

Amplitude Analyses

B Workshop Neckarzimmern
Jonas Rademacker

Why Amplitude Analyses?

- QM is intrinsically complex:

Wave functions/transition amplitudes etc: $\psi = a e^{i\alpha}$. Observable: $|\psi|^2$.

Only half the information. How do I get the rest?

- Note that the rest is very interesting - CP violation in the SM comes from phases!
- Answer: Interference effects:

$$\psi_{\text{total}} = a e^{i\alpha} + b e^{i\beta} + \dots$$

$$|\psi_{\text{total}}|^2 = |a e^{i\alpha} + b e^{i\beta} + \dots|^2 = a^2 + b^2 + 2ab \cos(\alpha - \beta) + \dots$$

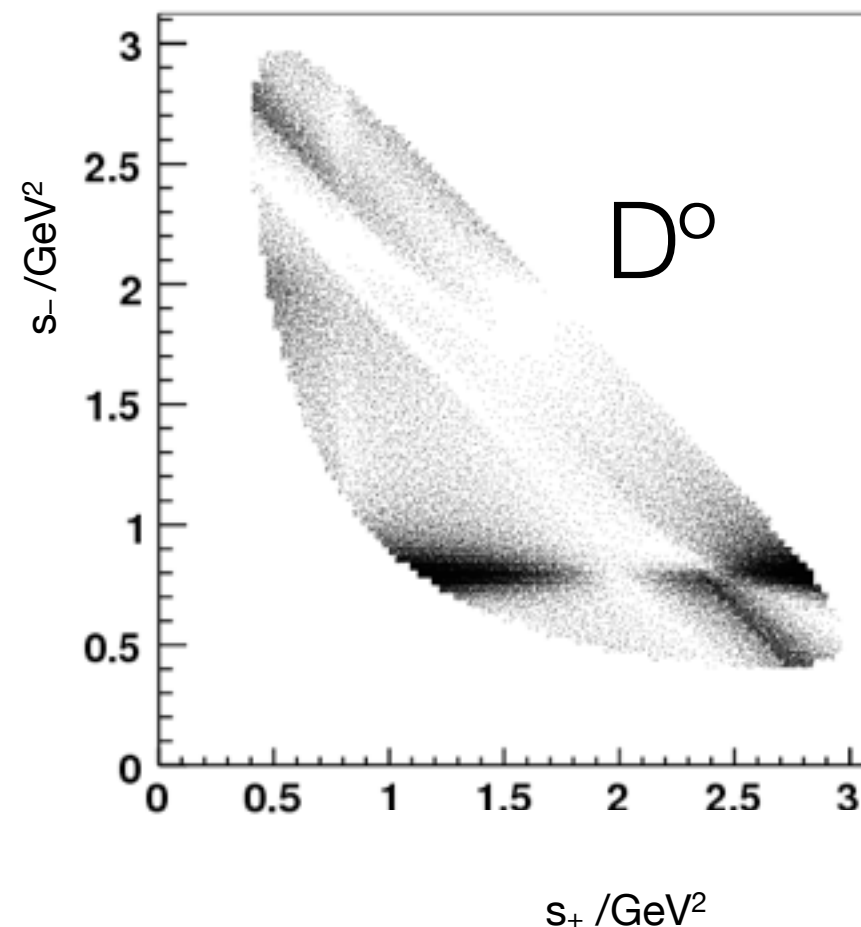
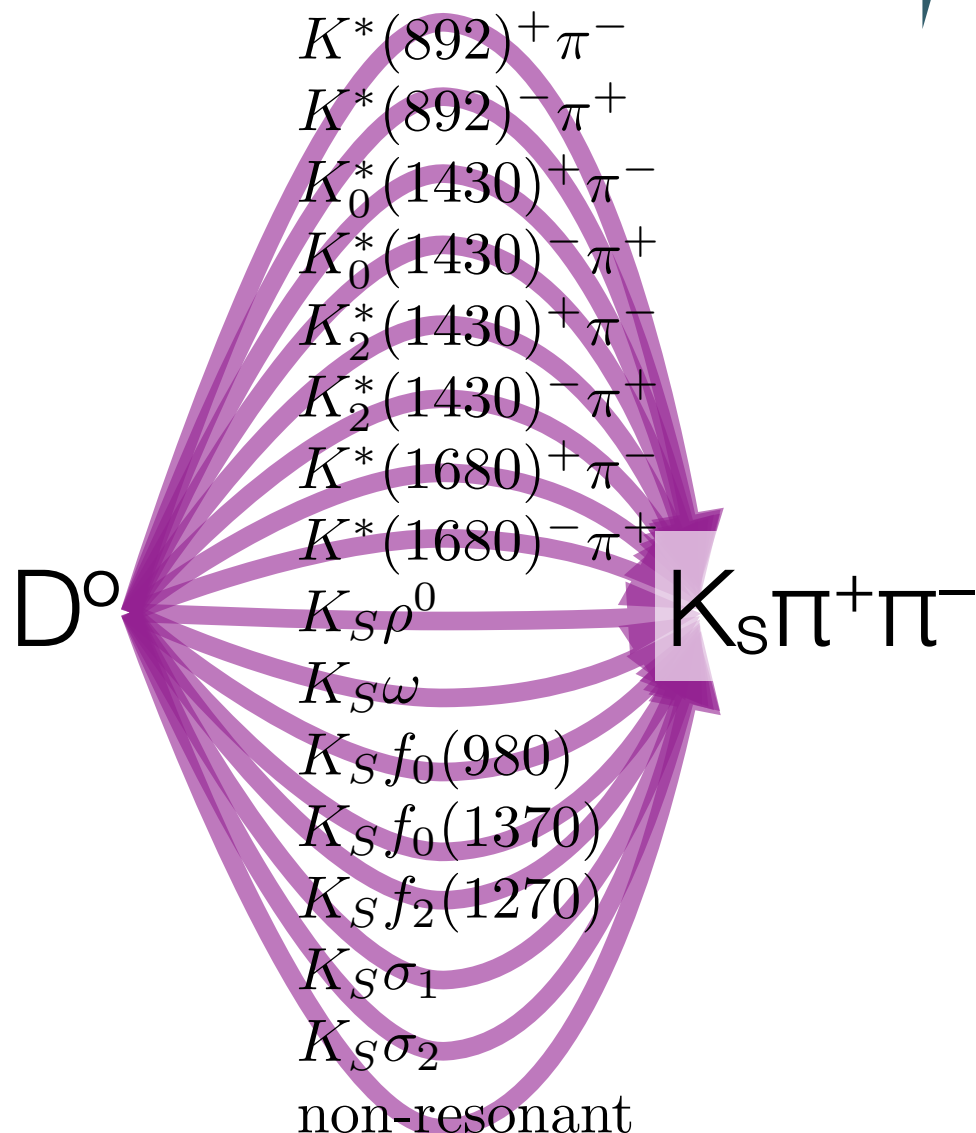
Dalitz plot analyses - lots of interfering amplitudes!

Many interfering decay paths contribute to the same final state



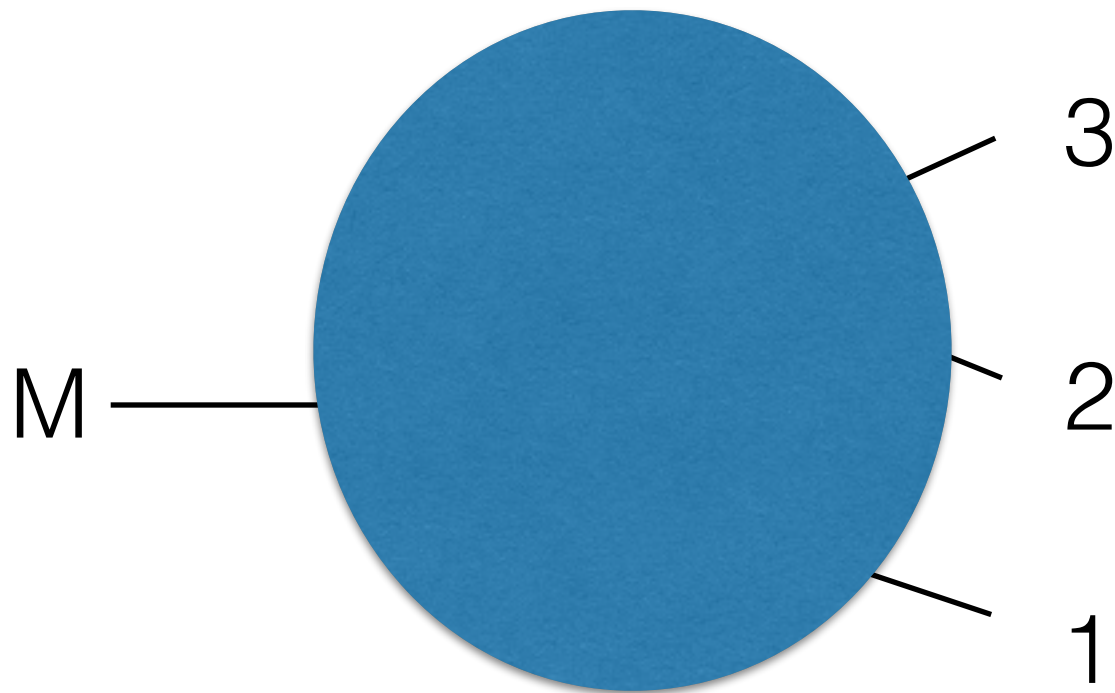
Described by a sum of complex amplitudes

$$A(s_+, s_-) = \sum_k a_k(s_+, s_-) e^{i\phi_k(s_+, s_-)}$$



$|A(s_+, s_-)|^2$
represented
in a Dalitz
plot

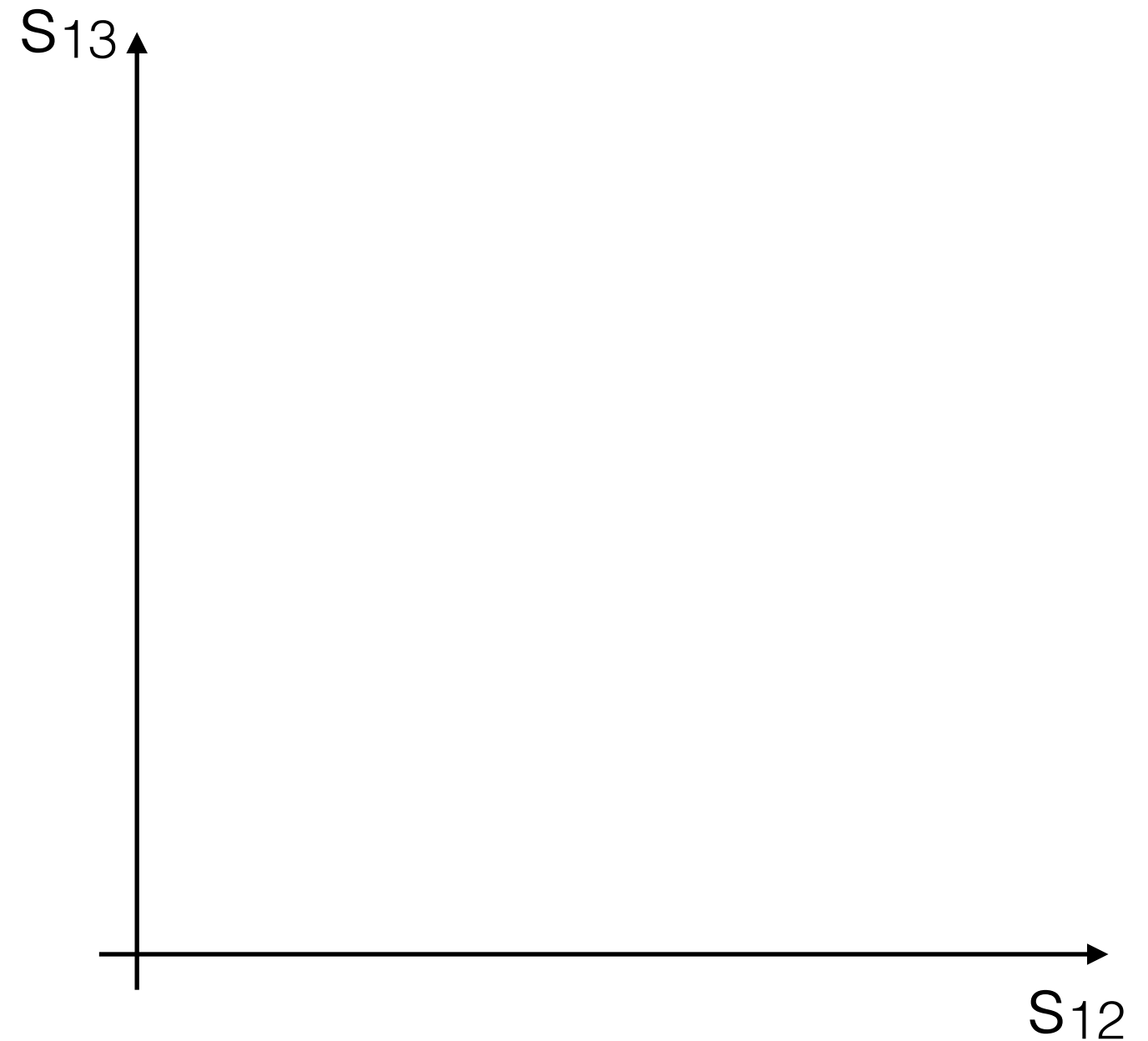
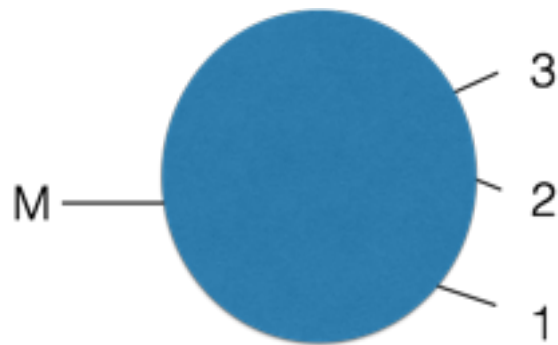
3 body decays



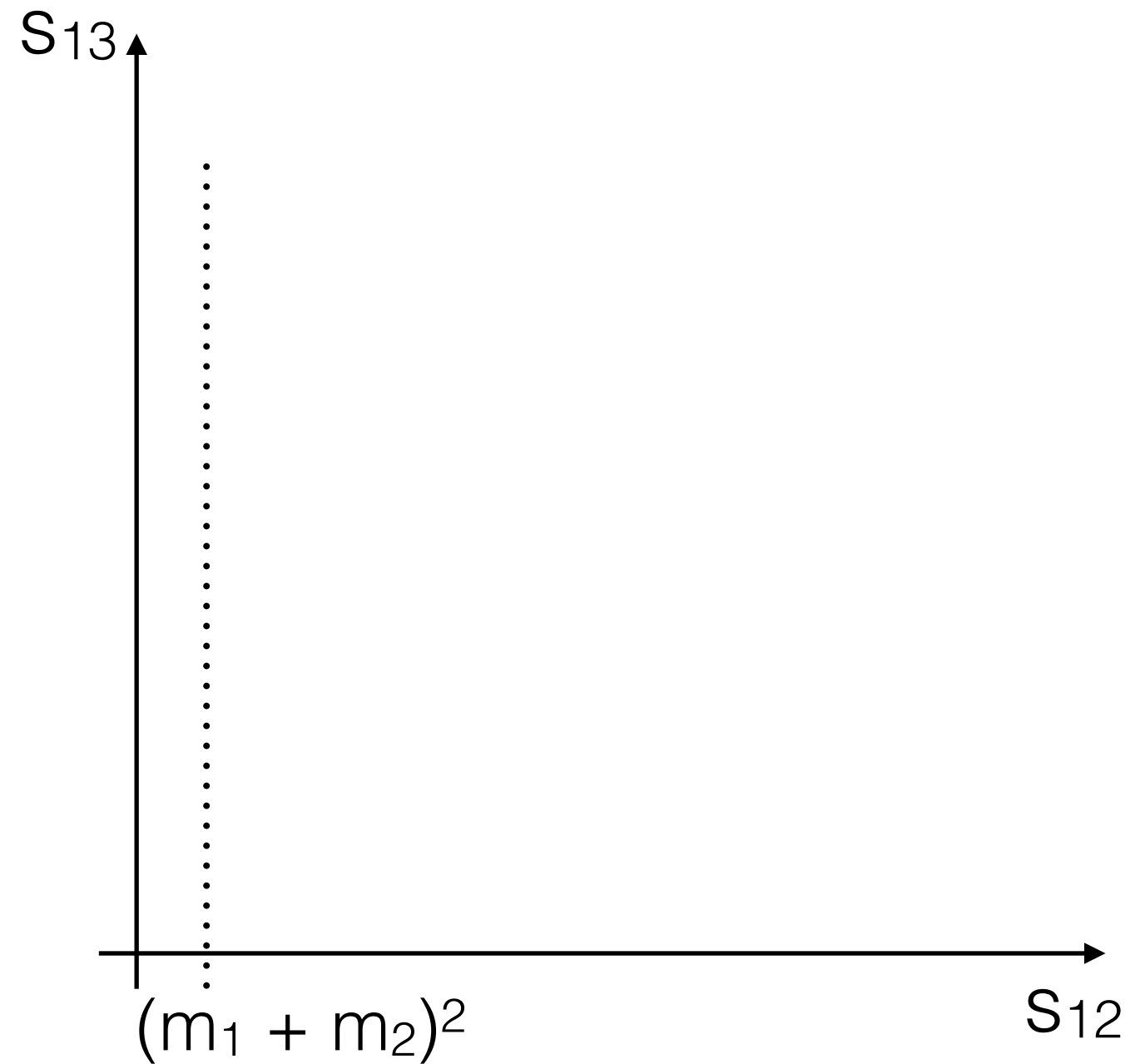
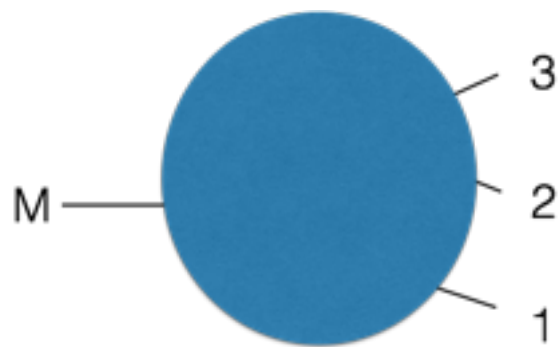
$$\begin{aligned} d\Gamma &= |\mathcal{M}_{fi}|^2 d\Phi \\ &= |\mathcal{M}_{fi}|^2 \left| \frac{\partial \Phi}{\partial (s_{12}, s_{13})} \right| ds_{12} ds_{13} \\ &= \frac{1}{(2\pi)^2 32M^3} |\mathcal{M}_{fi}|^2 ds_{12} ds_{13} \end{aligned}$$

$$s_{ij} \equiv (p_i + p_j)^2 \equiv m_{ij}^2$$

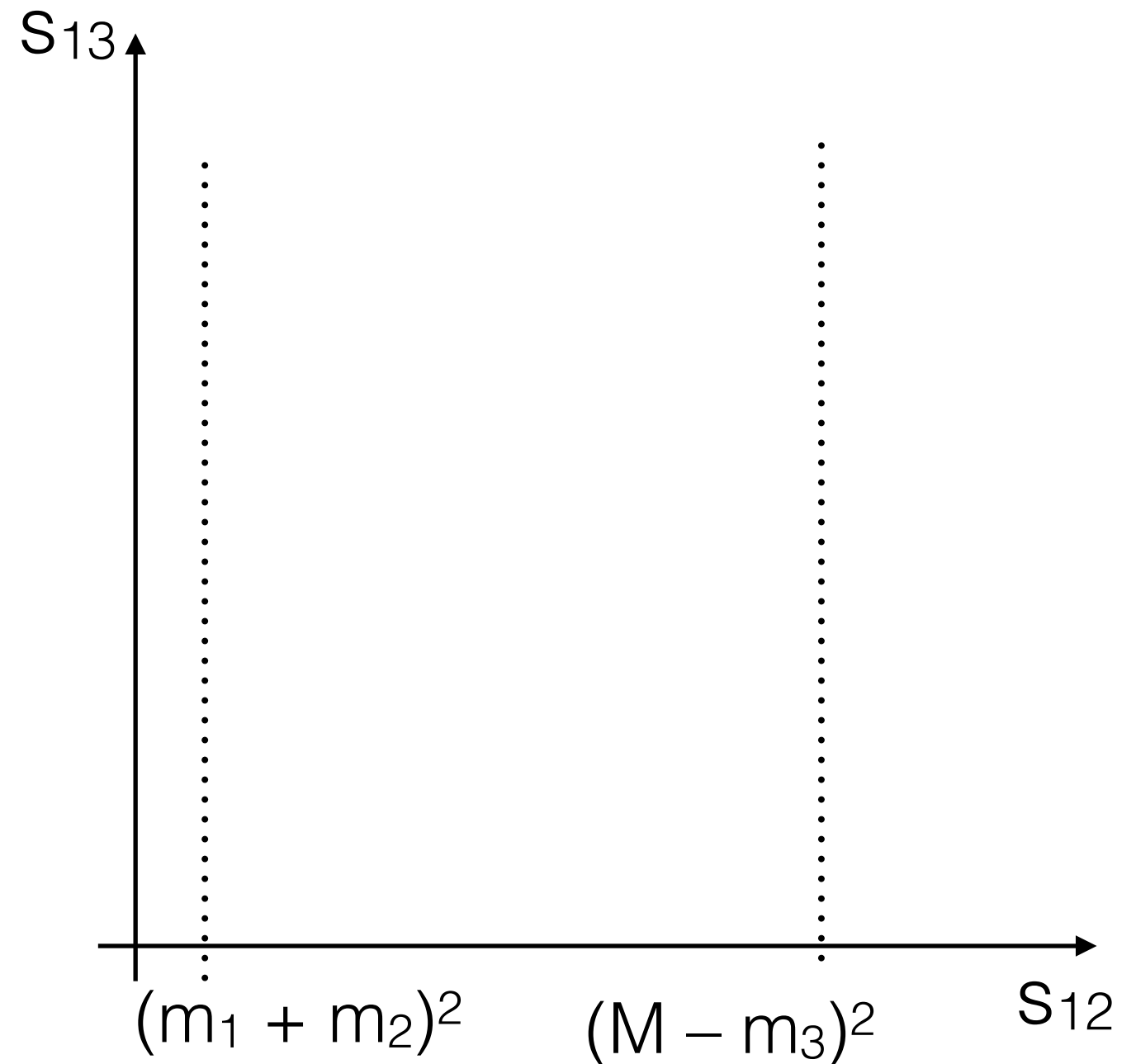
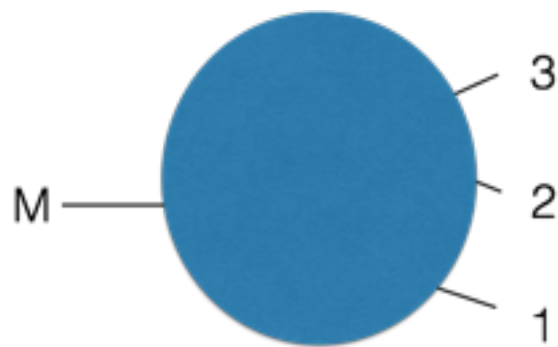
3-body phase space



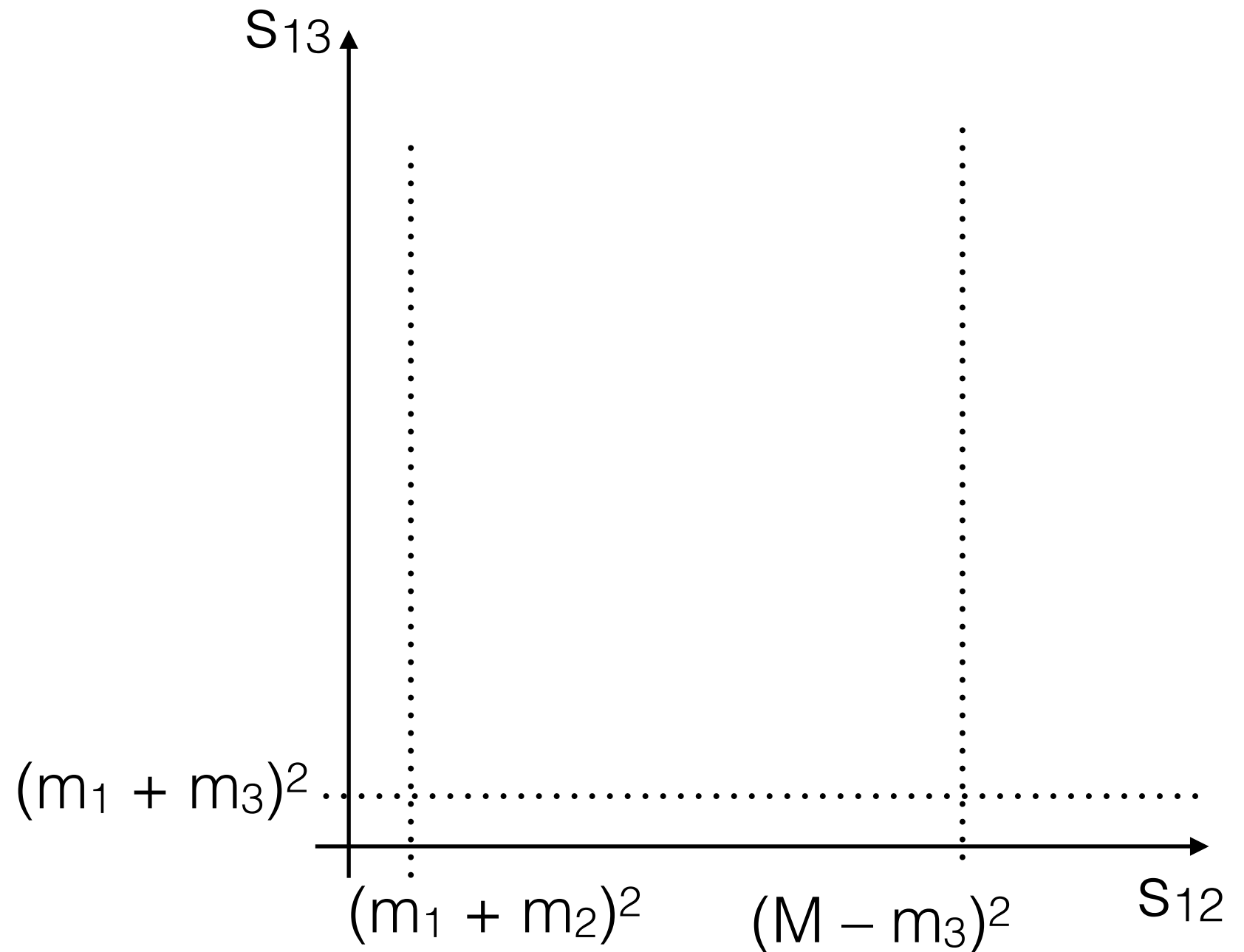
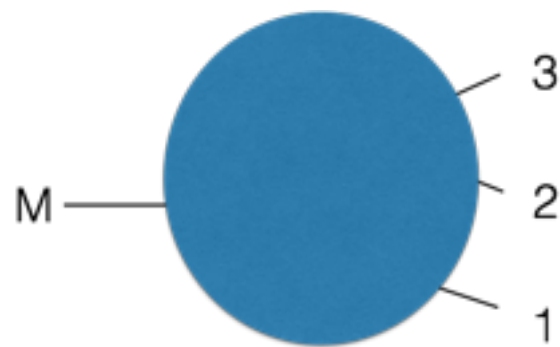
3-body phase space



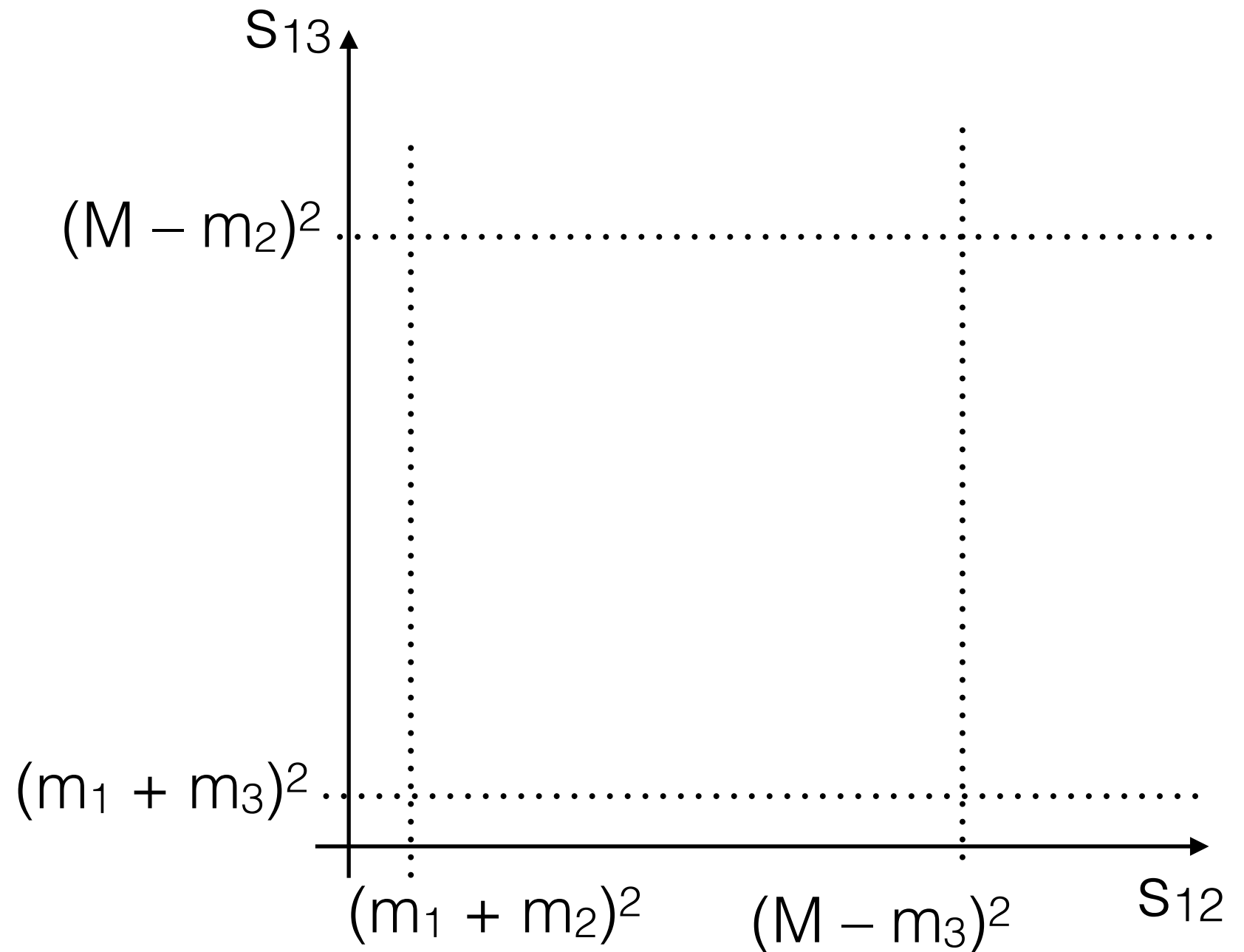
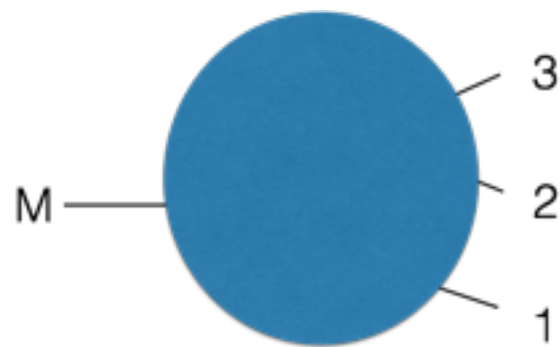
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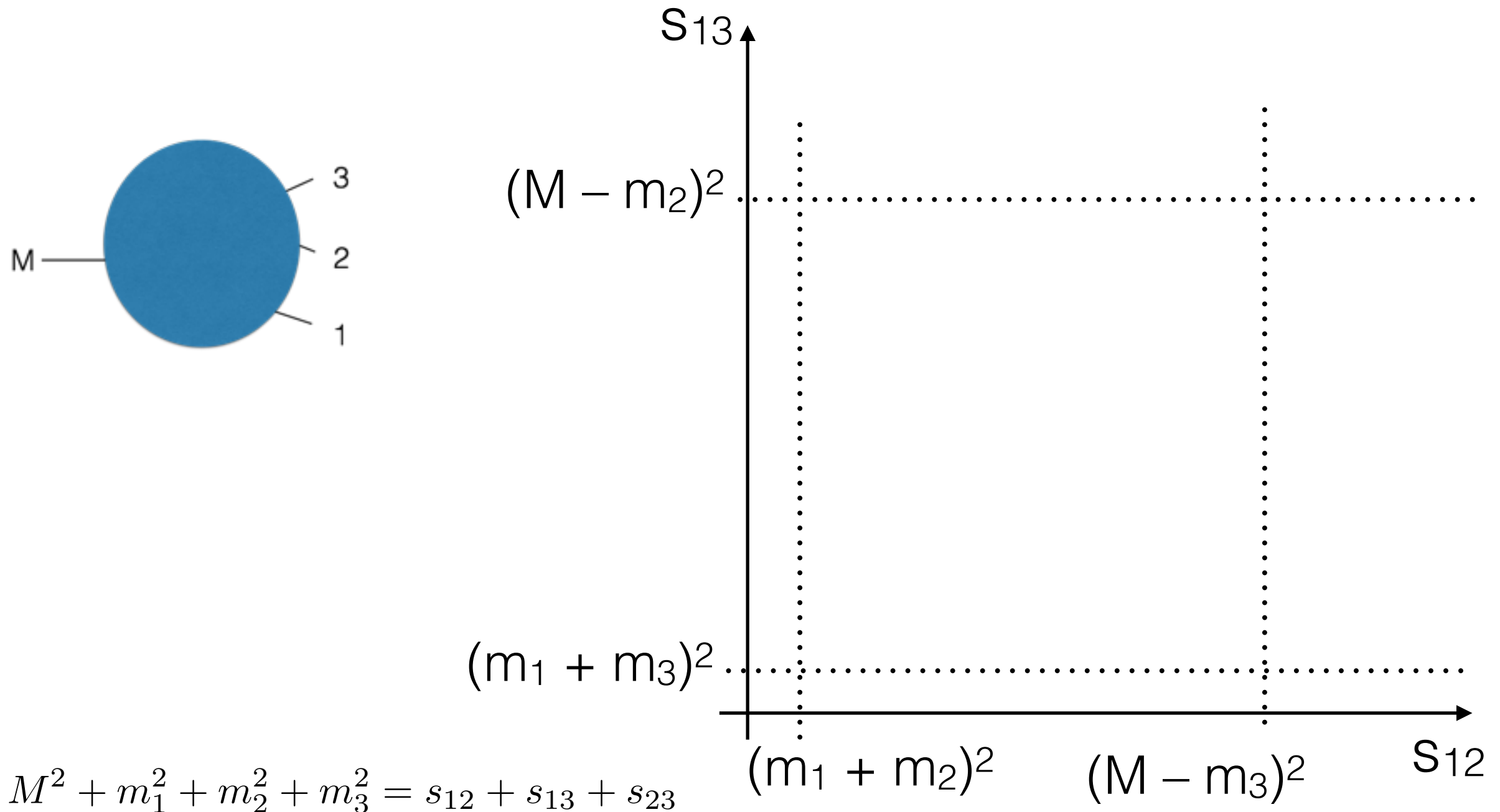
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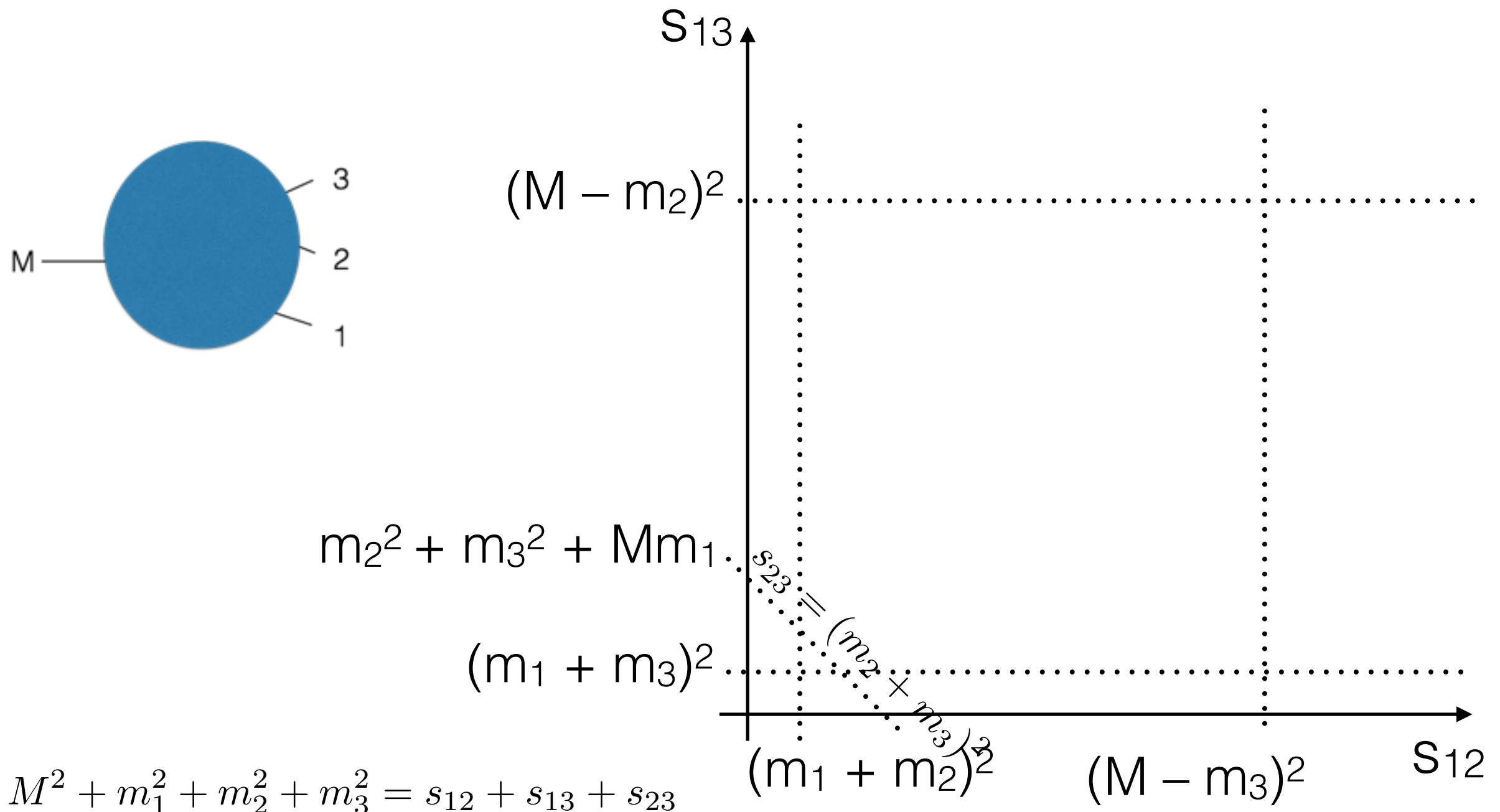
3-body phase space



