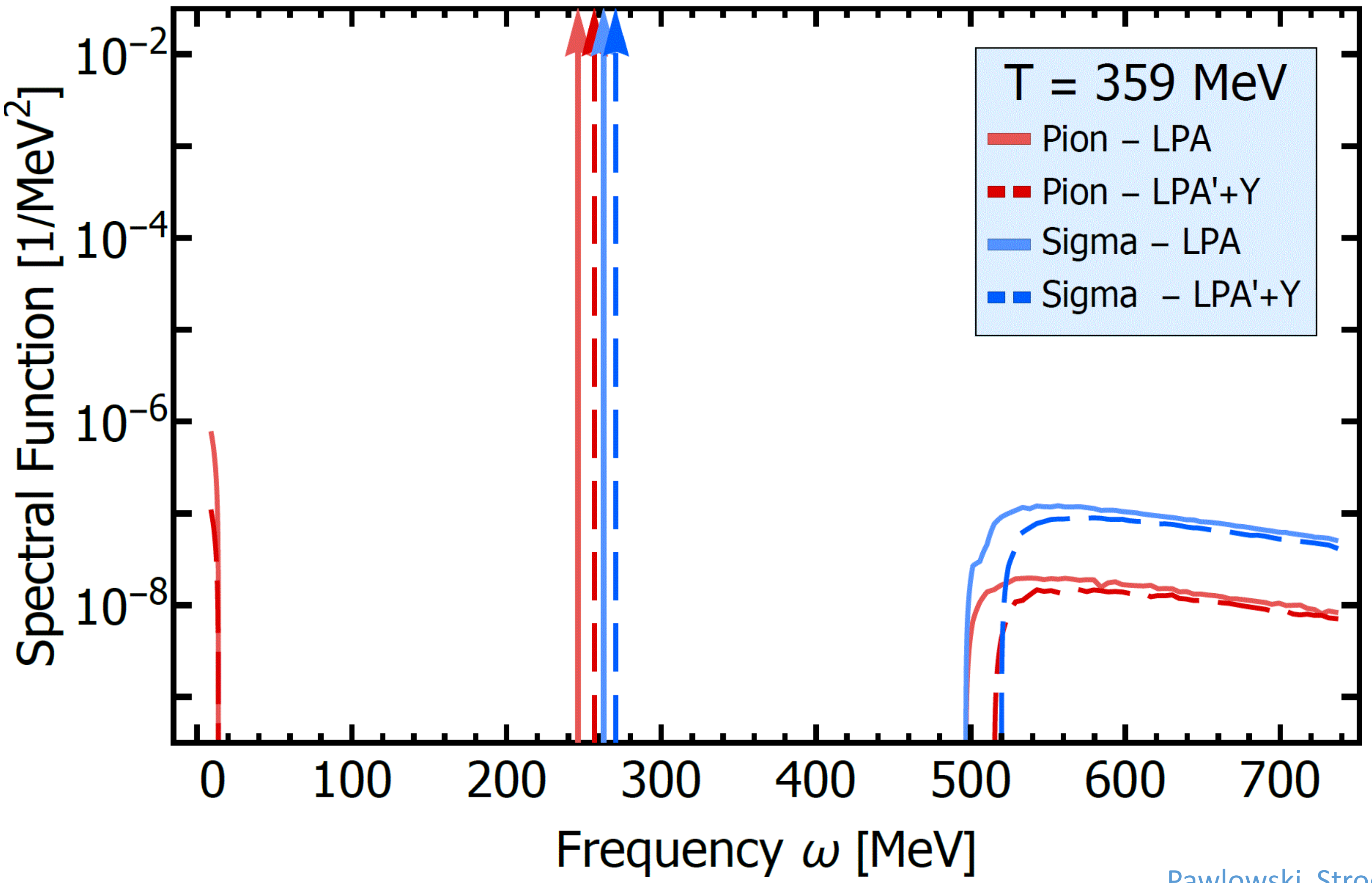
Finite temperature spectral functions

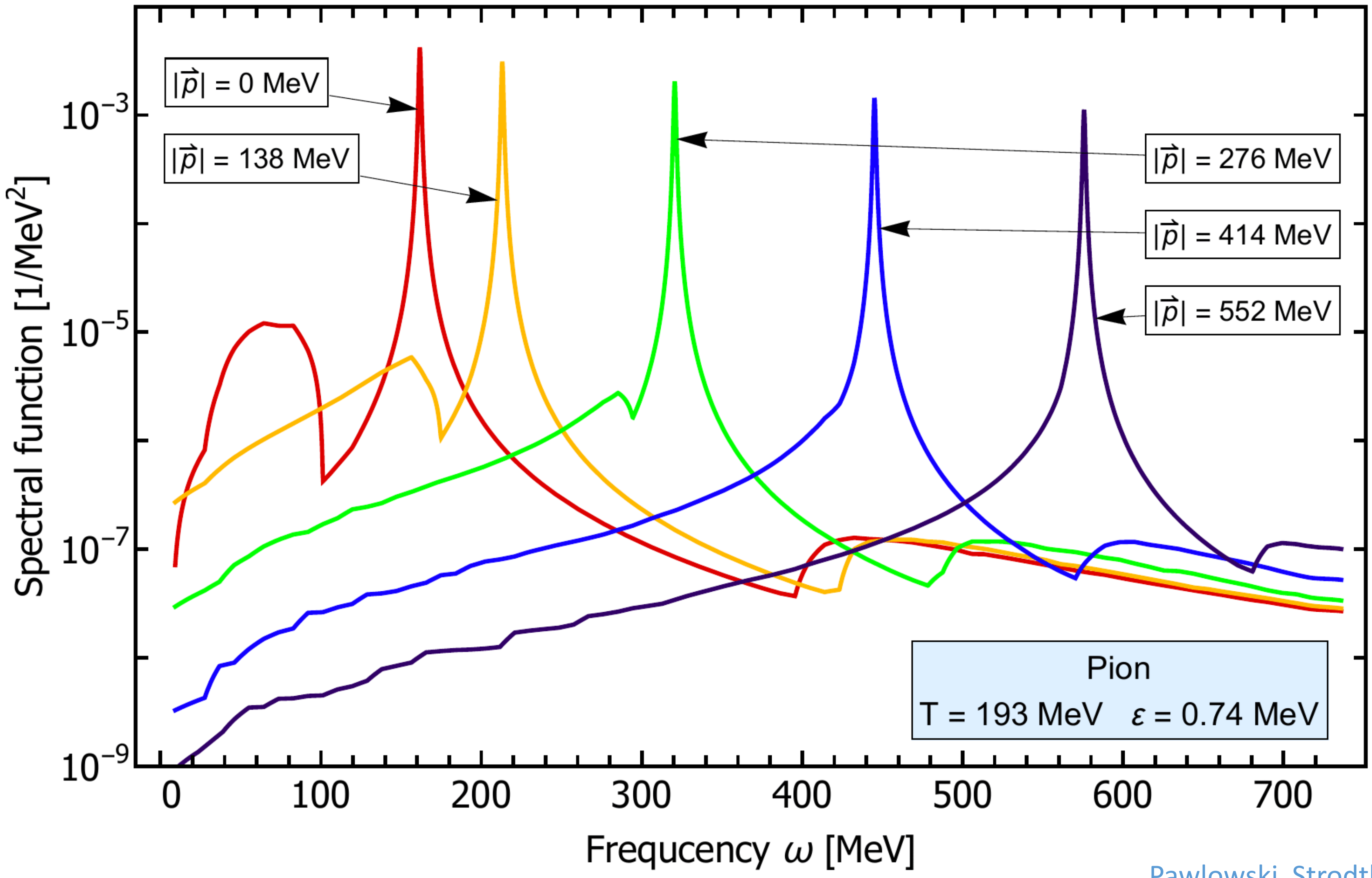


Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model

Pion meson

Finite temperature spectral function for various external momenta

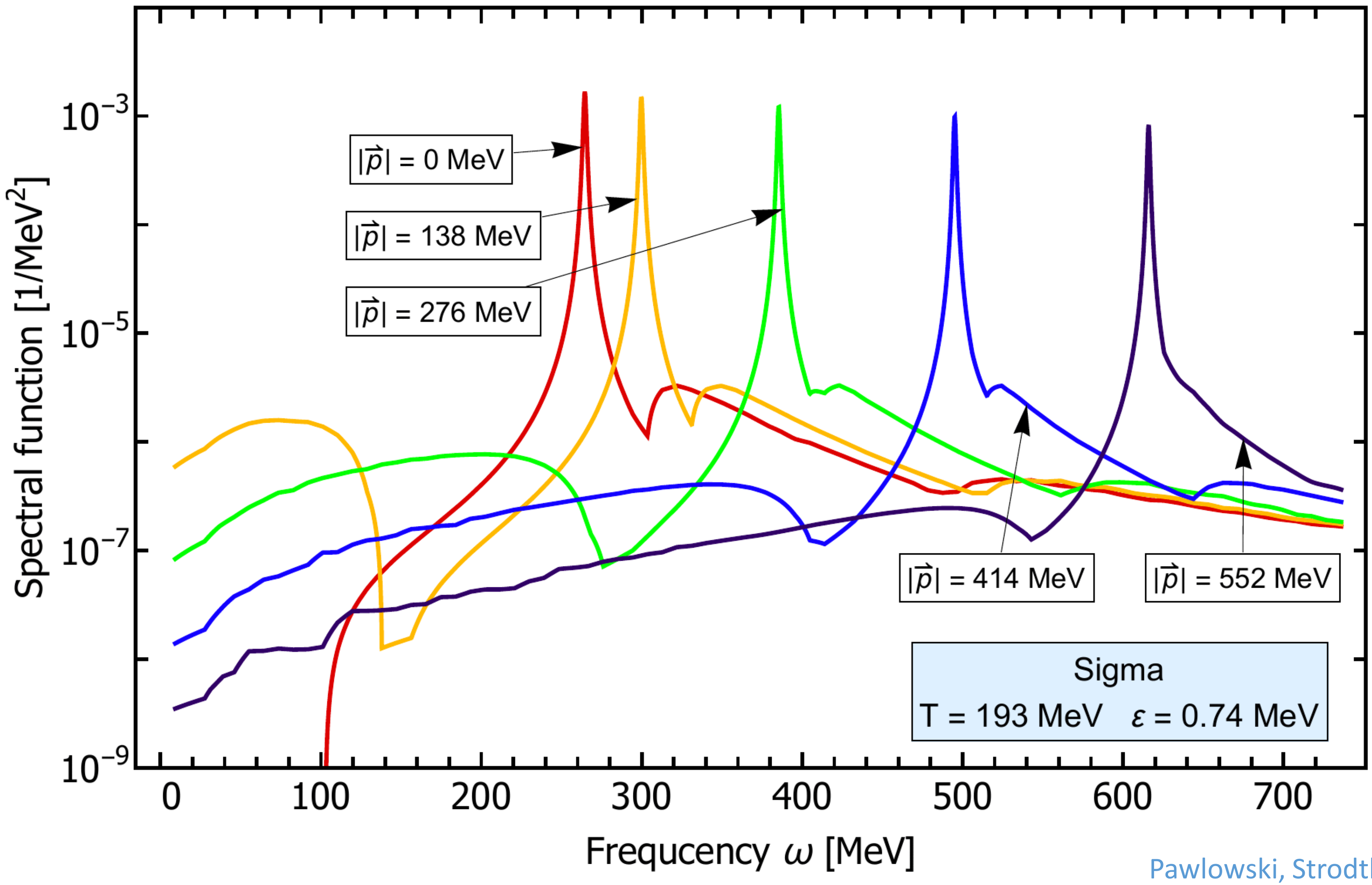


Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model

Sigma meson

Finite temperature spectral function for various external momenta



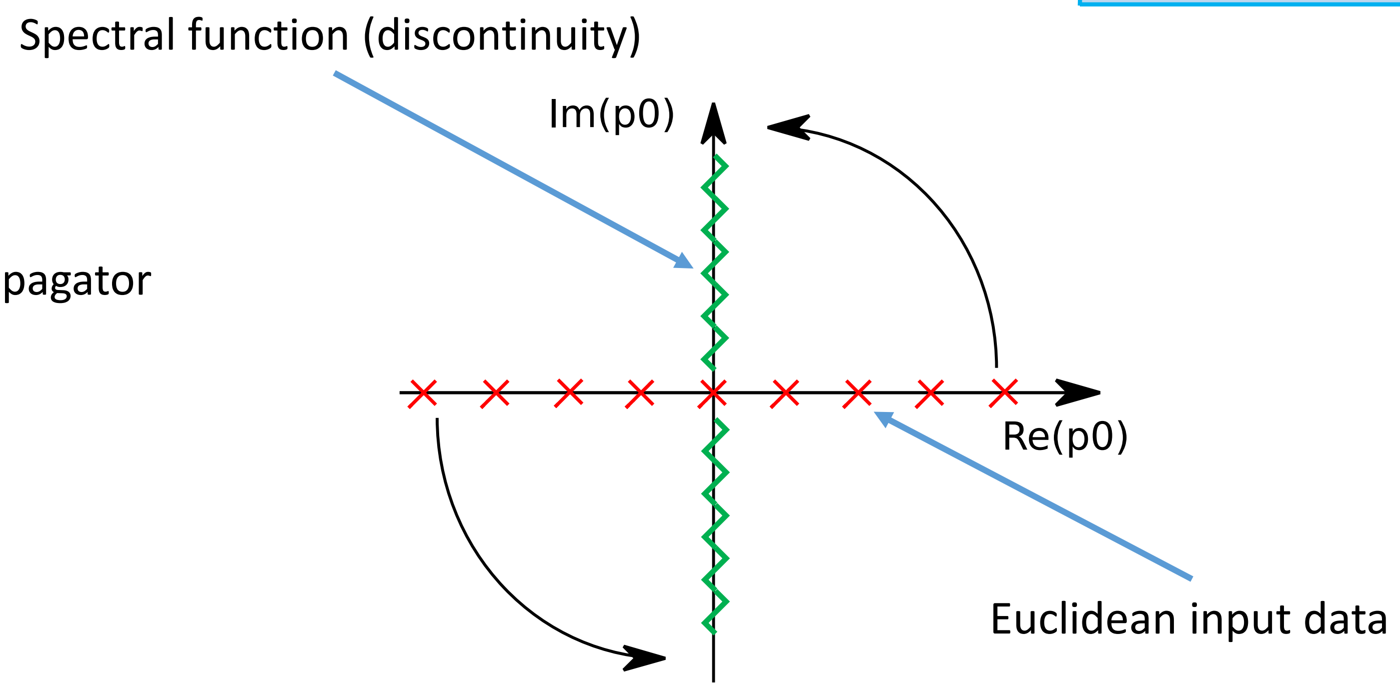
Pawlowski, Strodthoff, NW, arxiv:1711.07444

Reconstruction

Reconstructing spectral functions

Spectral function defined as the discontinuity of the propagator

$$\rho(\omega,\vec{p}) = 2 \lim_{\varepsilon\to 0} \operatorname{Im} \underbrace{G_{\mathrm{E}}(-\mathrm{i}(\omega+\mathrm{i}\varepsilon),\vec{p})}_{\text{retarded propagator}}$$