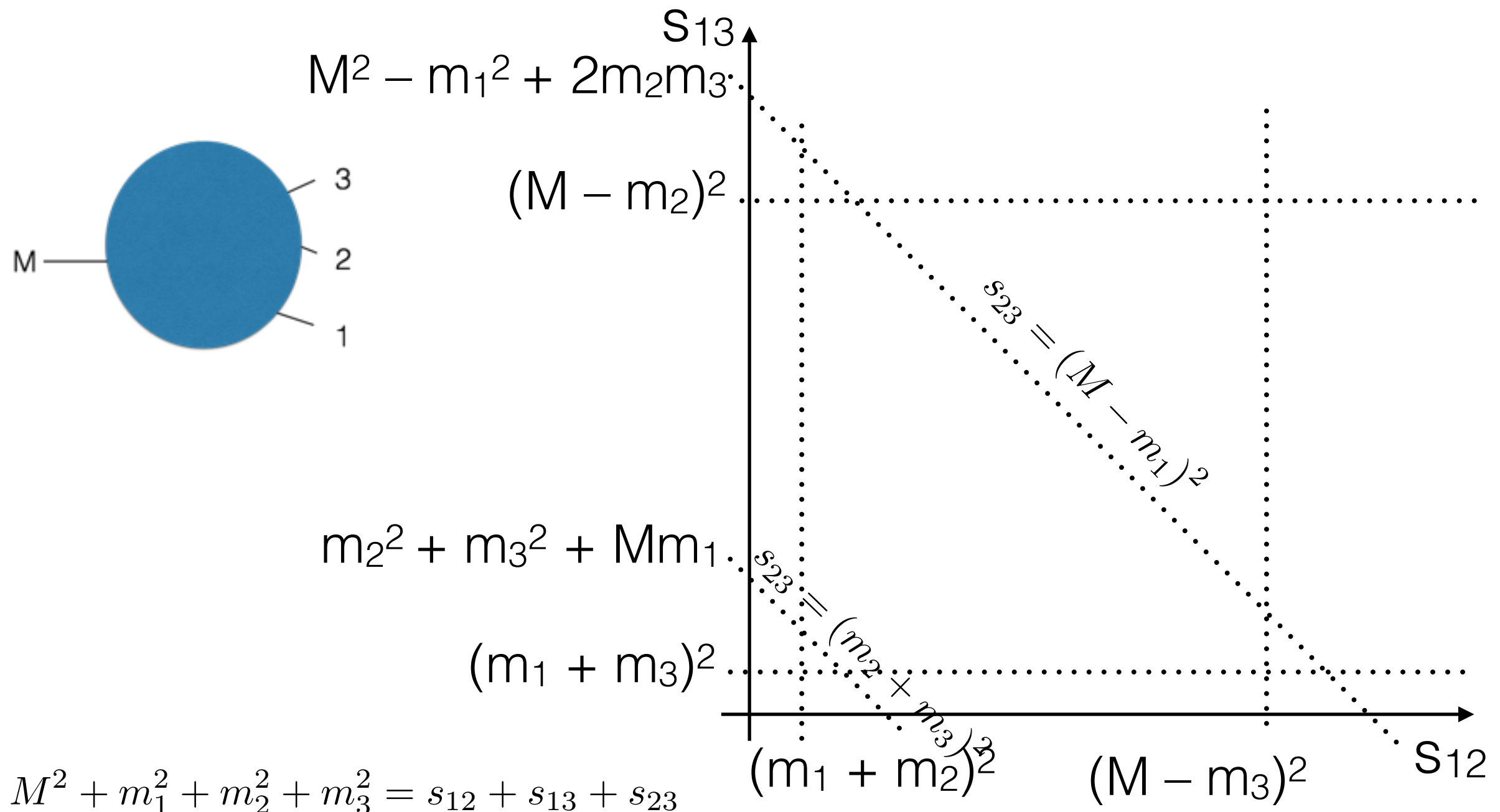
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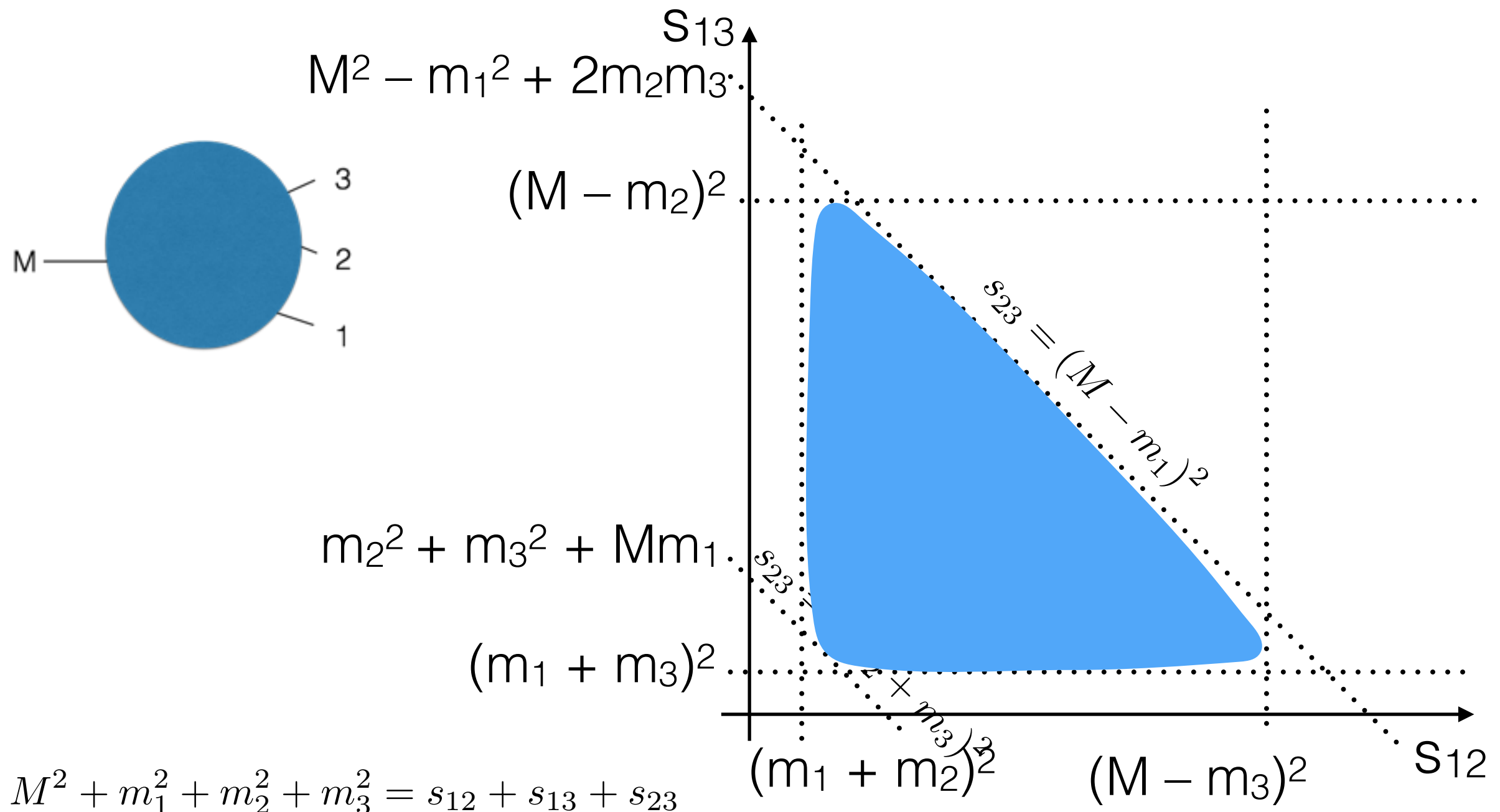
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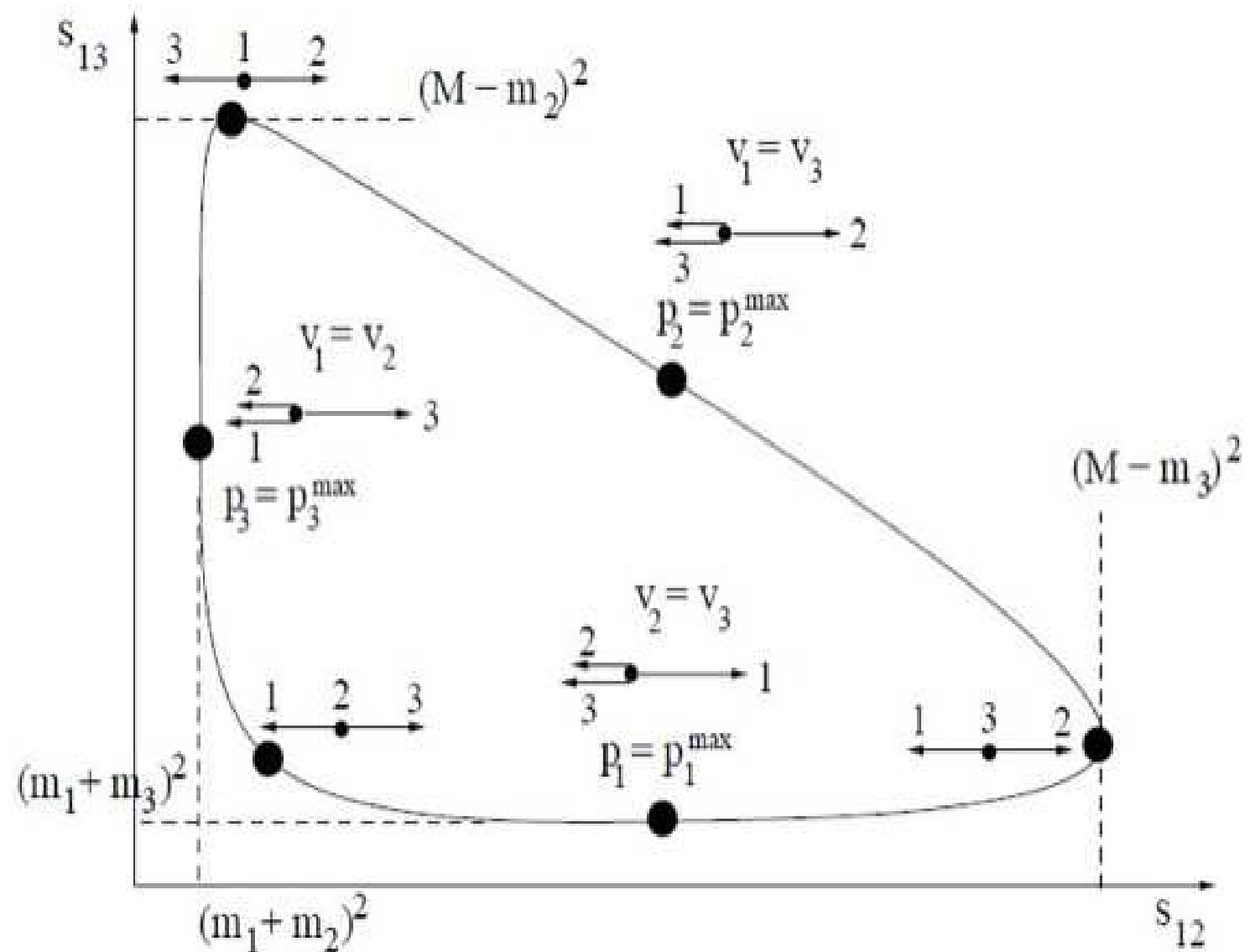
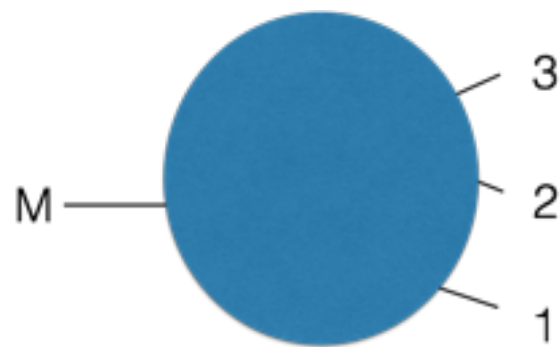
3-body phase space



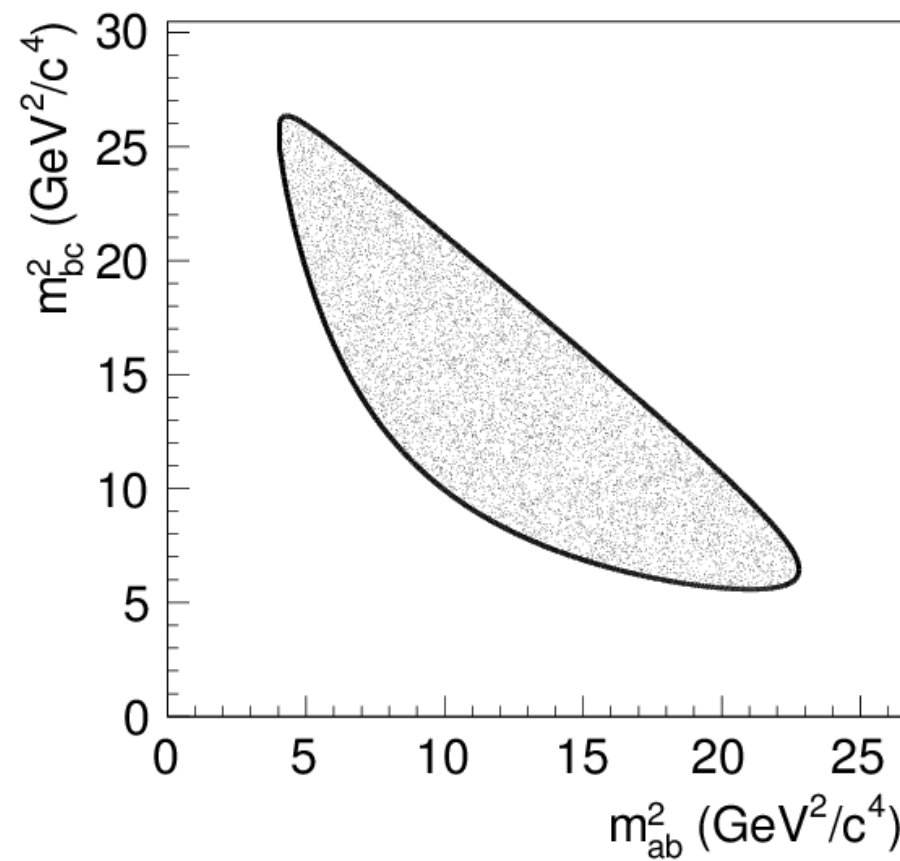
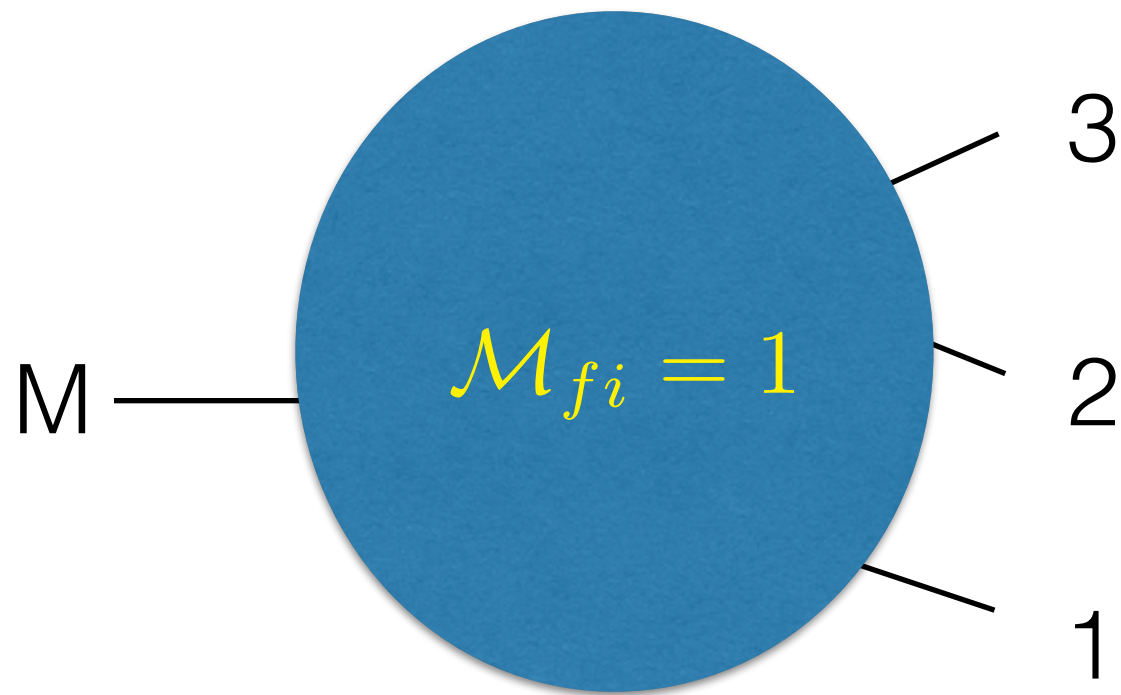
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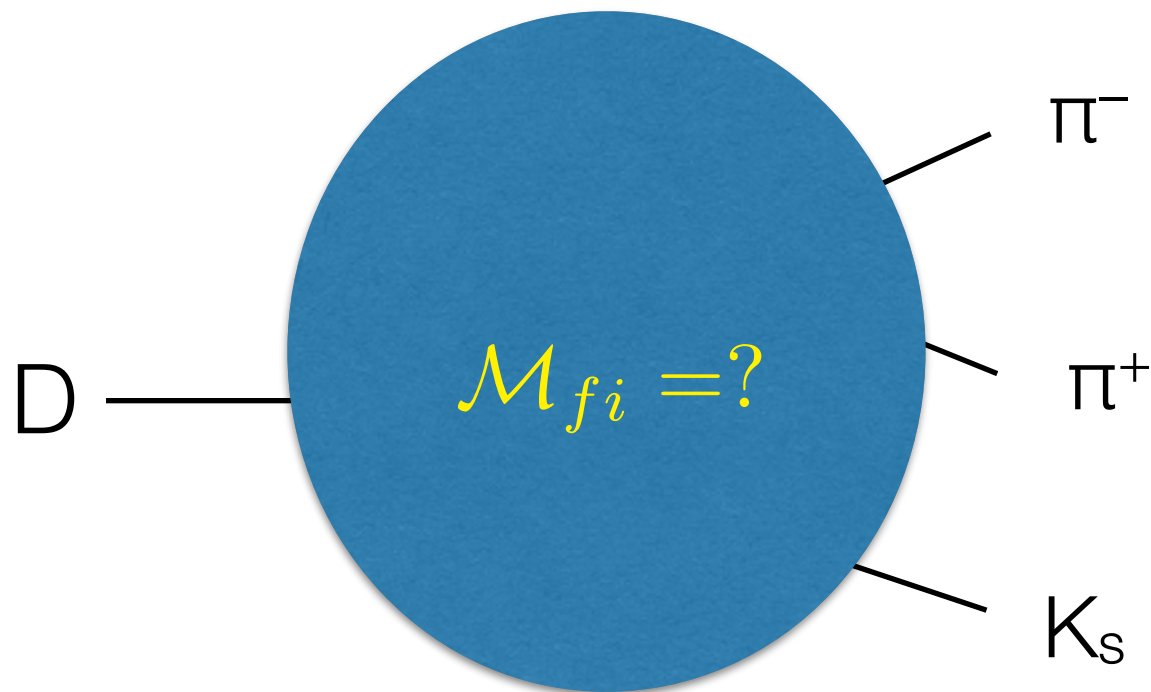
What happens if nothing happens



$$s_{ij} \equiv (p_i + p_j)^2 \equiv m_{ij}^2$$

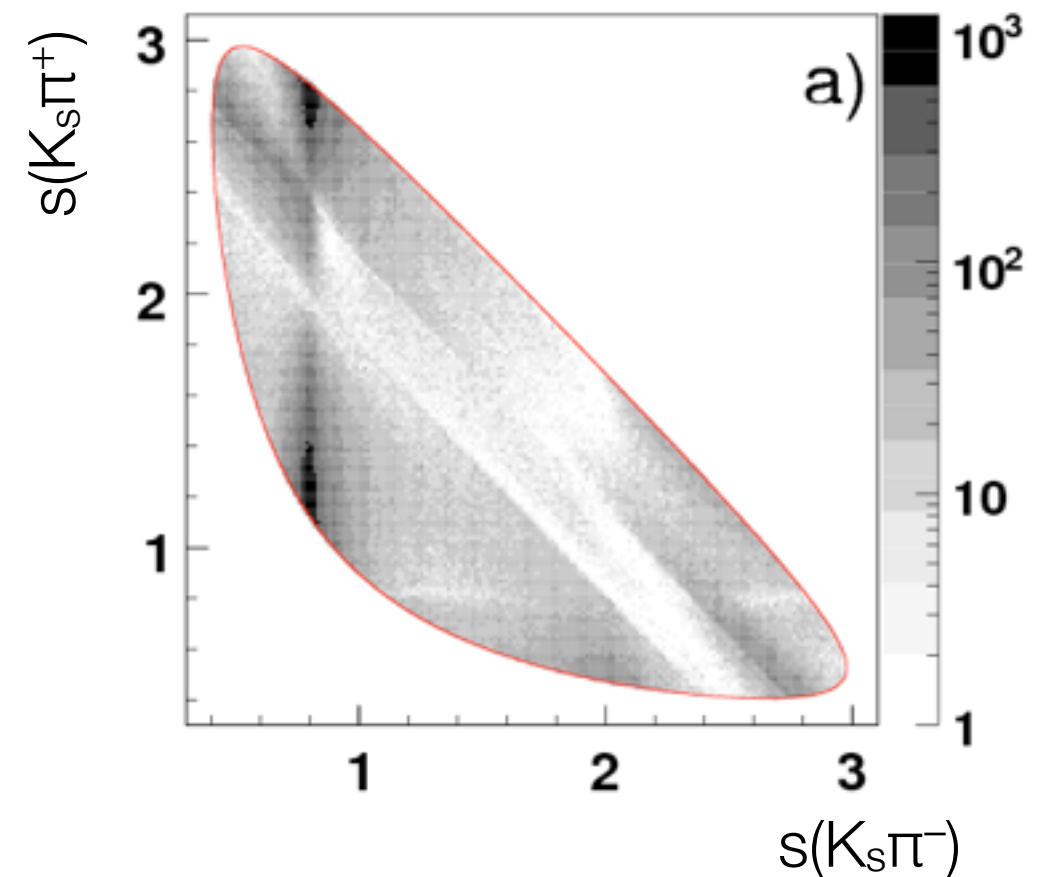
$$d\Gamma = \frac{1}{(2\pi)^2 32M^3} |\mathcal{M}_{fi}|^2 ds_{12} ds_{13}$$

What really happens



$$s_{ij} \equiv (p_i + p_j)^2 \equiv m_{ij}^2$$

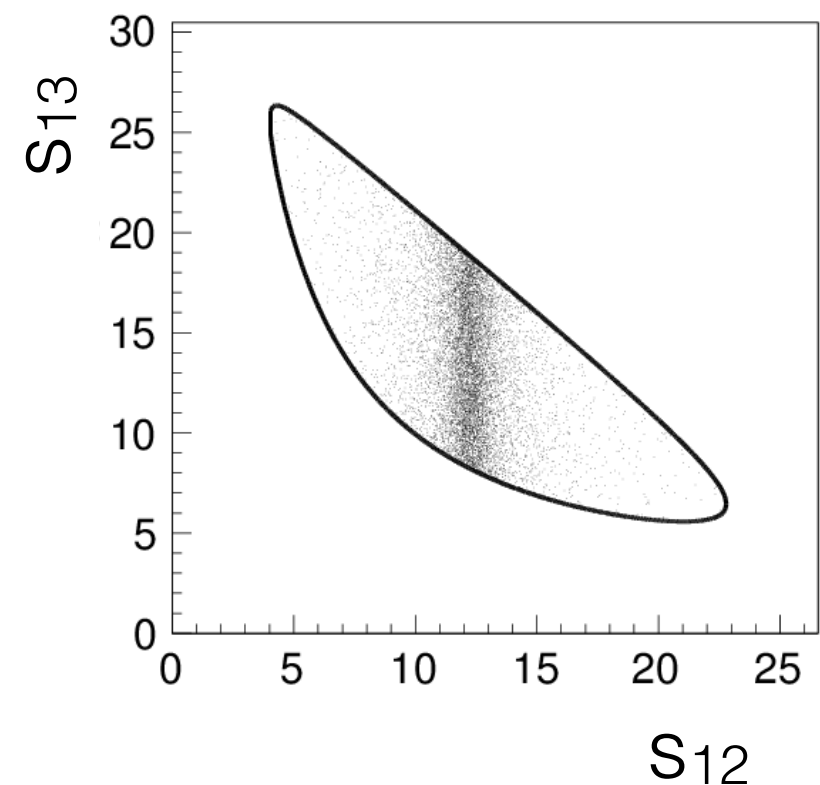
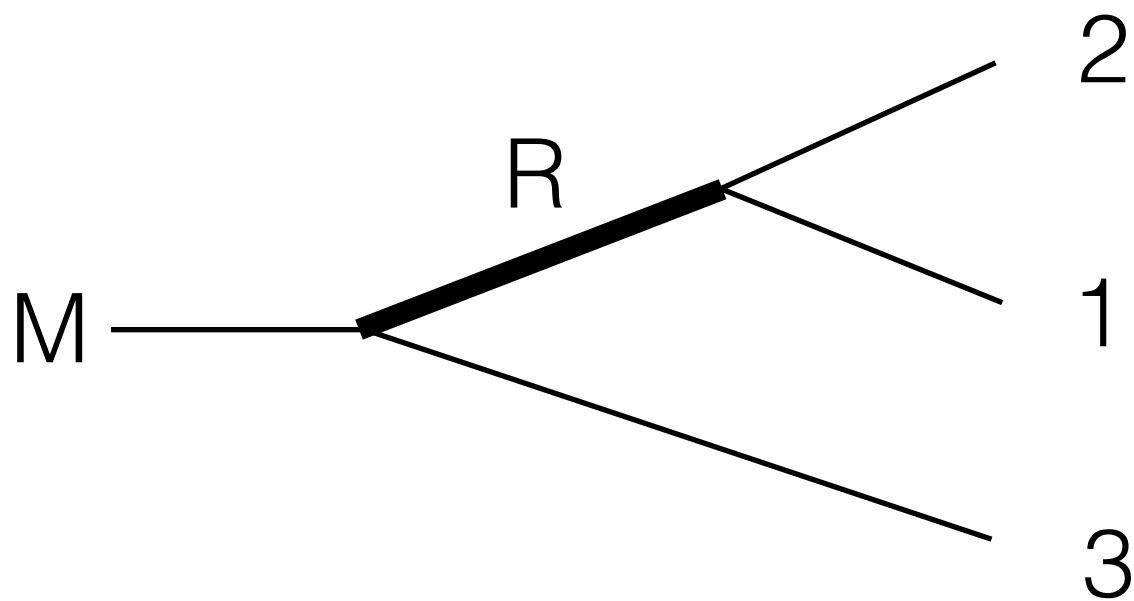
$D \rightarrow K_s \pi^+ \pi^-$



BaBar Phys. Rev. Lett. 105, 081803 (2010).

$$d\Gamma = \frac{1}{(2\pi)^2 32M^3} |\mathcal{M}_{fi}|^2 ds_{12} ds_{13}$$

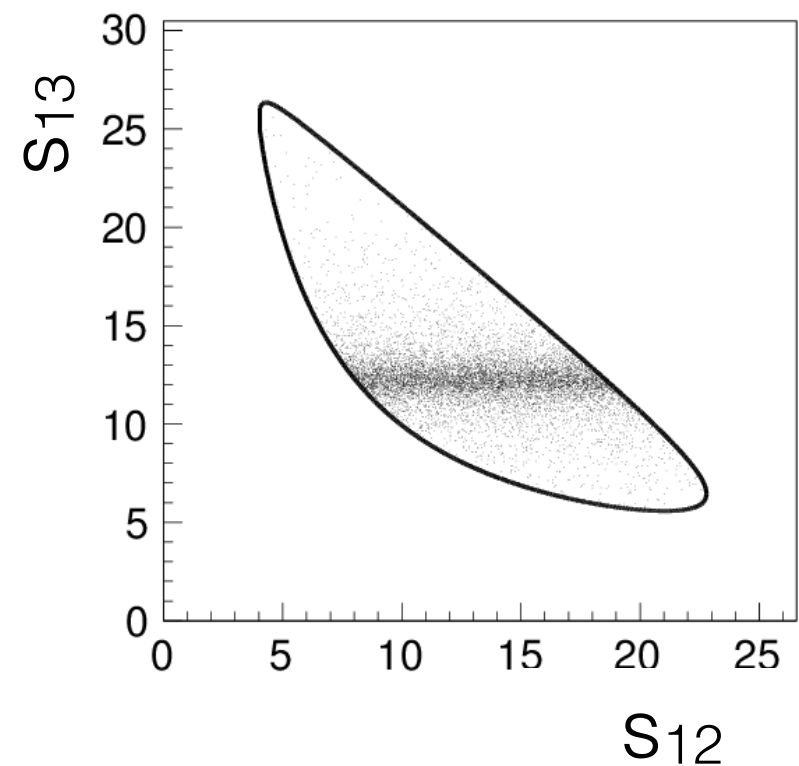
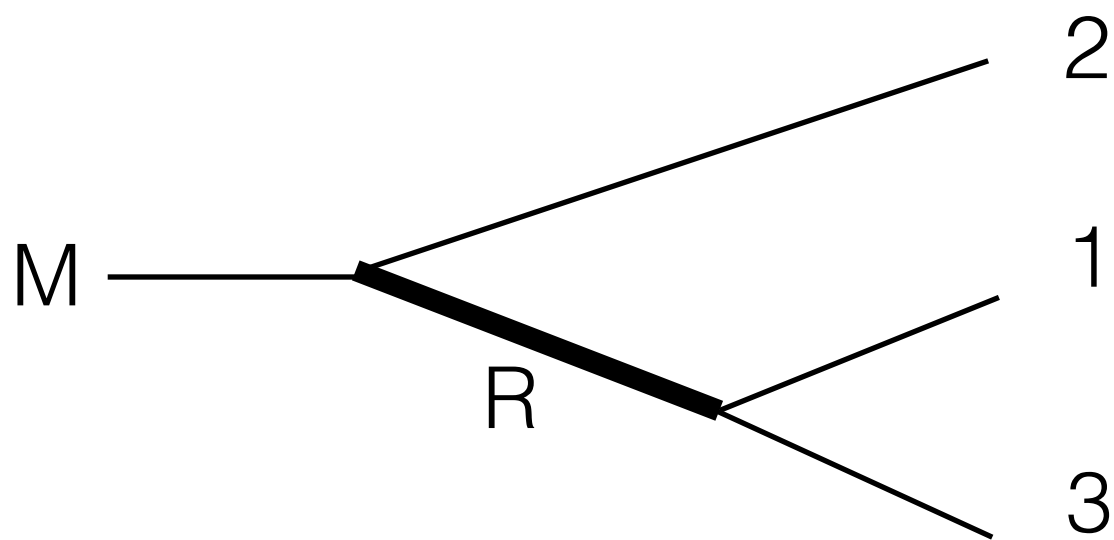
What happens if one thing happens



$$s_{ij} \equiv (p_i + p_j)^2 \equiv m_{ij}^2$$

$$d\Gamma = \frac{1}{(2\pi)^2 32M^3} |\mathcal{M}_{fi}|^2 ds_{12} ds_{13}$$

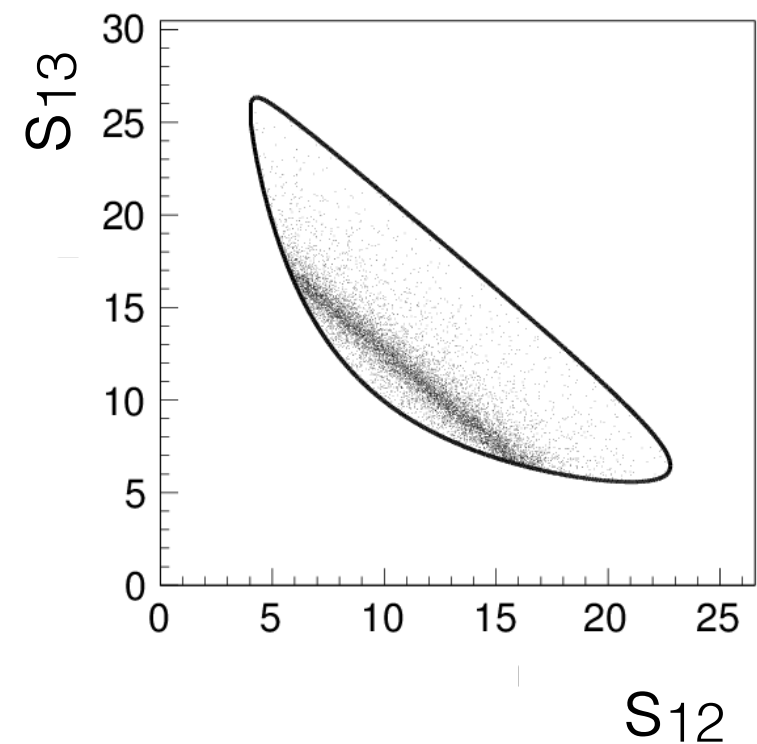
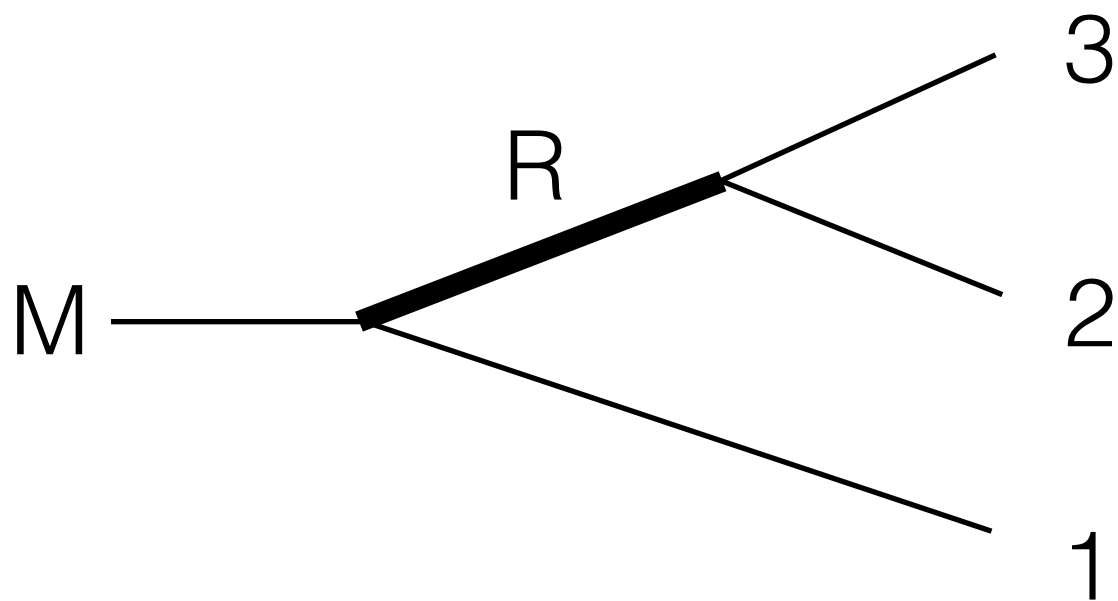
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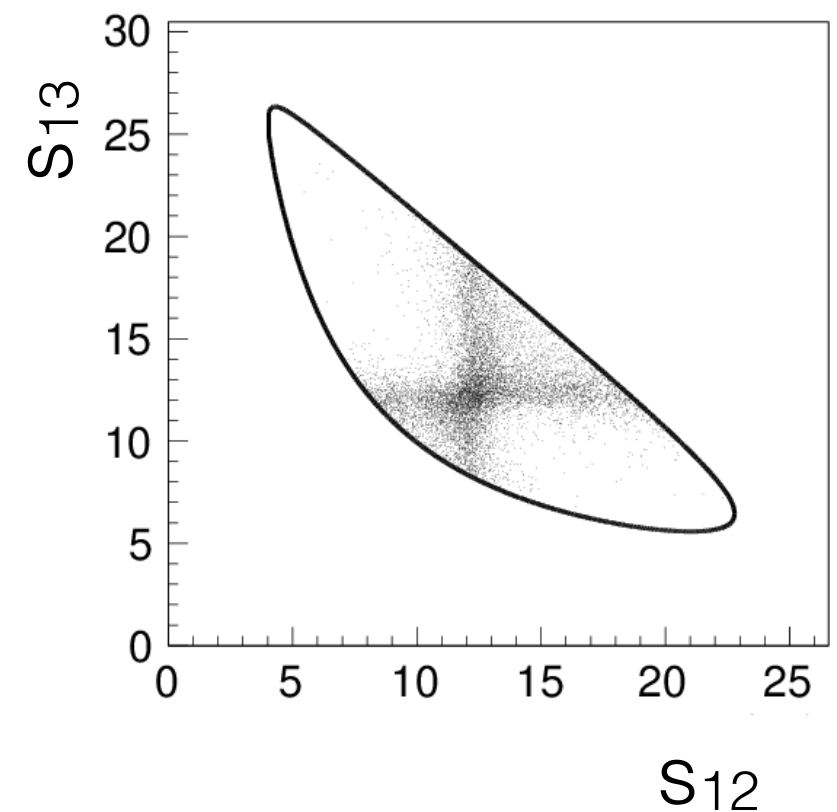
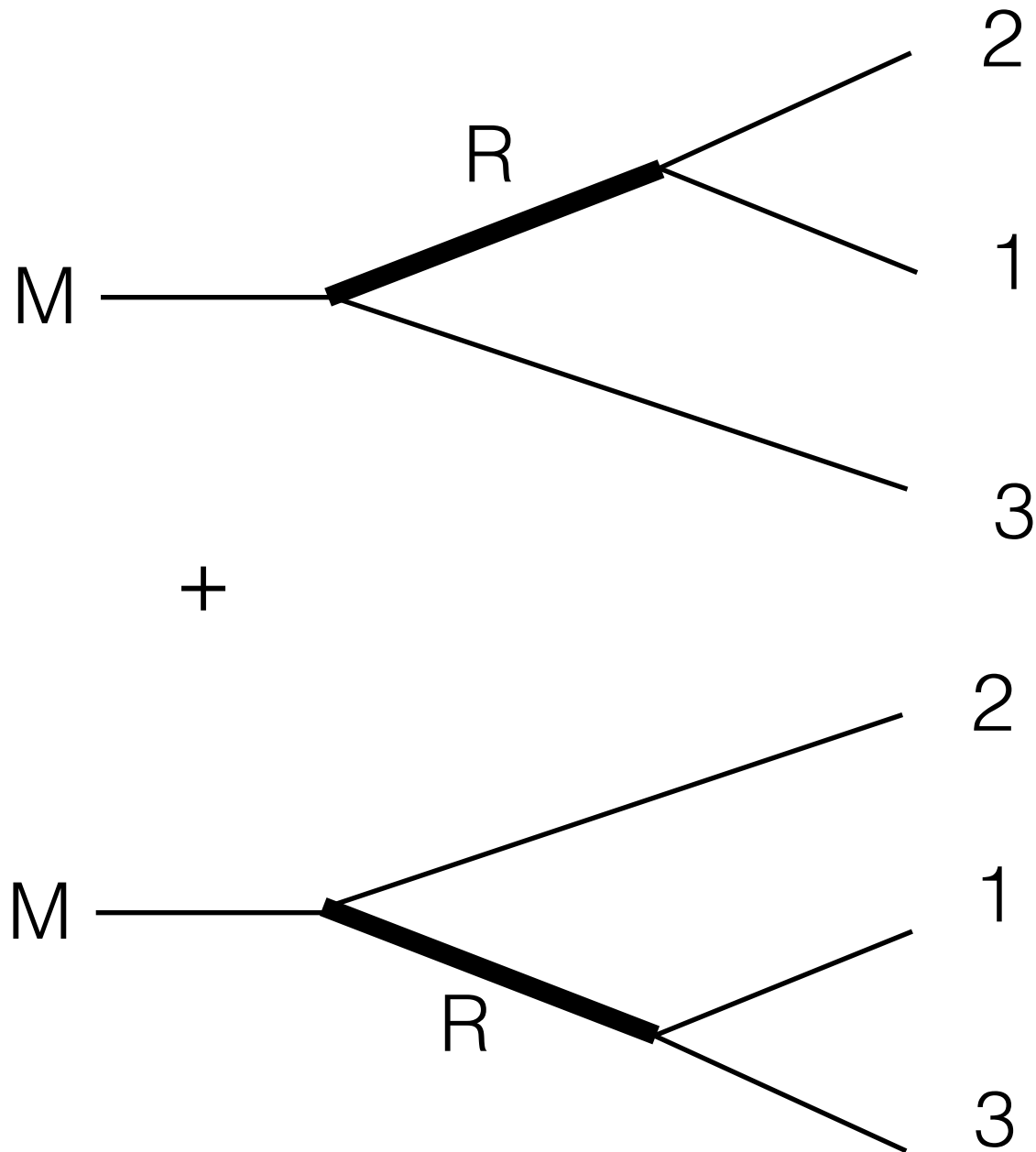
What happens if one thing happens



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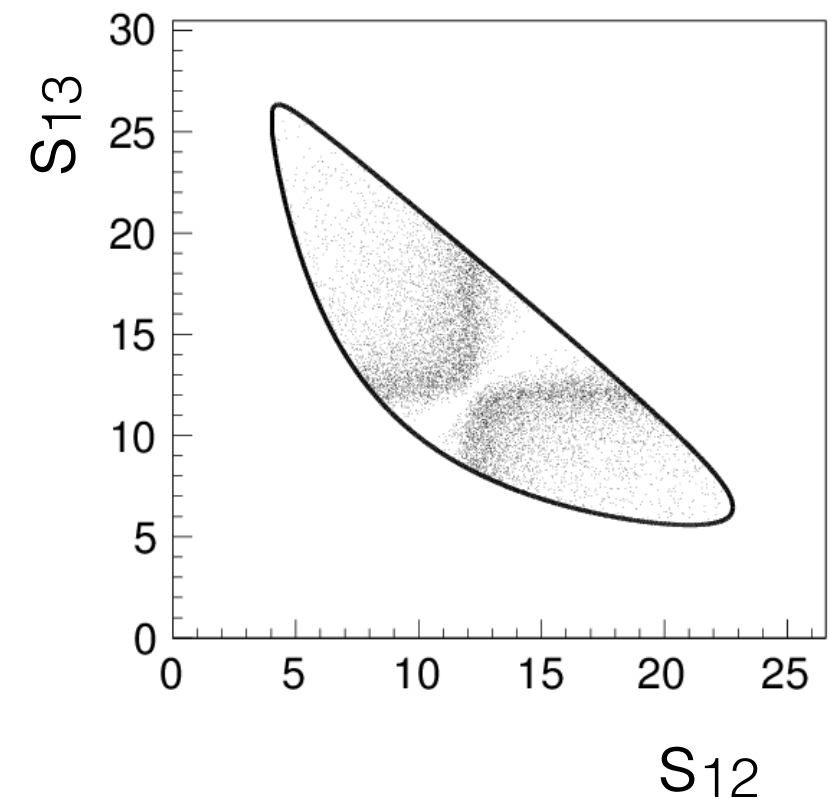
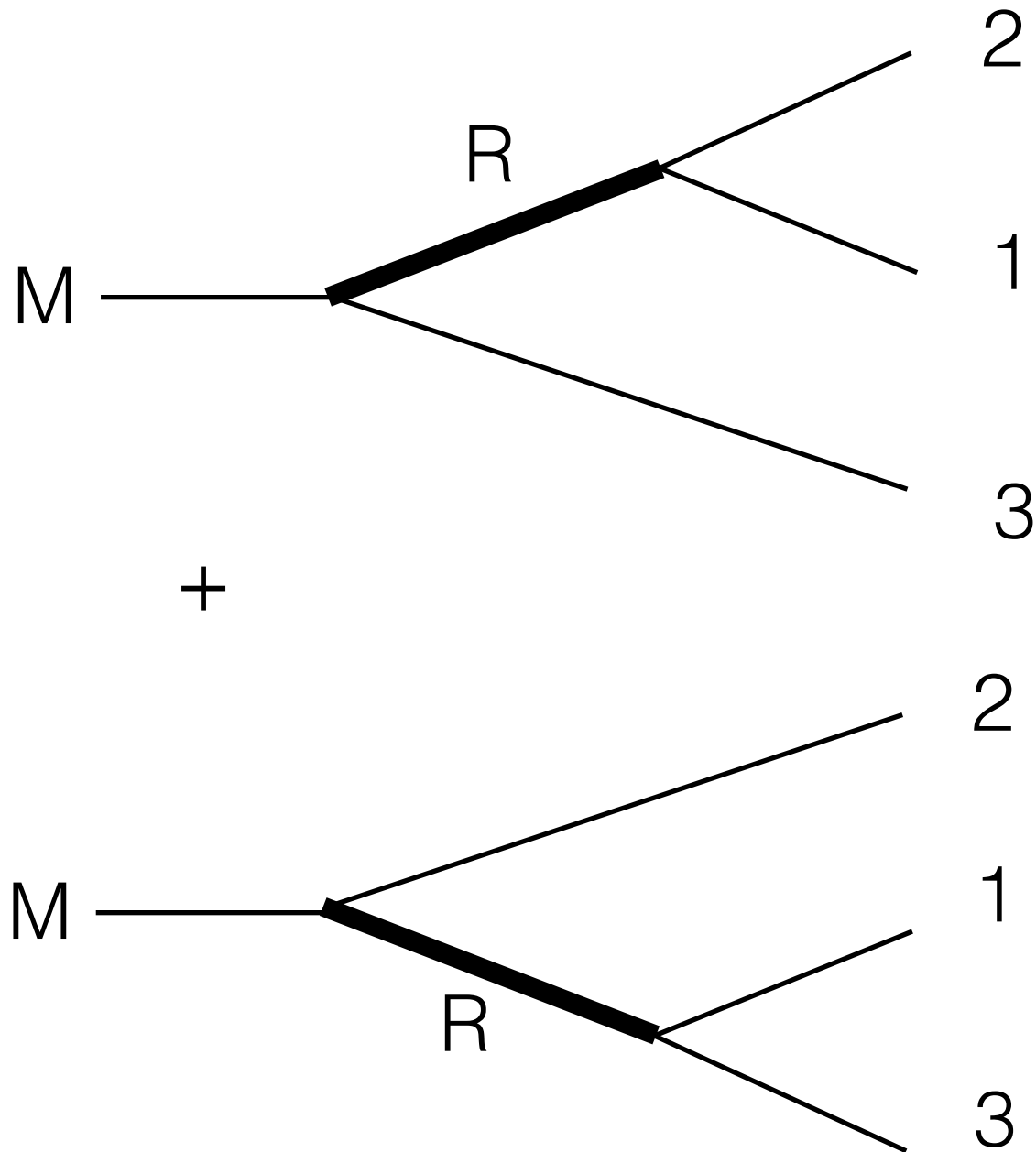
$$d\Gamma = \frac{1}{(2\pi)^2 32M^3} |\mathcal{M}_{fi}|^2 ds_{12} ds_{13}$$

What happens if two things happens



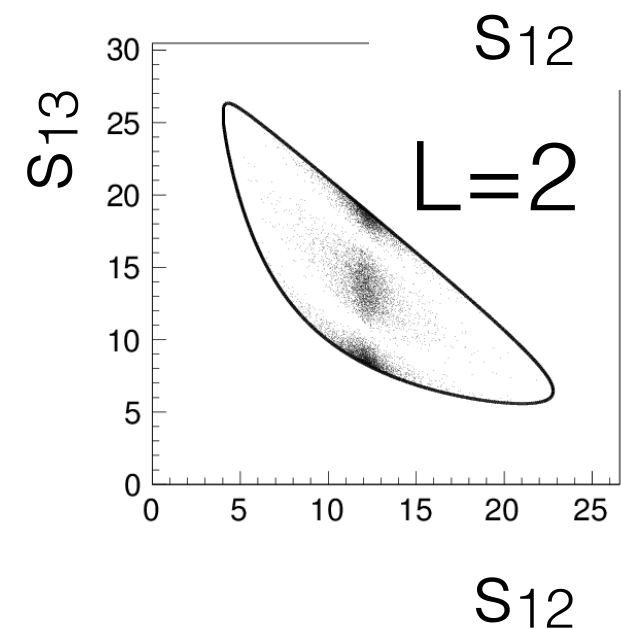
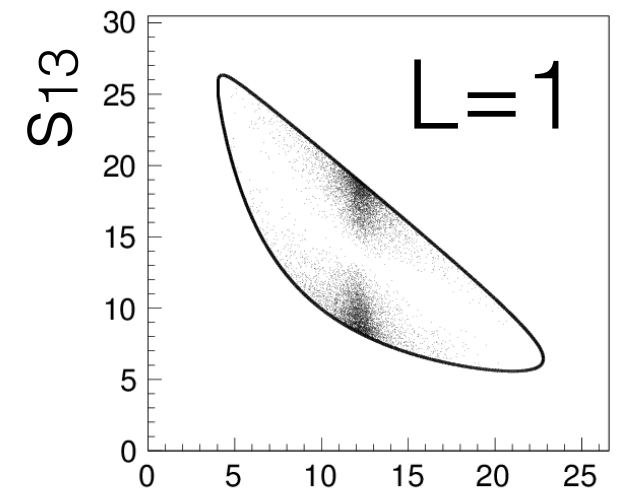
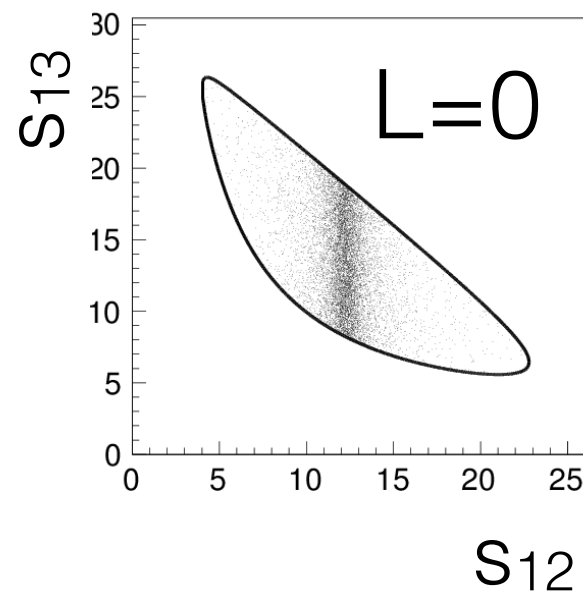
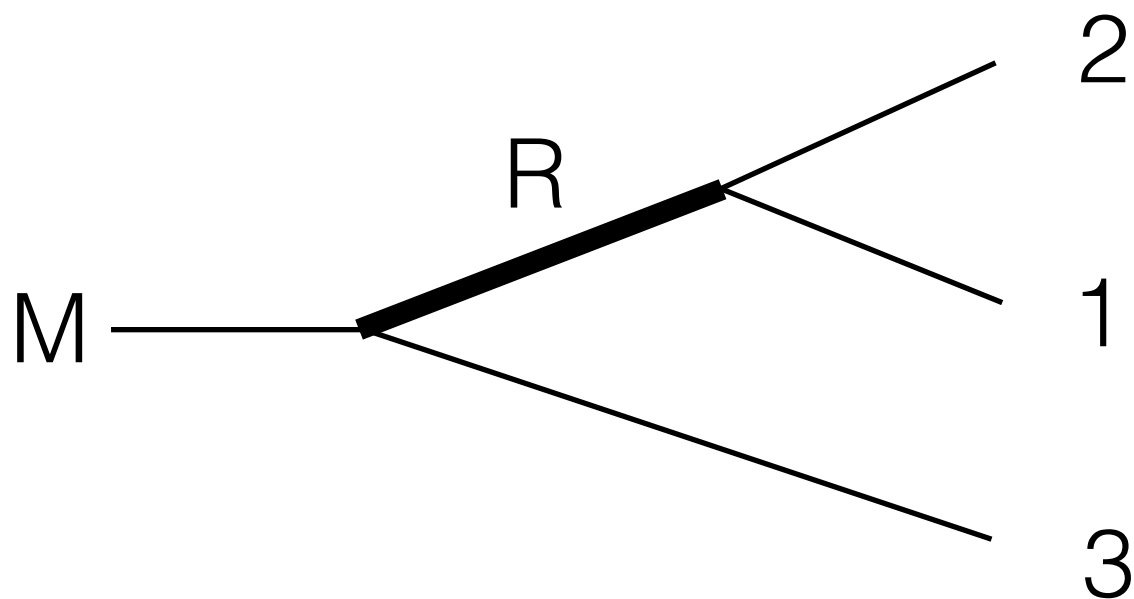
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What happens if two things happens



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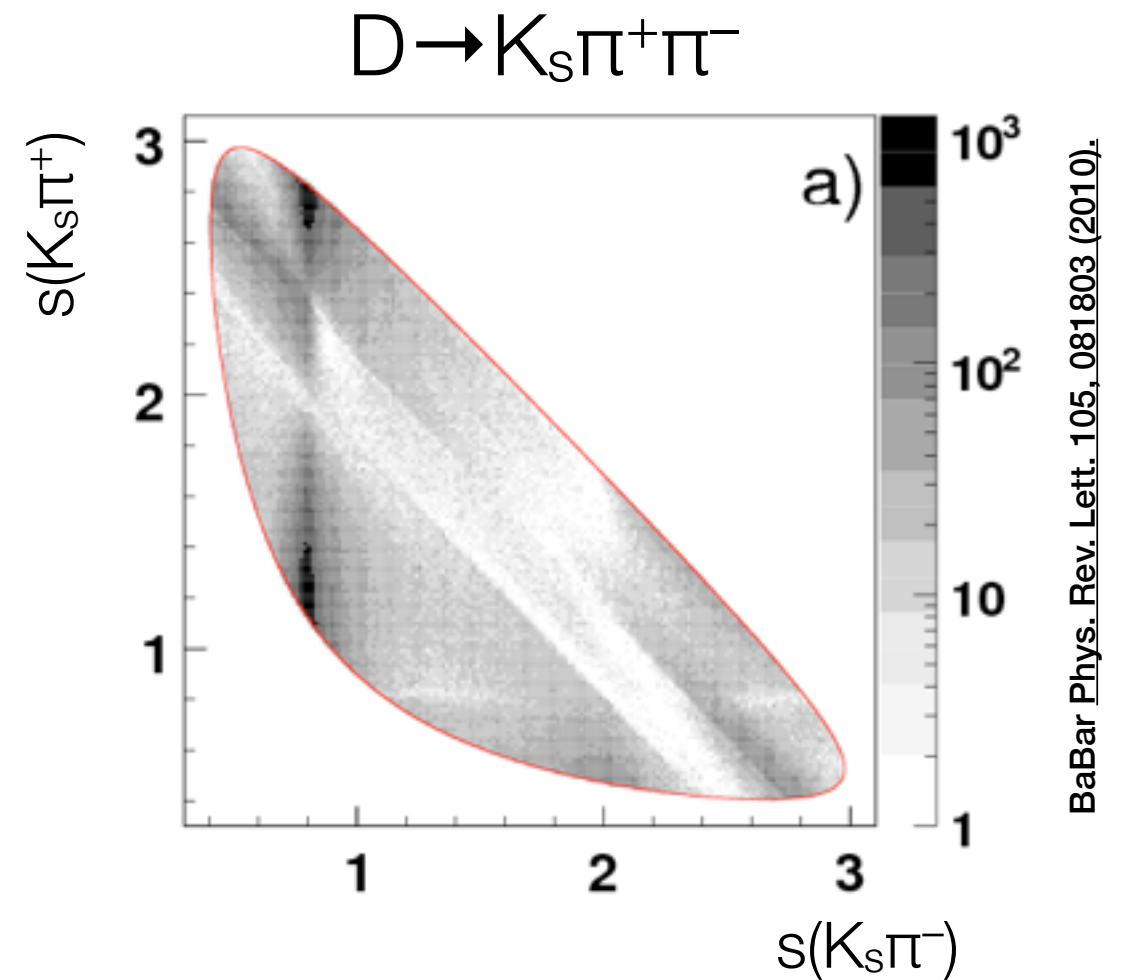
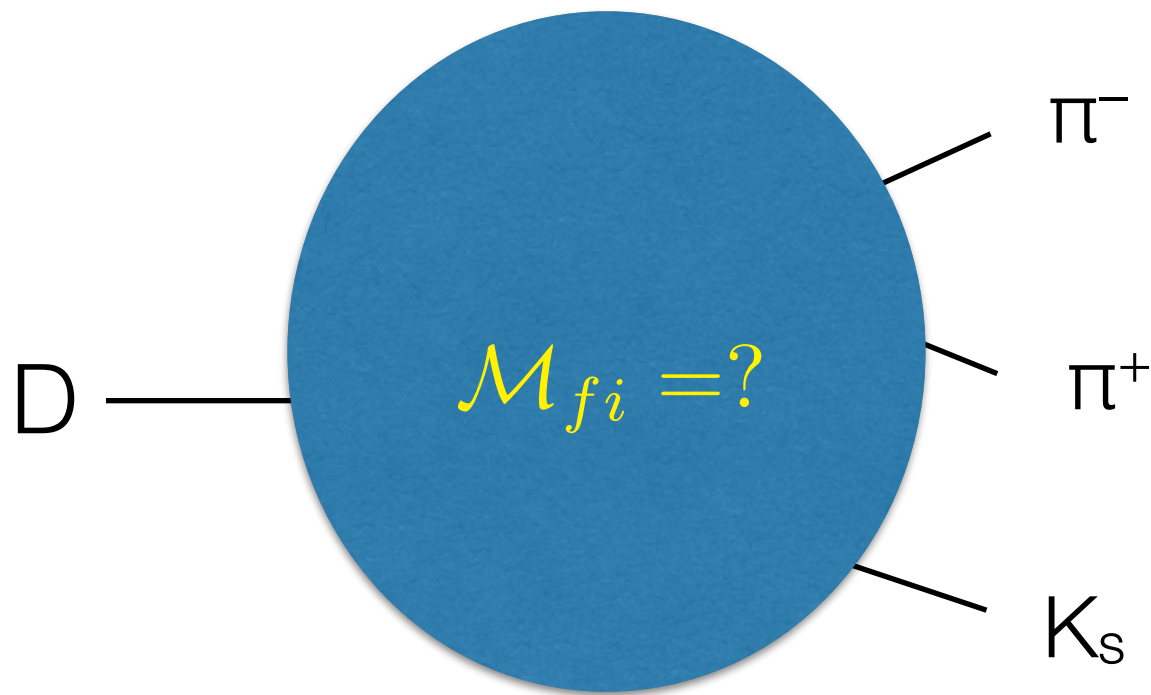
What happens if something with spin happens



$$s_{ij} \equiv (p_i + p_j)^2 \equiv m_{ij}^2$$

$$d\Gamma = \frac{1}{(2\pi)^2 32M^3} |\mathcal{M}_{fi}|^2 ds_{12} ds_{13}$$

Real dalitz plots

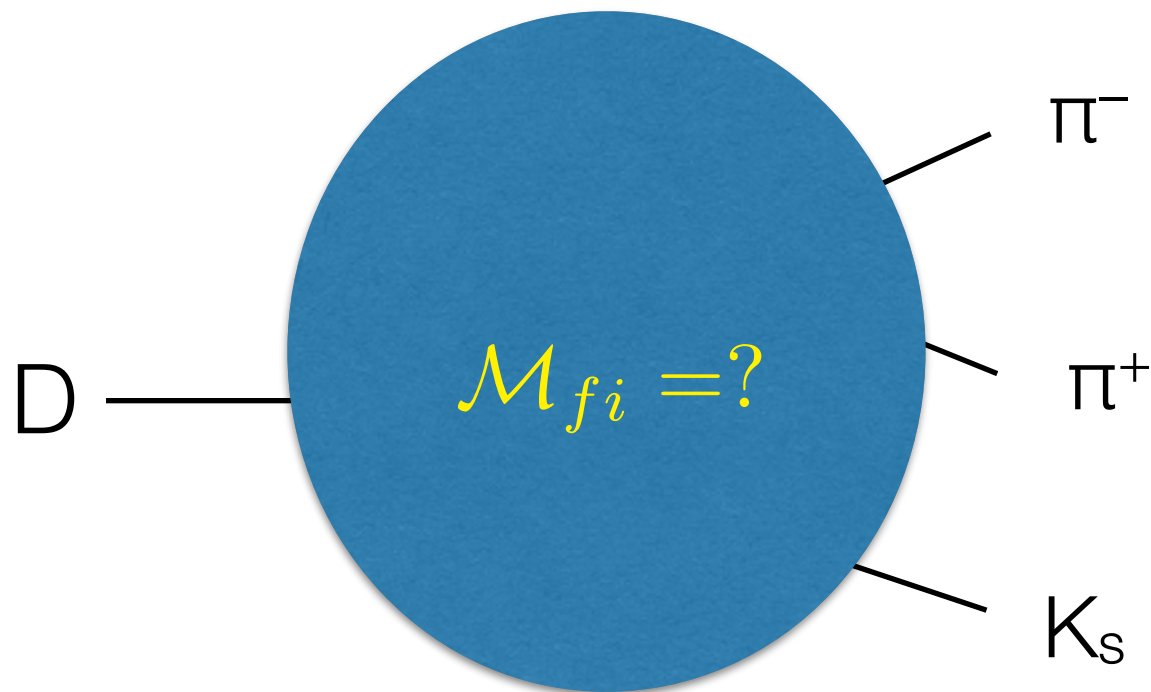


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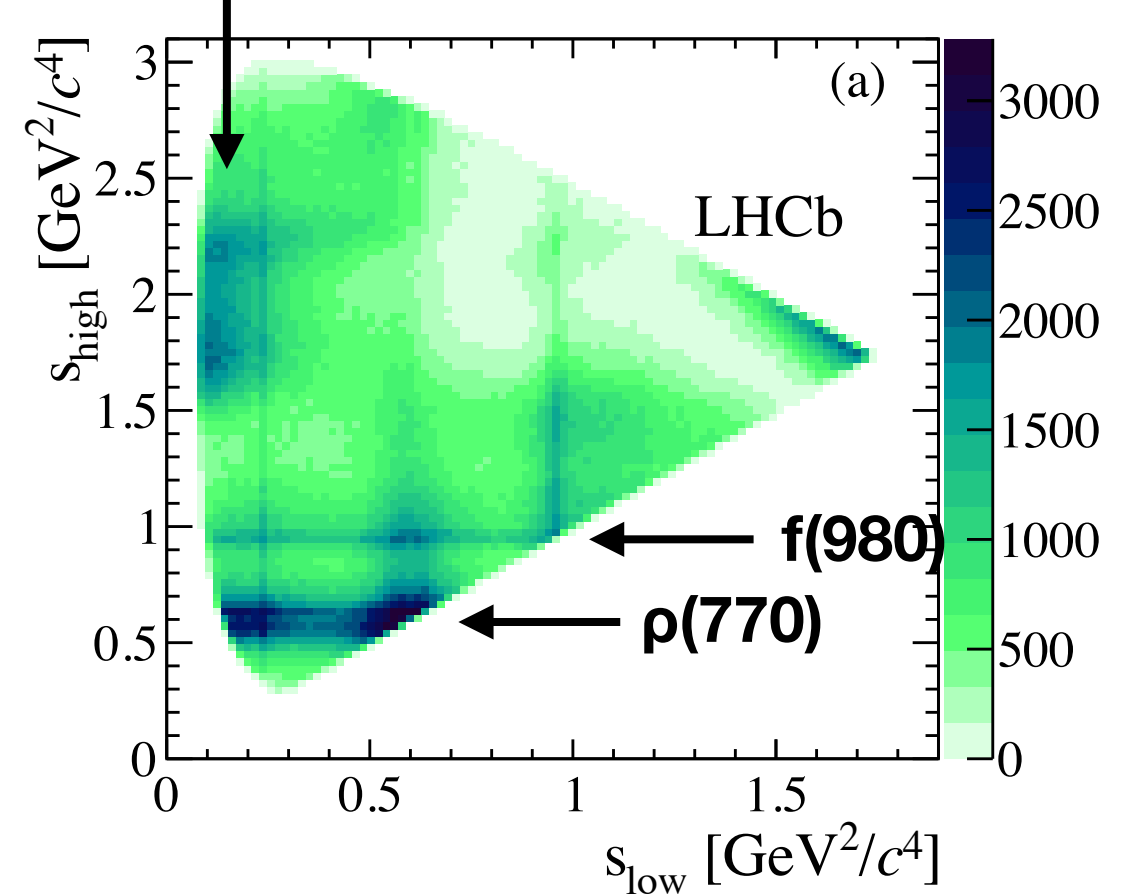
$$d\Gamma = \frac{1}{(2\pi)^2 32M^3} |\mathcal{M}_{fi}|^2 ds_{12} ds_{13}$$

Real Dalitz pots



2.4M $D^\pm \rightarrow \pi^\pm \pi^\mp \pi^\pm$ decays (LHCb)

$\sigma(500)$?

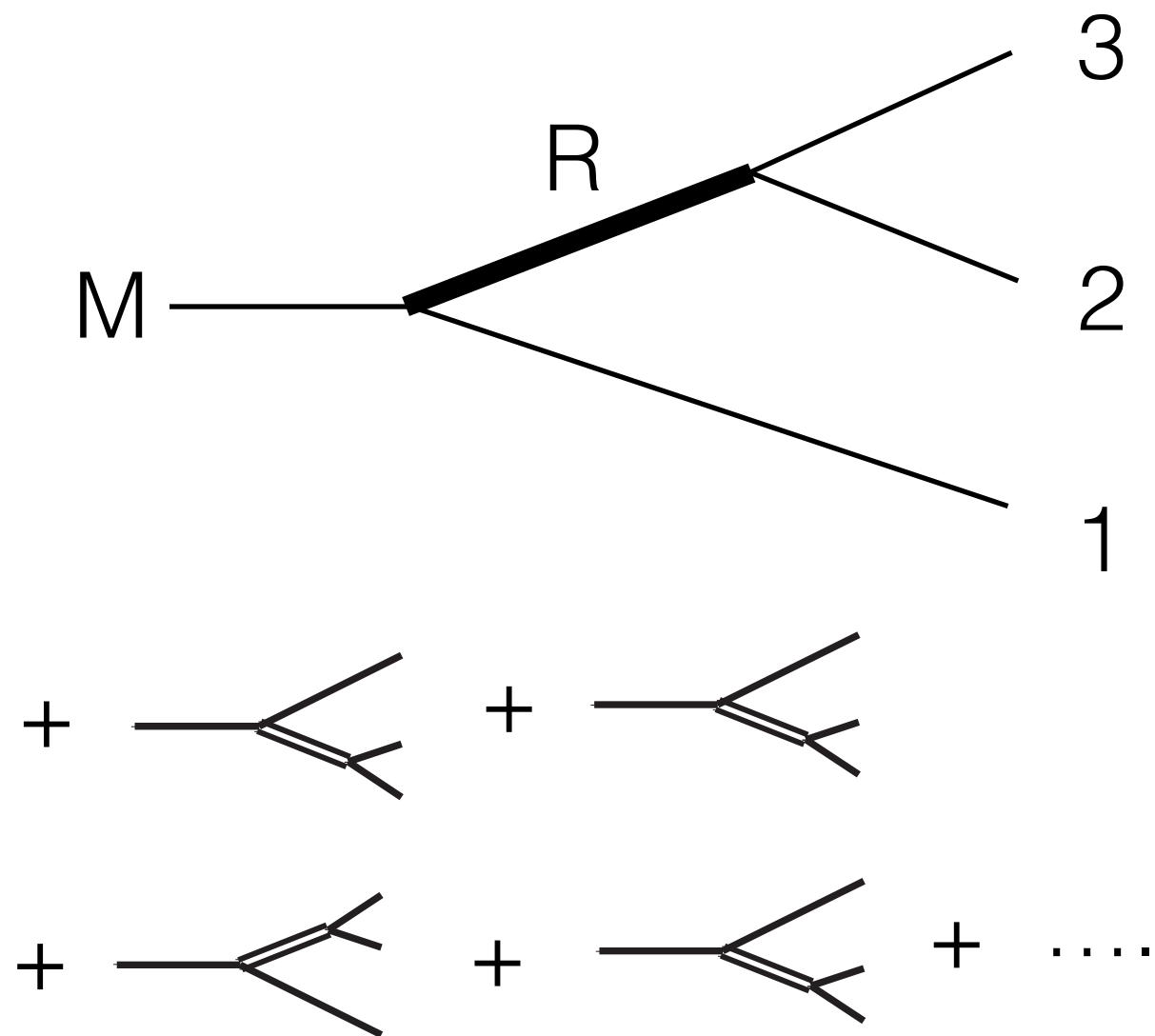


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$$s_{ij} \equiv (p_i + p_j)^2 \equiv m_{ij}^2$$

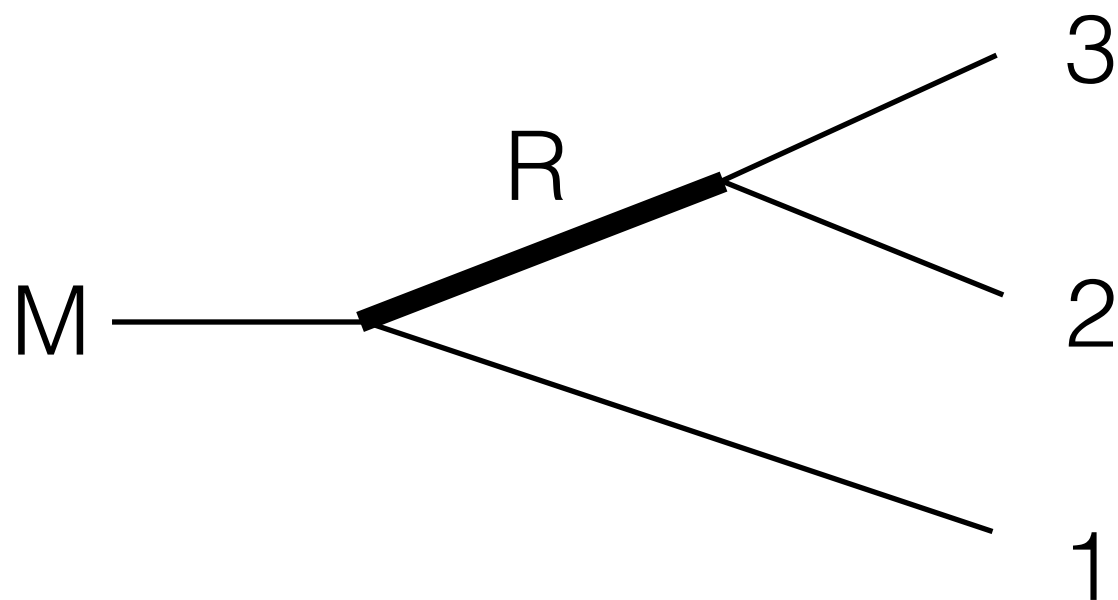
$$d\Gamma = \frac{1}{(2\pi)^2 32M^3} |\mathcal{M}_{fi}|^2 ds_{12} ds_{13}$$

Calculating amplitudes



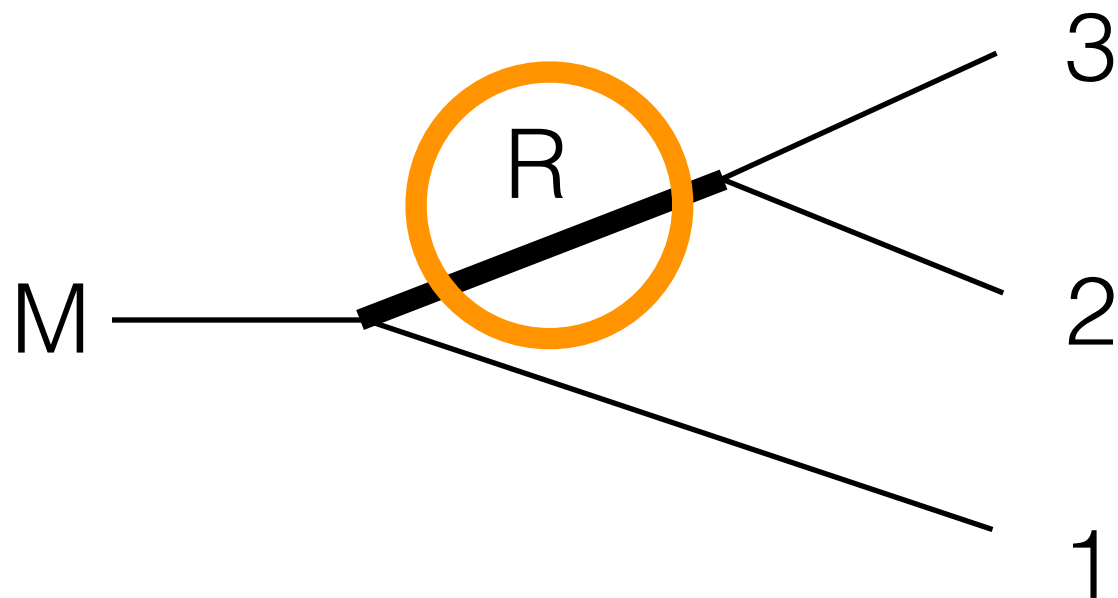
- Let us assume(!) that the full amplitude can be calculated as the sum of essentially independent two body processes.
- Doing this results in the so-called “isobar” model.

Calculating amplitudes

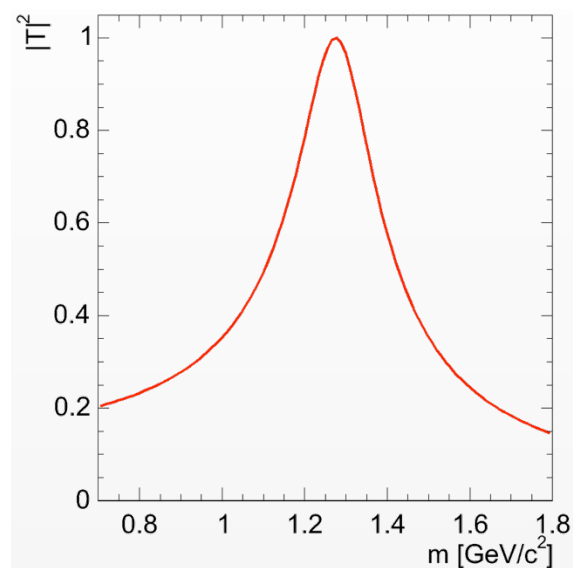


- We don't know anything about the strong interaction dynamics.
- As a first approximation, we treat each particle as point particle.
- We want a Lorentz-invariant matrix element...