Propagator in the complex plane

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

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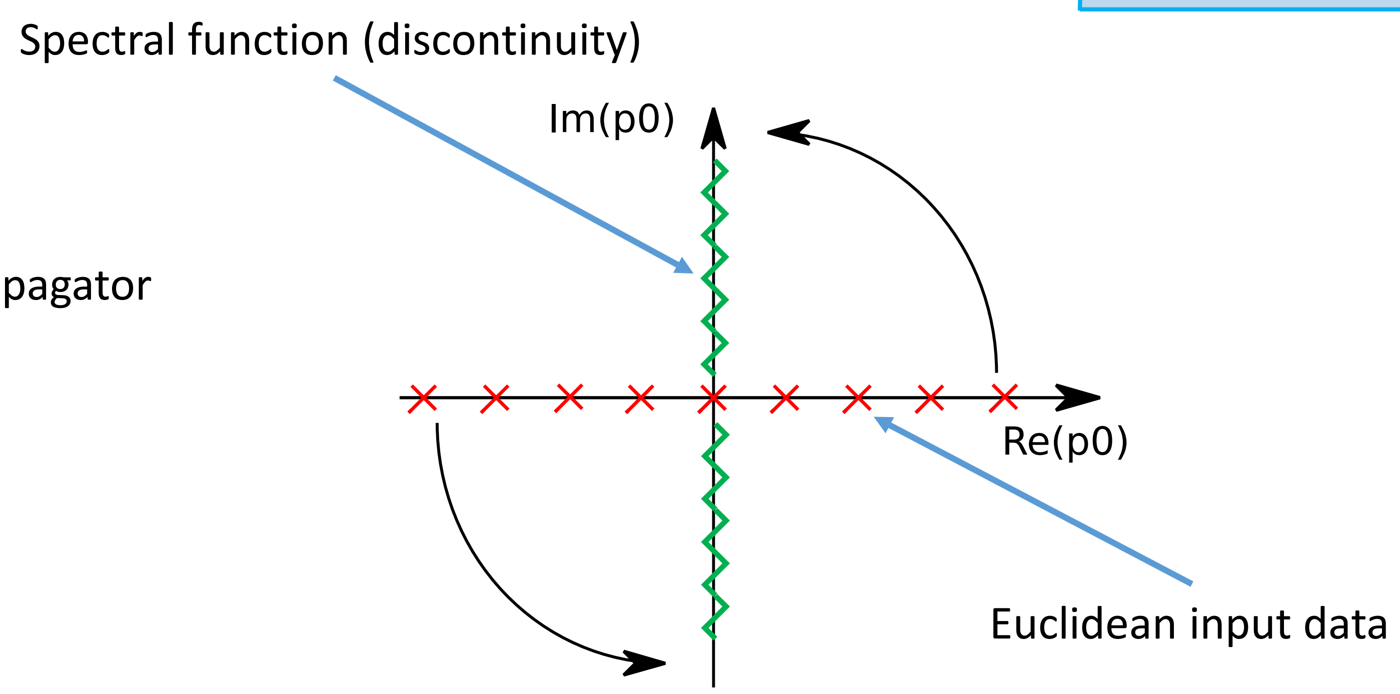
Gluon violates reflection-positivity

➡ Spectral function positive and negative



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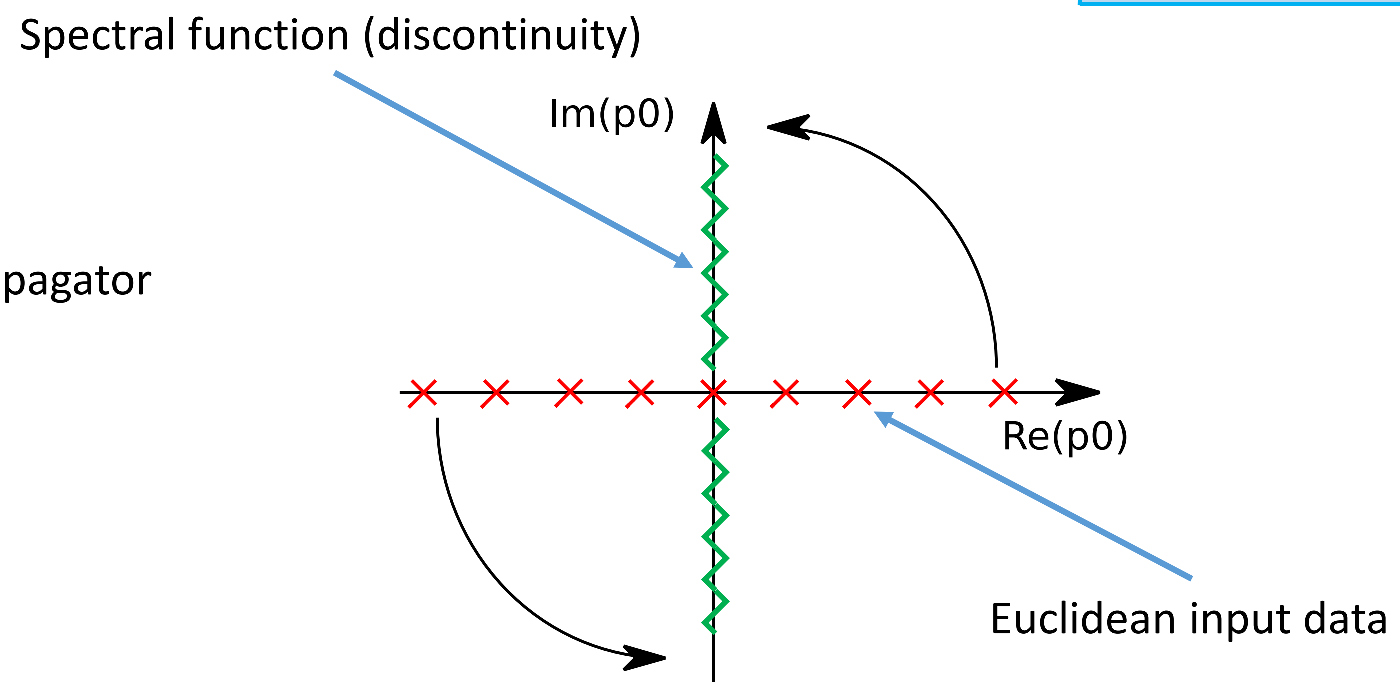
Use as much prior knowledge as possible!



Propagator in the complex plane

- Analytic properties of the gluon spectral function
- Existence of a spectral representation has strong implications on the complex structure

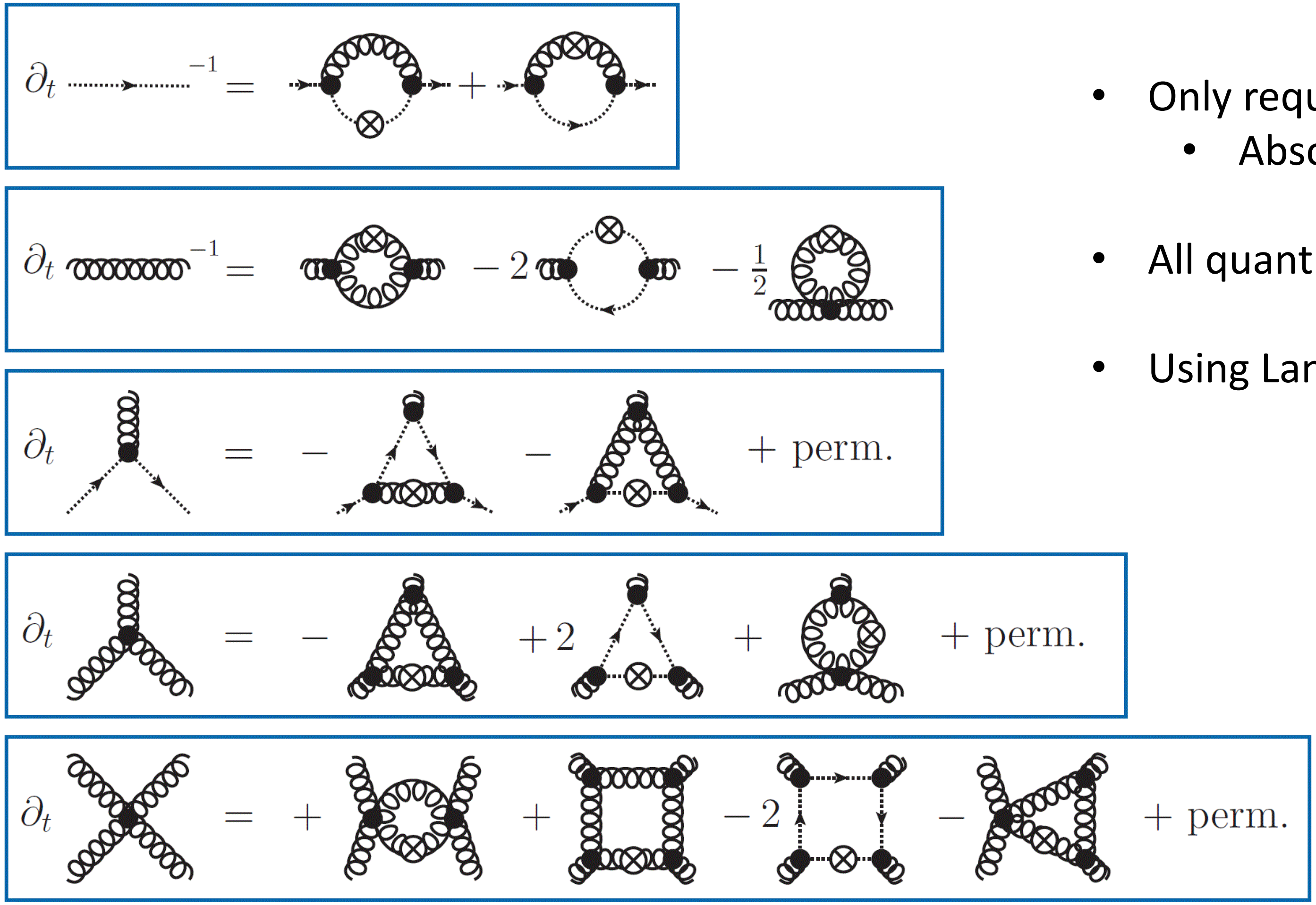
➡ Construct suitable functional basis

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

YM from the FRG

Aiming at apparent convergence

System of coupled equations



- Only requires coupling at a perturbative scale
  - Absorbed during scale setting
- All quantities are fully dressed and momentum dependent
- Using Landau gauge

cf. poster by Anton Cyrol

cf. talk by Mario Mitter

Vacuum Yang-Mills
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, Phys.Rev. D94 (2016)

Vacuum QCD
Cyrol, Mitter, Pawlowski, Strodthoff, arXiv:1706.06326

Finite temperature Yang-Mills
Cyrol, Mitter, Pawlowski, Strodthoff, arXiv:1708.03482

Finite temperature QCD, Extended truncations,…
Cyrol, Mitter, Pawlowski, NW, work in progress

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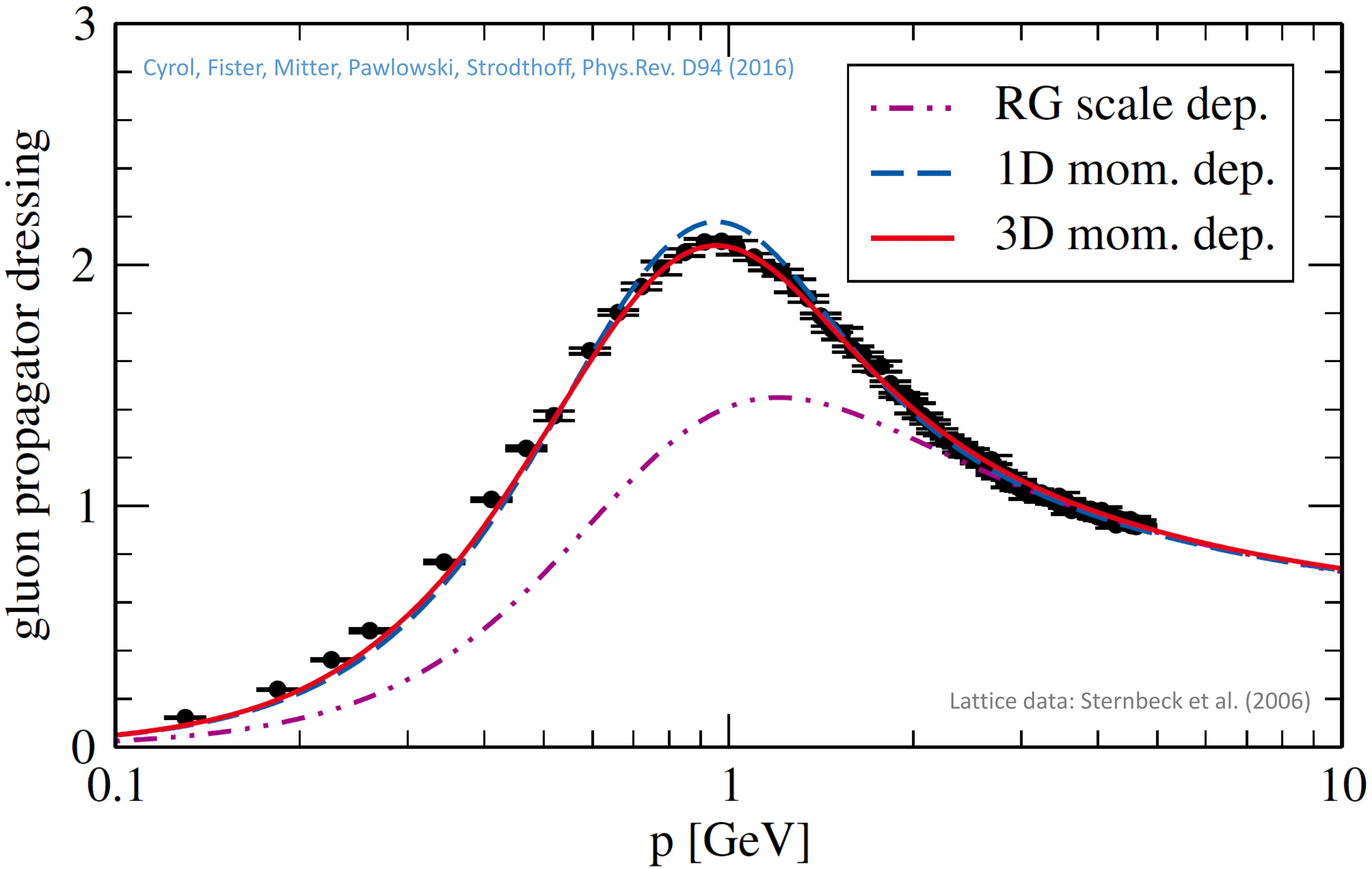


- Systematic improvement possible
- High numerical accuracy possible
- Direct computation of spectral functions possible in the near future

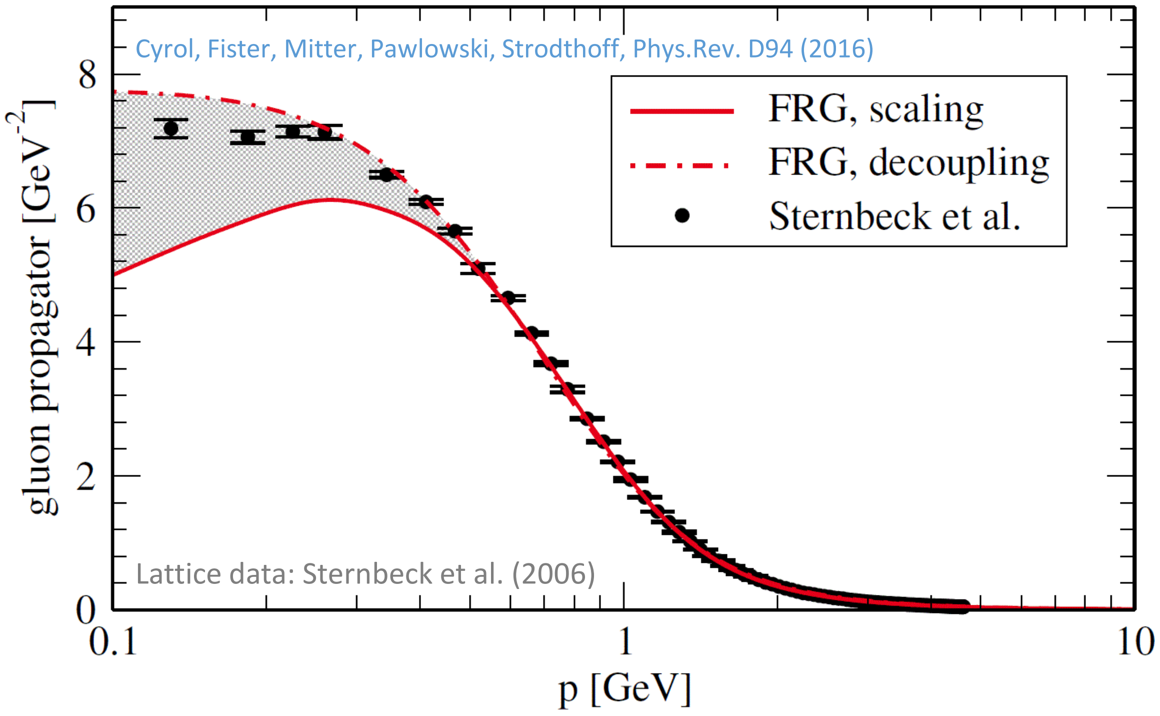
→ Suitable for reconstruction methods

cf. poster by Anton Cyrol

cf. talk by Mario Mitter

Reconstructing spectral functions

Use as much prior knowledge as possible!



Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

Reconstructing spectral functions

Use as much prior knowledge as possible!

1. Representation:

$$G_{\mathrm{E}}(p) = \int_{\mu>0} \frac{\mathrm{d}\mu}{2\pi} \frac{2\mu\,\rho(\mu)}{p^2+\mu^2}$$

2. Normalization: Super-convergence property

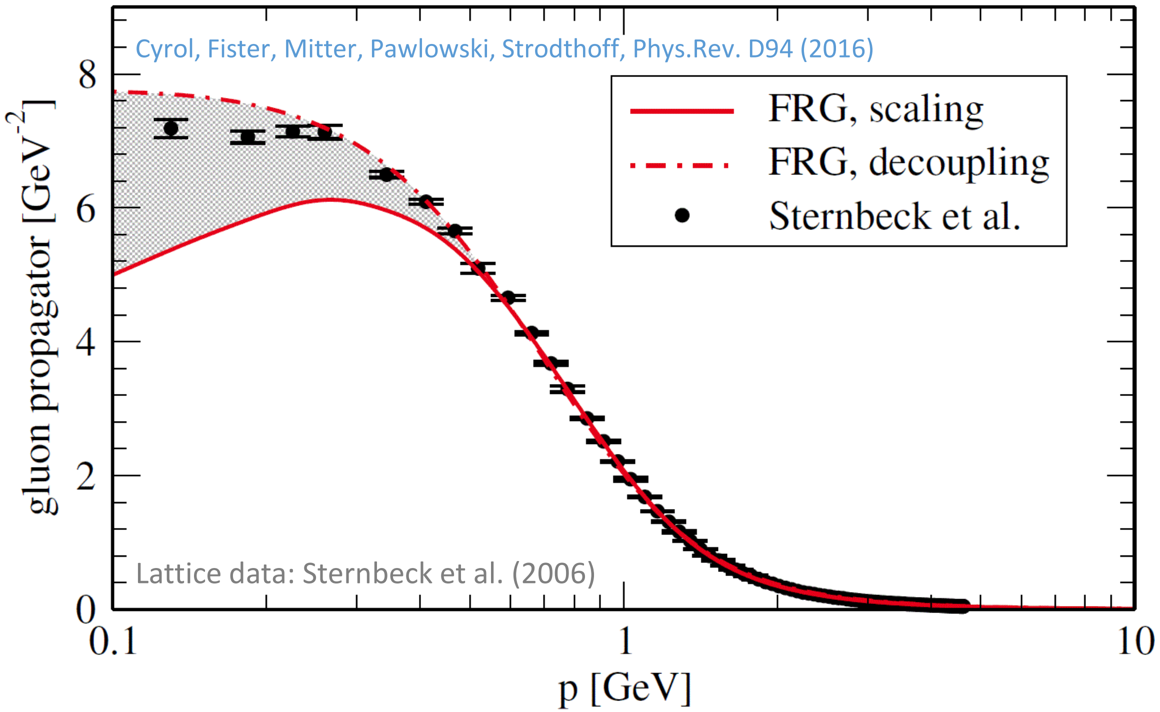
Oehme, Zimmermann, Phys.Rev. D21 (1980)

$$\int_{\mu>0} \mathrm{d}\mu\; \mu\,\rho(\mu) = 0$$

3. UV behavior: Perturbation theory

$$\rho(\omega) \sim -\frac{Z_{\mathrm{UV}}}{\omega^2 \ln(\omega^2)^{1+\gamma}}$$

4. IR behavior: new



Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

Reconstructing spectral functions

Infrared behavior of spectral functions

Start from

$$G(p_0) = \int_0^\infty \frac{\mathrm{d}\lambda}{\pi} \frac{\lambda\,\rho(\lambda)}{\lambda^2 + p_0^2}$$

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

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Derivative w.r.t. p_0 ⇒

$$\partial_{p_0} G(p_0) = -\int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{\pi} \lambda\, p_0 \frac{\rho(\lambda)}{(\lambda^2 + p_0^2)^2}$$

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

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Limit $p_0 \to 0$

$$\lim_{p_0 \to 0^+} \partial_{p_0} G(p_0) = -\frac{1}{2} \lim_{\omega \to 0^+} \partial_\omega \rho(\omega)$$

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

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Gluon:

Scaling solution to Yang-Mills

$$\hat{G}_{\mathrm{A}}^{(\mathrm{sca})}(p_0) \sim Z_{\mathrm{IR}} \left(\hat{p}_0^2\right)^{-1+2\kappa}$$

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

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Gluon spectral function is negative for small frequencies

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

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Similar result for decoupling scenarios, for more details see

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

Reconstructing spectral functions

Constructing a basis for the reconstruction:

→ Analytically continue the retarded propagator to the entire complex plane

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

Reconstructing spectral functions

$$\rho(\omega,\vec{p}) = 2 \lim_{\varepsilon\to 0} \operatorname{Im} G_{\mathrm{E}}(-\mathrm{i}(\omega+\mathrm{i}\varepsilon),\vec{p})$$

Constructing a basis for the reconstruction:

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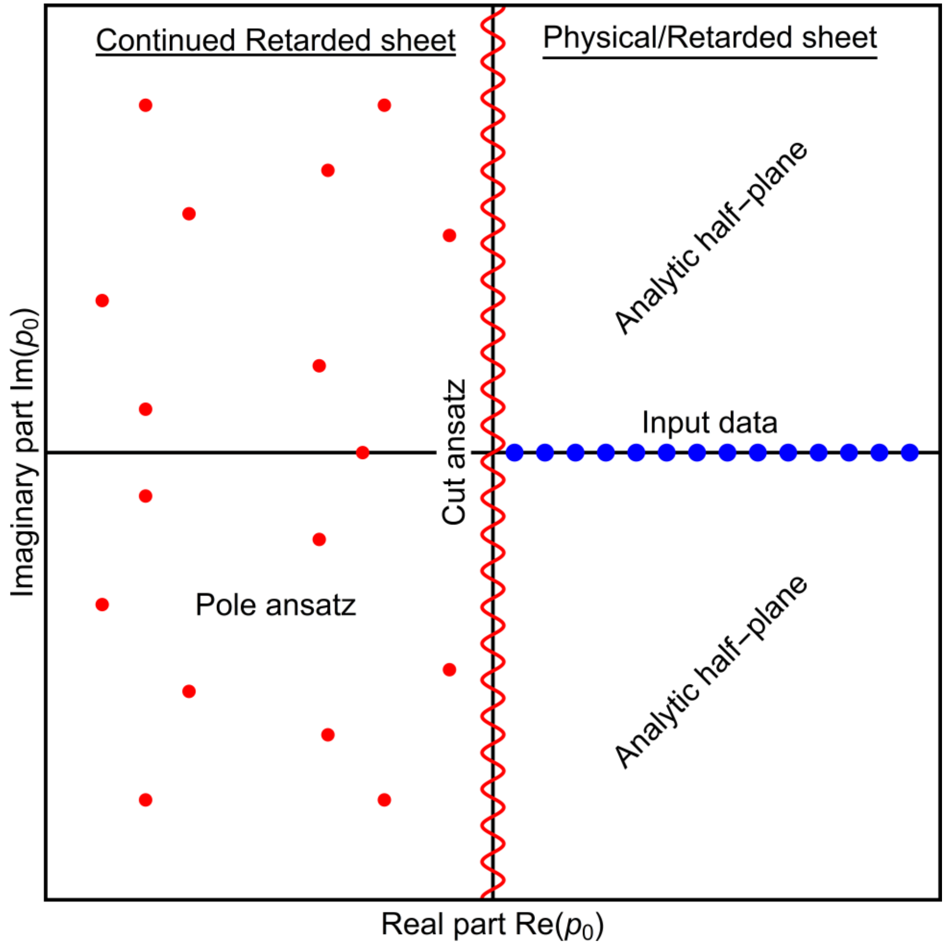

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

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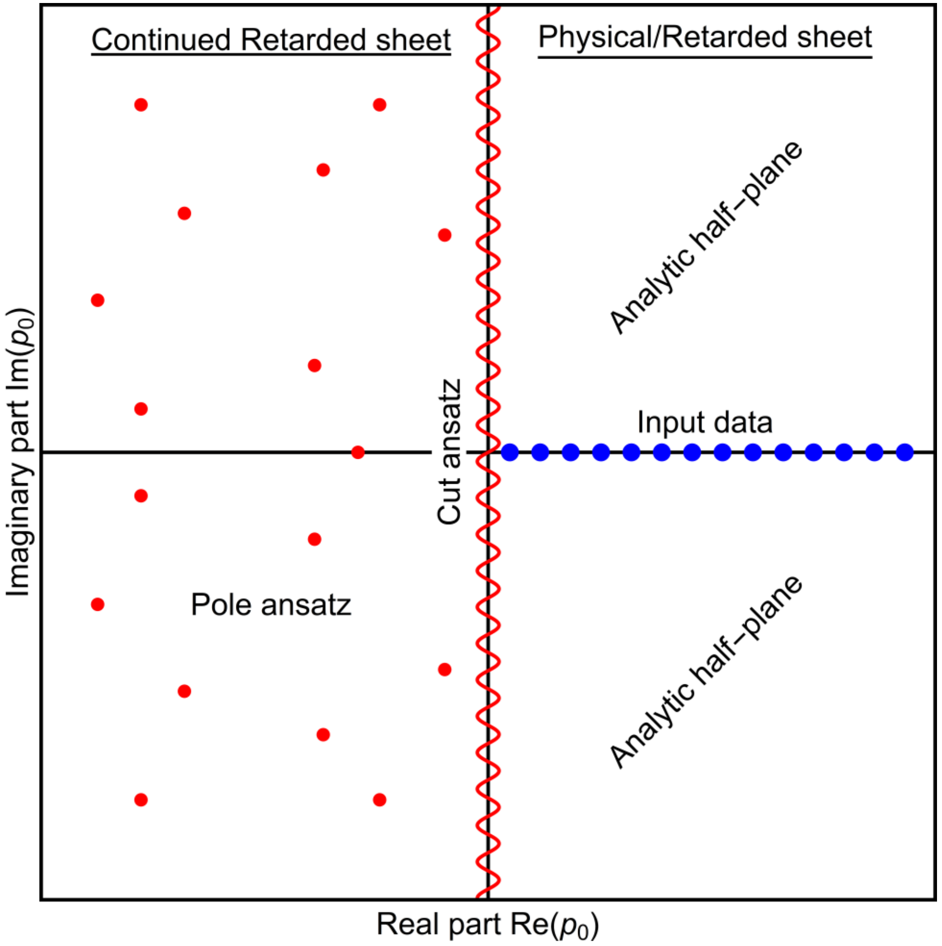
Constructing a basis for the reconstruction:

➡ Analytically continue the retarded propagator to the entire complex plane

Poles

$$\hat{G}^{\mathrm{pole}}_{\mathrm{Ans}}(p_0) = \sum_{k=1}^{N_{\mathrm{ps}}} \prod_{j=1}^{N^{(k)}_{\mathrm{pp}}} \left(\frac{\hat{\mathcal{N}}_k}{(\hat{p}_0 + \hat{\Gamma}_{k,j})^2 + \hat{M}^2_{k,j}} \right)^{\delta_{k,j}}$$



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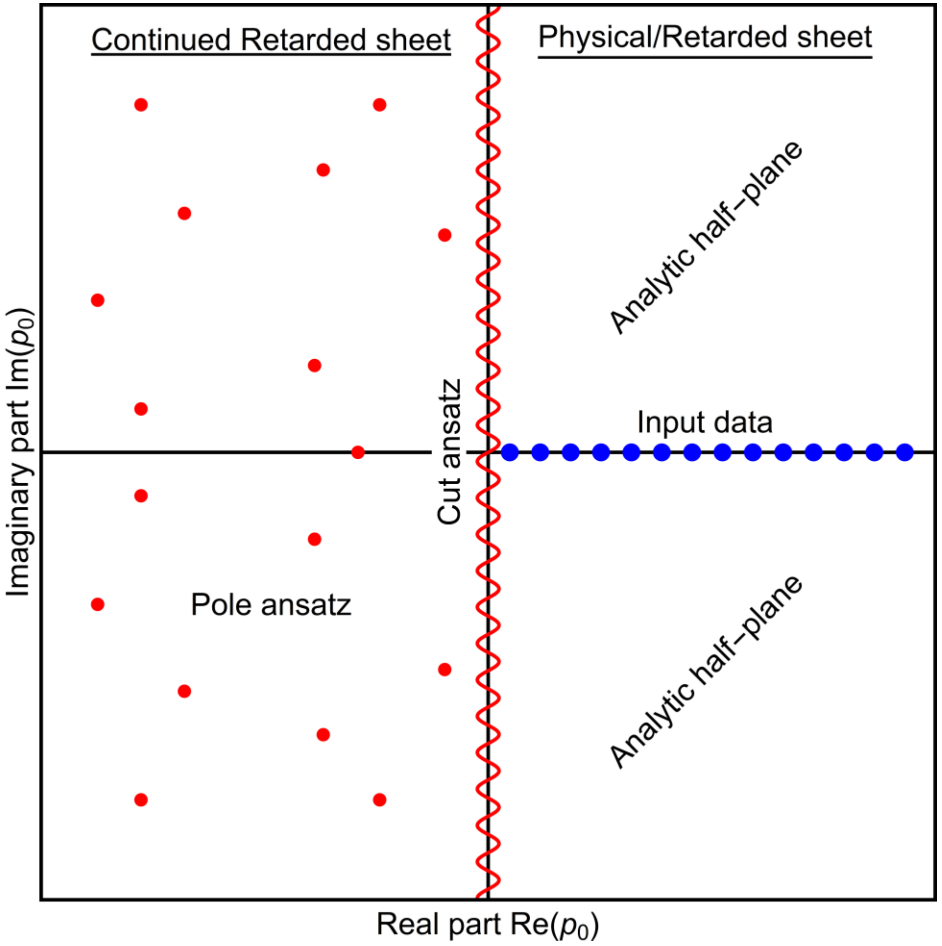
$$\hat{G}^{\mathrm{poly}}_{\mathrm{Ans}}(p_0) = \sum_{j=1}^{N_{\mathrm{poly}}} \hat{a}_k \left(\hat{p}_0^2\right)^{\frac{j}{2}}$$