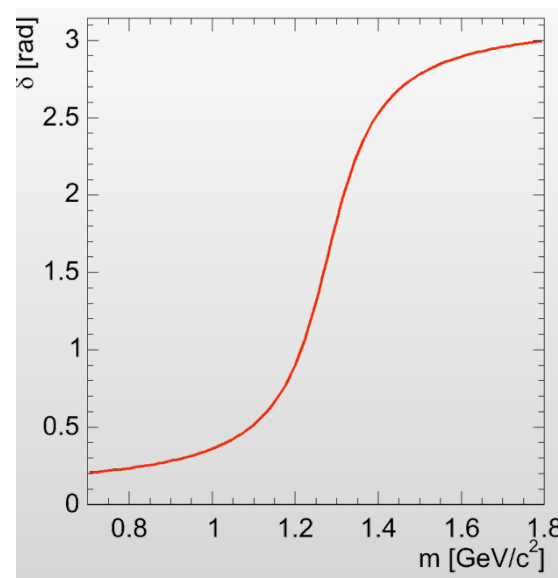
Calculating amplitudes



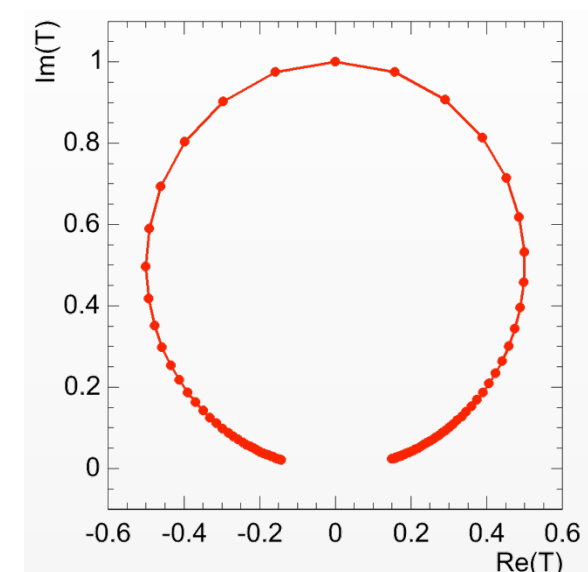
$$\frac{1}{s_{23} - m_R^2 - im_R\Gamma}$$



Jonas Rademacker: Amplitude Analyses



B-workshop



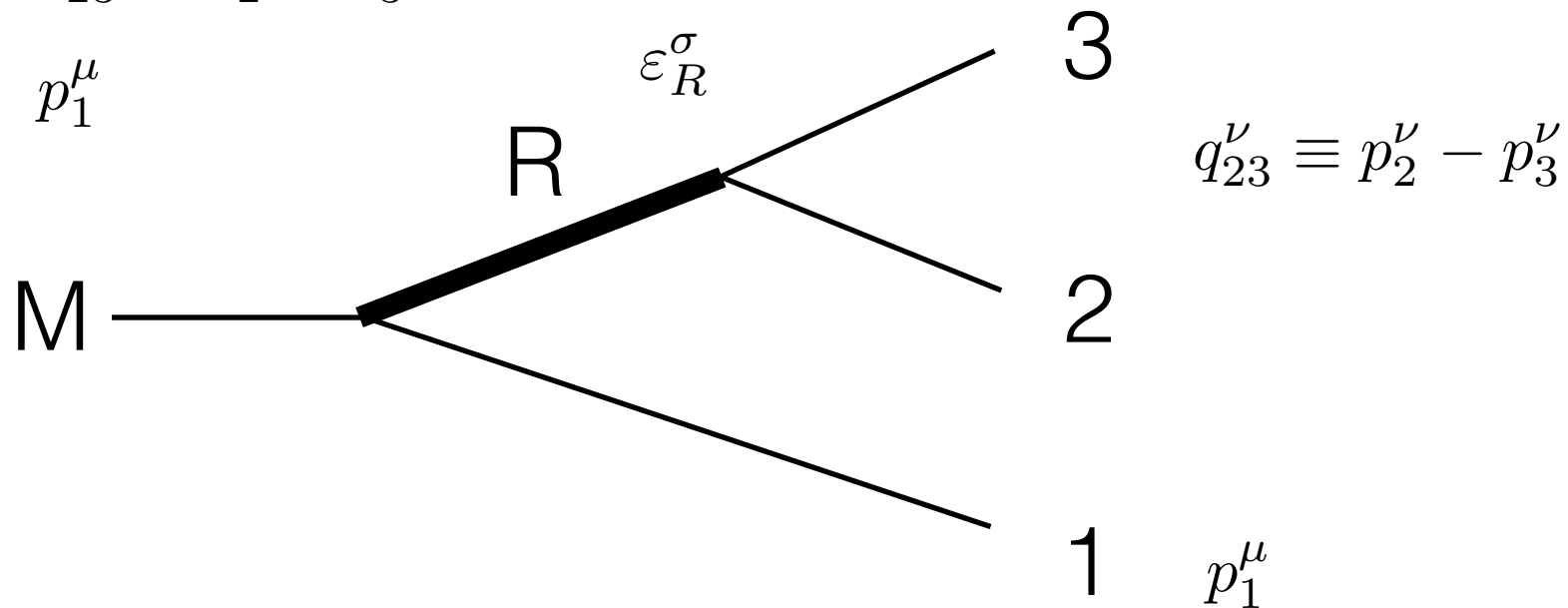
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Calculating the amplitudes

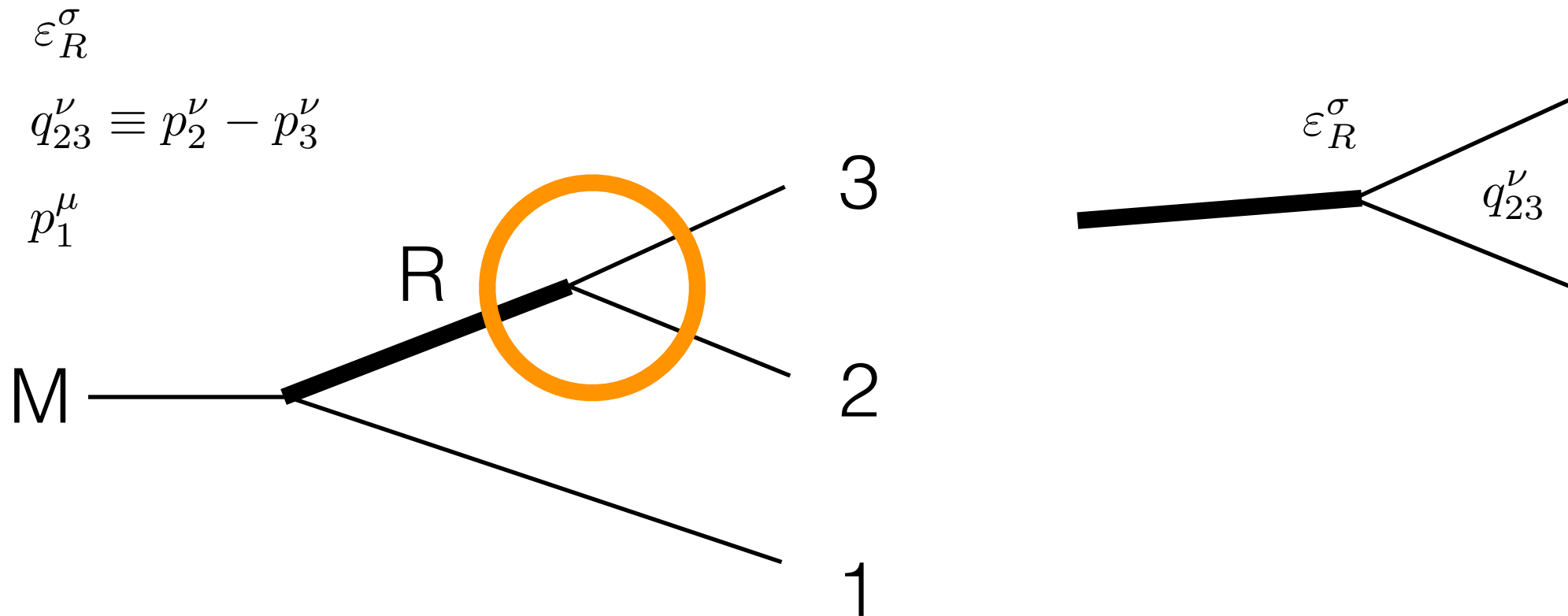
ε_R^σ say R has spin 1 (e.g. $K^*(892)$, $\rho(770)$ etc)

$$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$$

$$p_1^\mu$$

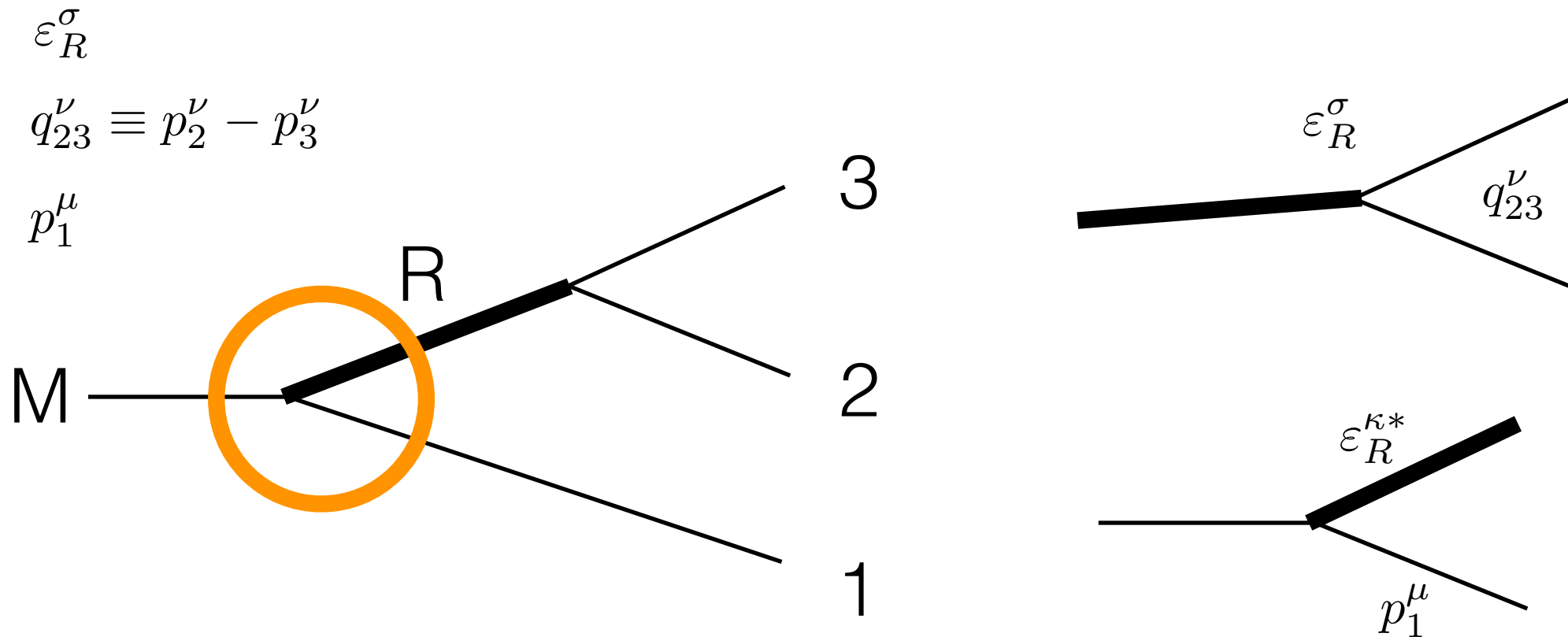


Calculating the amplitudes



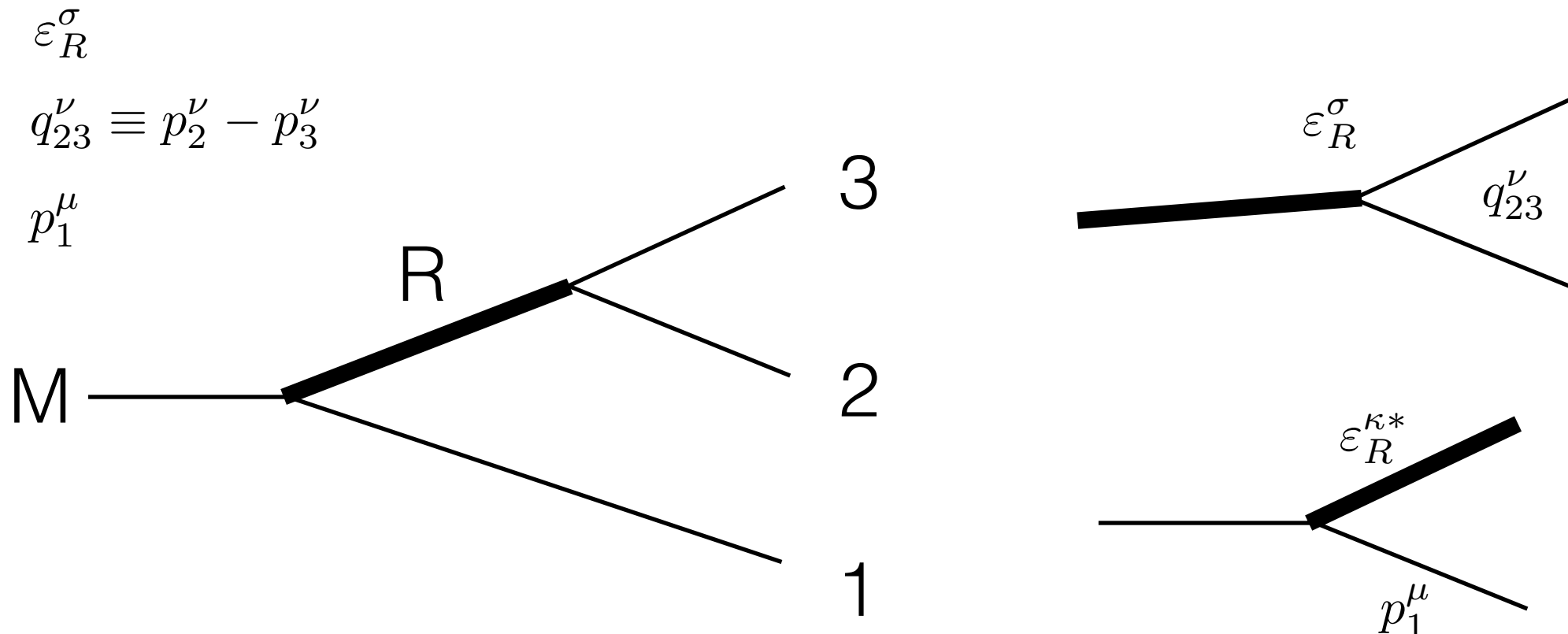
$$\frac{1}{s_{23} - m_R^2 - im_R\Gamma}$$

Calculating the amplitudes



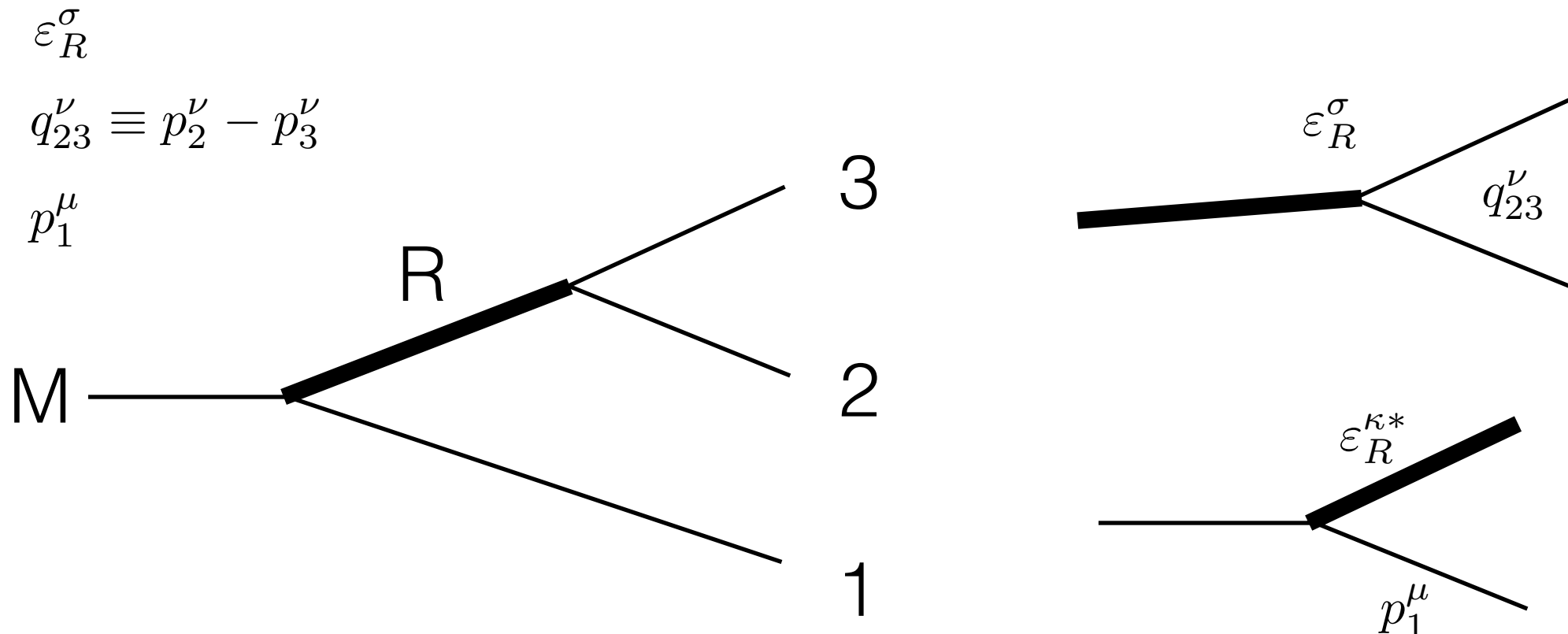
$$p_{1\mu} \varepsilon_R^{\mu*} \frac{1}{s_{23} - m_R^2 - im_R\Gamma} \varepsilon_R^\nu q_{23\nu}$$

Calculating the amplitudes



$$\sum_{\text{all } \lambda} p_{1\mu} \varepsilon_R^{\lambda\mu*} \frac{1}{s_{23} - m_R^2 - im_R\Gamma} \varepsilon_R^{\lambda\nu} q_{23\nu}$$

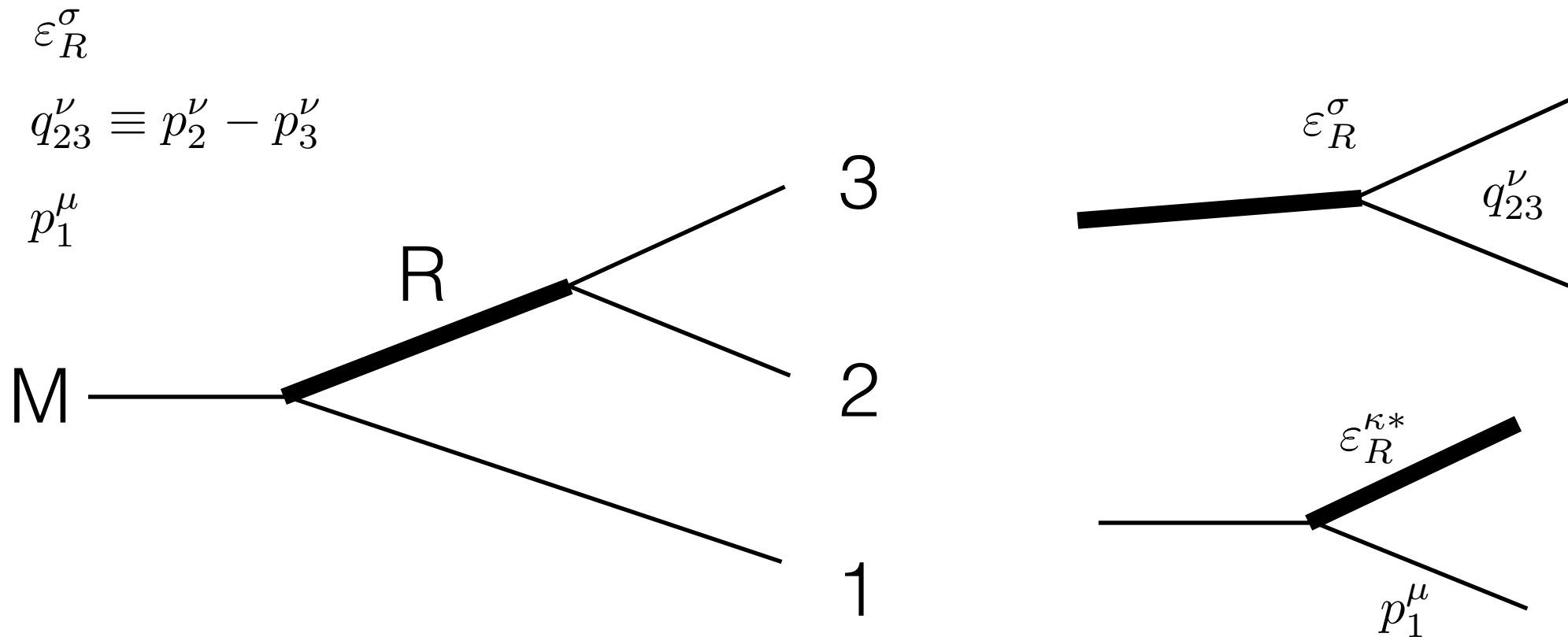
Calculating the amplitudes



$$\sum_{\text{all } \lambda} \varepsilon_R^{\lambda\mu*} \varepsilon_R^{\lambda\nu} = -g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}$$

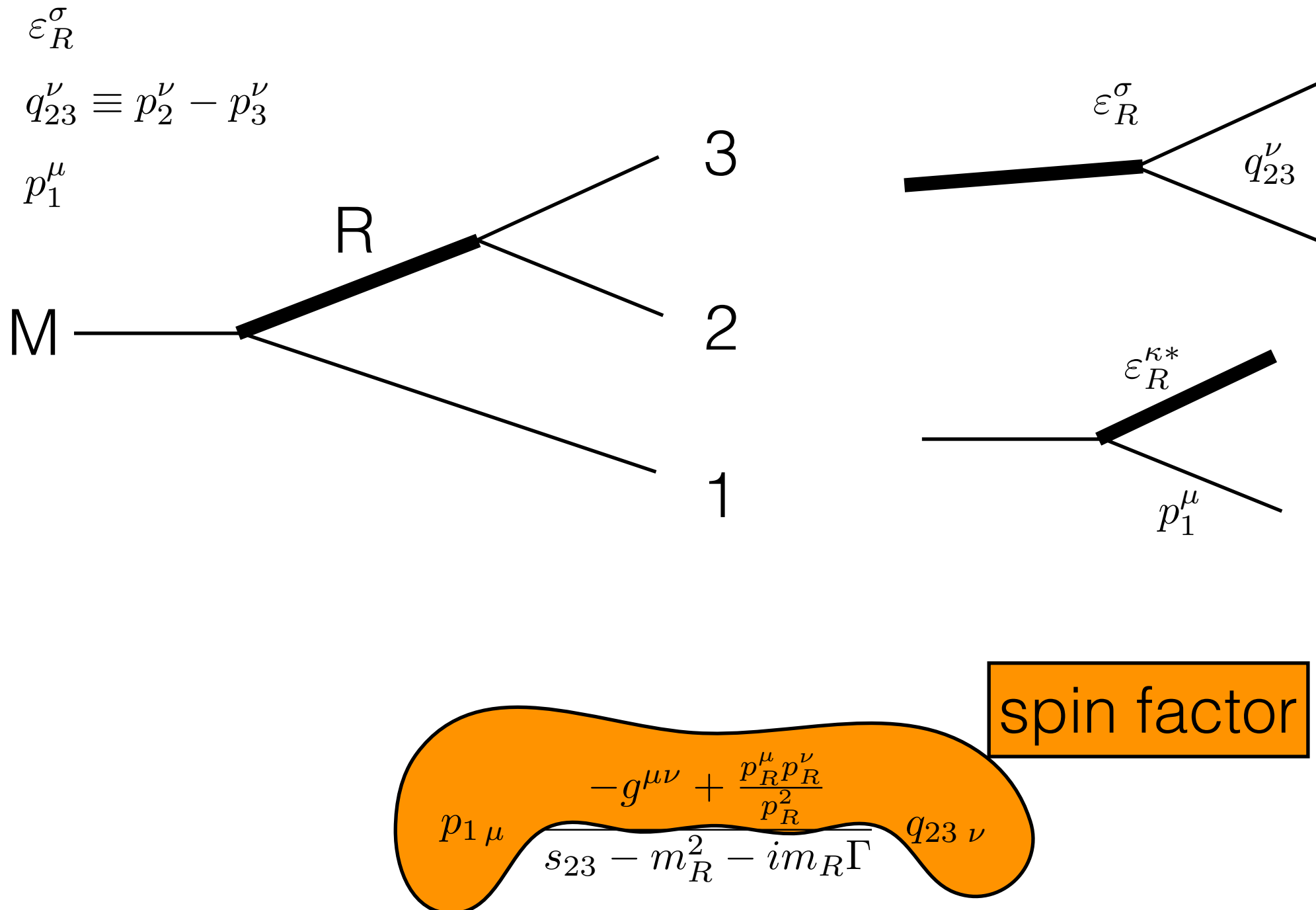
$$\sum_{\text{all } \lambda} p_{1\mu} \varepsilon_R^{\lambda\mu*} \frac{1}{s_{23} - m_R^2 - im_R\Gamma} \varepsilon_R^{\lambda\nu} q_{23\nu}$$

Calculating the amplitudes

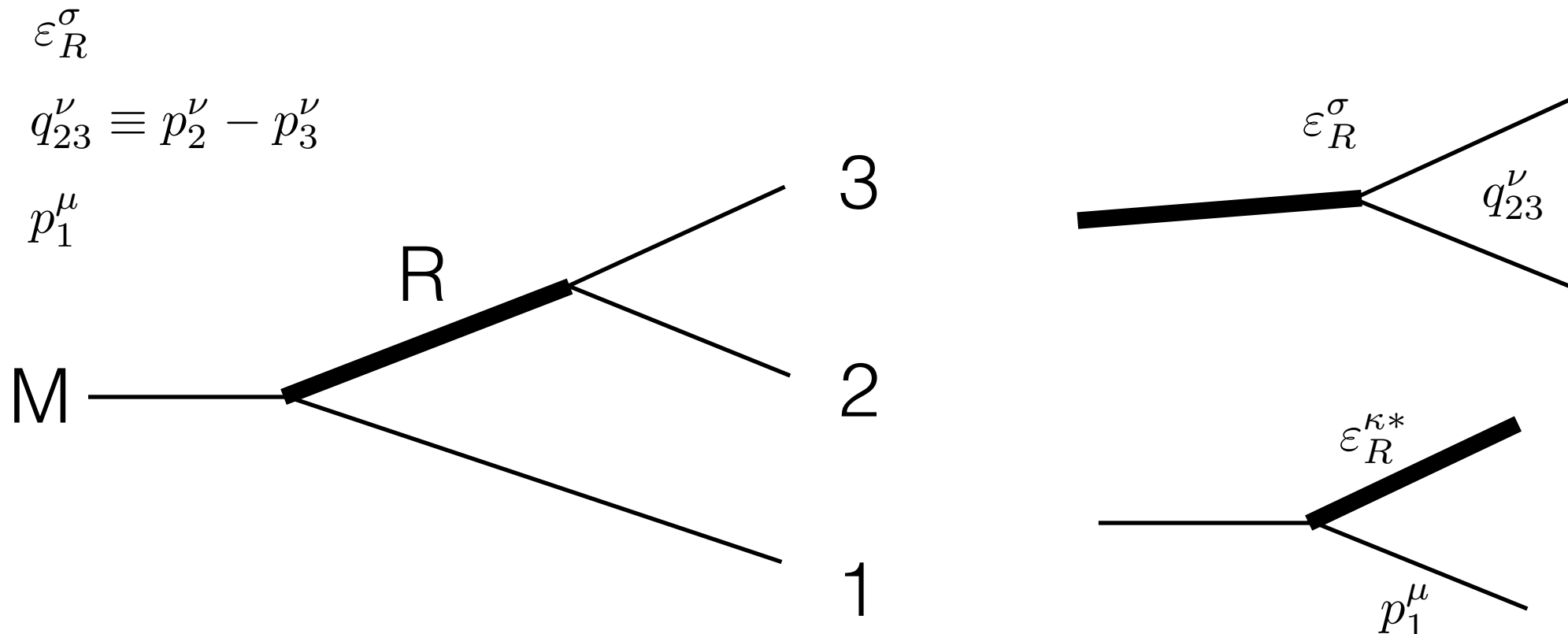


$$p_{1\mu} \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - im_R \Gamma} q_{23\nu}$$

Calculating the amplitudes



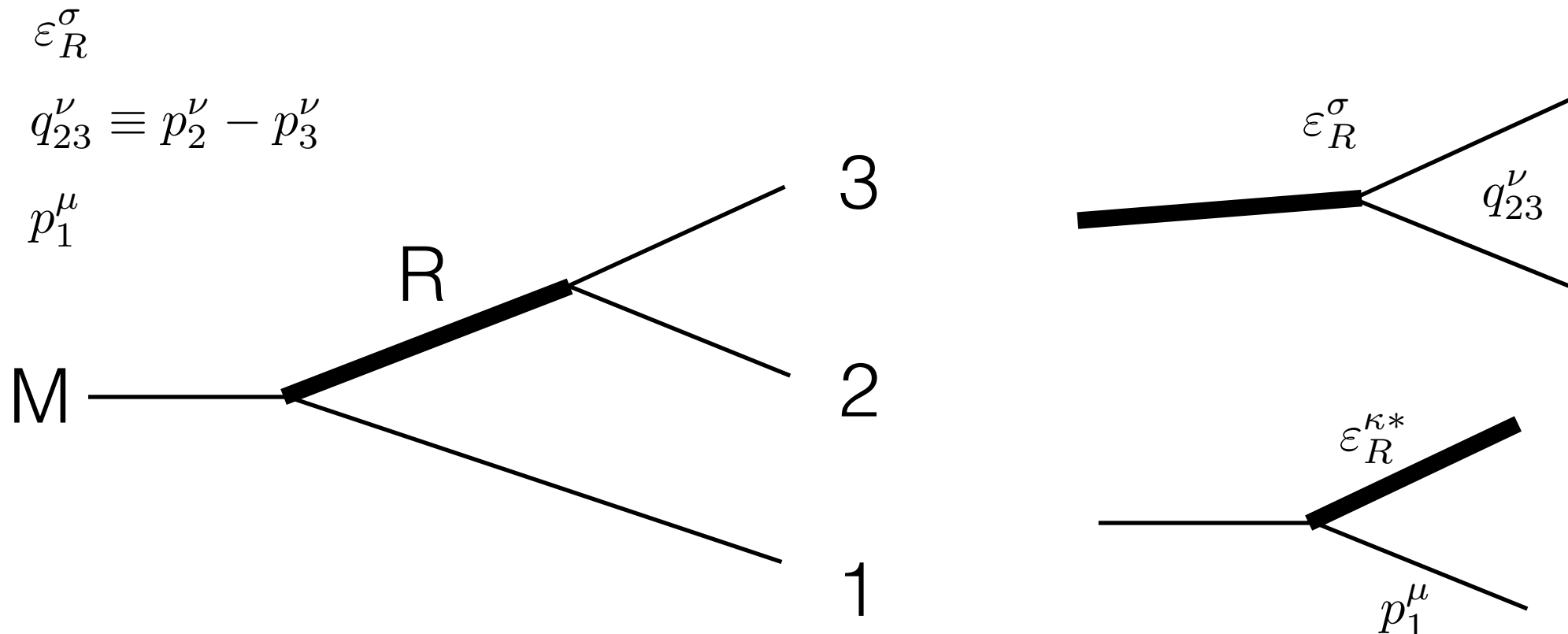
Calculating the amplitudes



$$\left(-p_1 \cdot q_{23} + \frac{(p_1 \cdot p_R)(q_{23} \cdot p_R)}{p_R^2} \right) \frac{1}{s_{23} - m_R^2 - im_R \Gamma}$$

spin factor

Calculating the amplitudes

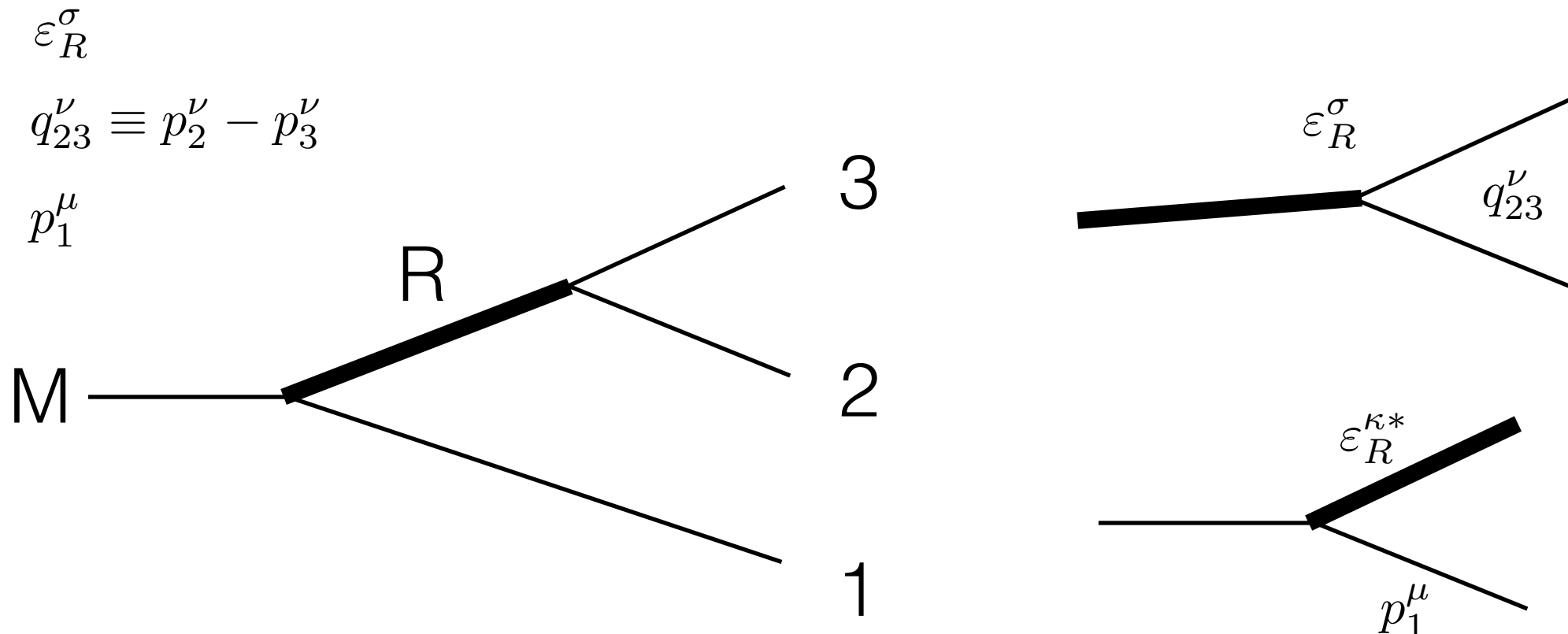


Express in terms of s_{ij} if you wish, using $p_i \cdot p_j = s_{ij} - m_i^2 - m_j^2$

$$\left(-p_1 \cdot q_{23} + \frac{(p_1 \cdot p_R)(q_{23} \cdot p_R)}{p_R^2} \right) \frac{1}{s_{23} - m_R^2 - im_R \Gamma}$$

spin factor

Calculating the amplitudes



$$p_{1\mu} \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - im_R \Gamma} q_{23\nu}$$

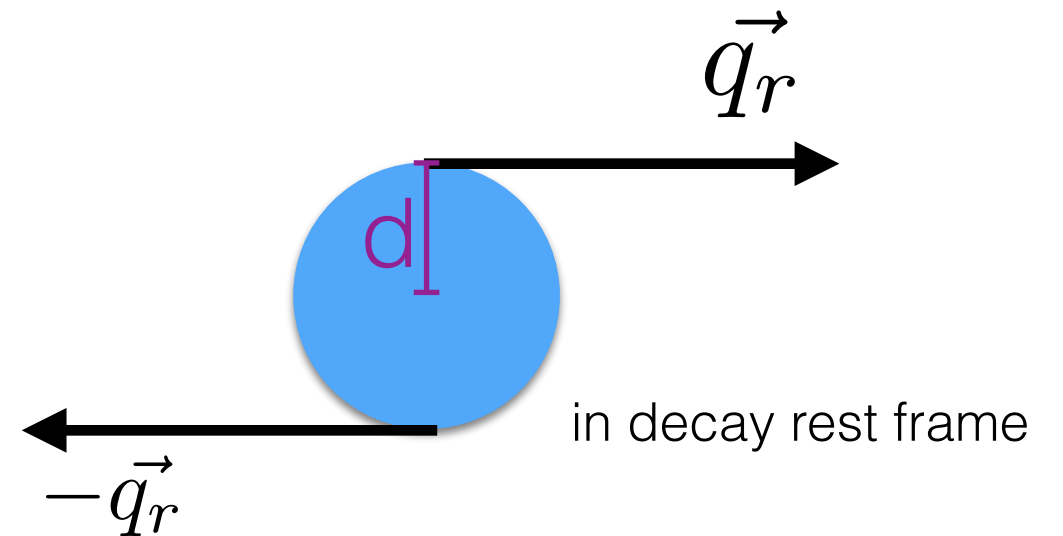
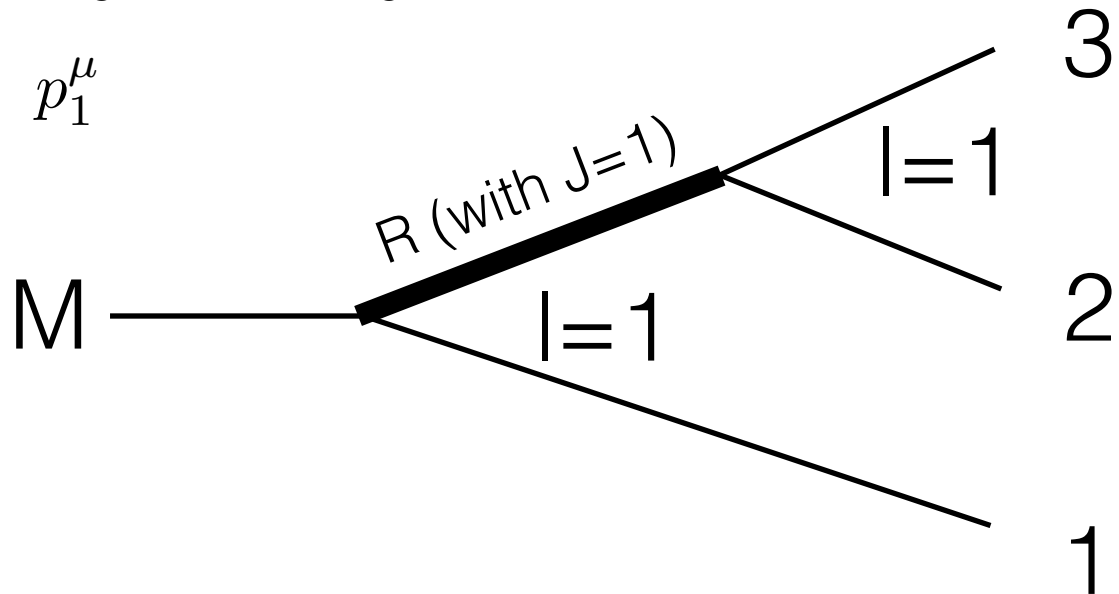
Calculating the amplitudes