Polynomial



Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

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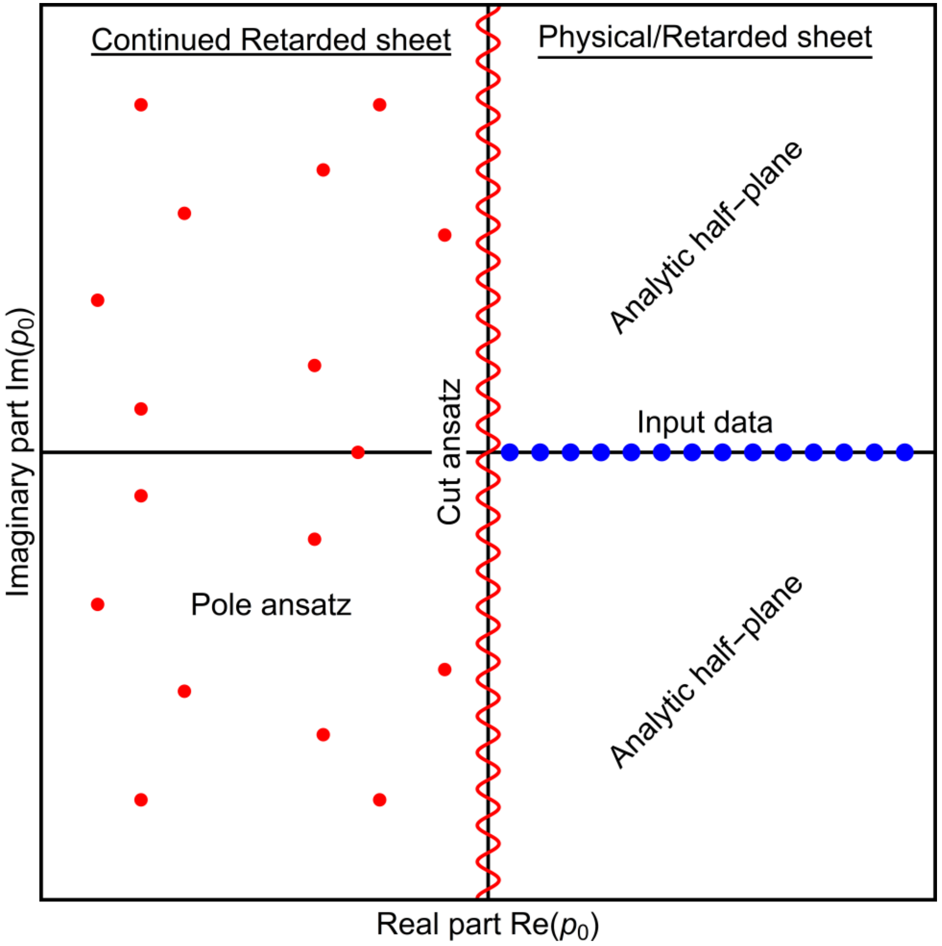
Polynomial

$$\hat{G}^{\text{asy}}_{\text{Ans}}(p_0) = (\hat{p}_0^2)^{-1-2\alpha} \left[\log\left(1 + \frac{\hat{p}_0^2}{\hat{\lambda}^2}\right)\right]^{-1-\beta}$$

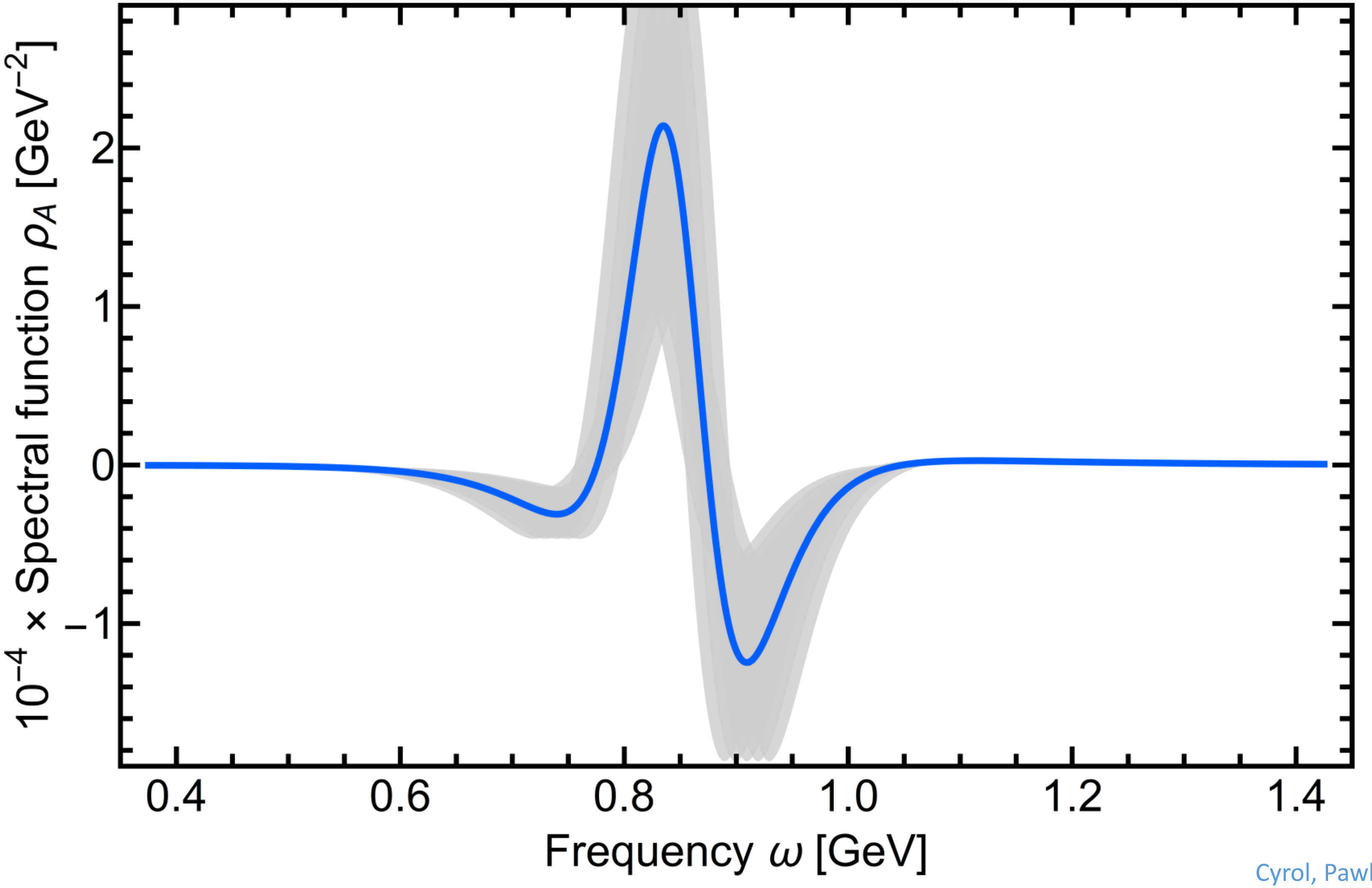
Cuts



Full ansatz $\quad G_{\text{Ans}}(p_0) = \mathcal{K}\,\hat{G}^{\text{pole}}_{\text{Ans}}(p_0)\,\hat{G}^{\text{poly}}_{\text{Ans}}(p_0)\,\hat{G}^{\text{asy}}_{\text{Ans}}(p_0)$

Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

Gluon spectral function



Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

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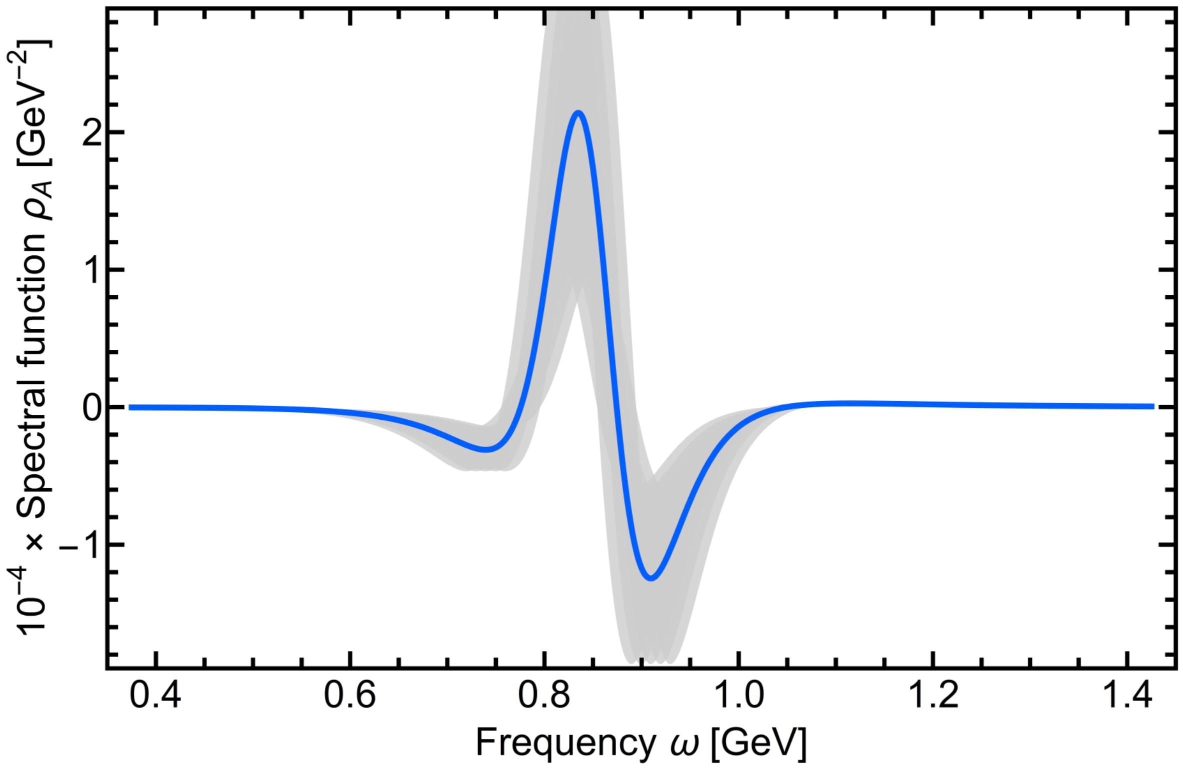
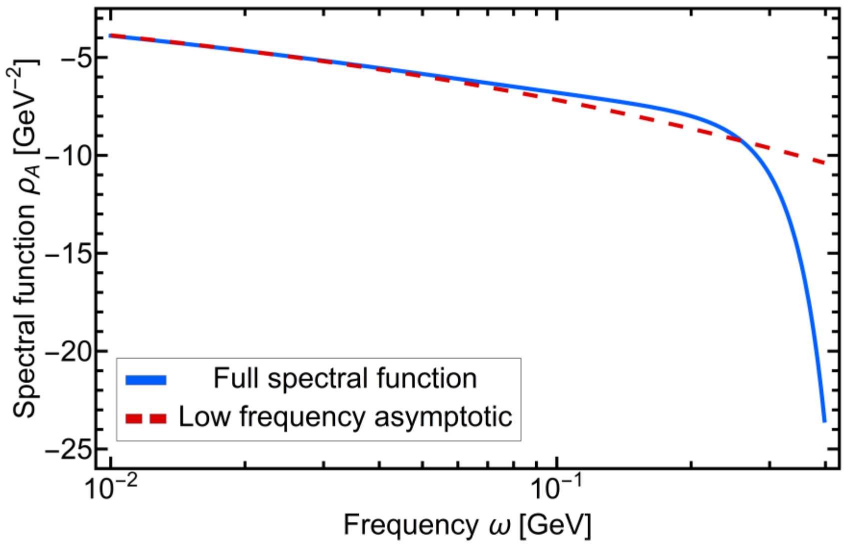
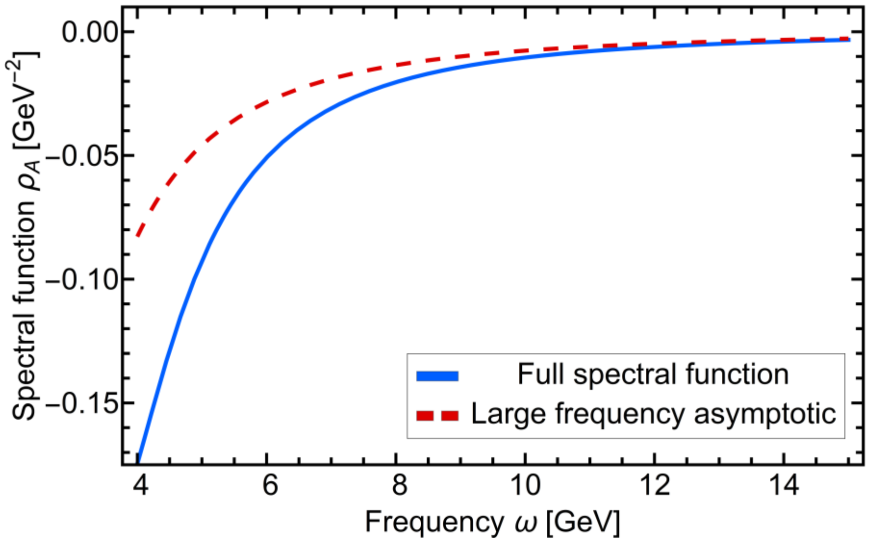


IR



UV



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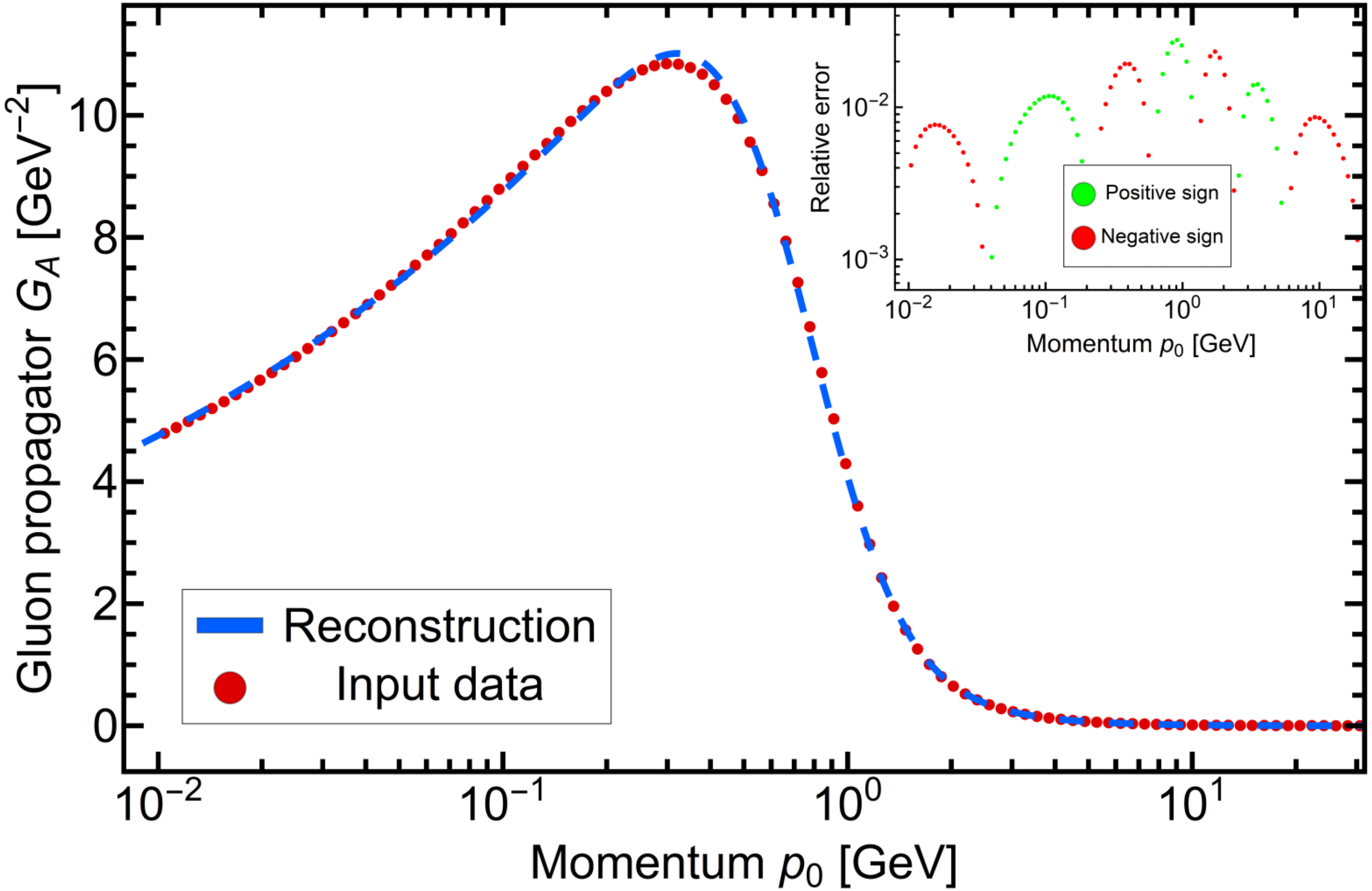


Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

Summary & Outlook

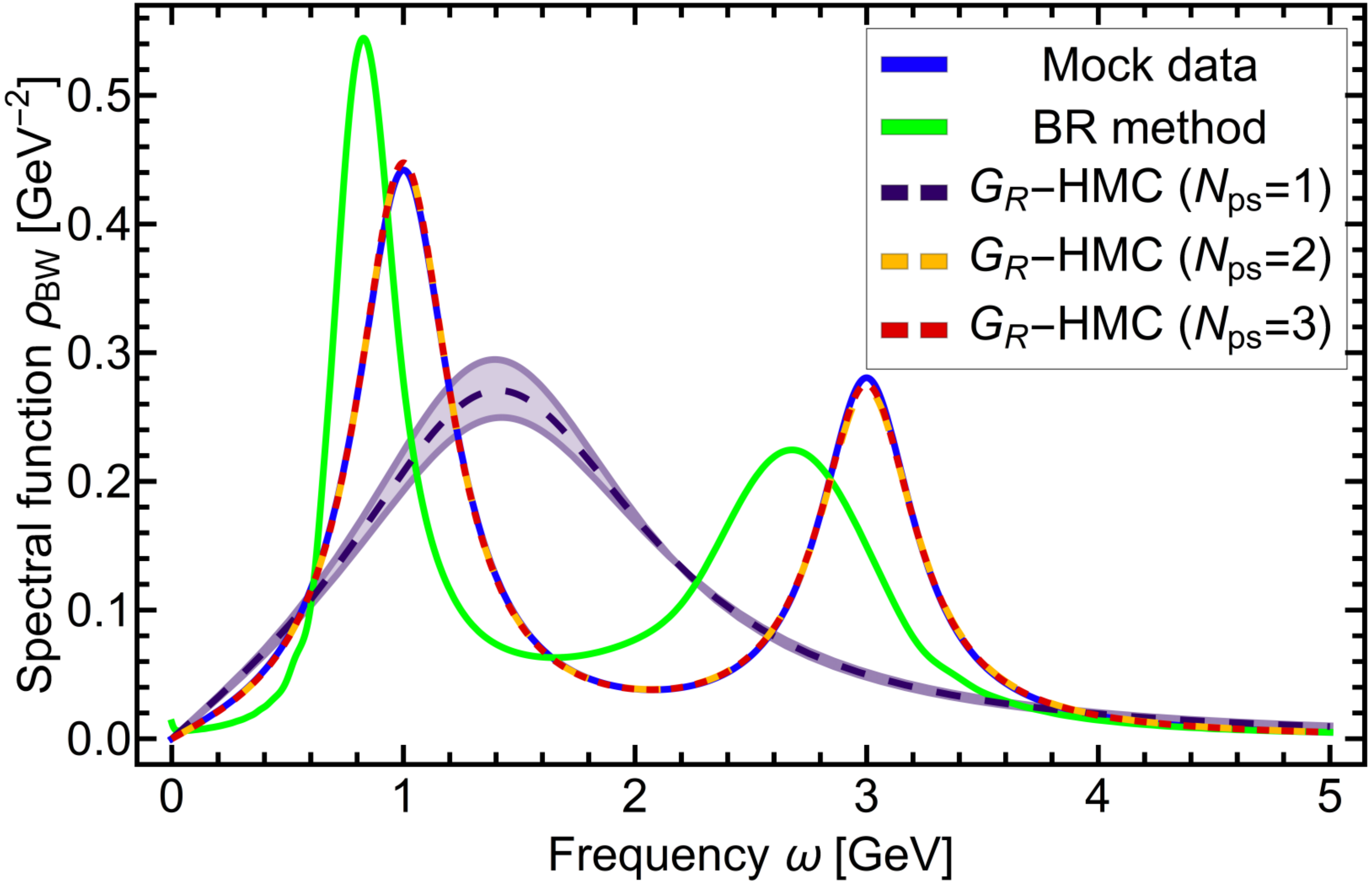
- Direct calculation of spectral functions in functional methods
- Reconstruction of the gluon spectral function with all priors

Thank you for your attention!

- Finite temperature gluon spectral functions
- Transport coefficients

Backup slides

Breit-Wigner benchmark



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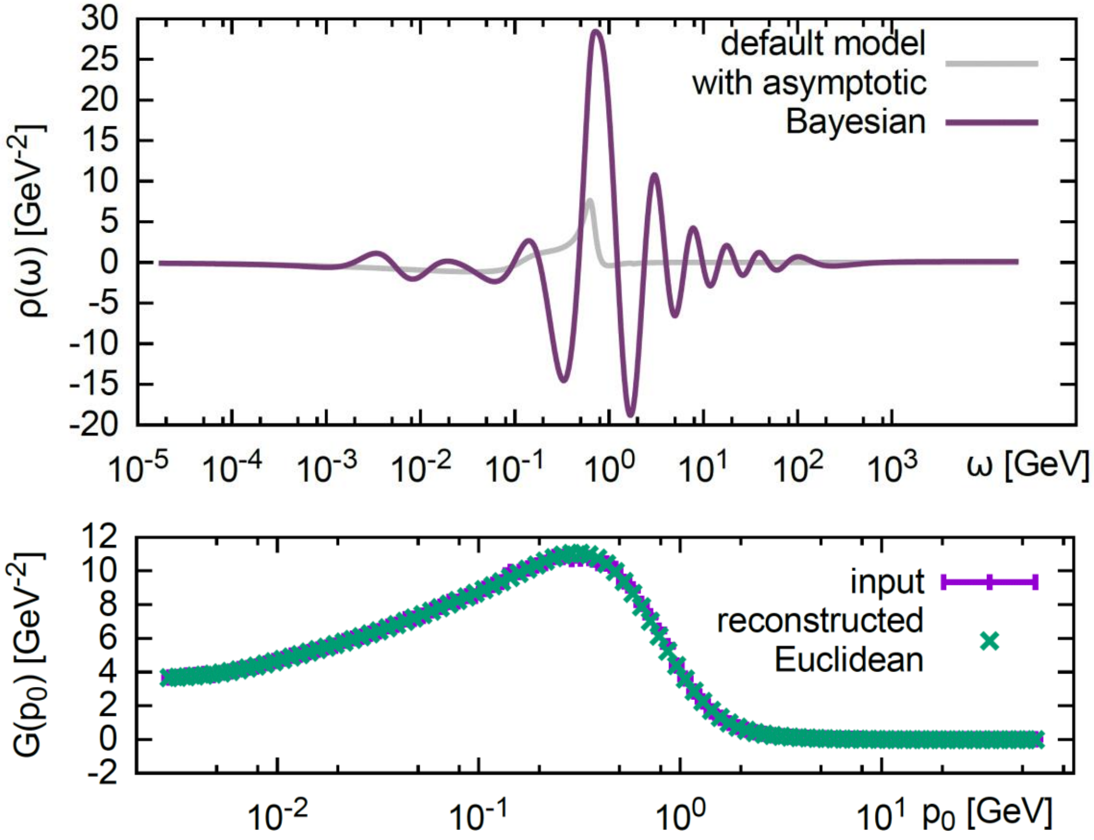
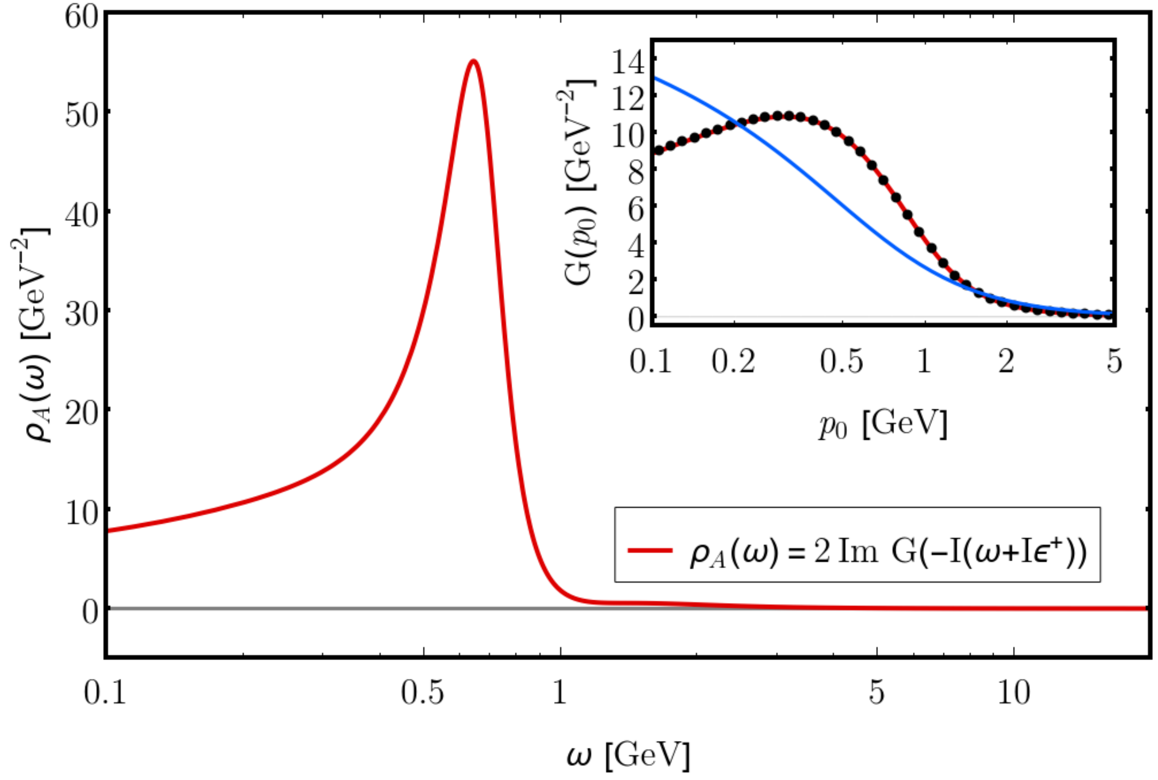
Comparison with other works