Angular Momenta require momenta

$$\varepsilon_R^\sigma$$

$$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$$

$$p_1^\mu$$



$$\vec{L} = 2 \vec{d} \times \vec{q}_r \quad \text{classical mechanics}$$

$$L = \sqrt{l(l+1)} \quad \text{QM}$$

$$p_{1\mu} \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - im_R\Gamma} q_{23\nu}$$

Blatt Weisskopf Penetration Factors

L	$B_L(q)$	$B'_L(q, q_0)$
0	1	1
1	$\sqrt{\frac{2z}{1+z}}$	$\sqrt{\frac{1+z_0}{1+z}}$
2	$\sqrt{\frac{13z^2}{(z-3)^2+9z}}$	$\sqrt{\frac{(z_0-3)^2+9z_0}{(z-3)^2+9z}}$
where $z = (q d)^2$ and $z_0 = (q_0 d)^2$		

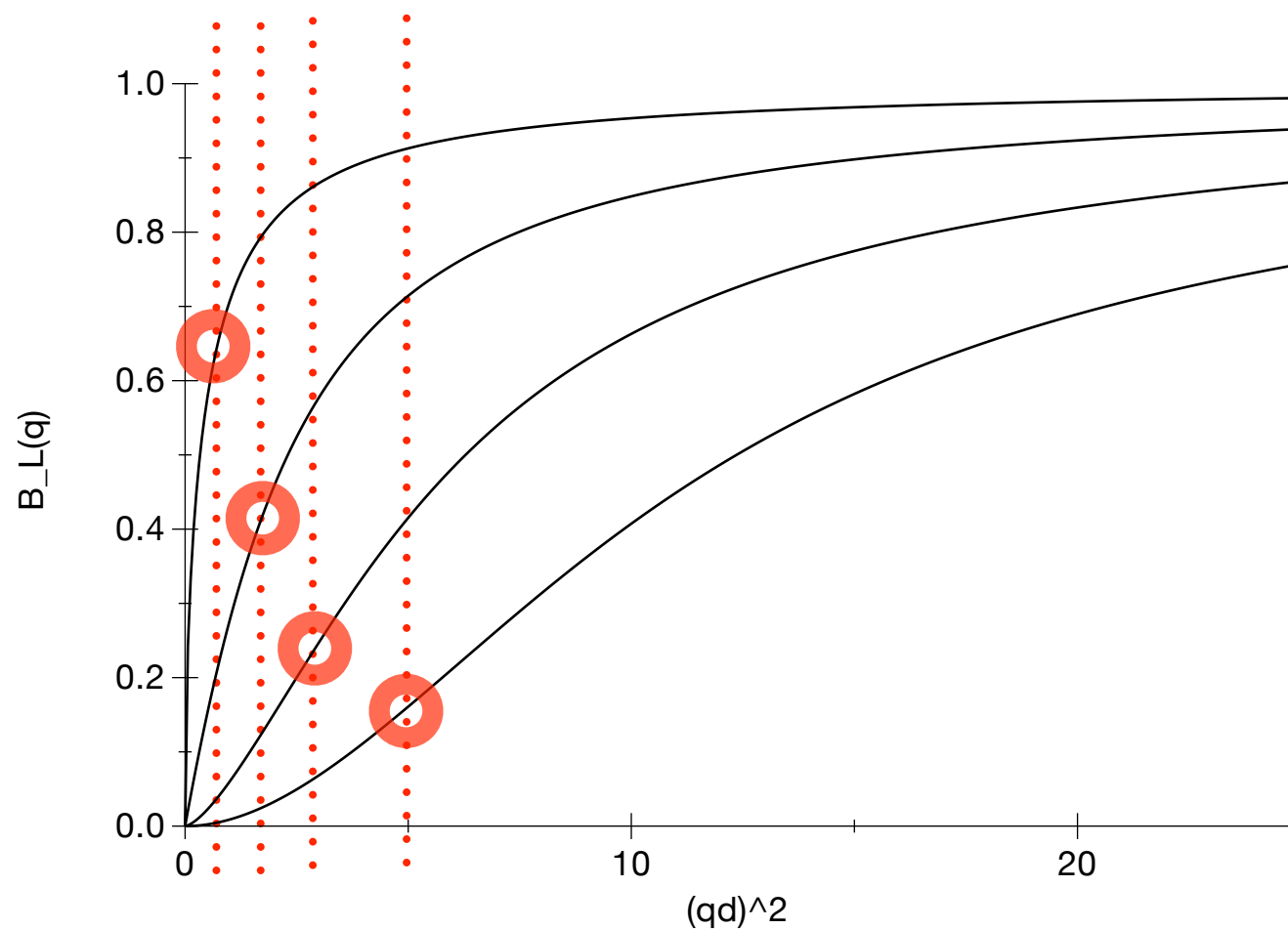
classical
mechanics:
 $L = 2 qd$

QM:
 $L^2 = l(l+1)$

Blatt Weisskopf Penetration Factors

$$1/4 L^2 = 1/4 l(l+1) \text{ for } l = \dots$$

1 2 3 4

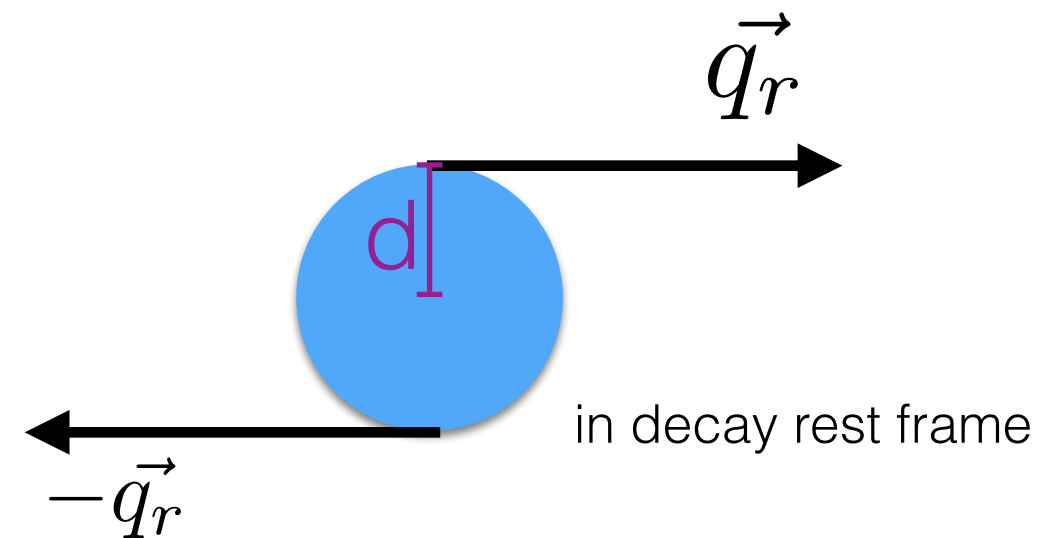
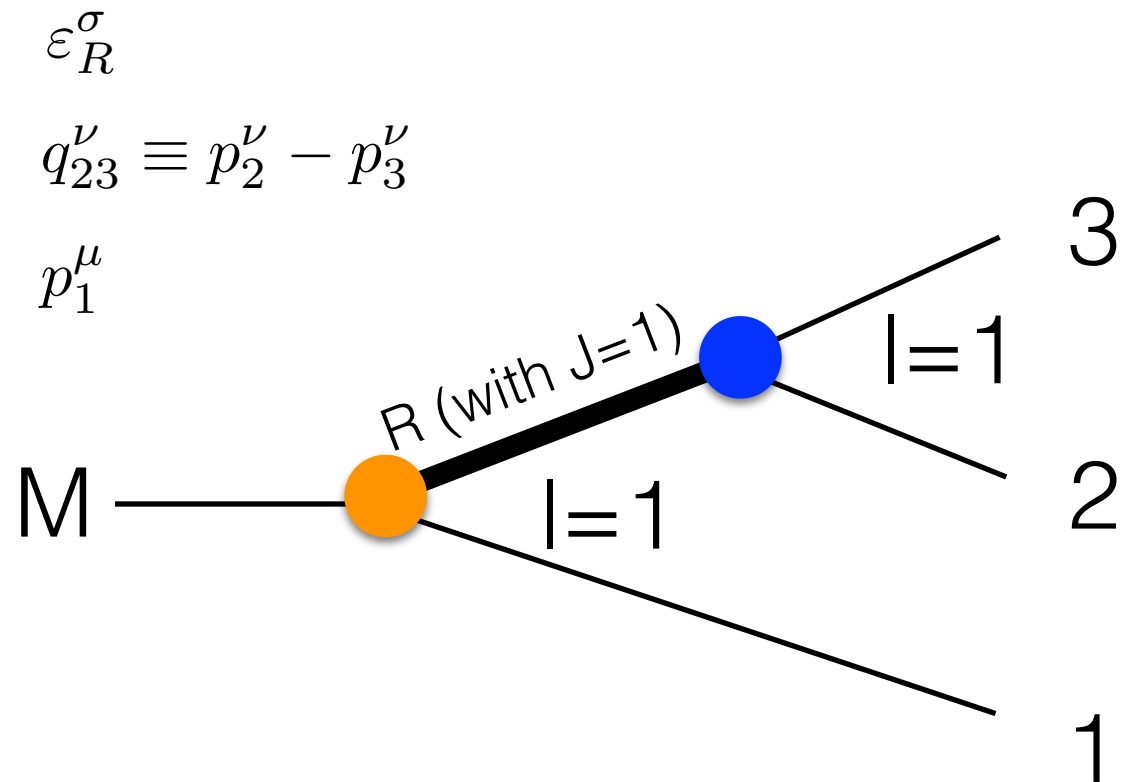


classical
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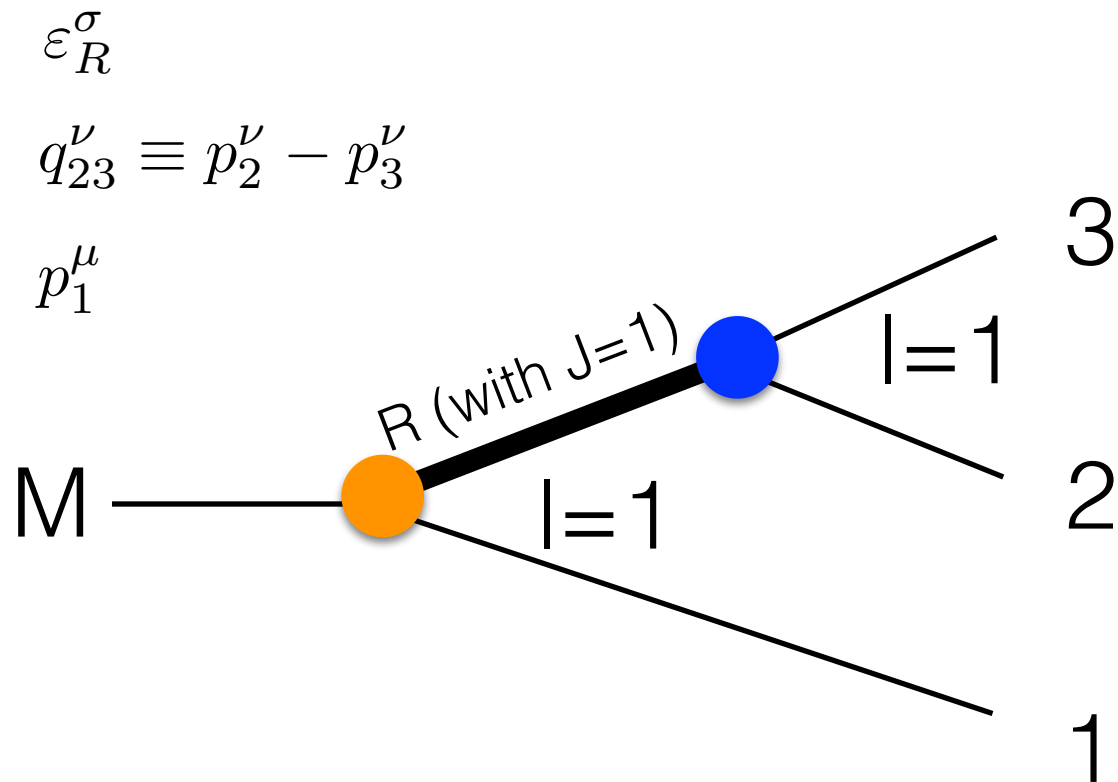
Calculating the amplitudes

Angular Momenta
require momenta



$$p_{1\mu} B_L(q_{rM}, d_M) \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - im_R \Gamma} B_L(q_{rR}, d_R) q_{23\nu}$$

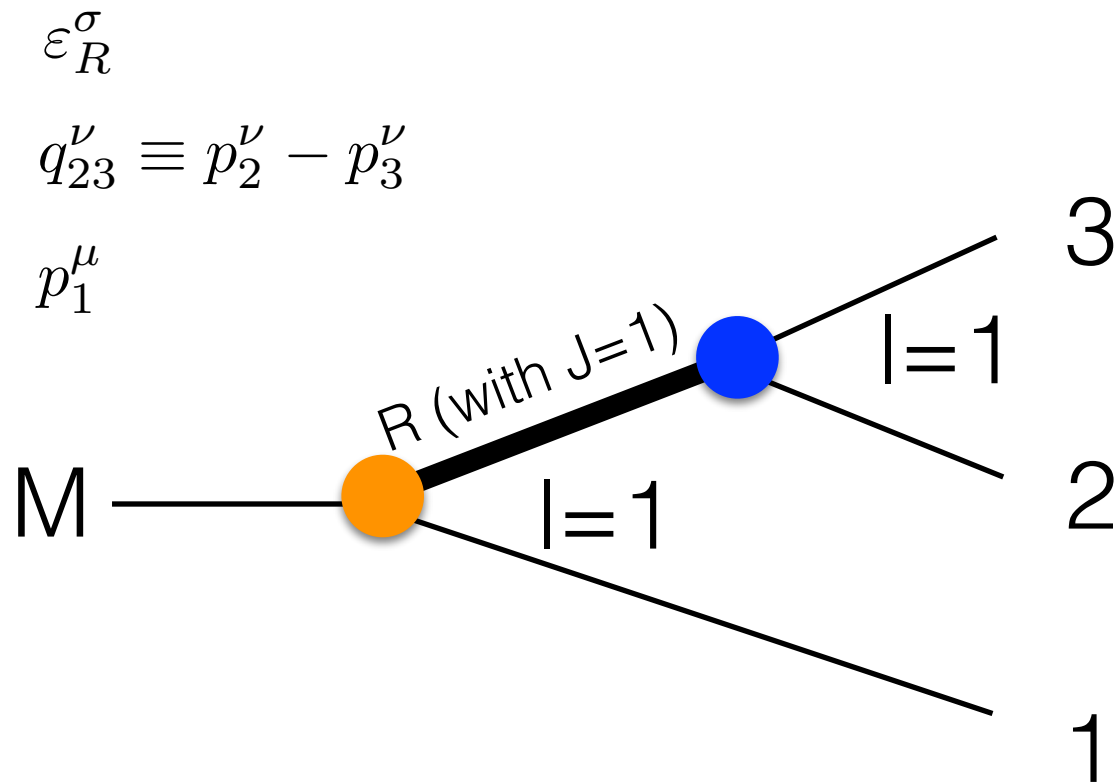
Calculating the amplitudes



- Width Γ = rate, depends on phase space = $2q/m$.
 ↑
 break-up momentum
- Rate also depends on B_L .

$$p_{1\mu} B_L(q_{rM}, d_M) \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - im_R \Gamma} B_L(q_{rR}, d_R) q_{23\nu}$$

Calculating the amplitudes



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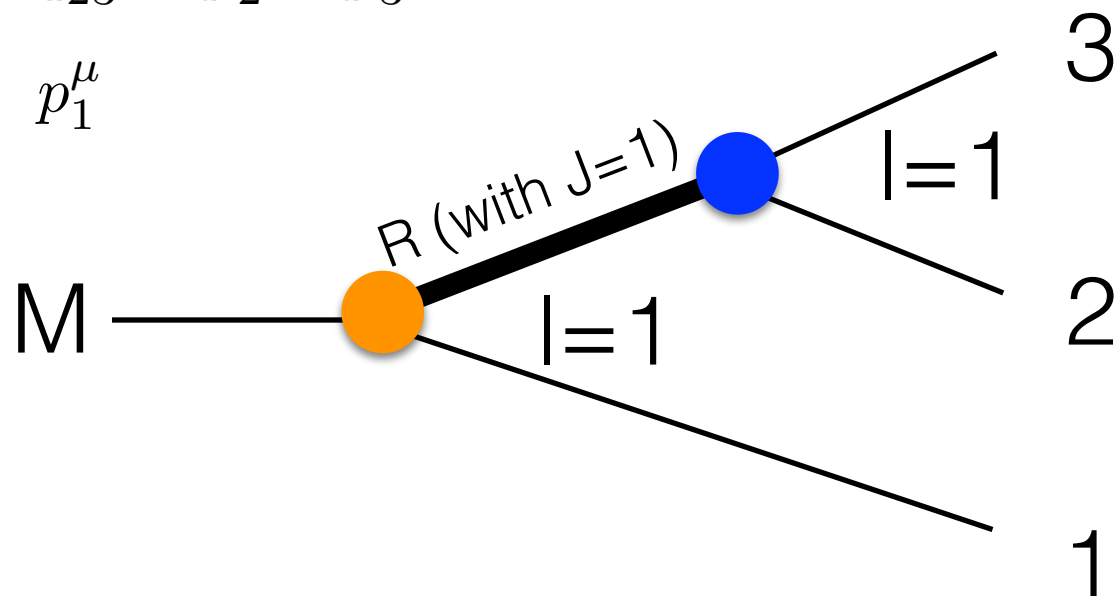
$$p_{1\mu} B_L(q_{rM}, d_M) \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - im_R \Gamma(m_{23})} B_L(q_{rR}, d_R) q_{23\nu}$$

Calculating the amplitudes

ε_R^σ

$$q_{23}^\nu \equiv p_2^\nu - p_3^\nu$$

p_1^μ

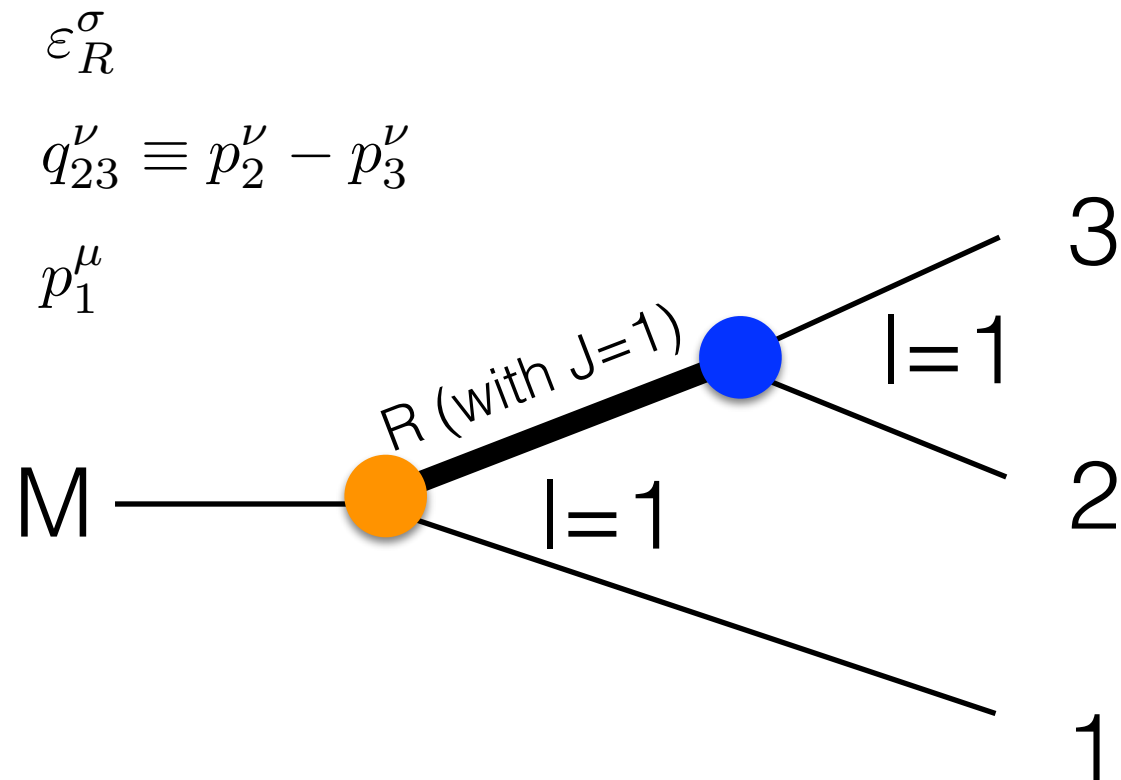


- Width Γ = rate, depends on phase space = $2q/m$.
↑
break-up momentum
- Rate also depends on B_L .

$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23}) B_L(q_{23})}{(q_0/m_R) B_L(q_0)}$$

$$p_{1\mu} B_L(q_{rM}, d_M) \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - im_R \Gamma(m_{23})} B_L(q_{rR}, d_R) q_{23\nu}$$

Calculating the amplitudes



- Width Γ = rate, depends on phase space = $2q/m$.
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- Rate also depends on B_L .

reconstructed mass $m_{23} \equiv \sqrt{s_{23}}$

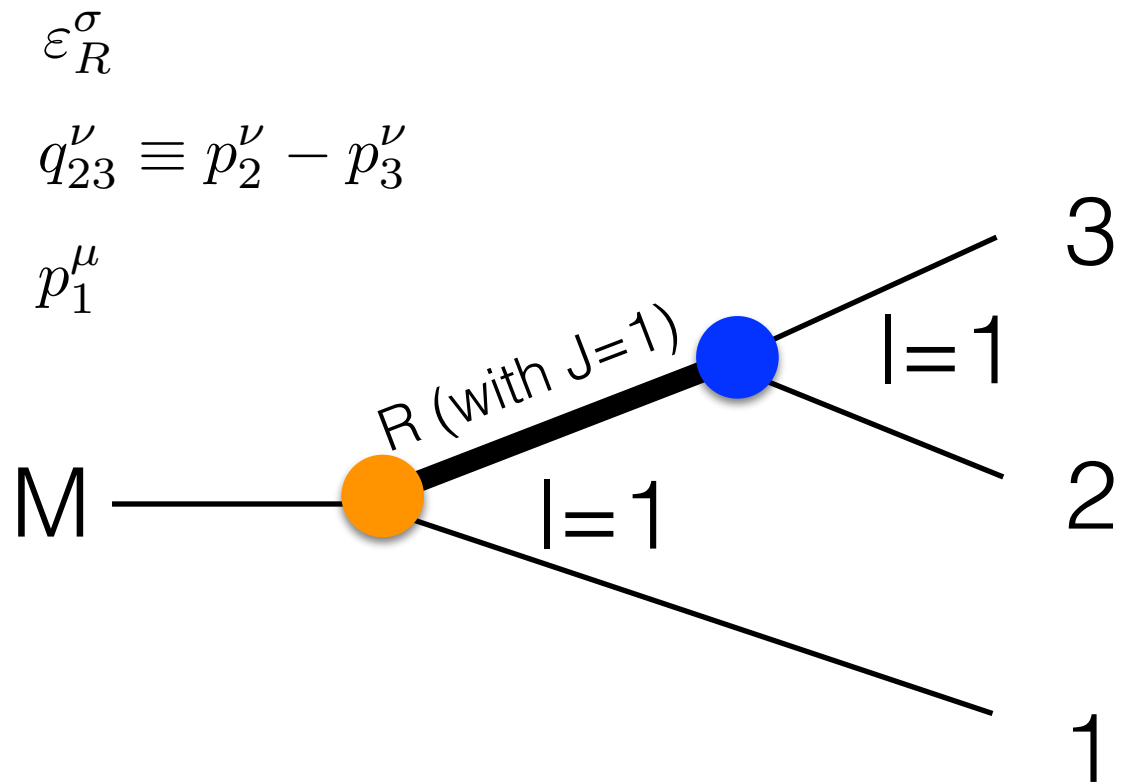
break-up momentum in restframe of decaying resonance

$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23}) B_L(q_{23})}{(q_0/m_R) B_L(q_0)}$$

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Calculating the amplitudes

- Width Γ = rate, depends on phase space = $2q/m$.
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break-up momentum in restframe of decaying resonance

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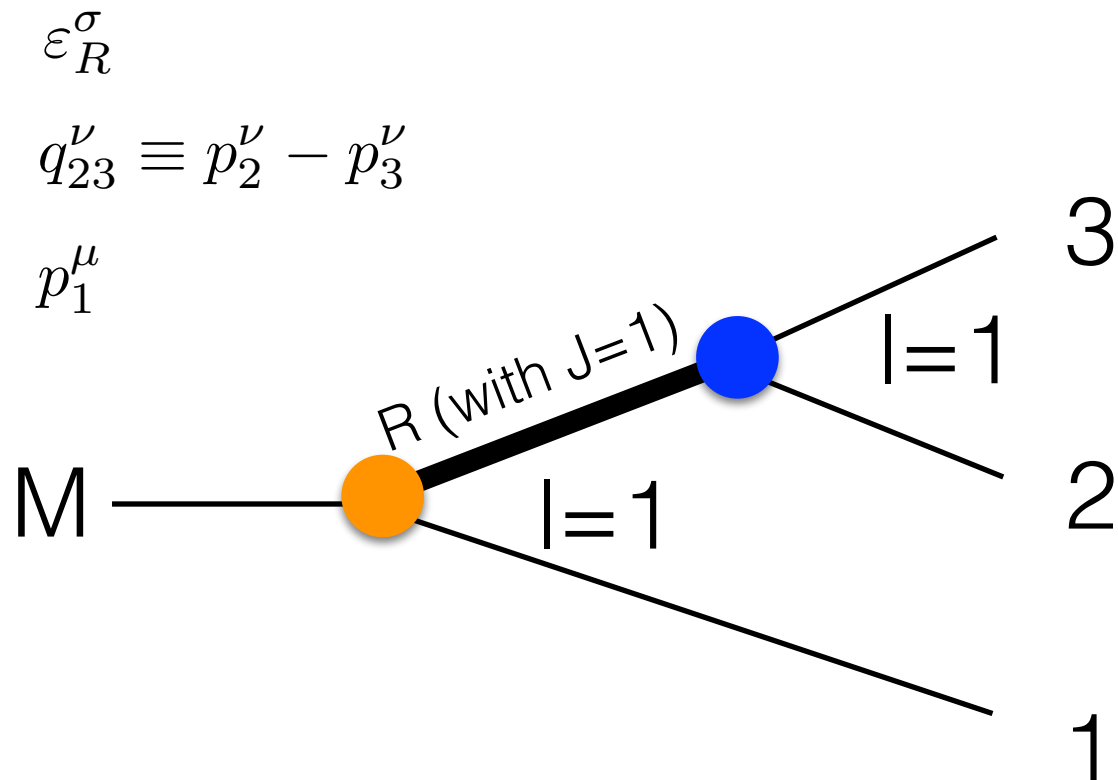
centrifugal barrier factor

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Calculating the amplitudes

- Width Γ = rate, depends on phase space = $2q/m$.
break-up momentum
- Rate also depends on B_L .



reconstructed mass $m_{23} \equiv \sqrt{s_{23}}$

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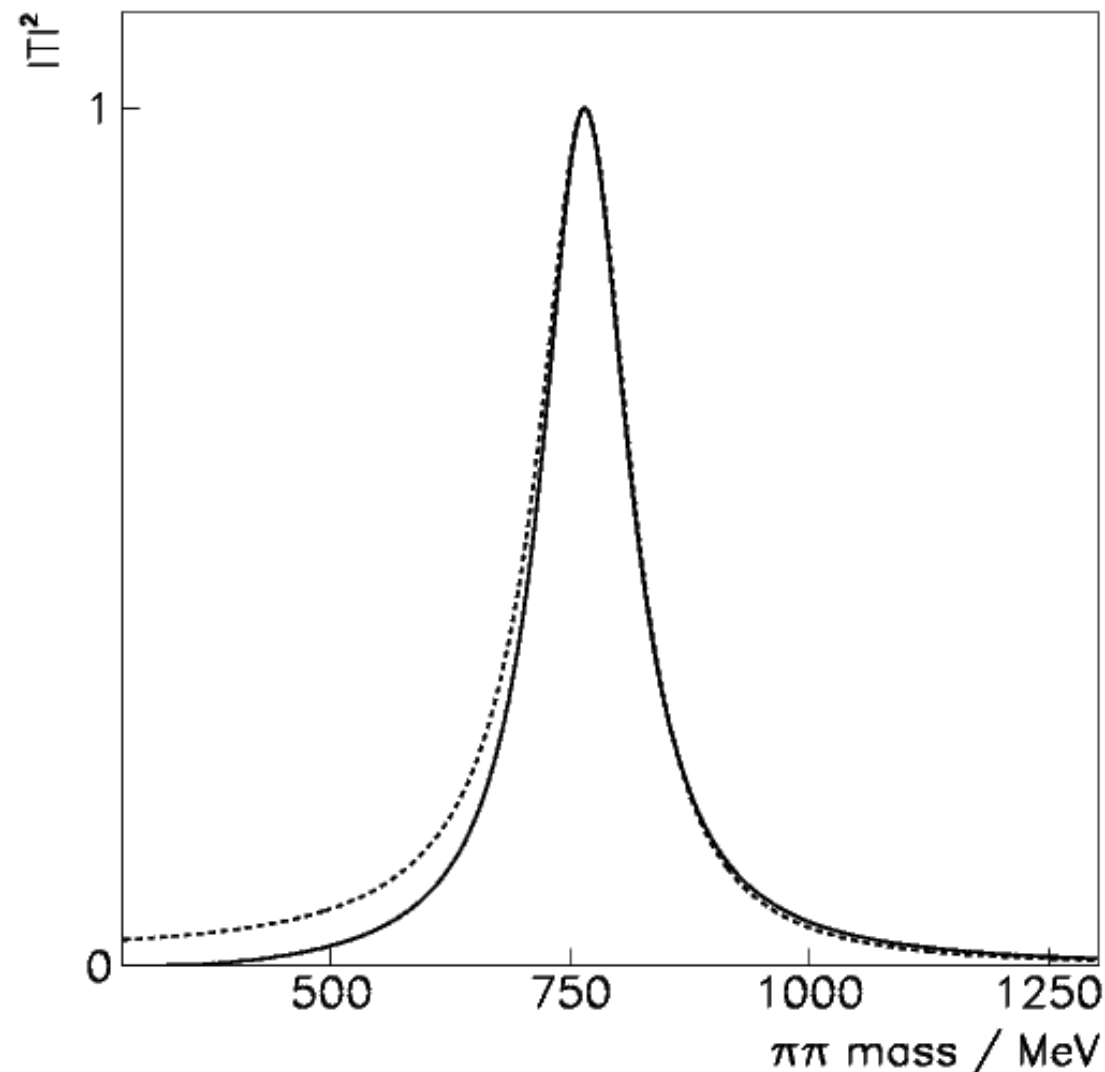
break-up momentum in restframe of decaying resonance

the same as numerator, but
calculated for “nominal” (peak)
resonance mass.

$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23}) B_L(q_{23})}{(q_0/m_R) B_L(q_0)}$$

$$p_{1\mu} B_L(q_{rM}, d_M) \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - im_R \Gamma(m_{23})} B_L(q_{rR}, d_R) q_{23\nu}$$

Mass dependent width (ignoring ang. mom)

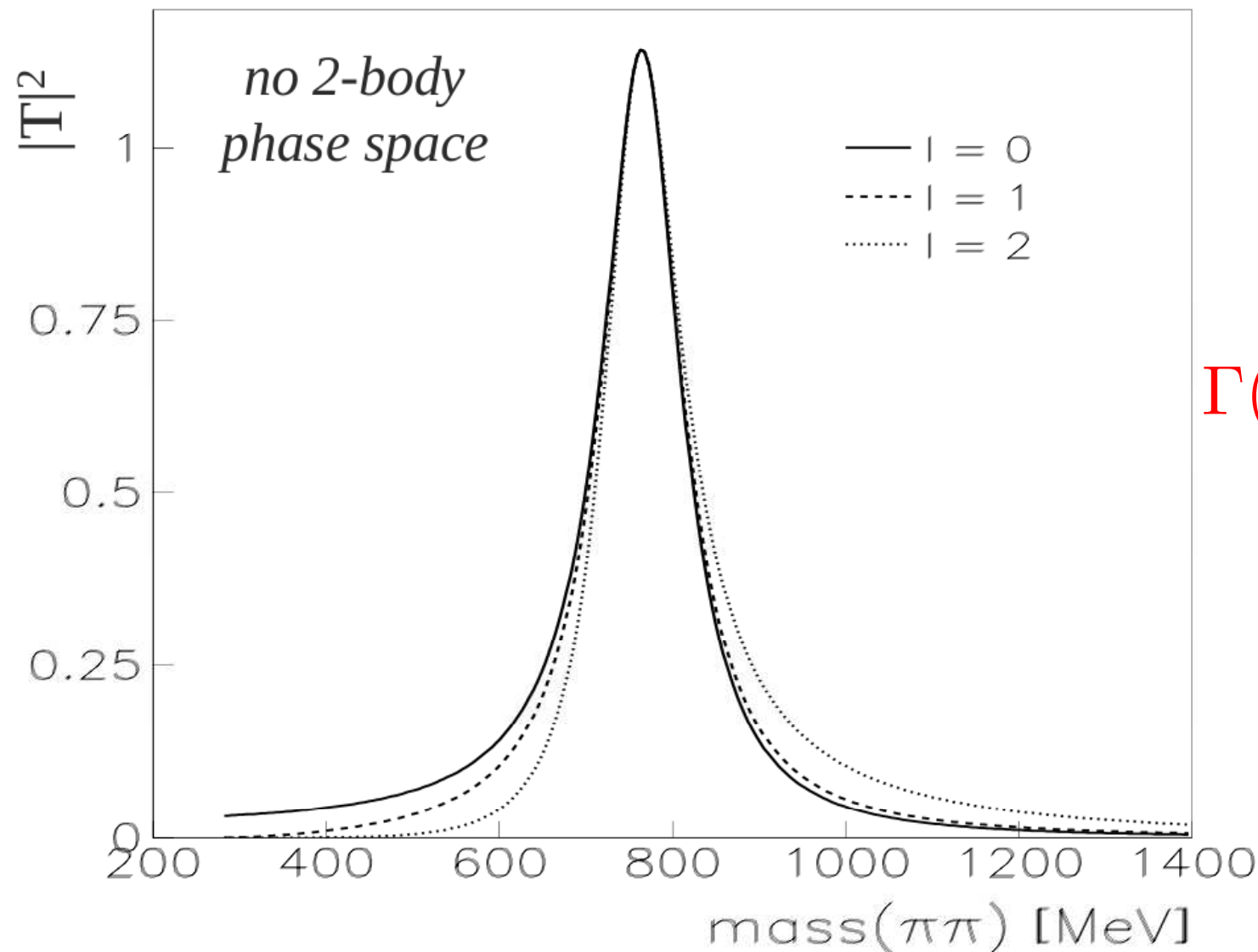


dashed: fixed width

solid: mass dependent width

$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23}) B_L(q_{23})}{(q_0/m_R) B_L(q_0)}$$

Breit Wigner with angular momentum effects (only)



$$\Gamma(m_{23}) = \Gamma_0 \frac{(q_{23}/m_{23}) B_L(q_{23})}{(q_0/m_R) B_L(q_0)}$$

Amplitude Model

$$A_R = p_{1\mu} B_L(q_{rM}, d_M) \frac{-g^{\mu\nu} + \frac{p_R^\mu p_R^\nu}{p_R^2}}{s_{23} - m_R^2 - im_R \Gamma(m_{23})} B_L(q_{rR}, d_R) q_{23\nu}$$

$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23})$$

sensitivity to phases is one of the key reasons amplitude analyses are so interesting.

$$P(s_{12}, s_{23}) = \frac{|\mathcal{M}_{fi}|^2 \left| \frac{d\Phi}{ds_{12} ds_{23}} \right|}{\int |\mathcal{M}_{fi}|^2 \left| \frac{d\Phi}{ds_{12} ds_{23}} \right| ds_{12} ds_{23}}$$

$$= \frac{|\mathcal{M}_{fi}|^2}{\int_{\text{within kin boundary}} |\mathcal{M}_{fi}|^2 ds_{12} ds_{23}}$$

Fitters frequently used at LHCb:
 MINT (esp for >3 body)
 Laura++
 GooFit-based fitter
 and others.