Bayesian reconstruction

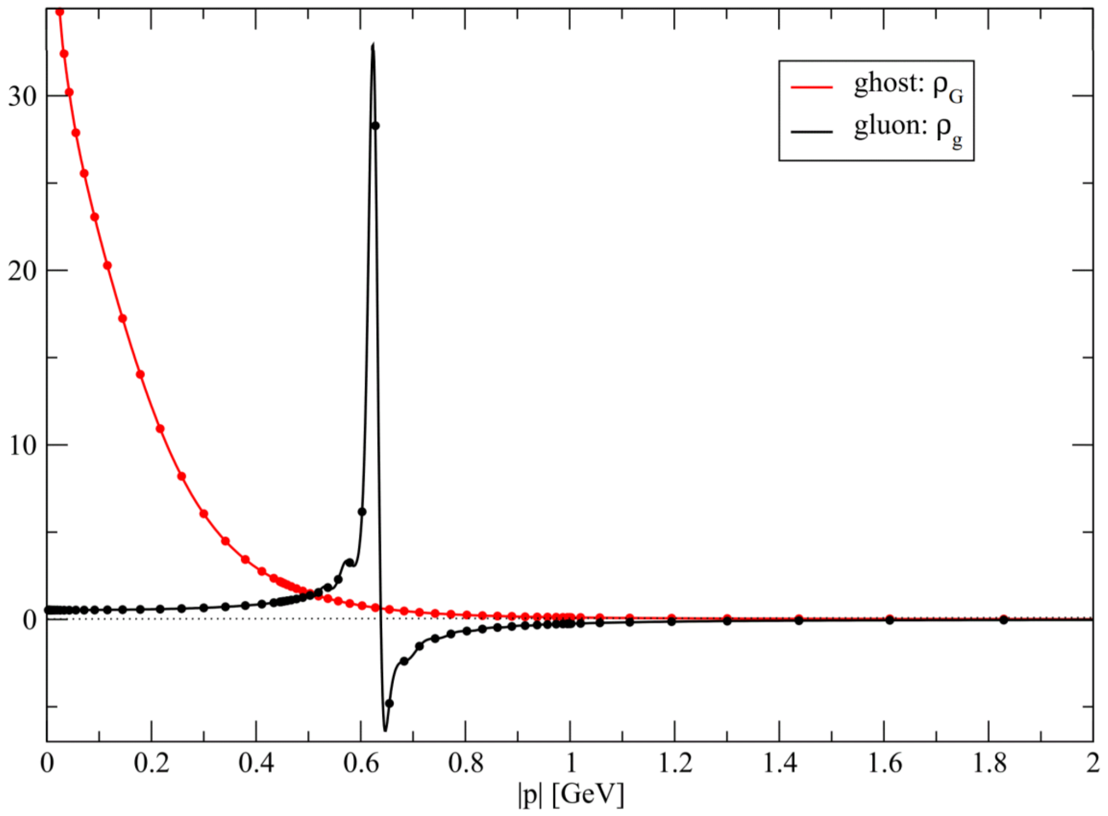


Pade/Schlessinger point

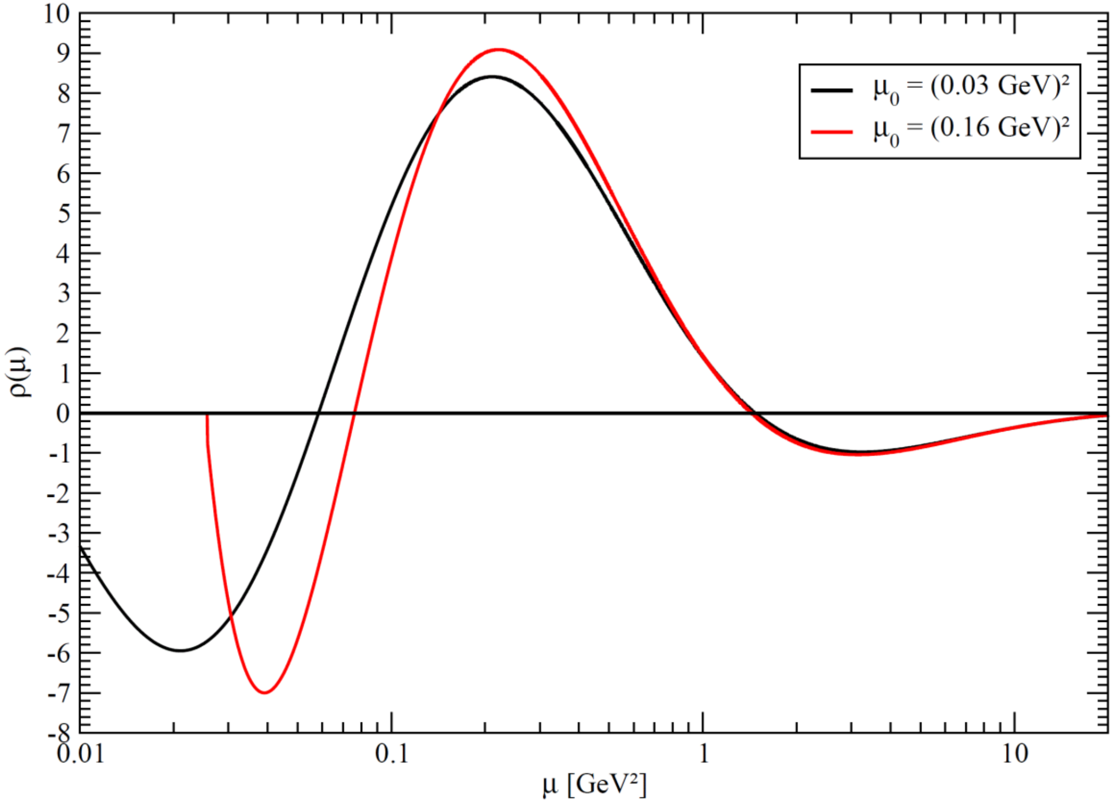


Cyrol, Pawlowski, Rothkopf, NW arxiv:1804.today

Comparison with other works



Strauss, Fischer, Kellermann, Phys.Rev.Lett. 109 (2012)

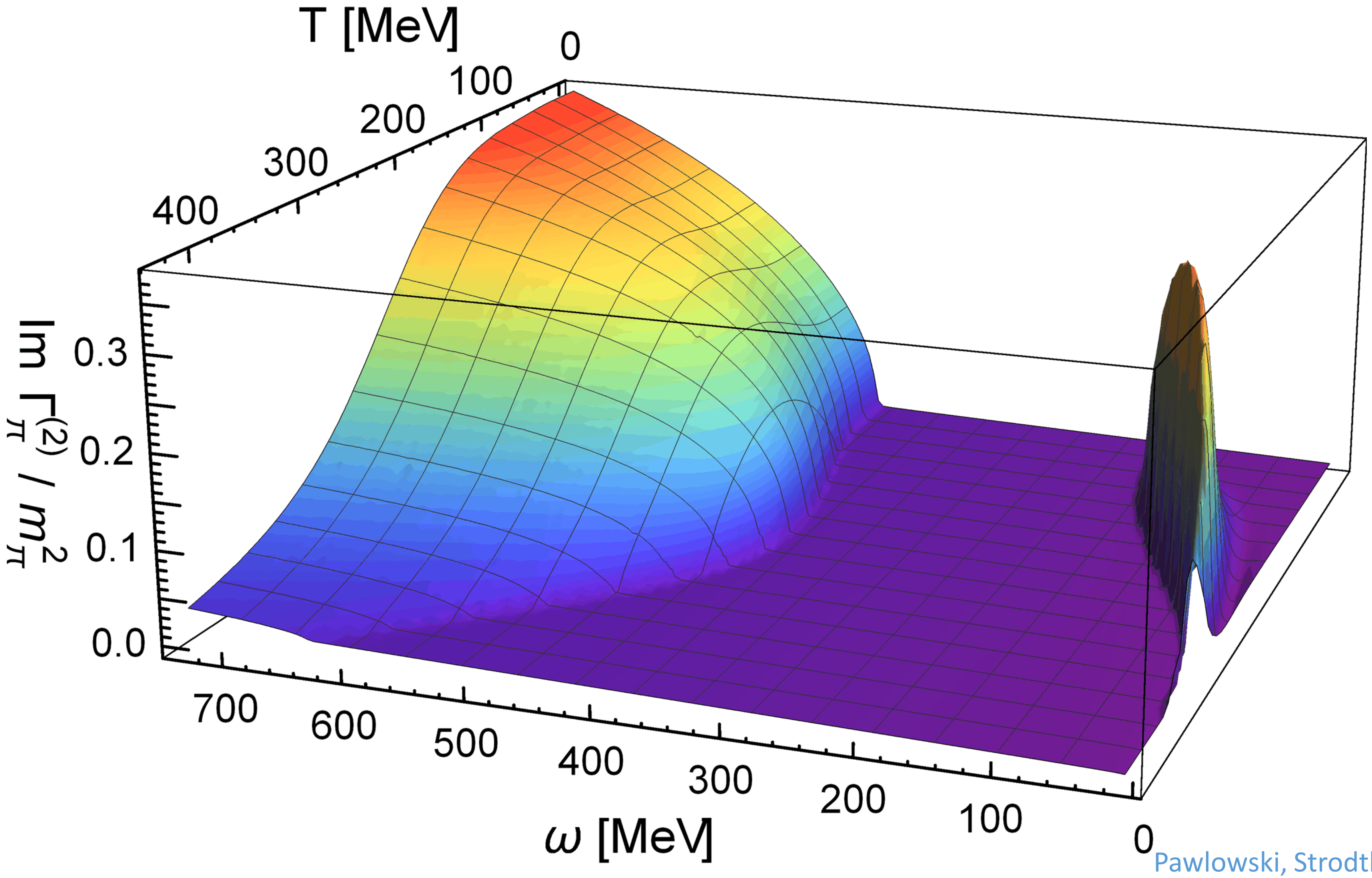


Dudal, Oliveira, Silva, Phys.Rev. D89 (2014)