Amplitude Model

$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23})$$

example:

CDF: PHYSICAL REVIEW D 86, 032007 (2012)

Resonance	a	δ [°]	Fit fractions [%]
$K^*(892)^\pm$	1.911 ± 0.012	132.1 ± 0.7	61.80 ± 0.31
$K_0^*(1430)^\pm$	2.093 ± 0.065	54.2 ± 1.9	6.25 ± 0.25
$K_2^*(1430)^\pm$	0.986 ± 0.034	308.6 ± 2.1	1.28 ± 0.08
$K^*(1410)^\pm$	1.092 ± 0.069	155.9 ± 2.8	1.07 ± 0.10
$\rho(770)$	1	0	18.85 ± 0.18
$\omega(782)$	0.038 ± 0.002	107.9 ± 2.3	0.46 ± 0.05
$f_0(980)$	0.476 ± 0.016	182.8 ± 1.3	4.91 ± 0.19
$f_2(1270)$	1.713 ± 0.048	329.9 ± 1.6	1.95 ± 0.10
$f_0(1370)$	0.342 ± 0.021	109.3 ± 3.1	0.57 ± 0.05
$\rho(1450)$	0.709 ± 0.043	8.7 ± 2.7	0.41 ± 0.04
$f_0(600)$	1.134 ± 0.041	201.0 ± 2.9	7.02 ± 0.30
σ_2	0.282 ± 0.023	16.2 ± 9.0	0.33 ± 0.04
$K^*(892)^\pm$ (DCS)	0.137 ± 0.007	317.6 ± 2.8	0.32 ± 0.03
$K_0^*(1430)^\pm$ (DCS)	0.439 ± 0.035	156.1 ± 4.9	0.28 ± 0.04
$K_2^*(1430)^\pm$ (DCS)	0.291 ± 0.034	213.5 ± 6.1	0.11 ± 0.03
Nonresonant	1.797 ± 0.147	94.0 ± 5.3	1.64 ± 0.27
Sum			107.25 ± 0.65

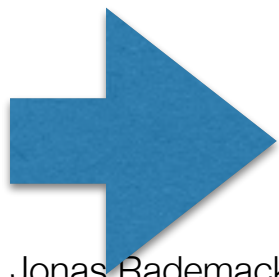
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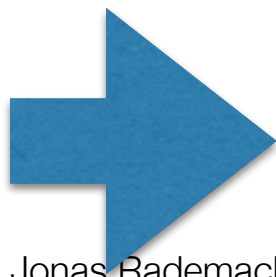
Amplitude Model

$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23}) + a_0 e^{i\theta_0}$$

example:

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Amplitude Model

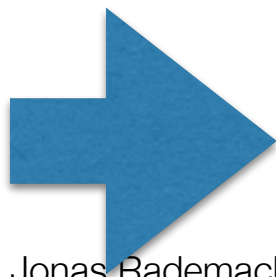
$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23}) + a_0 e^{i\theta_0}$$

$$FF_R = \frac{\int |c_R e^{i\theta_R} A_R(s_{12}, s_{23})|^2 ds_{12} ds_{23}}{\int \left| \sum_j c_j e^{i\theta_j} A_j(s_{12}, s_{23}) \right|^2 ds_{12} ds_{23}}$$

example:

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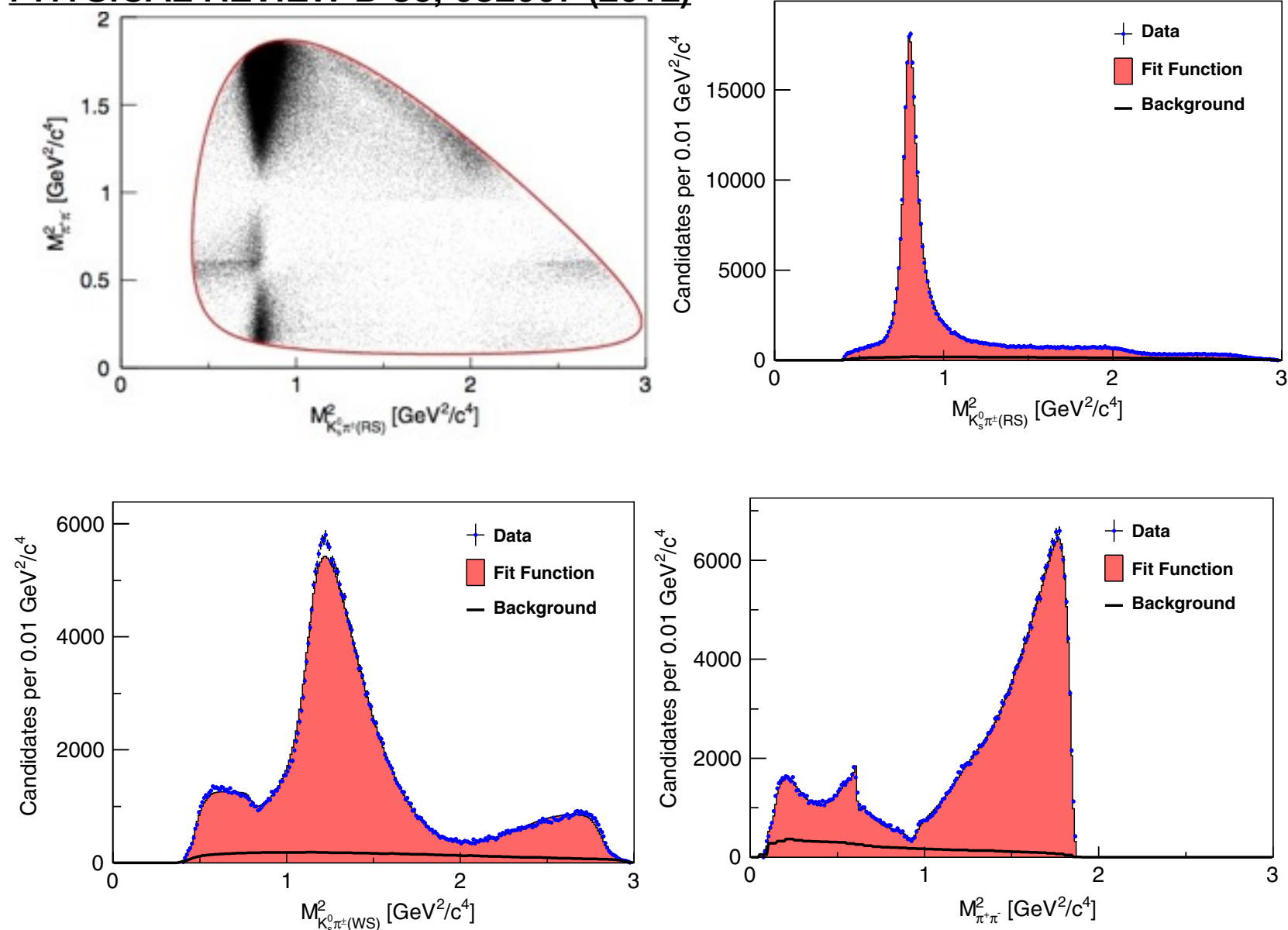


Amplitude Model

$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23})$$

example:

CDF: PHYSICAL REVIEW D 86, 032007 (2012)

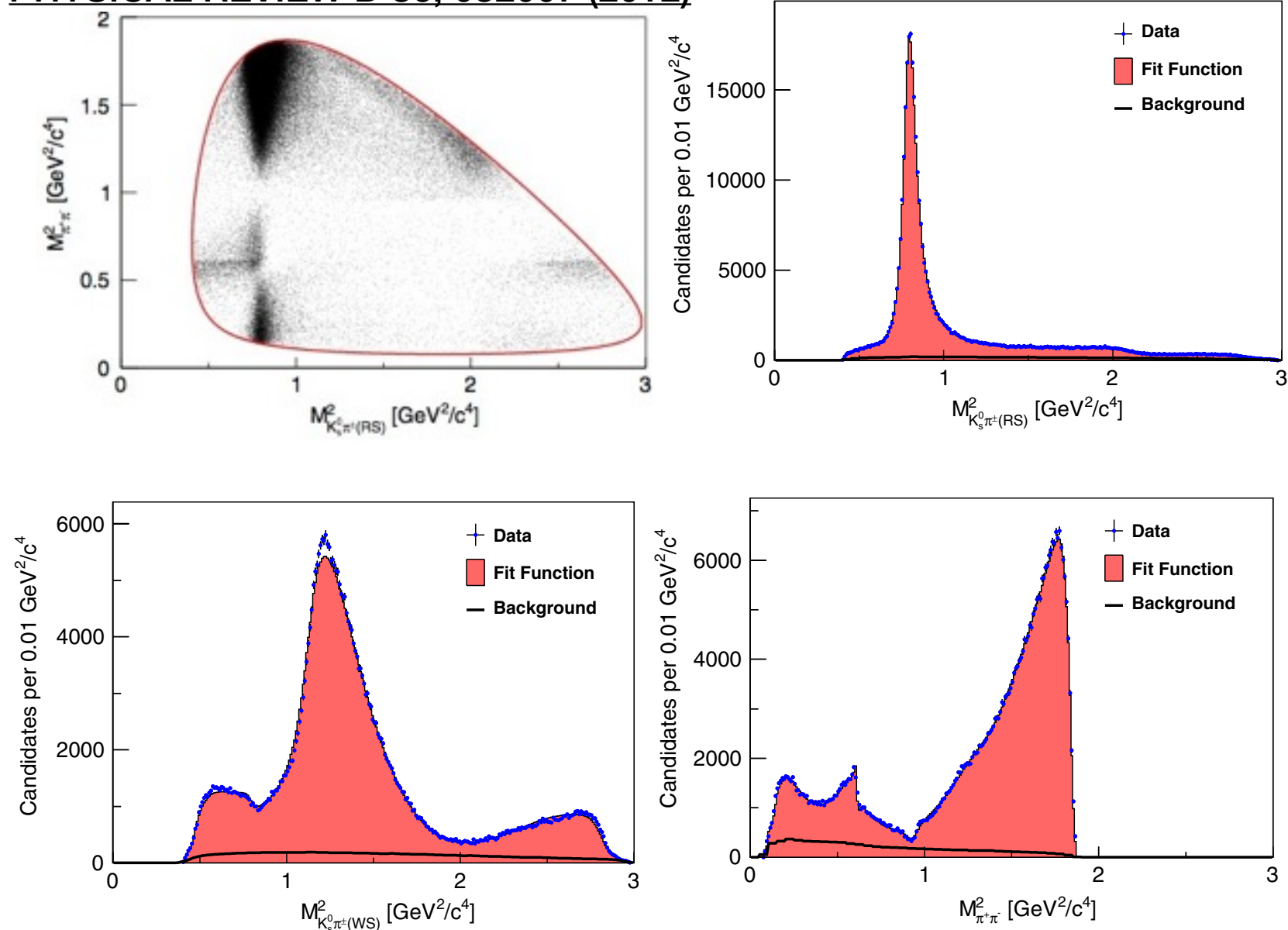


Amplitude Model

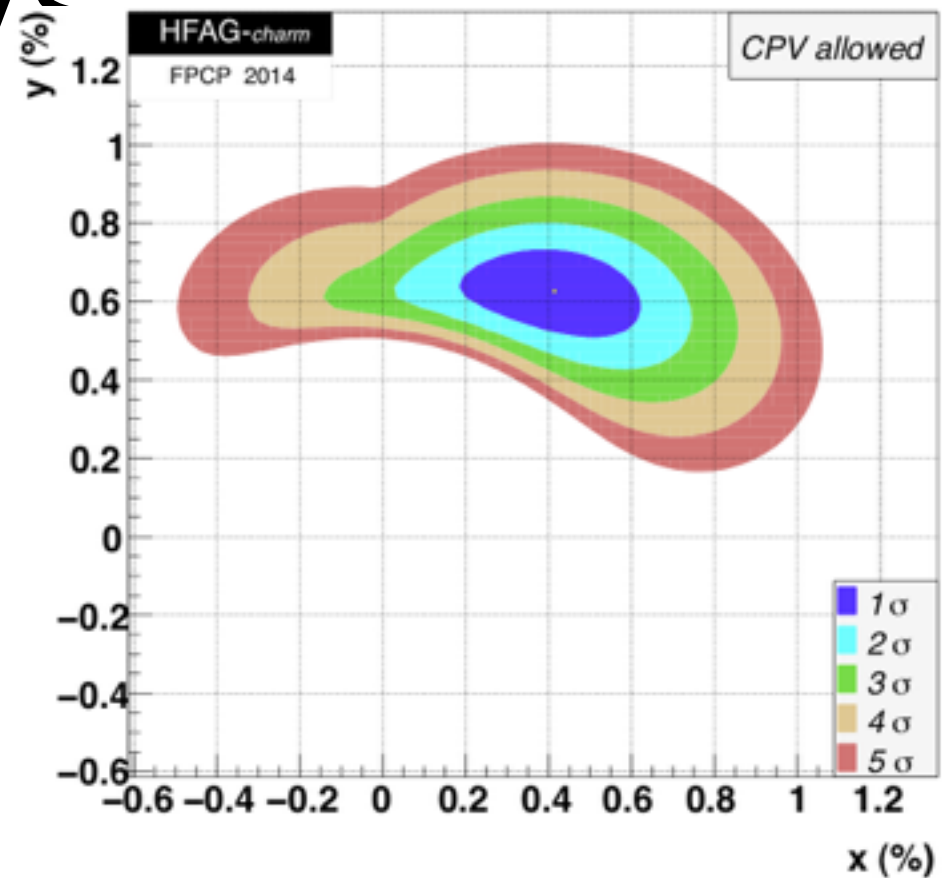
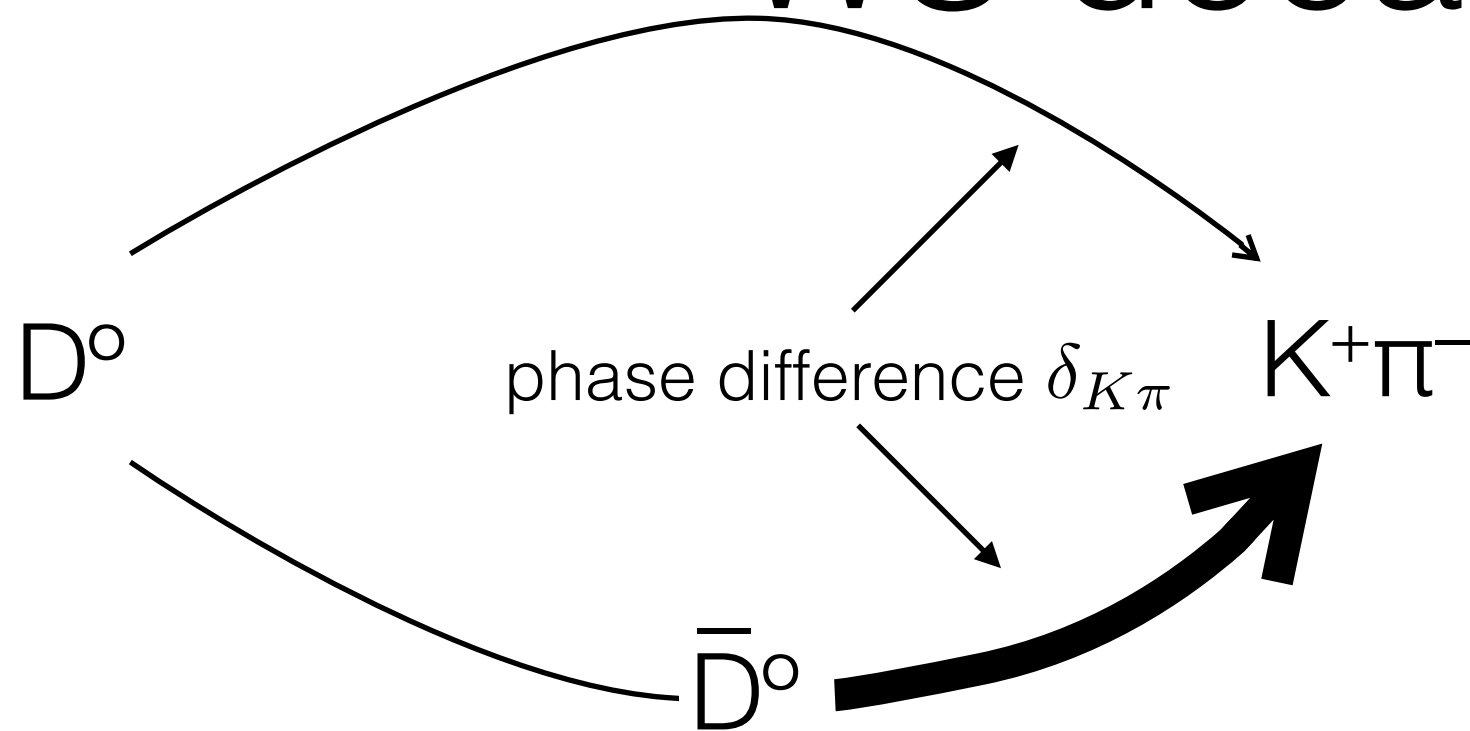
$$\mathcal{M}_{fi} = \sum_R c_R e^{i\theta_R} A_R(s_{12}, s_{23}) + a_0 e^{i\theta_0}$$

example:

CDF: PHYSICAL REVIEW D 86, 032007 (2012)



Mixing formalism for 2-body WS decays

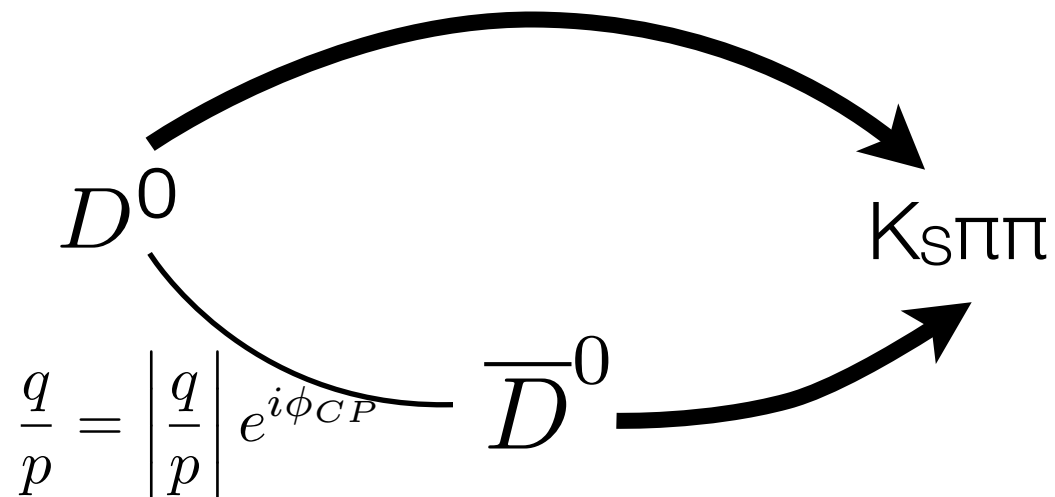


$$\frac{\Gamma(D^0 \rightarrow K^+\pi^-)}{\Gamma(D^0 \rightarrow K^-\pi^+)}(t) \approx (r_D^{K\pi})^2 + r_D^{K\pi} y'_{K\pi} \Gamma t + \frac{x'^2_{K\pi} + y'^2_{K\pi}}{4} (\Gamma t)^2$$

$$\text{where } \begin{pmatrix} x'_{K\pi} \\ y'_{K\pi} \end{pmatrix} = \begin{pmatrix} \cos \delta_{K\pi} & \sin \delta_{K\pi} \\ \cos \delta_{K\pi} & -\sin \delta_{K\pi} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

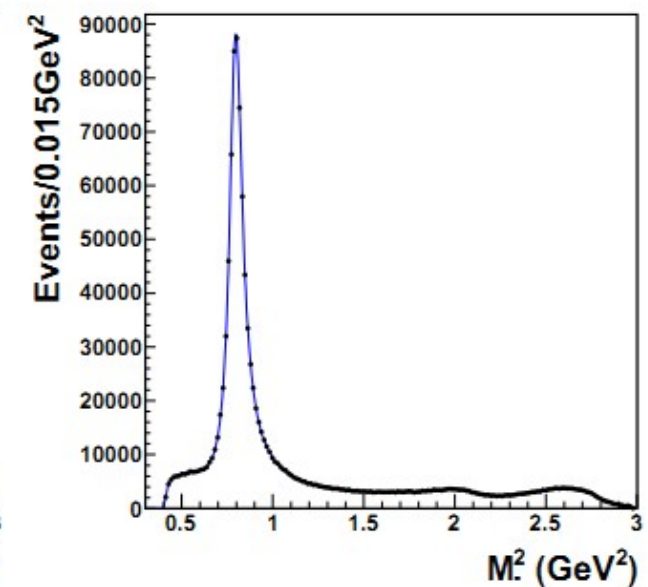
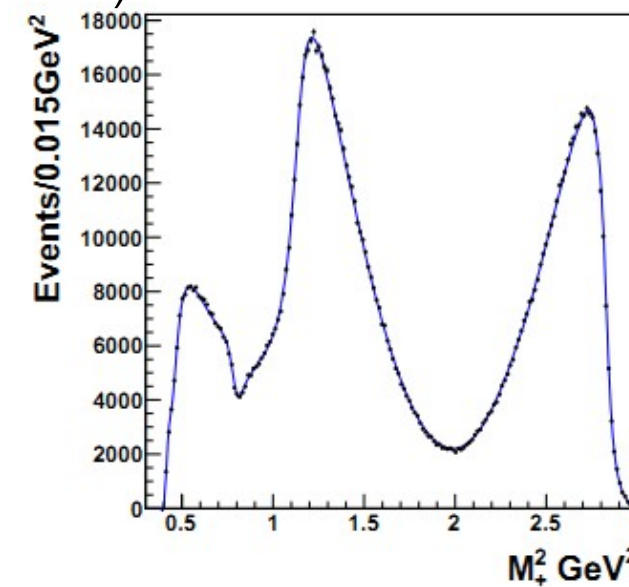
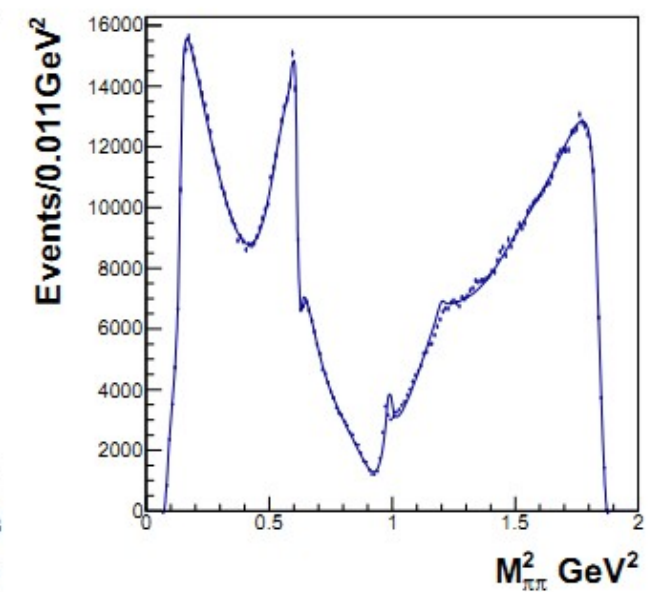
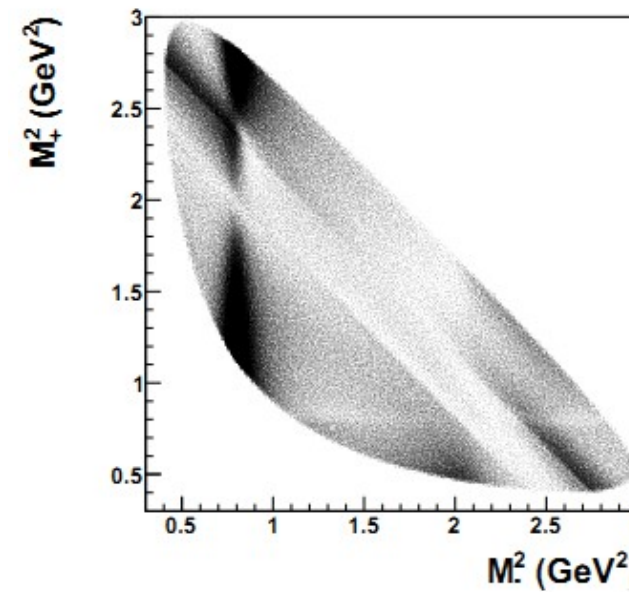
Time-dependent CPV $D^0 \rightarrow K_S \pi \pi$

see also previous result: [Phys. Rev. Lett. 99, 131803 \(2007\)](#).



(Belle preliminary) (by now published)

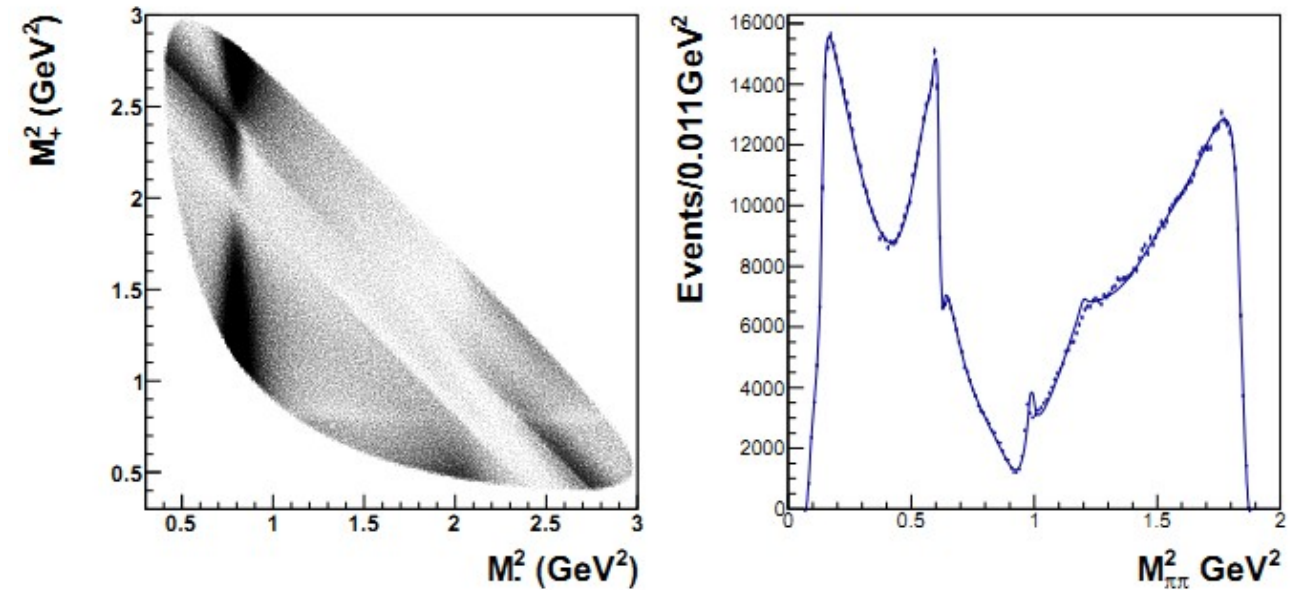
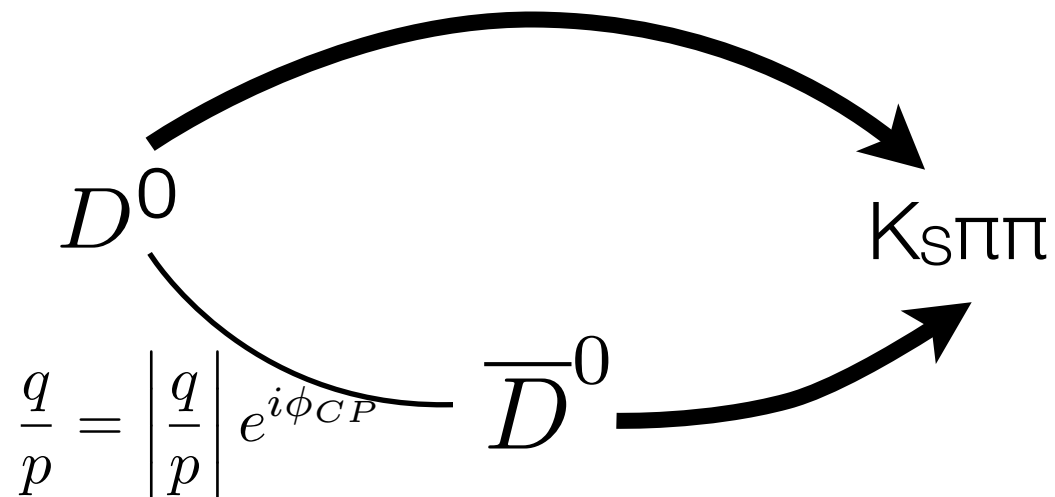
Fit case	Parameter	Fit new result
No CPV	$x(\%)$	$0.56 \pm 0.19^{+0.03+0.06}_{-0.09-0.09}$
	$y(\%)$	$0.30 \pm 0.15^{+0.04+0.03}_{-0.05-0.06}$
No dCPV	$ q/p $	$0.90^{+0.16+0.05+0.06}_{-0.15-0.04-0.05}$
	$\arg q/p(^{\circ})$	$-6 \pm 11^{+3+3}_{-3-4}$



see also BaBar [Phys. Rev. Lett. 105, 081803 \(2010\)](#) and CLEO-c [Phys. Rev. D 72, 012001 \(2005\)](#).

Time-dependent CPV $D^0 \rightarrow K_S \pi \pi$

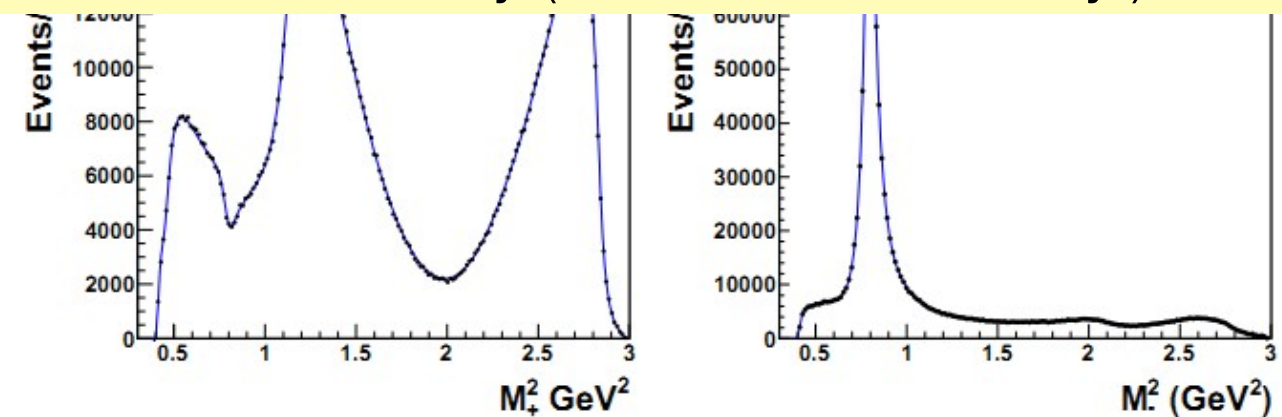
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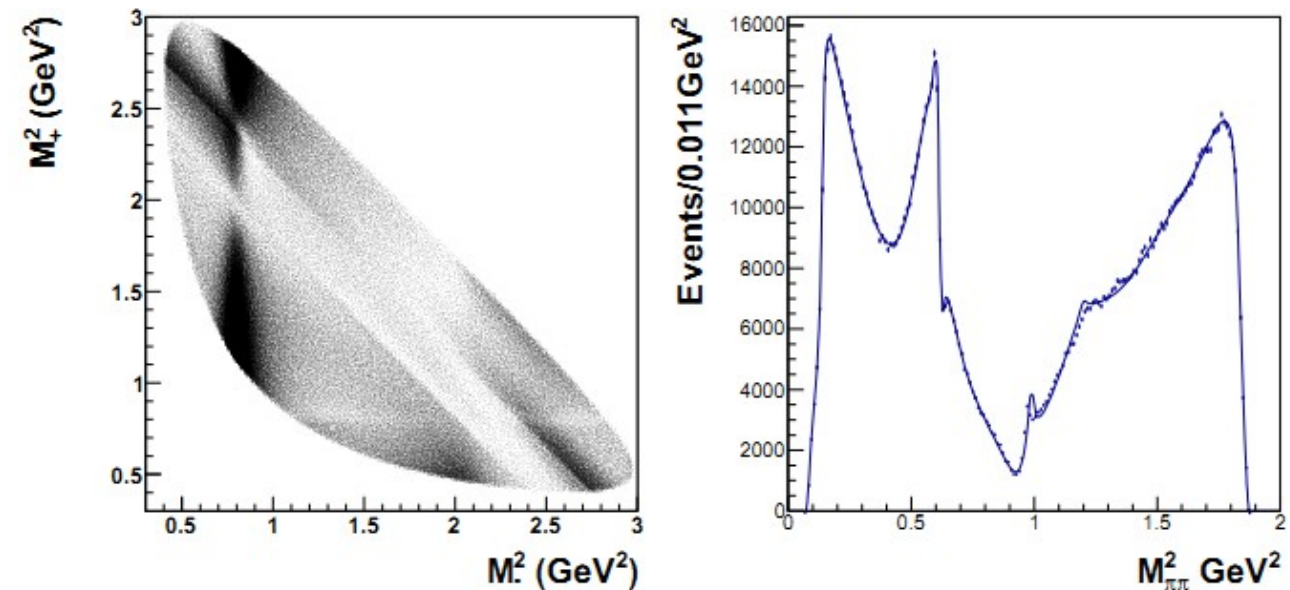
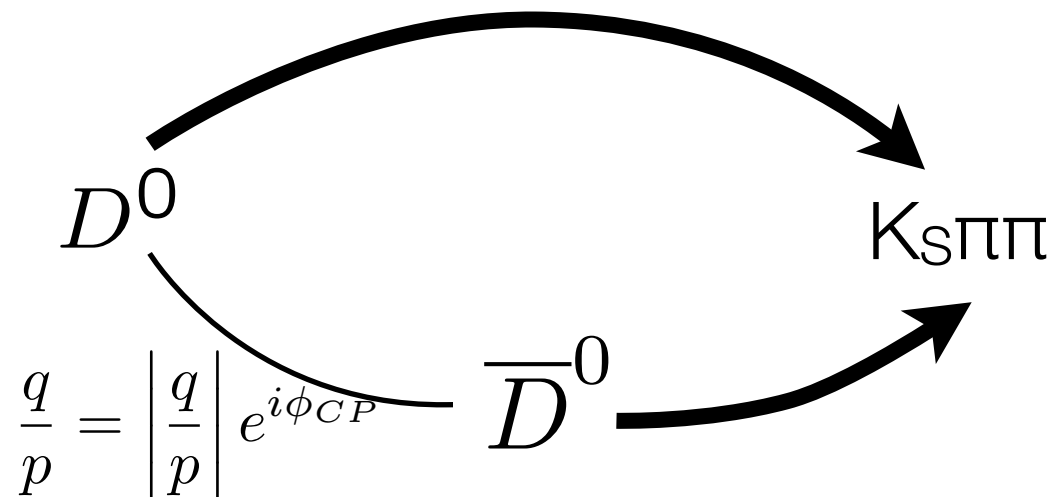
Magic of Dalitz plot (sensitivity to phases) gives access to x, y (rather than x'^2 and y')



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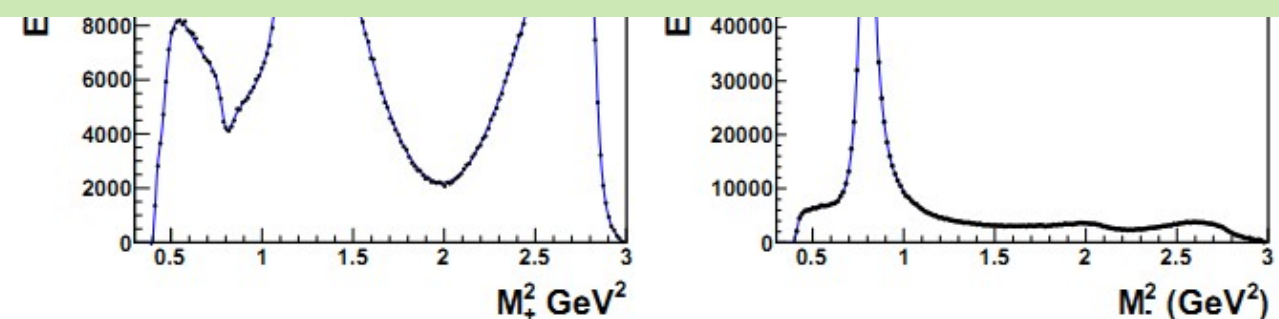


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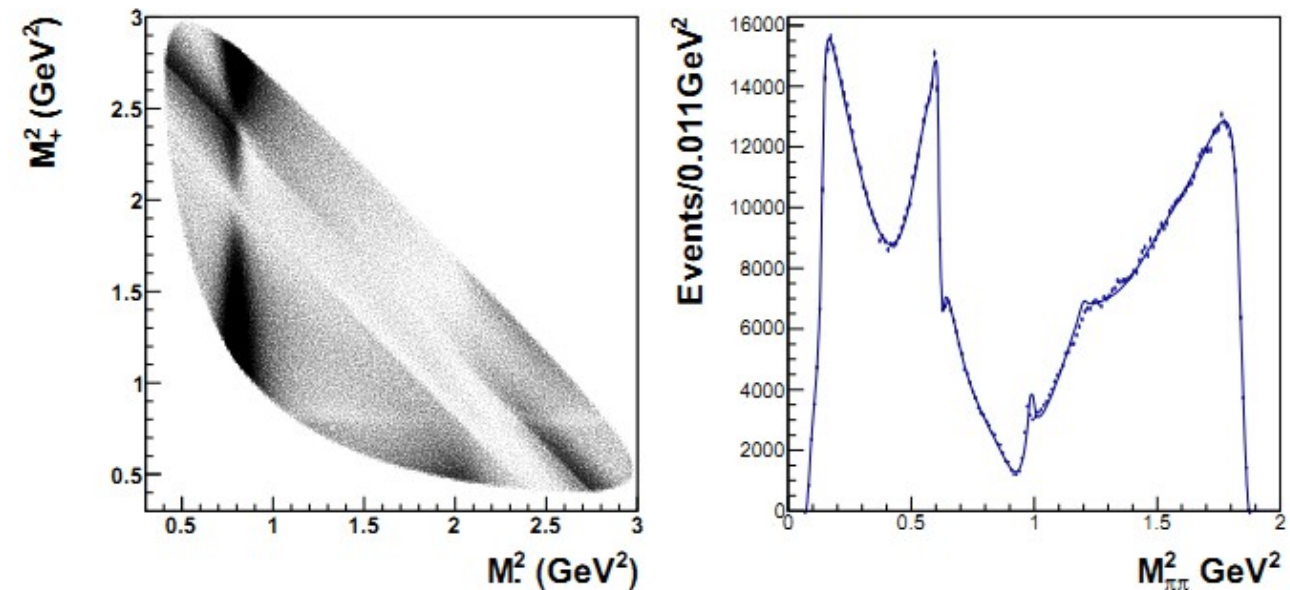
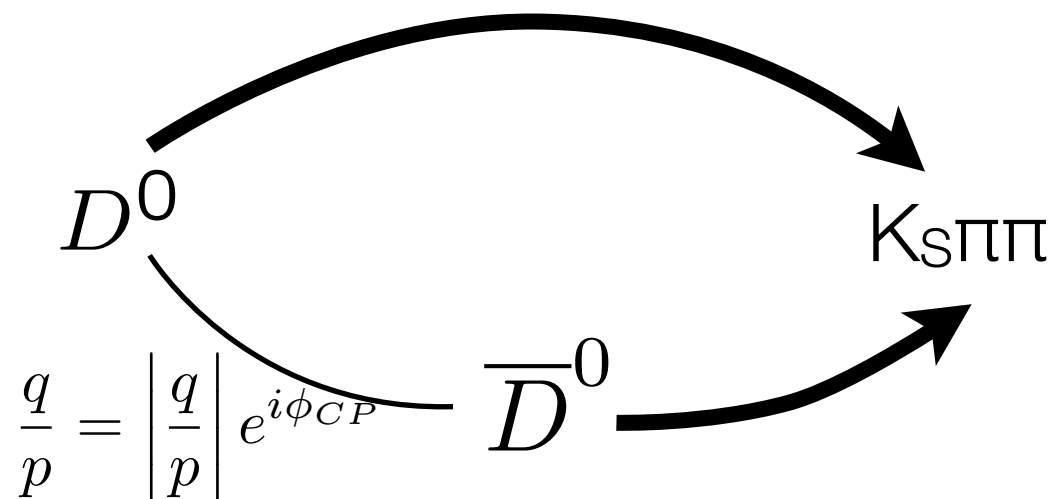
No evidence of CP violation



see also BaBar [Phys. Rev. Lett. 105, 081803 \(2010\)](#) and CLEO-c [Phys. Rev. D 72, 012001 \(2005\)](#).

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Magic of Dalitz plot (sensitivity to phases) gives access to x, y (rather than x'^2 and y')

No evidence of CP violation

Significant systematic uncertainty from amplitude model dependence. (Could be limiting with future LHCb/upgrade statistics.)

see also BaBar [Phys. Rev. Lett. 105, 081803 \(2010\)](#) and CLEO-c [Phys. Rev. D 72, 012001 \(2005\)](#).

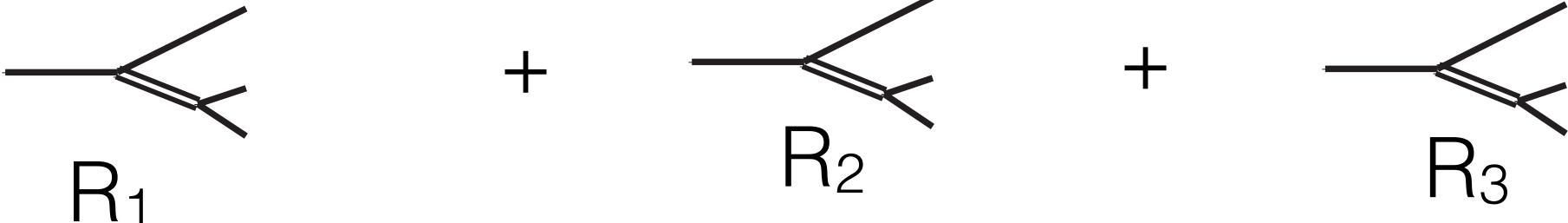
“Isobar” Model

- “Isobar”: Describe decay as series of 2-body processes.



- Usually: each resonance described by Breit Wigner lineshape (or similar) times factors accounting for spin.
- Popular amongst experimentalists, less so amongst theorists: violates unitarity. But not much as long as resonances are reasonably narrow, don't overlap too much.
- General consensus: Isobar OK for P, D wave, but problematic for S-wave. Alternatives exist, e.g. K-matrix formalism, which respects unitarity.

Isobar Model with sum of Breit Wigners

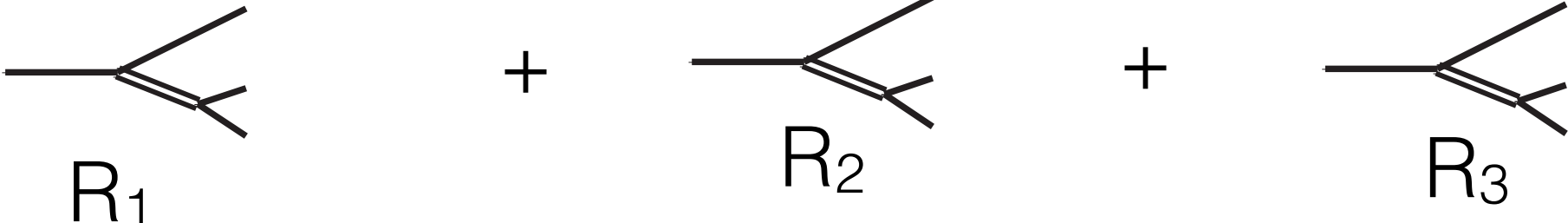


The diagram shows three Feynman diagrams representing resonances R_1 , R_2 , and R_3 . Each diagram consists of a horizontal line on the left (incoming particle) and two lines on the right (outgoing particles). The lines on the right are slightly curved, indicating a resonance. The diagrams are labeled R_1 , R_2 , and R_3 below them. They are separated by plus signs and followed by an ellipsis.

$$\frac{1}{s_{12} - m_1^2 - im_1\Gamma_1(s_{12})} + \frac{1}{s_{12} - m_2^2 - im_2\Gamma_2(s_{12})} + \frac{1}{s_{12} - m_3^2 - im_3\Gamma_3(s_{12})} + \dots$$

- Single resonance well described by Breit Wigner
- Overlapping resonances not so. Theoretically problematic: violates unitarity. From a practical point of view problematic as you might get the wrong phase motion.

Isobar Model with sum of Breit Wigners



The diagrams show three types of resonances, labeled R_1 , R_2 , and R_3 . Each diagram consists of a horizontal line on the left representing an incoming particle, which then splits into two outgoing particles. The splitting is represented by a vertex with two lines extending to the right. The diagrams are arranged in a row, separated by plus signs, and followed by an ellipsis.

$$\frac{1}{s_{12} - m_1^2 - im_1\Gamma_1(s_{12})} + \frac{1}{s_{12} - m_2^2 - im_2\Gamma_2(s_{12})} + \frac{1}{s_{12} - m_3^2 - im_3\Gamma_3(s_{12})} + \dots$$

4 resonances

32 resonances

Flatté Formula

- Consider $f_0(980)$ (width $\Gamma \approx 40\text{-}100$ MeV). Decays to $\pi\pi$ and KK . To KK only above ~ 987.4 MeV.
- The availability of the KK final state above 987.4 MeV increases the phase space and thus the width above this threshold.
- Need to take this into account even if I only look at $f_0(980) \rightarrow \pi\pi$.

$$\Gamma_{f_0}(s) = \Gamma_{\pi}(s) + \Gamma_K(s).$$

$$\Gamma_{\pi}(s) = g_{\pi} \sqrt{s/4 - m_{\pi}^2},$$

$$\Gamma_K(s) = \frac{g_K}{2} \left(\sqrt{s/4 - m_{K^+}^2} + \sqrt{s/4 - m_{K^0}^2} \right)$$

K-matrix

$$S_{fi} = \langle f|S|i\rangle = I + 2iT$$

$$T = K(I - iK)^{-1}$$

$$K_{ij} = \sum_{\alpha} \frac{\sqrt{m_{\alpha}\Gamma_{\alpha i}}\sqrt{m_{\alpha}\Gamma_{\alpha j}}}{m_{\alpha}^2 - m^2}$$

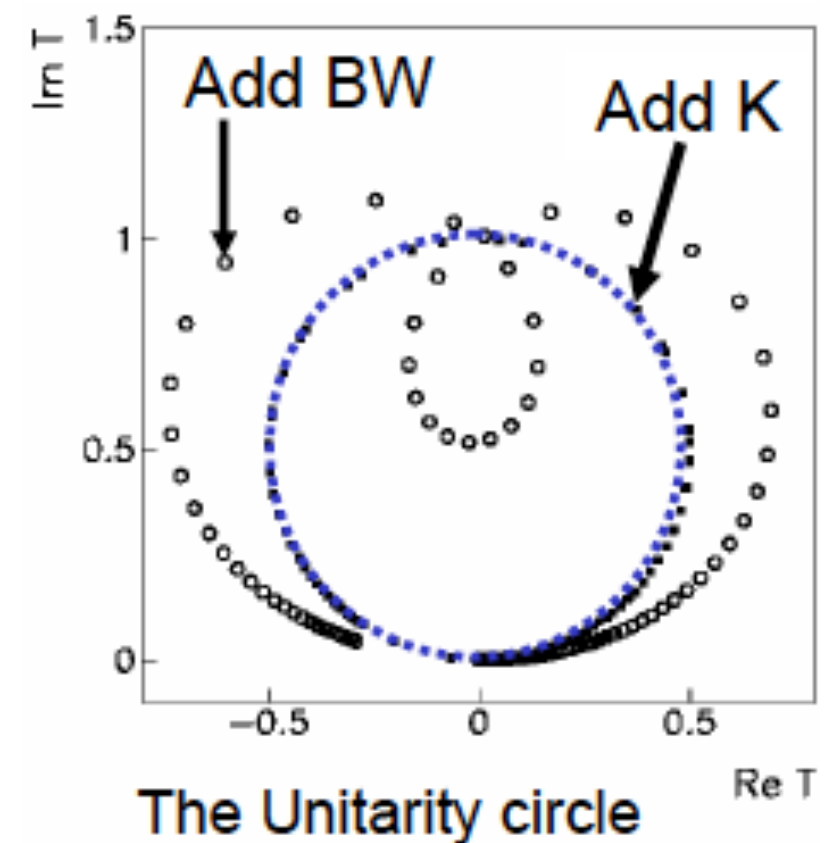
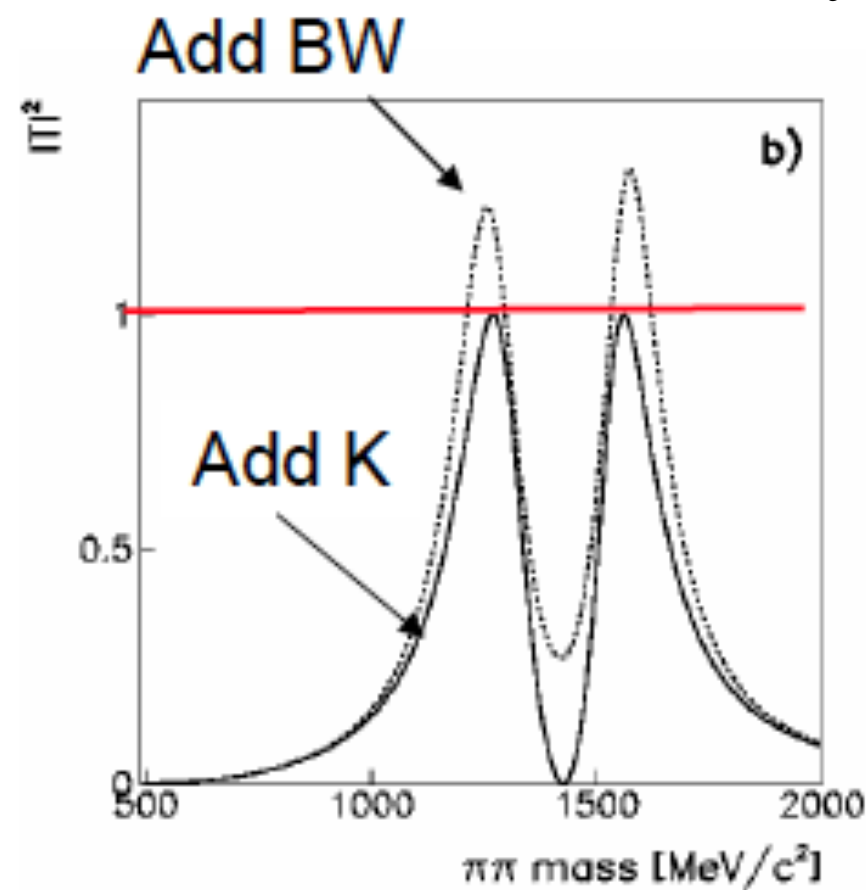
- For single channel: Reproduces Breit Wigner
- For single resonance that can decay to different final state: Reproduces Flatté.

K-matrix

$$S_{fi} = \langle f|S|i\rangle = I + 2iT$$

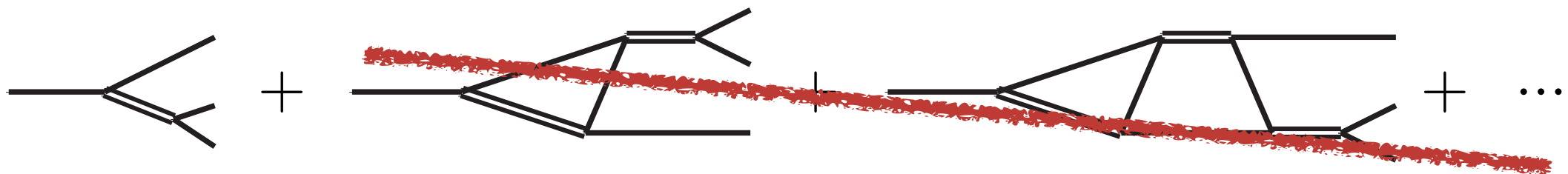
$$T = K(I - iK)^{-1}$$

$$K_{ij} = \sum_{\alpha} \frac{\sqrt{m_{\alpha}\Gamma_{\alpha i}}\sqrt{m_{\alpha}\Gamma_{\alpha j}}}{m_{\alpha}^2 - m^2}$$



K-matrix

- Note that the K-matrix approach is still an approximation.
- While it ensures unitarity (by construction), it is not completely theoretically sound/motivated (and violates analyticity).
- And it does not in any way address this:



What theorists think of all this

(a few slides from a recent LHCb Amplitude Analysis Workshop with experimentalists and theorists)

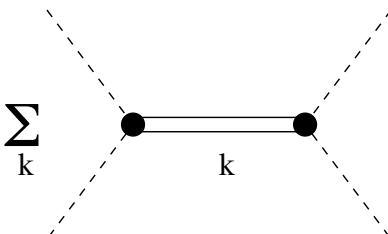
Modeling hadron physics



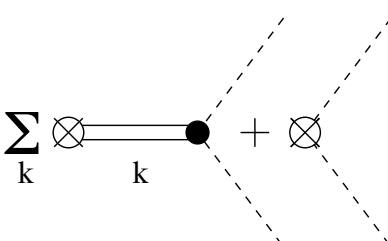
Standard treatment: **sum of Breit-Wigners**

Propagator: $iG_k(s) = \overline{\hspace{1cm}}_k = i/(s - M_k^2 + iM_k\Gamma_k)$

Scattering: $\sum_k \text{[diagram]} = \sum_k ig_k^2 G_k(s)$



Production: $\sum_k \text{[diagram]} = (\sum_k ig_k G_k(s) \alpha_k) + i\beta$



Problems:

- Wrong threshold behavior (cured by $\Gamma = \Gamma(s)$)
- Violates unitarity → **wrong phase motion**
- Parameters reaction dependent
only pole positions and residues universal!

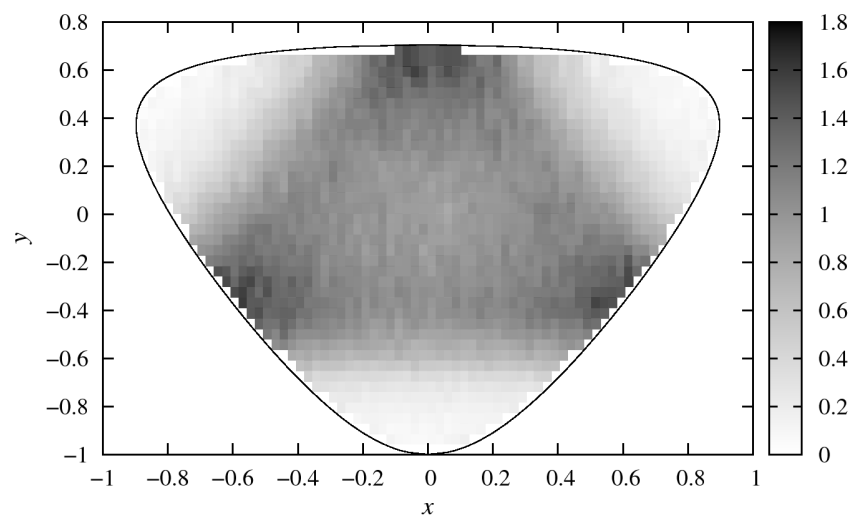
Sum of Breit Wigners



3-body Dalitz plot (theory)

Bastian Kubis

A simple Dalitz plot: $\phi \rightarrow 3\pi$



- 2×10^6 events in 1834 bins
KLOE 2003
- analyzed in terms of:
sum of 3 Breit–Wigners (ρ^\pm, ρ^0)
+ constant background term



Problem:

- unitarity fixes Im/Re parts
- adding a contact term destroys this relation

Sum of Breit Wigners with non-resonant term



Last Judgement (Detail) by Fra Angelico

Factorising the form factor into universal and reaction-specific parts

Christoph Hanhart

$$F(s) = P(s)\Omega(s)$$

- $\Omega(s)$ is **universal and fixed** in elastic regime (**Omnès function**)
- $P(s)$ **reaction specific** and contains e.g.
 - ▷ higher thresholds
 - ▷ inelastic resonances

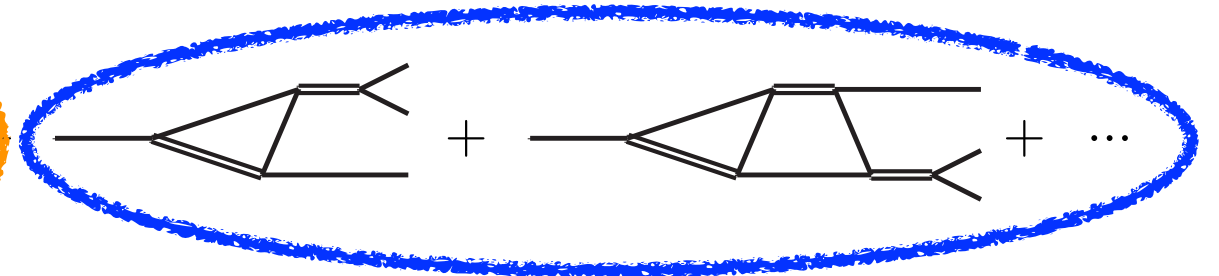
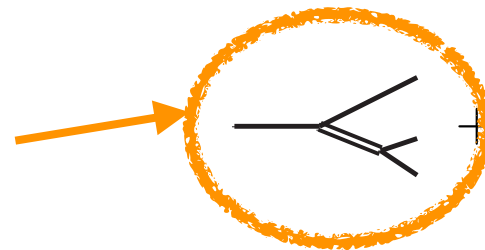
3-body Dalitz plot (theory)

Bastian Kubis

$$\mathcal{F}(s) = a \Omega(s) \left\{ 1 + \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\sin \delta_1^1(s') \hat{\mathcal{F}}(s')}{|\Omega(s')|(s' - s)} \right\}$$

takes into account
this

Omnès
takes into
account just this



3-body Dalitz plot (theory)

Bastian Kubis

calculable (but interaction-dependent)



$$\mathcal{F}(s) = \Omega(s) \left\{ \underset{\uparrow}{a} + \underset{\uparrow}{b} s + \frac{s^2}{\pi} \int_{4M_\pi^2}^{\infty} \frac{ds'}{s'^2} \frac{\sin \delta_1^1(s') \hat{\mathcal{F}}(s')}{|\Omega(s')|(s' - s)} \right\}$$

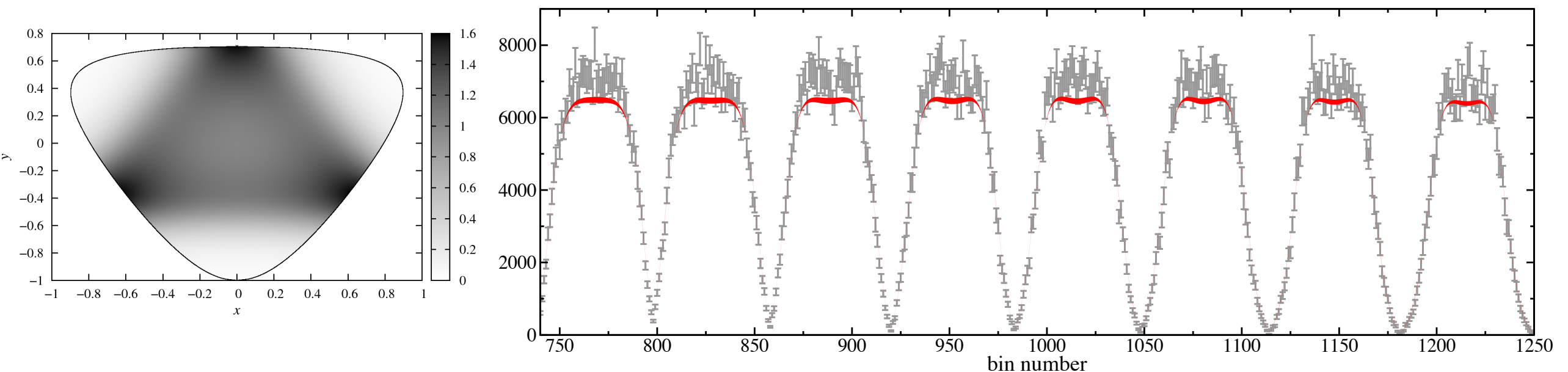
fit to data

Formalism applied to $\phi \rightarrow \pi\pi\pi\pi^0$

Bastian Kubis

Experimental comparison to $\phi \rightarrow 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012



χ^2/ndof 1.7...2.1

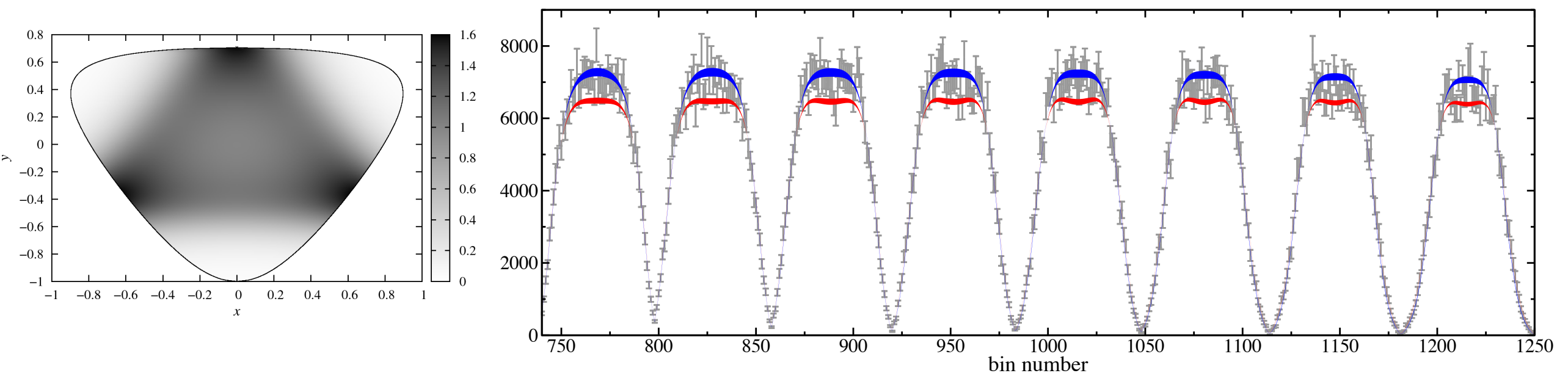
→ pairwise interaction only (with correct $\pi\pi$ scattering phase)

Formalism applied to $\phi \rightarrow \pi\pi\pi\pi^0$

Bastian Kubis

Experimental comparison to $\phi \rightarrow 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012



χ^2/ndof	1.7...2.1	1.2...1.5
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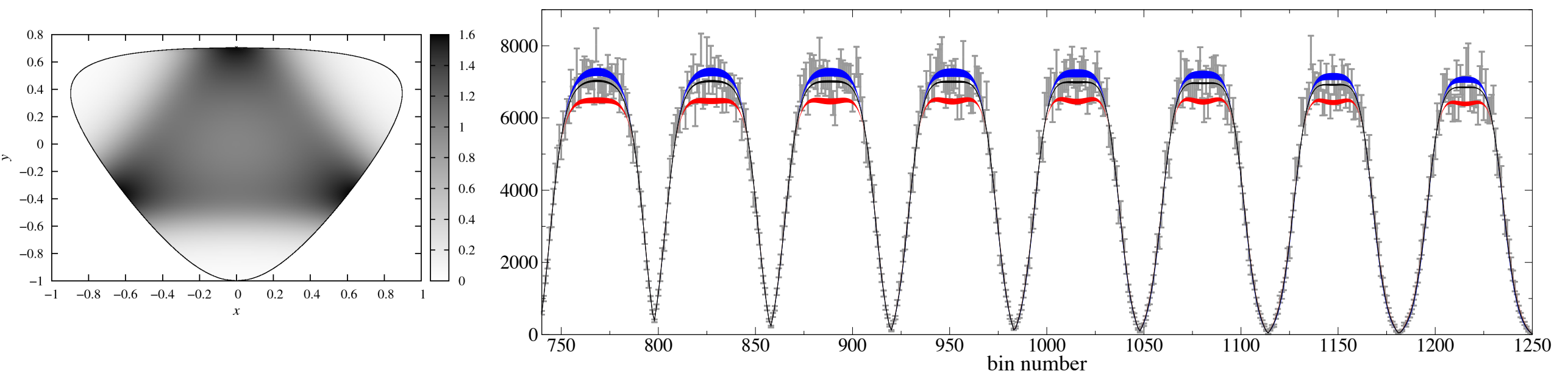
→ full 3-particle rescattering, only overall normalization adjustable

Formalism applied to $\phi \rightarrow \pi\pi\pi^0$

Bastian Kubis

Experimental comparison to $\phi \rightarrow 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012



χ^2/ndof	1.7...2.1	1.2...1.5	1.0
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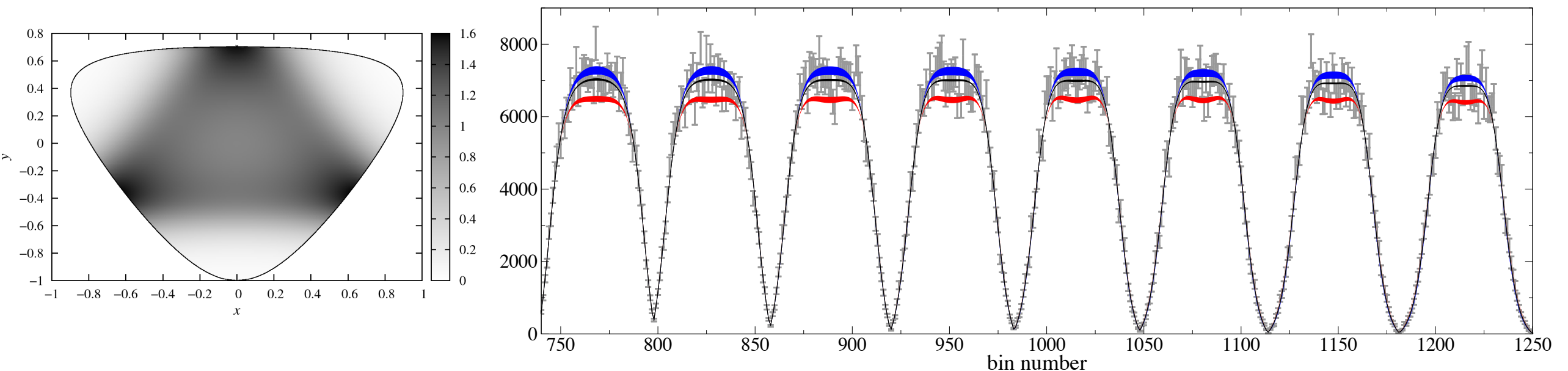
→ full 3-particle rescattering, 2 adjustable parameters
(additional "subtraction constant" to suppress inelastic effects)

Formalism applied to $\phi \rightarrow \pi\pi\pi^0$

Bastian Kubis

Experimental comparison to $\phi \rightarrow 3\pi$

- successive slices through Dalitz plot: Niecknig, BK, Schneider 2012

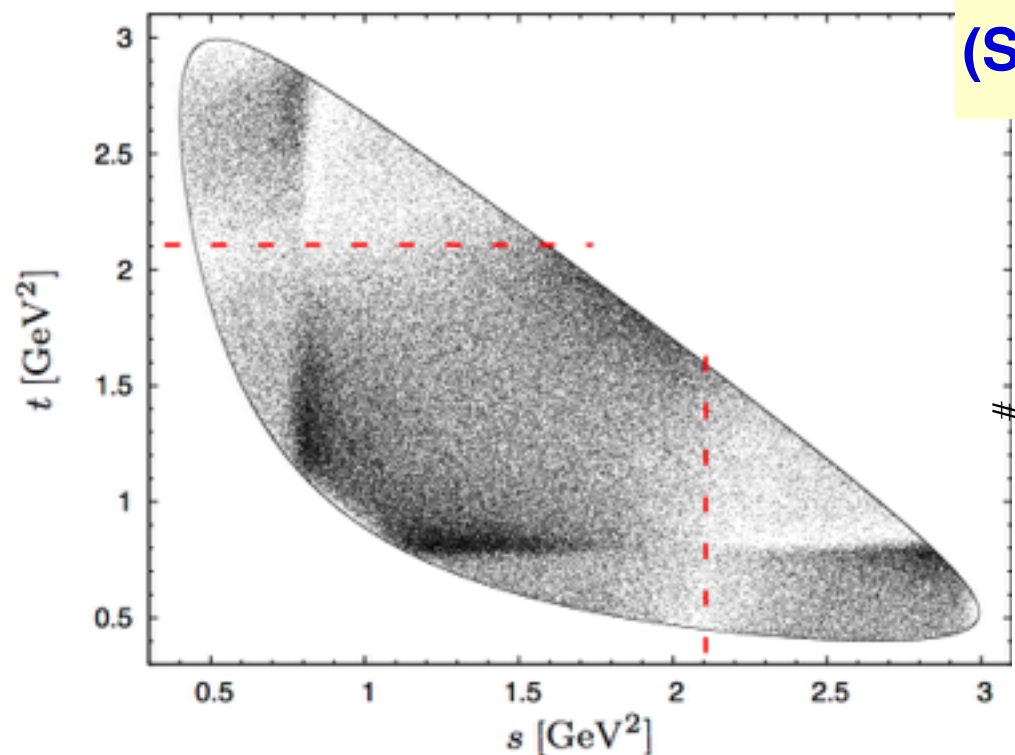


χ^2/ndof	1.7...2.1	1.2...1.5	1.0
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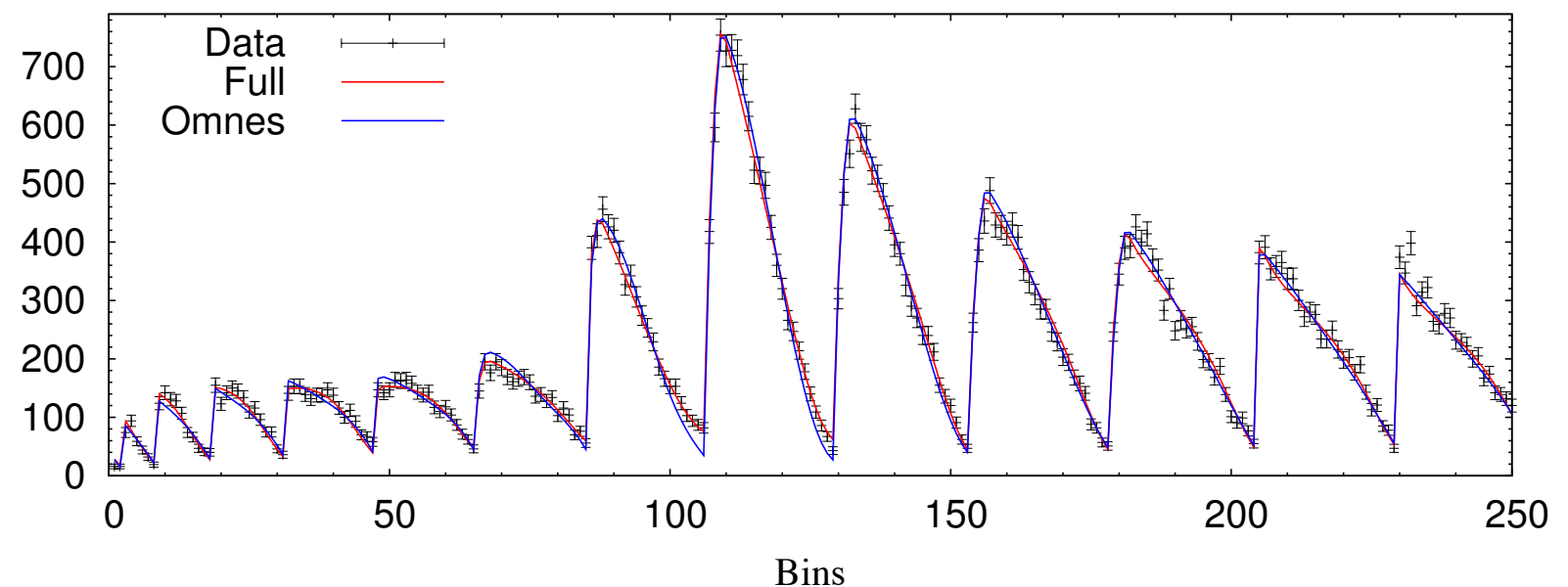
- perfect fit respecting analyticity and unitarity possible
- contact term emulates neglected rescattering effects
- no need for "background" — inseparable from "resonance"

Formalism applied to $D \rightarrow \pi\pi K$

Bastian Kubis



(Slices through) Dalitz plot $D^+ \rightarrow \pi^+ \pi^+ K^-$




Fit limited to
 $M(K\pi) < M(\eta') + M(K) \approx 1.45 \text{ GeV}$
 elastic approximation breaks down
 beyond.

- **Omnès fit:** $\chi^2/\text{ndof} \approx 1.42$
 ("isobar model" + non-resonant background waves)
- **full dispersive solution:** $\chi^2/\text{ndof} \approx 1.11$
 → visible improvement similar to $\phi \rightarrow 3\pi$
- full fit in terms of 7 complex subtraction constants
 (−1 phase, −1 overall normalisation)

7 fit parameters

Niecknig, BK *in progress*

Dalitz plot analyses

- rigorous using modern phase shift input
- allow to understand ad-hoc "background" 
- ideal demonstration case: $\phi \rightarrow 3\pi$ (elastic, one partial wave)
- implementation: + linear combination of basis functions
– basis functions different for each decay

Open questions / problems

- inelastic effects
 - ▷ we understand $I = 0$ S-wave $\pi\pi \leftrightarrow K\bar{K} \leftrightarrow f_0(980)$
→ may attempt $D \rightarrow 3\pi / \pi K\bar{K}$
 - ▷ how to parametrise "small" inelastic effects ($\eta' K$ in πK)?
- complex subtractions — can we understand imaginary parts?
- uncertainties in πK phase shifts? can we learn about them?
- high-energy extensions ($B \rightarrow 3h$ Dalitz plots??)

Das Model

The Model

- There are, for most cases we care about, no theoretically sound amplitude models...
- However, there are “good enough” models. What’s good enough depends on the purpose.
- So what to do? Suggest a mix of....
 - model-independent approaches
 - “good enough” models of various levels of sophistication
 - improve models (there is - and that’s fairly new - real, tangible, progress!)

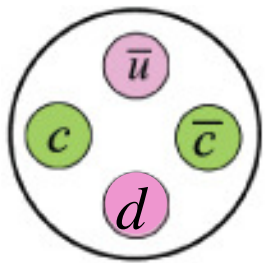


A few recent applications of amplitude analyses.

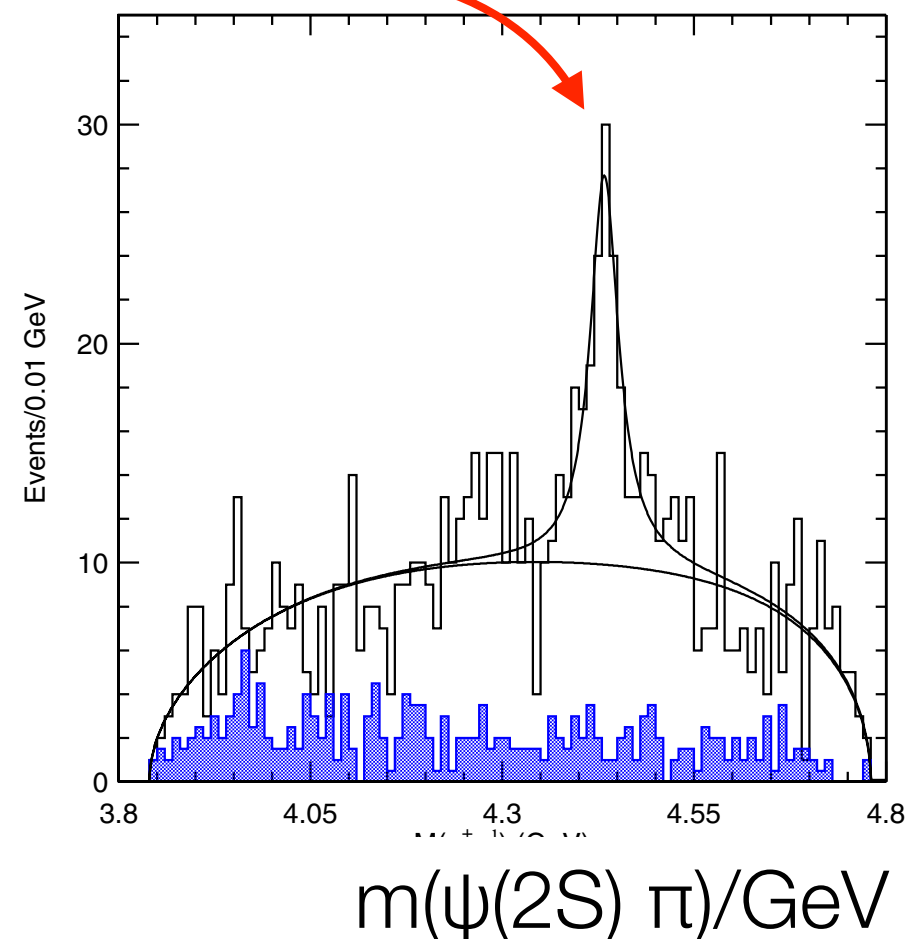
The Z(4430) question:

Is **this** peak in the $\psi(2S)\pi^-$ invariant mass, seen first by BELLE in 2008 when analysing $B \rightarrow \psi(2S)\pi^- K^+$, really a resonance?