Application to the O(N)-Model

Pion

Imaginary part of the retarded two-point function

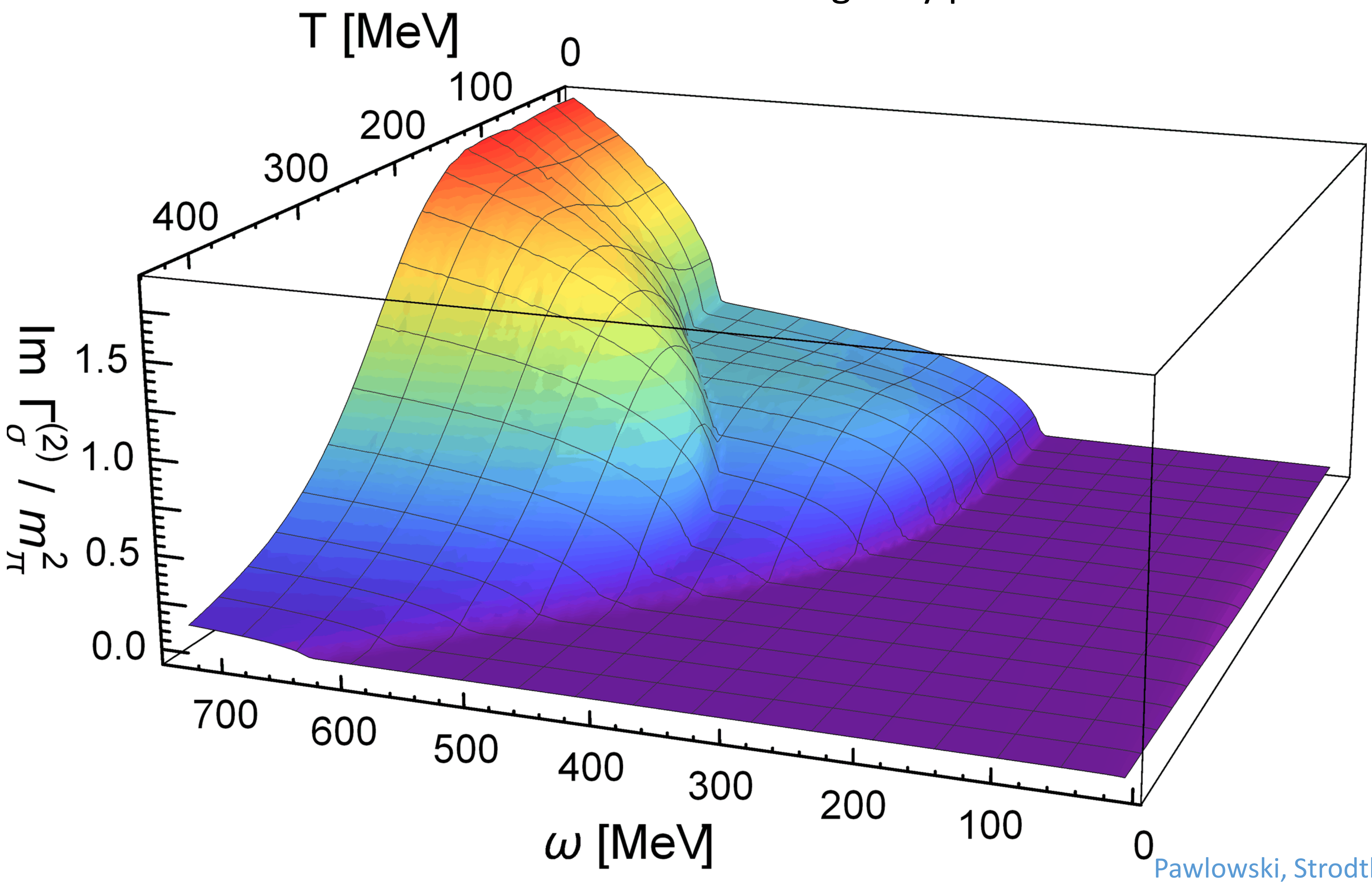


Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model

Sigma meson

Imaginary part of the retarded two-point function

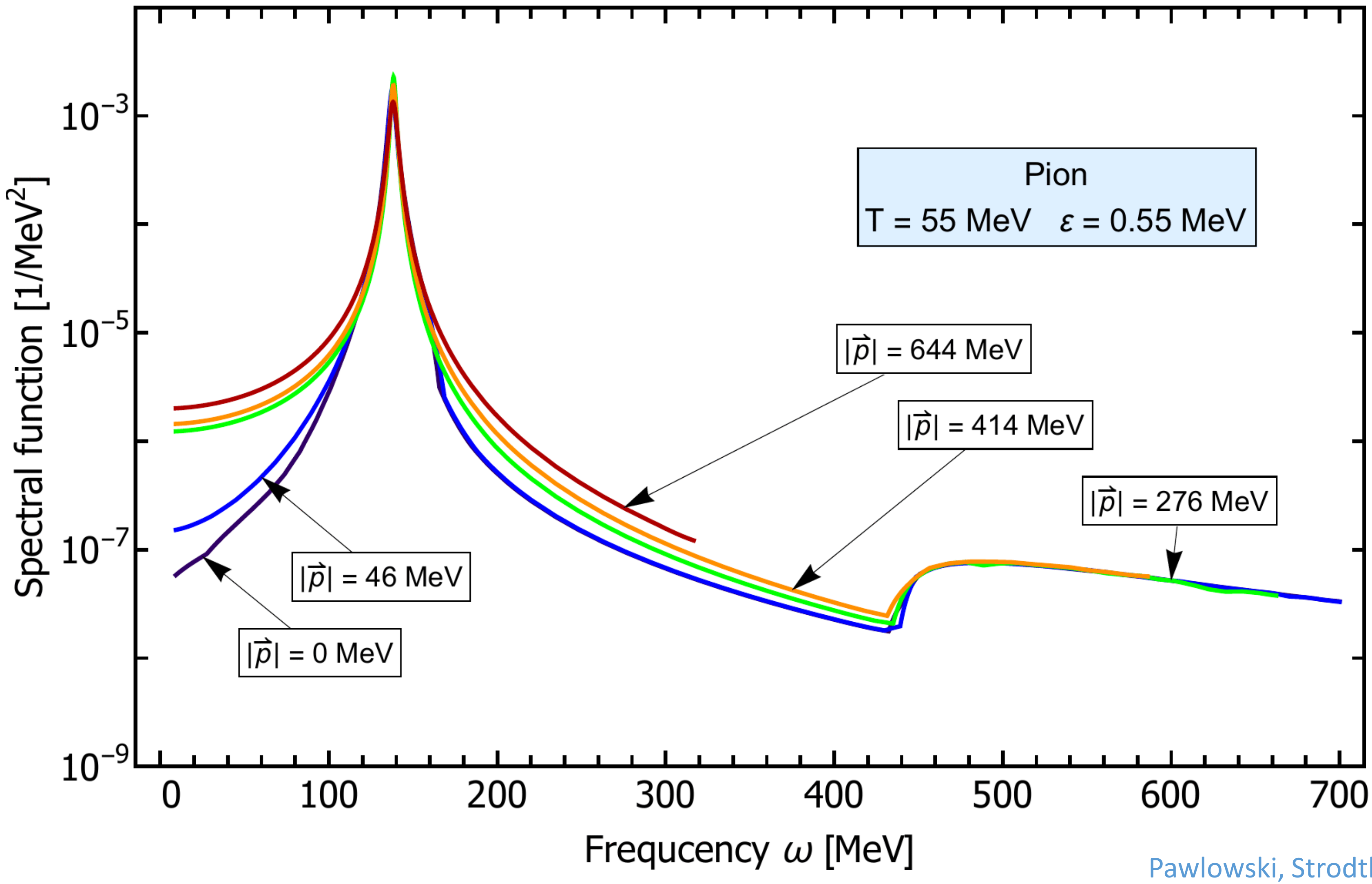


Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model

Pion meson

Finite temperature spectral function for various external momenta

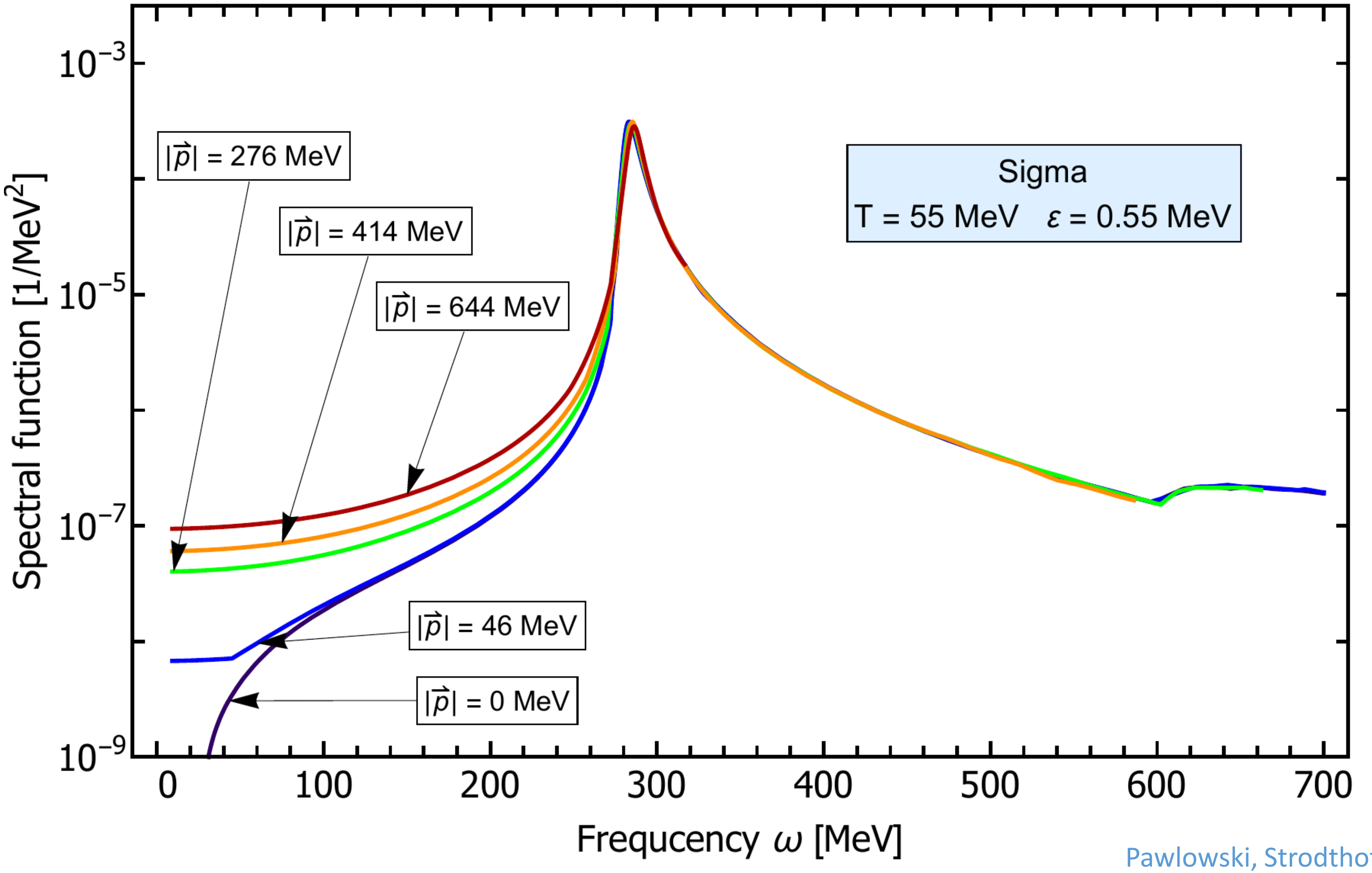


Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model

Sigma meson

Finite temperature spectral function for various external momenta

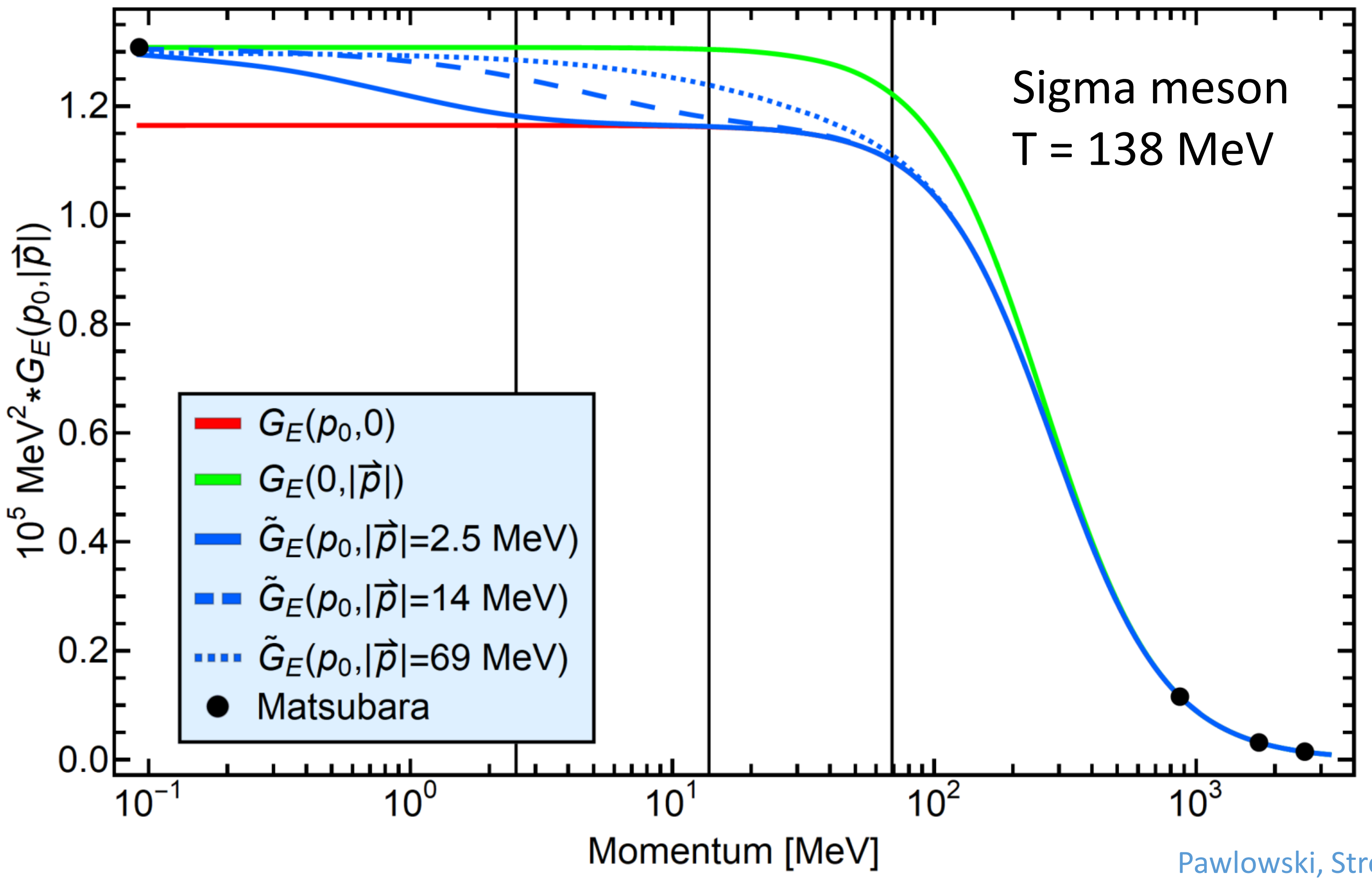


Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model

In medium non-commuting limits

$$\lim_{\vec{p}\to 0}\lim_{p_0\to 0}\Gamma^{(2)}(p_0,\vec{p}) \neq \lim_{p_0\to 0}\lim_{\vec{p}\to 0}\Gamma^{(2)}(p_0,\vec{p})$$



Pawlowski, Strodthoff, NW, arxiv:1711.07444

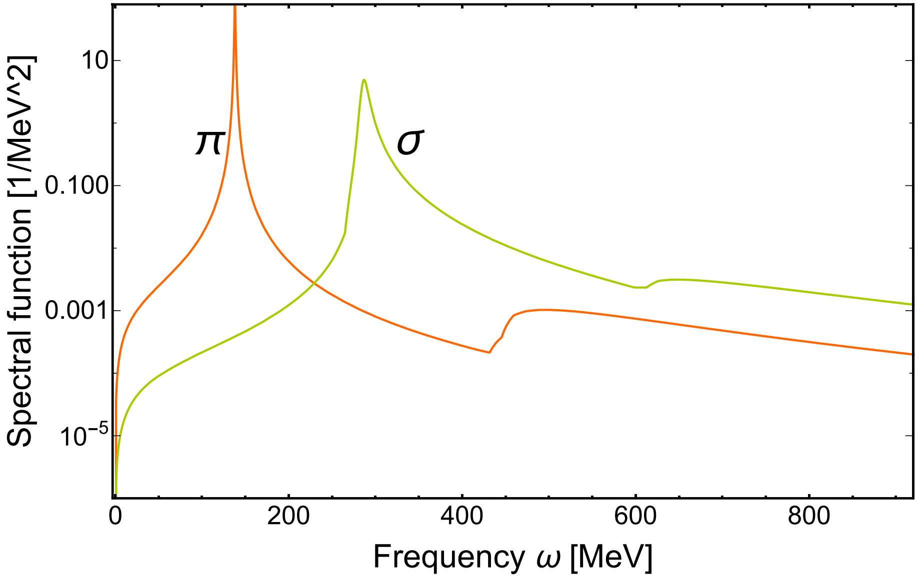
Retarded Greens function

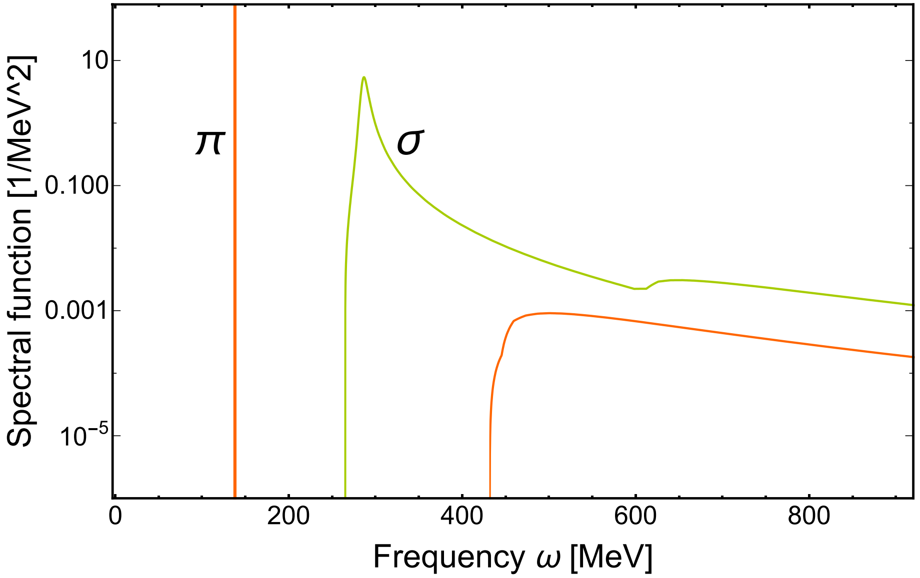
Retarded Greens function $\lim_{\varepsilon \to 0} G(-i(\omega + i\varepsilon))$

→ Take limit analytically

→ Numerical extrapolation

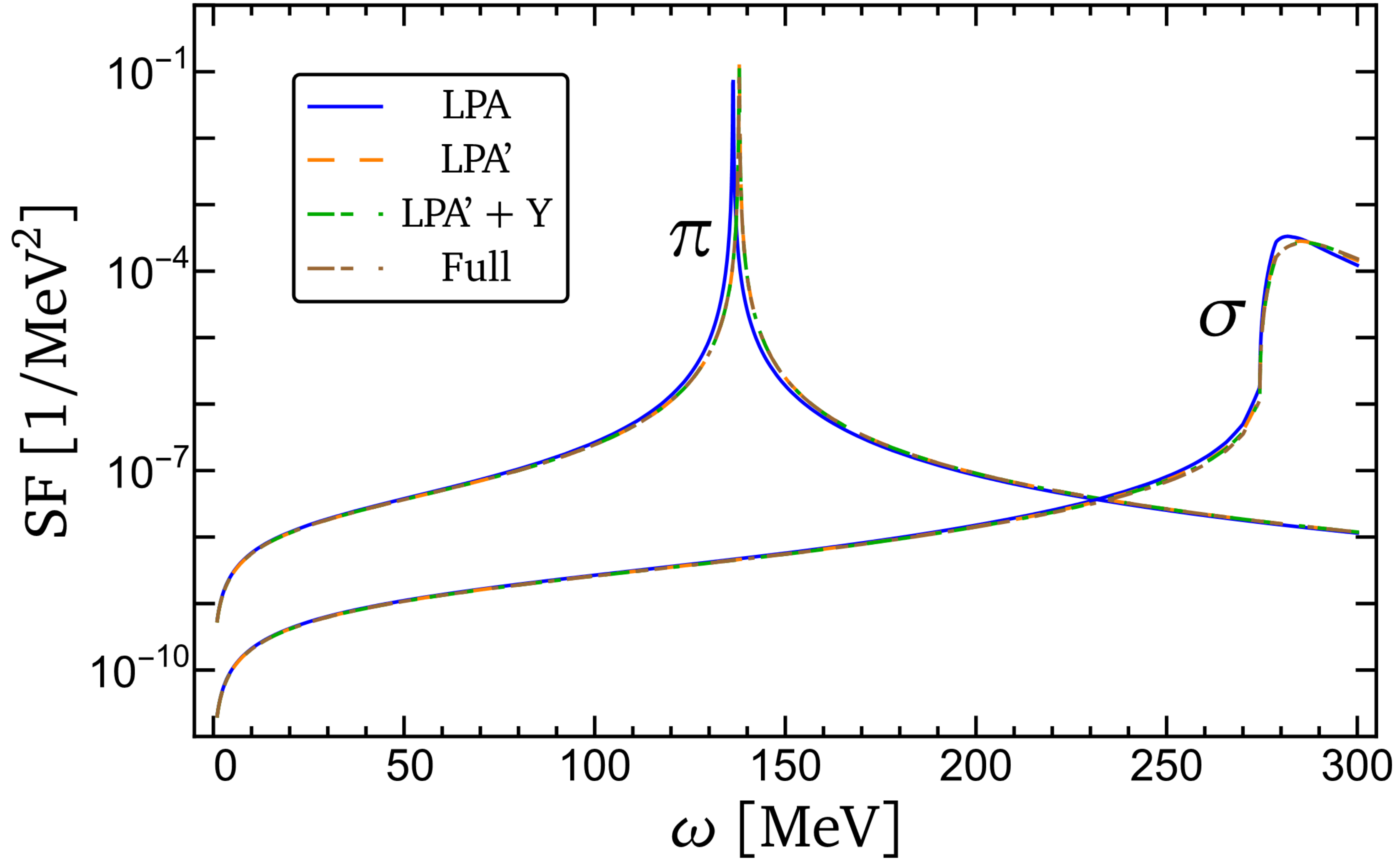
$$\rho(\omega, \vec{p}) = -2 \, \mathrm{Im} \, G_{\mathrm{R}}(\omega, \vec{p})$$





Application to the O(N)-Model

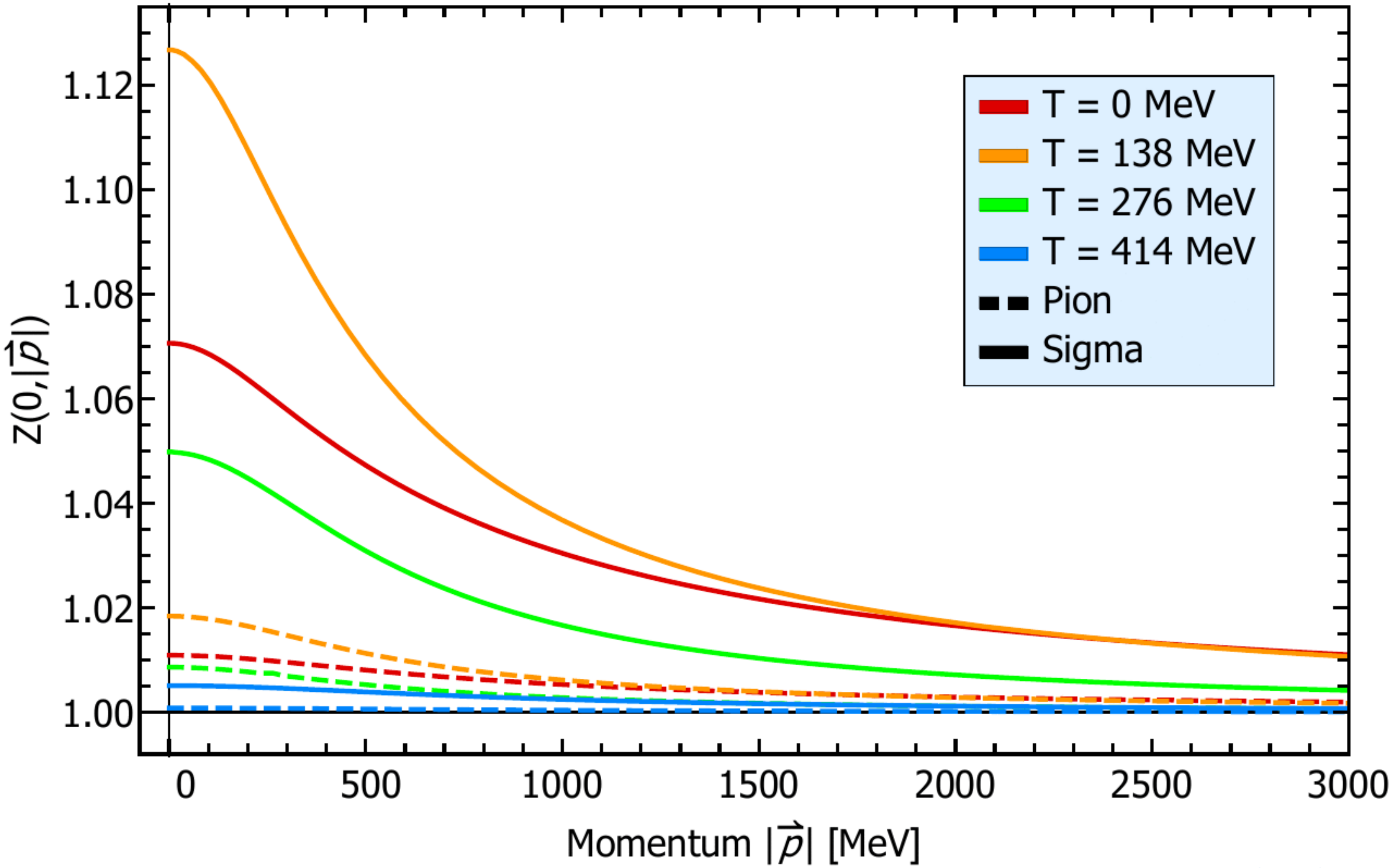
Vacuum spectral function



NW, Master thesis

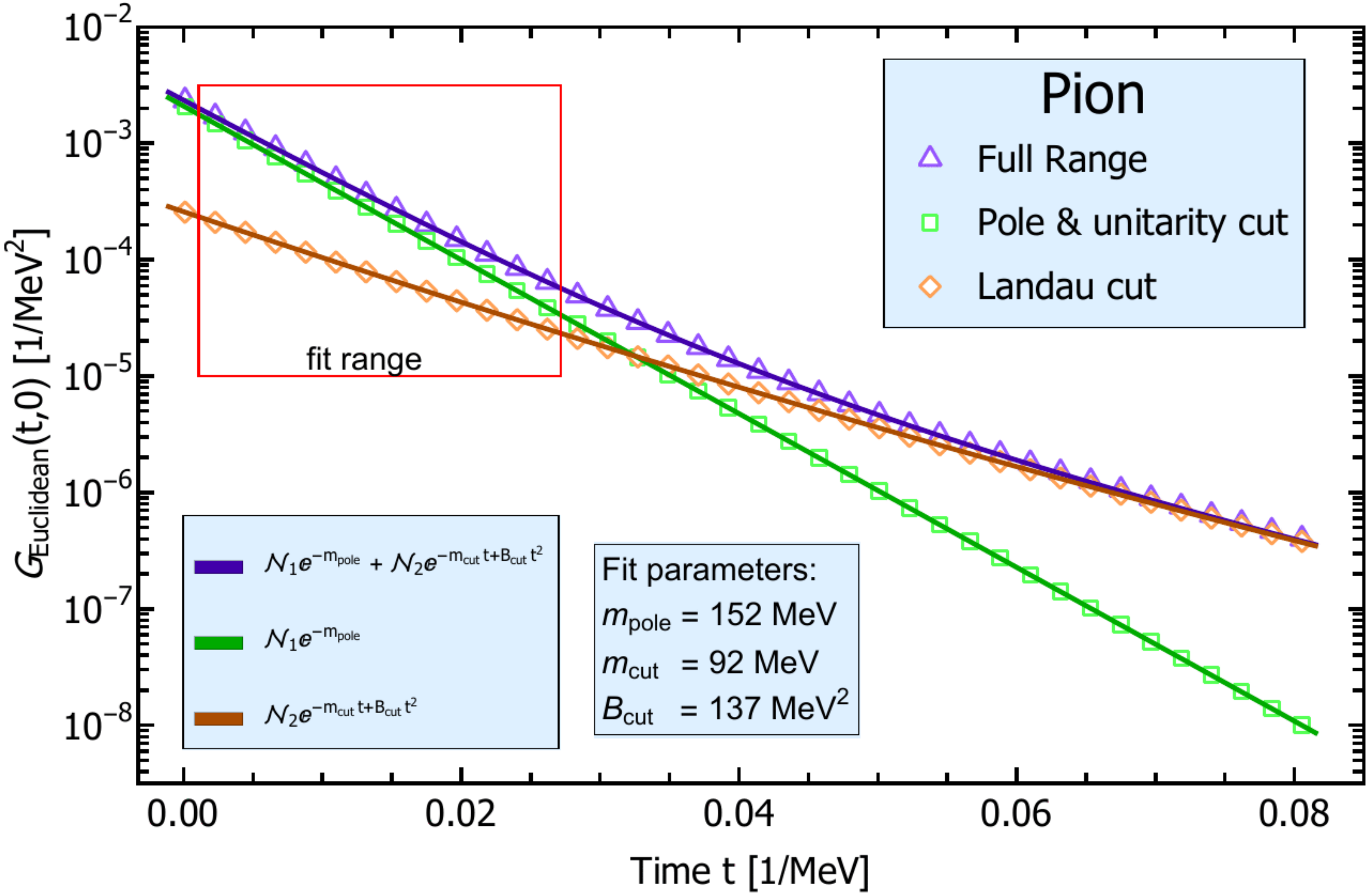
Application to the O(N)-Model

Euclidean momentum dependent dressings



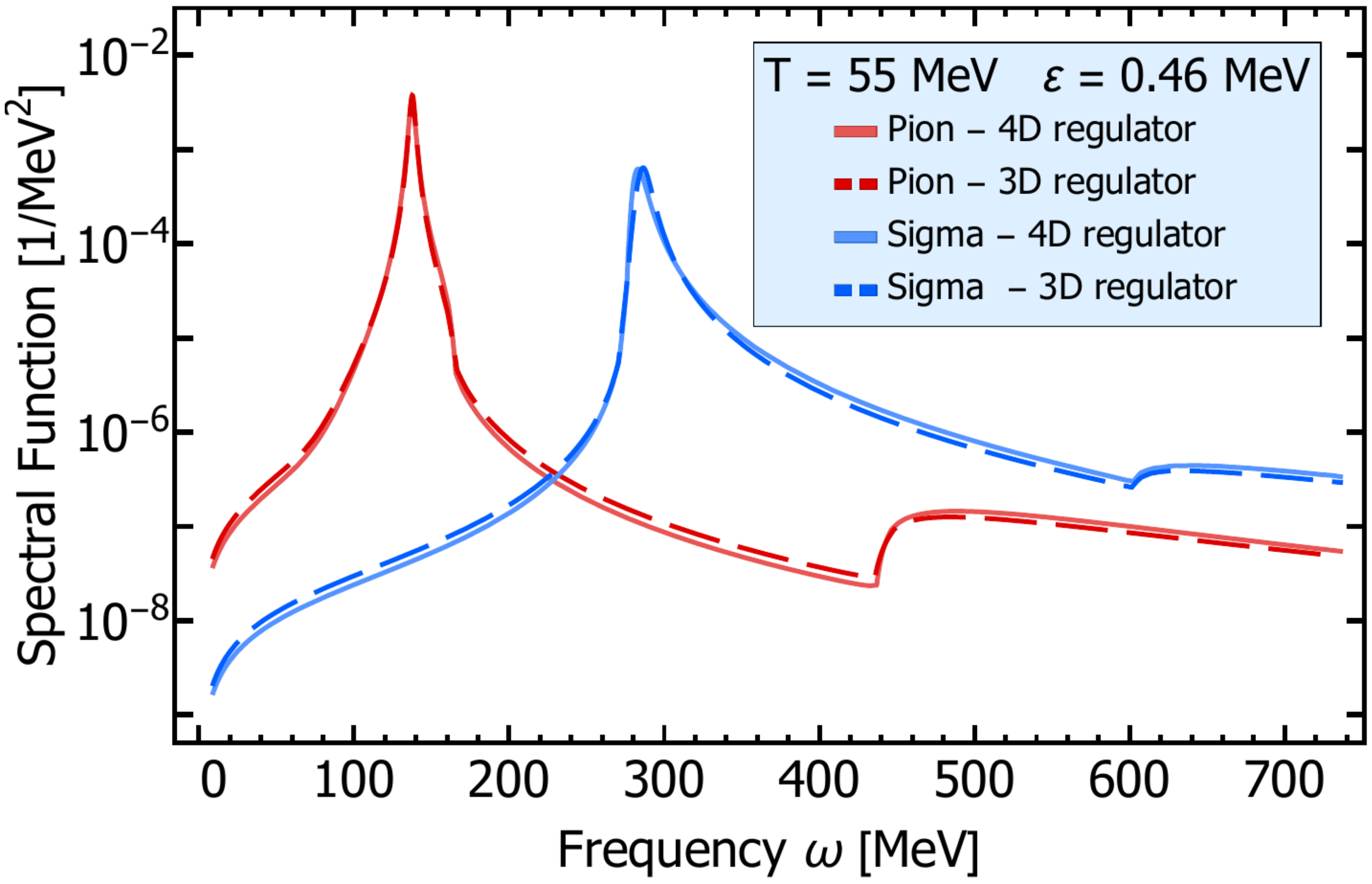
Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model



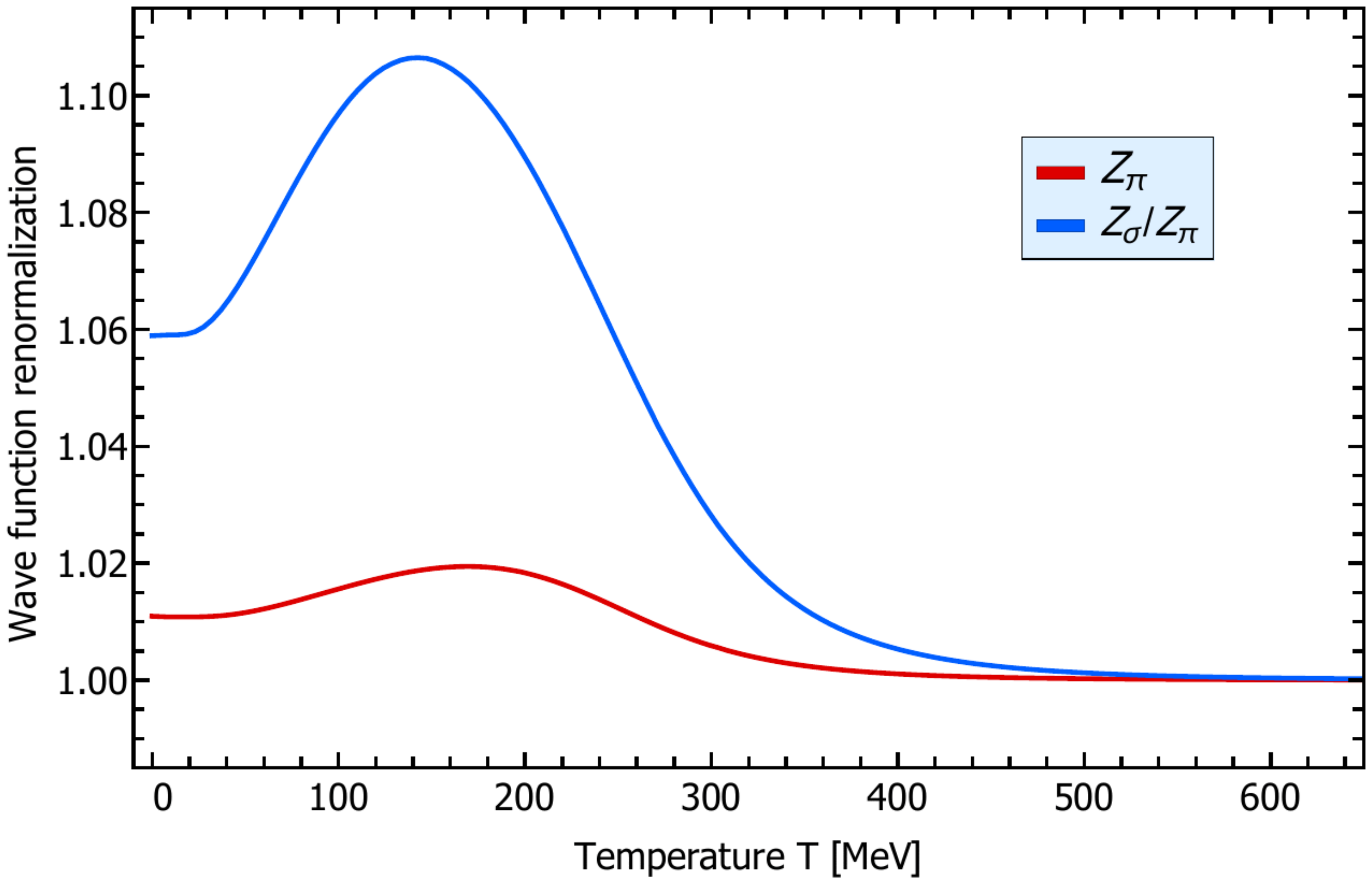
Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model



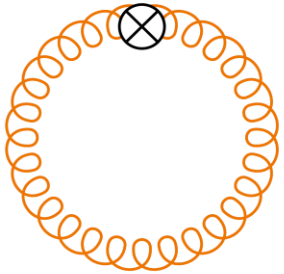
Pawlowski, Strodthoff, NW, arxiv:1711.07444

Application to the O(N)-Model



Pawlowski, Strodthoff, NW, arxiv:1711.07444

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}$$



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