Big thing - charged 4-quark state



The problem is that this is just the 1-D projection of a 4-D distribution...



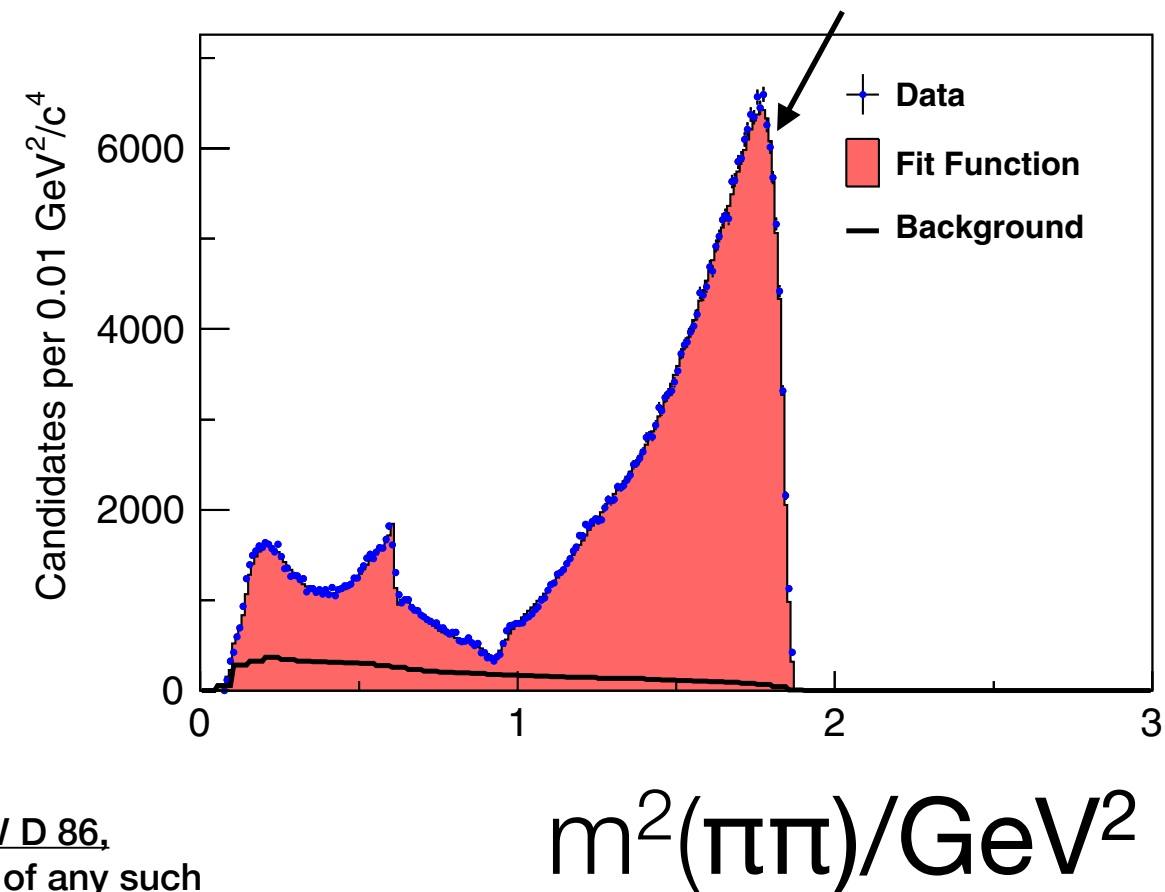
BELLE, Phys. Rev. Lett. 100 (2008) 142001, arXiv:0708.1790.

The 2-D illustration of this 4-D question

$$D^0 \rightarrow K_S \pi \pi$$

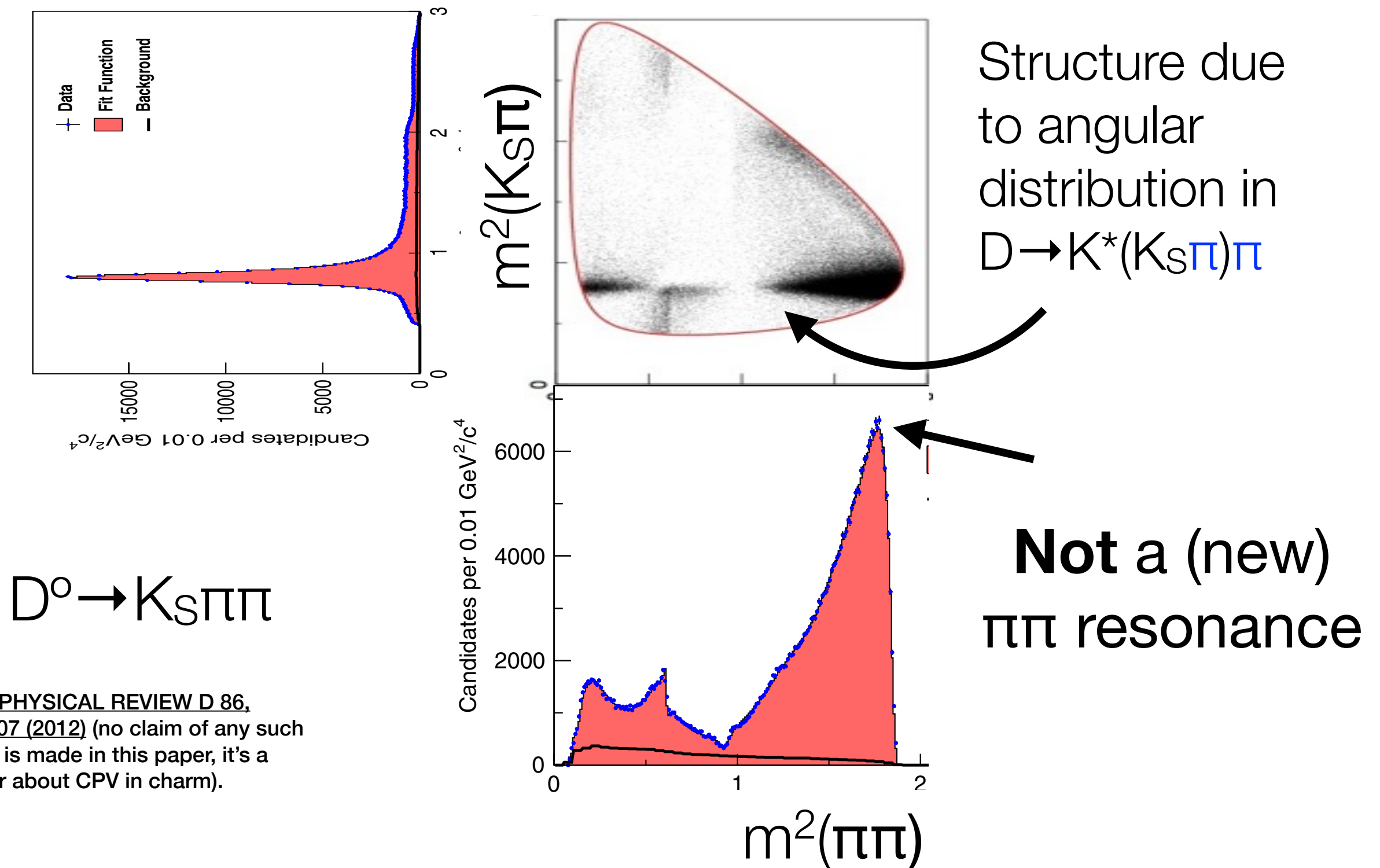
$\pi\pi$

resonance
near 2GeV^2 ?

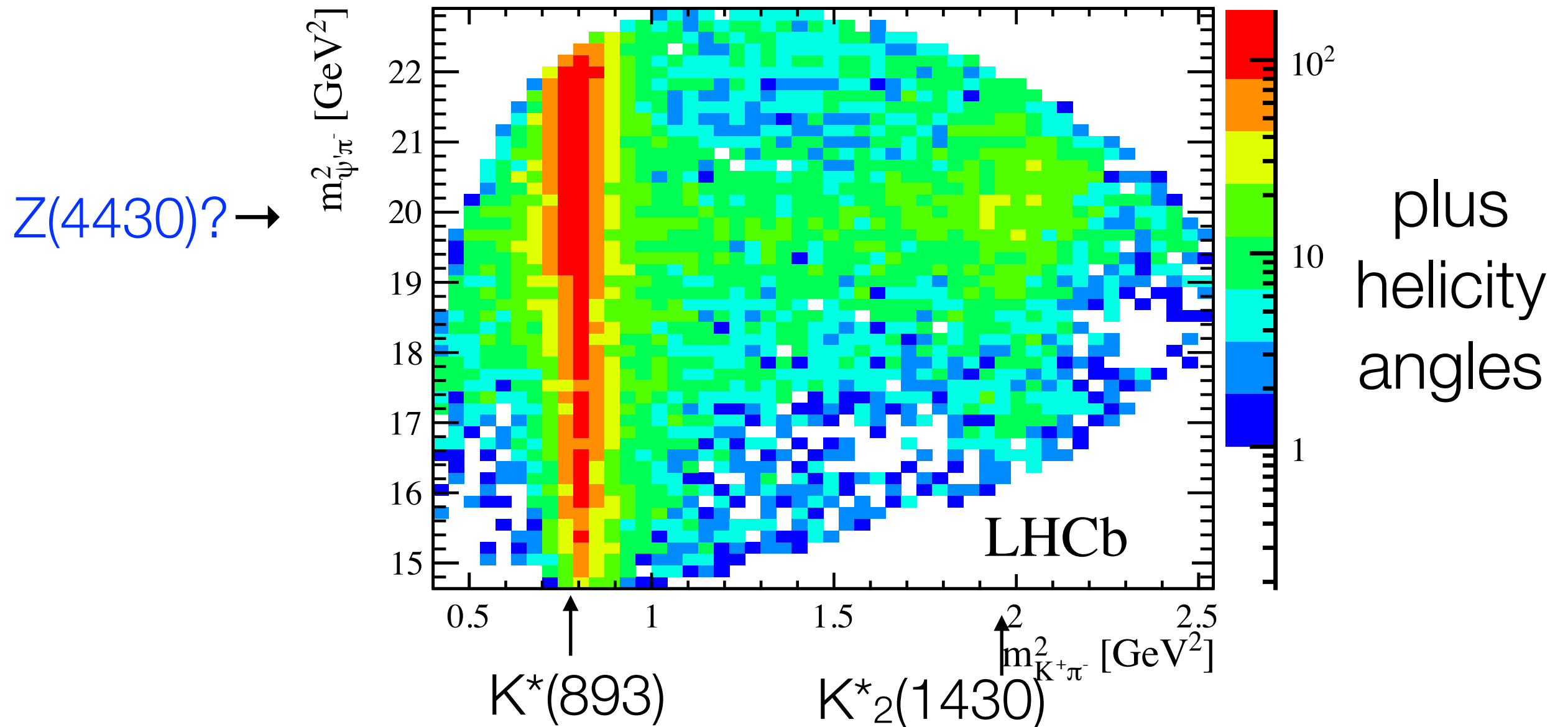


CDF PHYSICAL REVIEW D 86,
032007 (2012) (no claim of any such
thing is made in this paper, it's a
paper about CPV in charm).

The 2-D illustration of this 4-D question



$Z(4430) \rightarrow \psi(2S)\pi^-$ in $B \rightarrow \psi(2S)\pi^- K^+$?

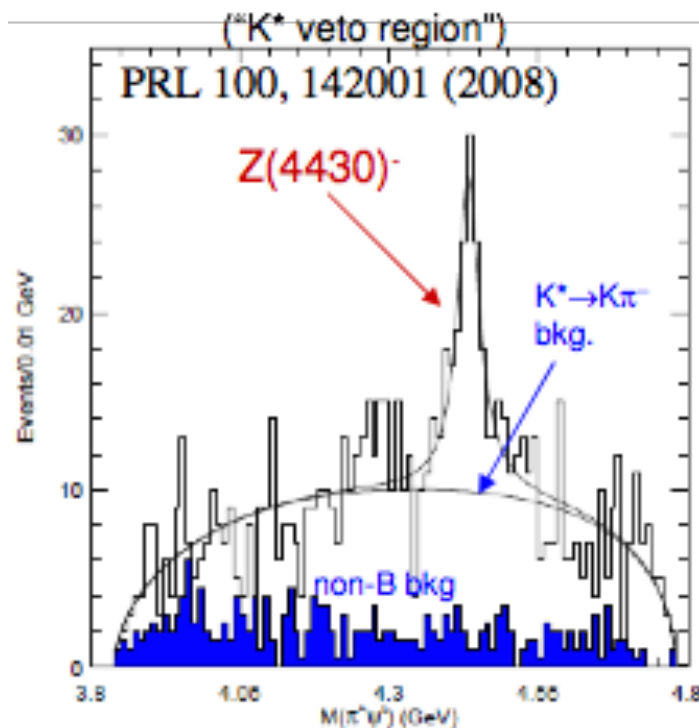


...and many other K^* resonances

$Z(4430) \rightarrow \psi(2S)\pi^-$ in $B \rightarrow \psi(2S)\pi^- K^+$?

Belle 2008

1D



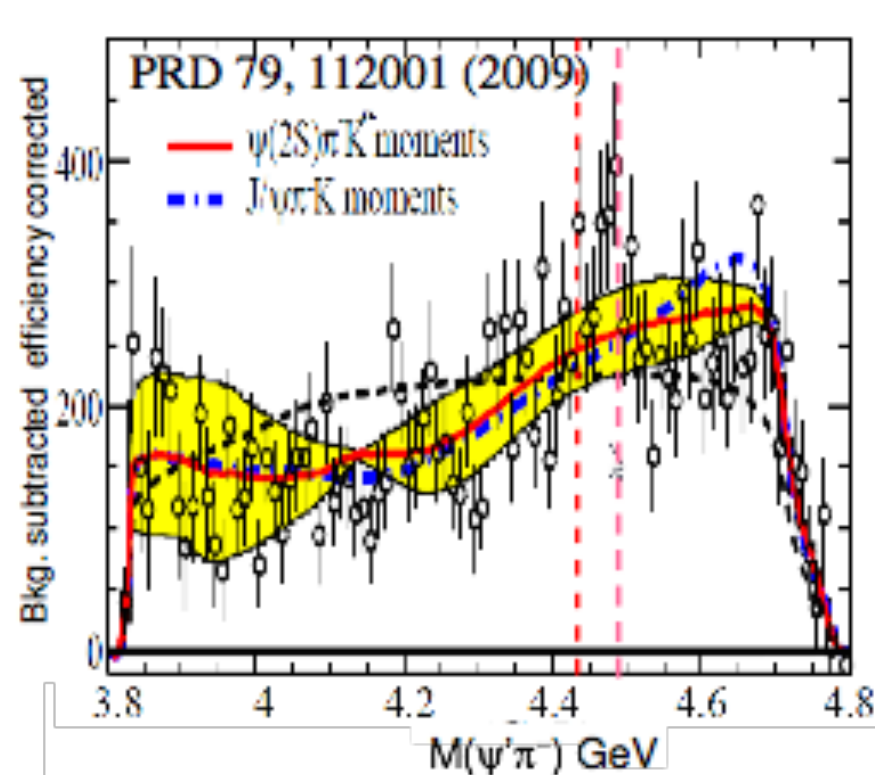
$$M(Z) = 4433 \pm 4 \pm 2 \text{ MeV}$$

$$\Gamma(Z) = 45_{-13}^{+18} \text{ MeV}$$

significance 6.5σ

BaBar 2009

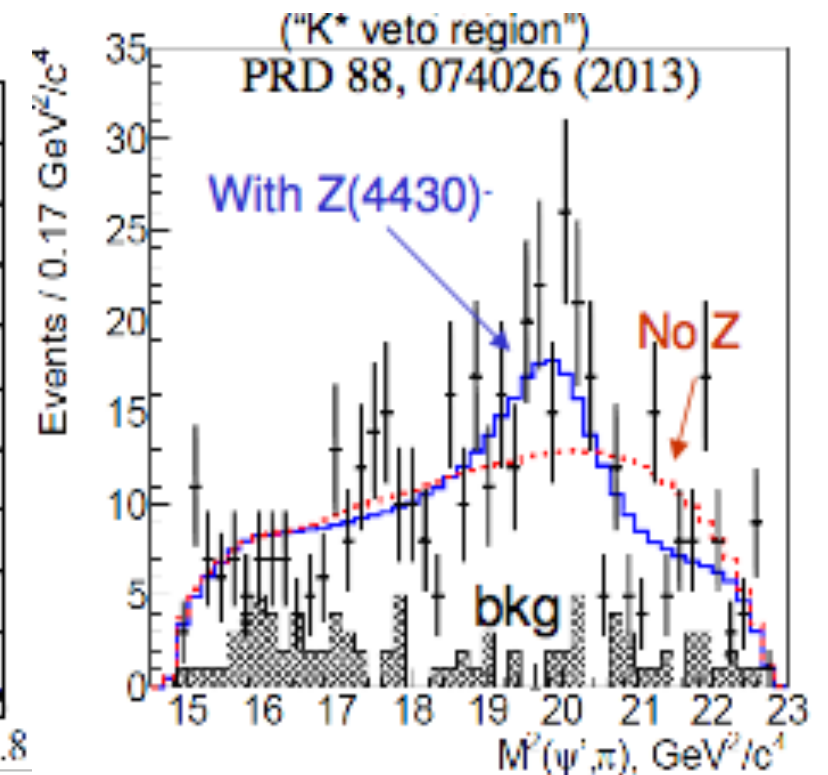
model-independent



Takes Kπ mass distribution as is (no K* model) and attempts to reproduce structure in $\psi(2S)\pi^-$ from angular momentum effects. Works within statistics.

Belle 2013

4D



$$M(Z) = 4485_{-22}^{+22} \text{ MeV}$$

$$\Gamma(Z) = 200_{-46}^{+41} \text{ MeV}$$

6.4σ (5.6σ with sys.)

J^P=1⁺ preferred by >3.4σ

LHCb's evidence for the $Z(4430)$ in $B \rightarrow \psi(2S)\pi^-K^+$

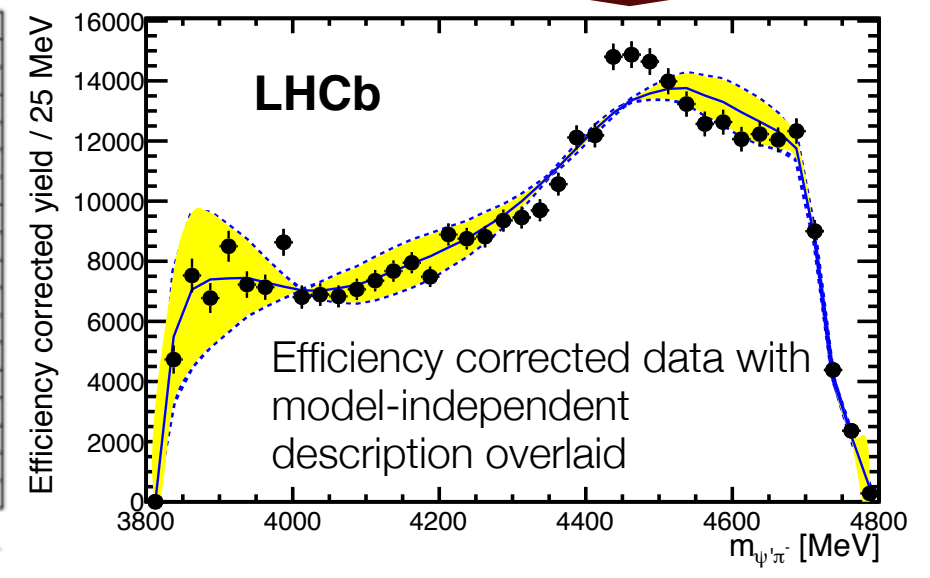
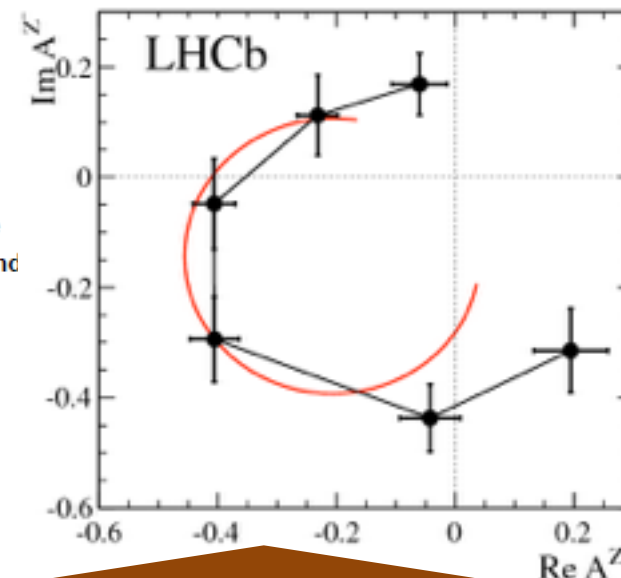
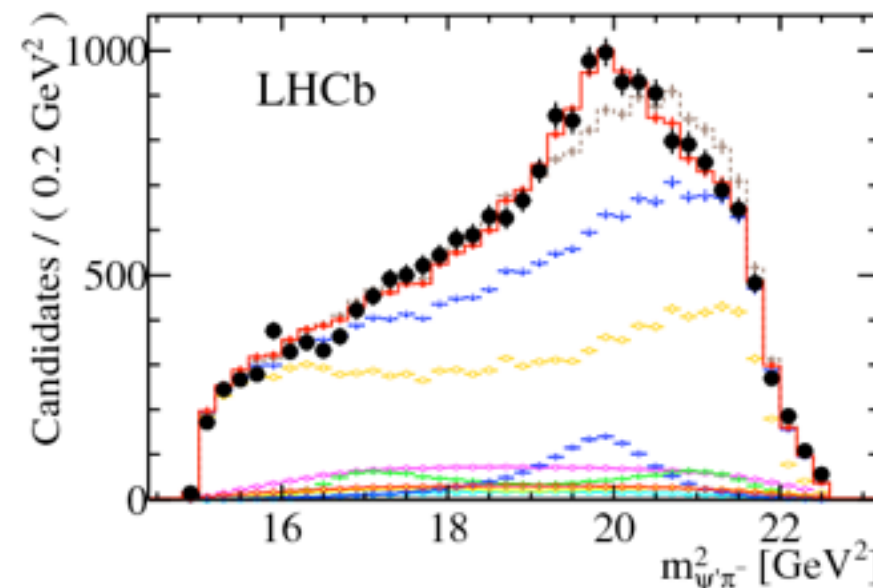
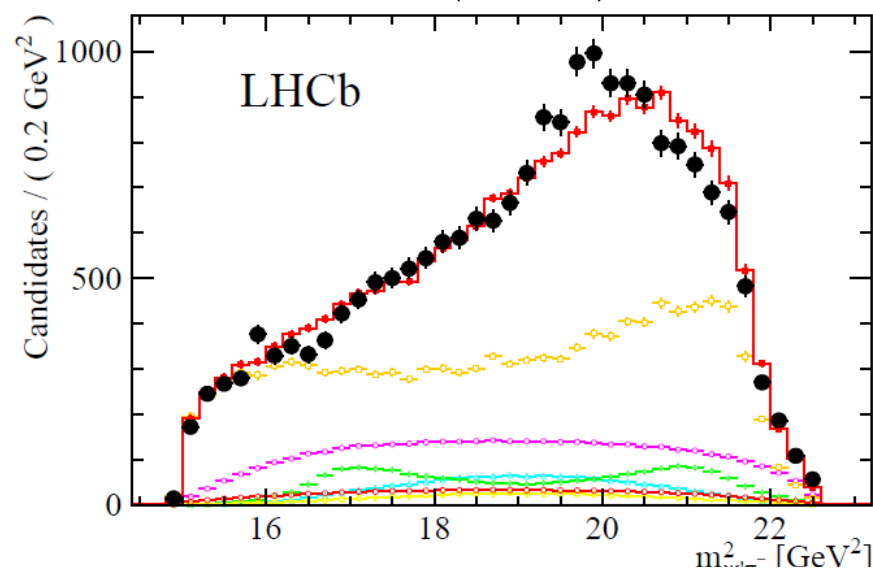
Amplitude fit:

**>13.9 σ in amplitude fit for $Z(4430)$ (and
>9.7 σ for 1^+ relative other J^P assignments)**

Model-independent

Model-indep. description of K^* resonances (w/o Z)
incompatible with data, clear excess in $Z(4430)$ region

("all data")



Phase Motion

Fit where K^* amplitudes are allowed to float,
but Z amplitude is described model-
independently by complex numbers in 6
bins of $m(\psi(2S)\pi)$ confirms resonance-like
phase motion

Tetraquark candidate travels



LHCb confirms existence of exotic hadrons

How CERN's Discovery of Exotic Particles May Affect Astrophysics

by BRIAN KOBERLEIN on APRIL 10, 2014

大型强子对撞机捕获到神秘粒子Z(4430)

或许成为物质形式“四夸克态”存在的有力证据

2014/04/13 15:46

LHCb実験を行っている国際研究チームが、4個のクォークが結合した粒子である「Z(4430)⁻」を合成したと発表した。Z(4430)としては、初発見から7年目にしようやく別の研究チームが存在を立証した事になる。

นักฟิสิกส์ยืนยันพบฮาดรอนสองควาร์กสองแอนติควาร์ก

WRITTEN BY NATTY_SCI ON APRIL 13, 2014. POSTED IN ฟิสิกส์, วิทยาศาสตร์

ล่าสุด เครื่อง LHCb ได้มีการศึกษาอีกครั้งและใช้ข้อมูลอนุภาคจากเครื่องโดยตรงมาวิเคราะห์ แต่เอาเทคนิคการวิเคราะห์ของศูนย์ปฏิบัติการวิจัยเบลล์และ BaBar มาใช้ ศาสตราจารย์วารินันท์และทีมงานได้ยืนยันแล้วว่า Z(4430) นั้นมีอยู่จริง และ exotic hadron ก็มีอยู่จริงด้วย

Nowa forma materii: potwierdzono istnienie egzotycznych hadronów

13-04-2014 13:08 TO TRZECI RODZAJ HADRONÓW, DOTYCZĄCY WYRÓŻNIANO BARIONY I MEZONY

"המובקה (לאותות של Z(4430) מדויקת – לפחות 13.9 סגמטה – דבר המאשר את קיומו של מצב זה" אמר דובר LHCb פייר-ולארי קמפנה. "ניתוח ה-LHCb חשף את הטבע המהדהד של המבנים הנצפים, והוכיח כי זהו באמת חלקיק, ולא תכונה מיוחדת של הנתונים."

CONFIRMADA L'EXISTÈNCIA D'UNA NOVA PARTÍCULA SUBATÒMICA

Эксперимент LHCb окончательно доказал реальность экзотического мезона Z(4430)

PISTOLA FUMANTE DI UNA PARTICELLA A QUATTRO QUARK

LHCb kinnitas tetrakvargi olemasolu

LHC Beauty Tangkap Z (4430)
Mungkin Tetraquark

Mystisk partikel udfordrer fysikernes kvarkmodel

Các nhà nghiên cứu tại LHC xác nhận sự tồn tại của hạt Tetraquark: tổ hợp tạo thành từ 4 quark

Thảo luận trong 'Khoa học' bắt đầu bởi ndminhduc, 15/4/14.

ISNA

خبرگزاری دانشجویان ایران، ایسنا
Iranian Students' News Agency

اخبار غیب، منید و امید بخش مجاز و کنیه، سلام سلام

تاکنون کشف ذره Z(4430) در سال 2007 به شدت جنجال برانگیز بود و فیزیکدانان بر سر موجودیت یا عدم موجودیت آن اختلاف نظر داشتند
تأیید کنونی ذره با استفاده از آشکارساز LHCb ماورای هرگونه تردید منطقی موجود است.

SU professors test boundaries of 'new physics' with discovery of four-quark hadron

Physicist Tomasz Skwarnicki confirms existence of exotic hadron with two quarks, two anti-quarks

Apr 10, 2014 | Article by: Rob Enslin

De LHCb heeft 't bevestigd: er bestaan exotische hadronen

10 APRIL 2014 DOOR ARIE NOUWEN • REAGEER

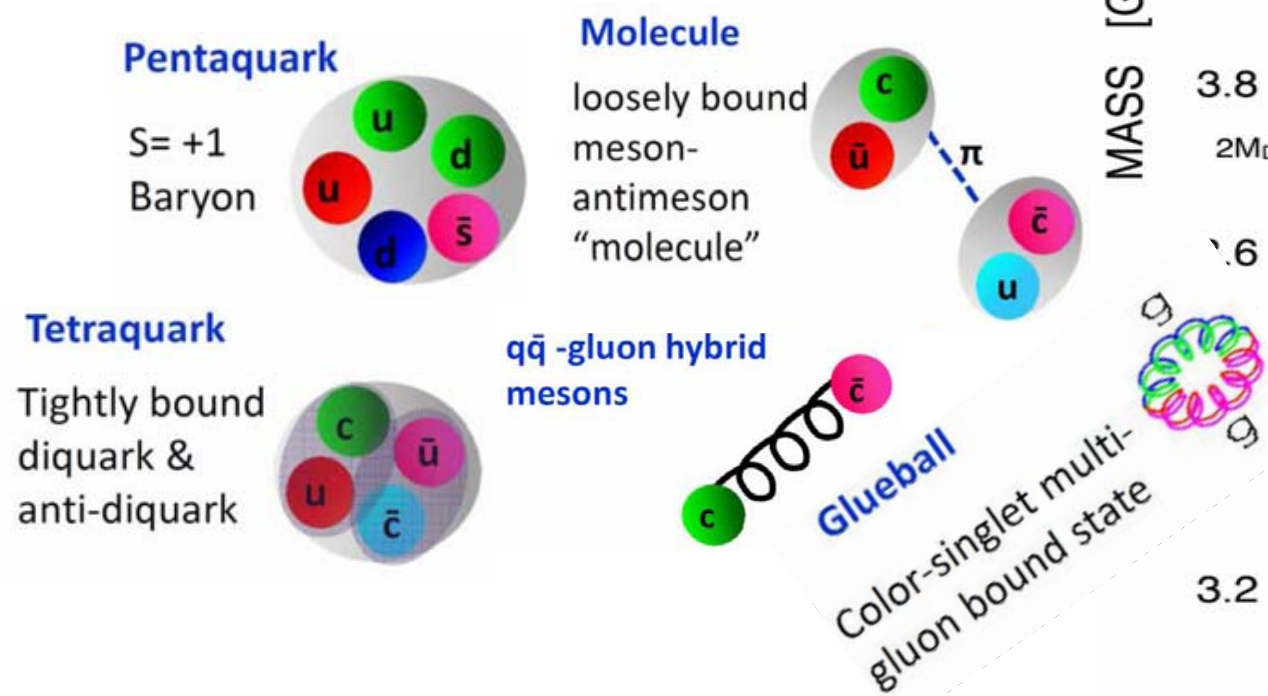
LHCb confirma la existencia de la partícula Z(4430) formada por cuatro quarks

Παρασκευή, 11 Απριλίου 2014

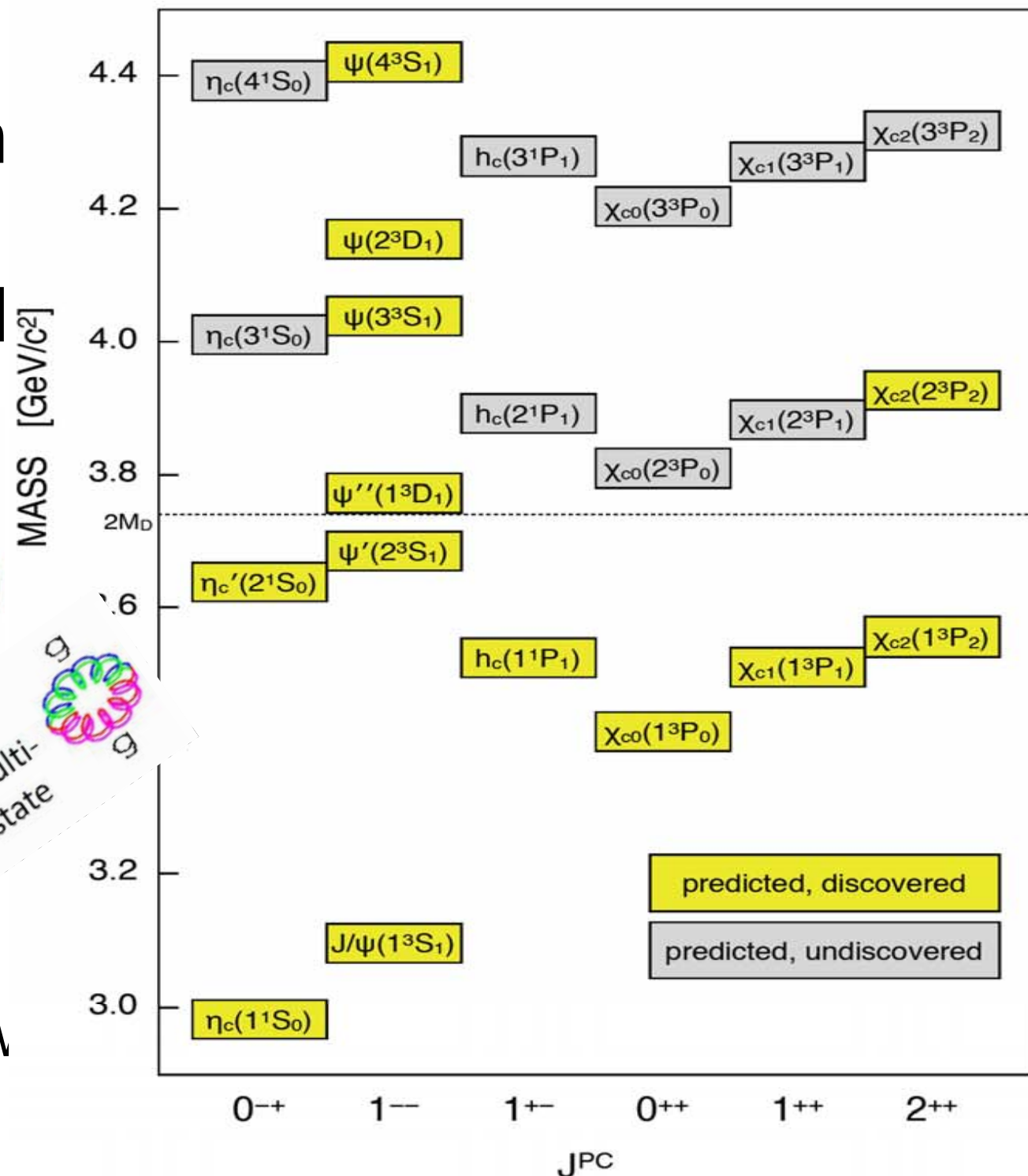
Ο LHCb επιβεβαιώνει την ύπαρξη εξωτικού σωματιδίου, LHCb confirms existence of exotic hadrons

XYZ like states

- Plenty of new charm
- What are they? (And



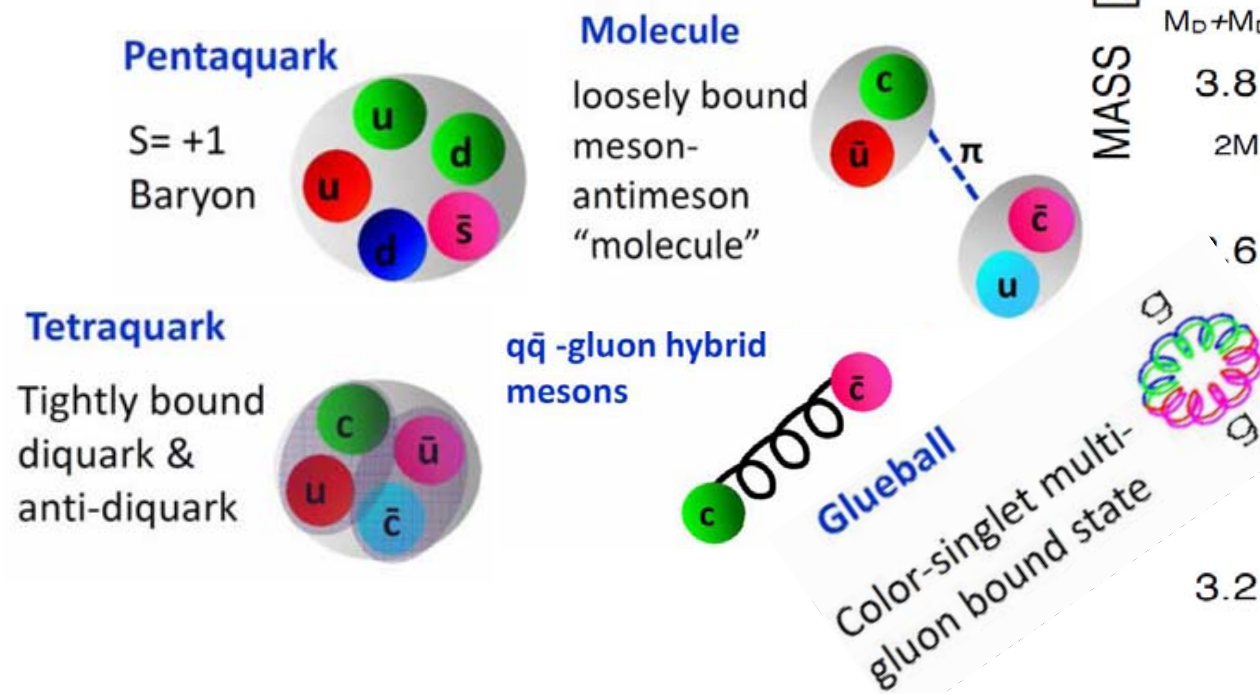
- and how/where do w



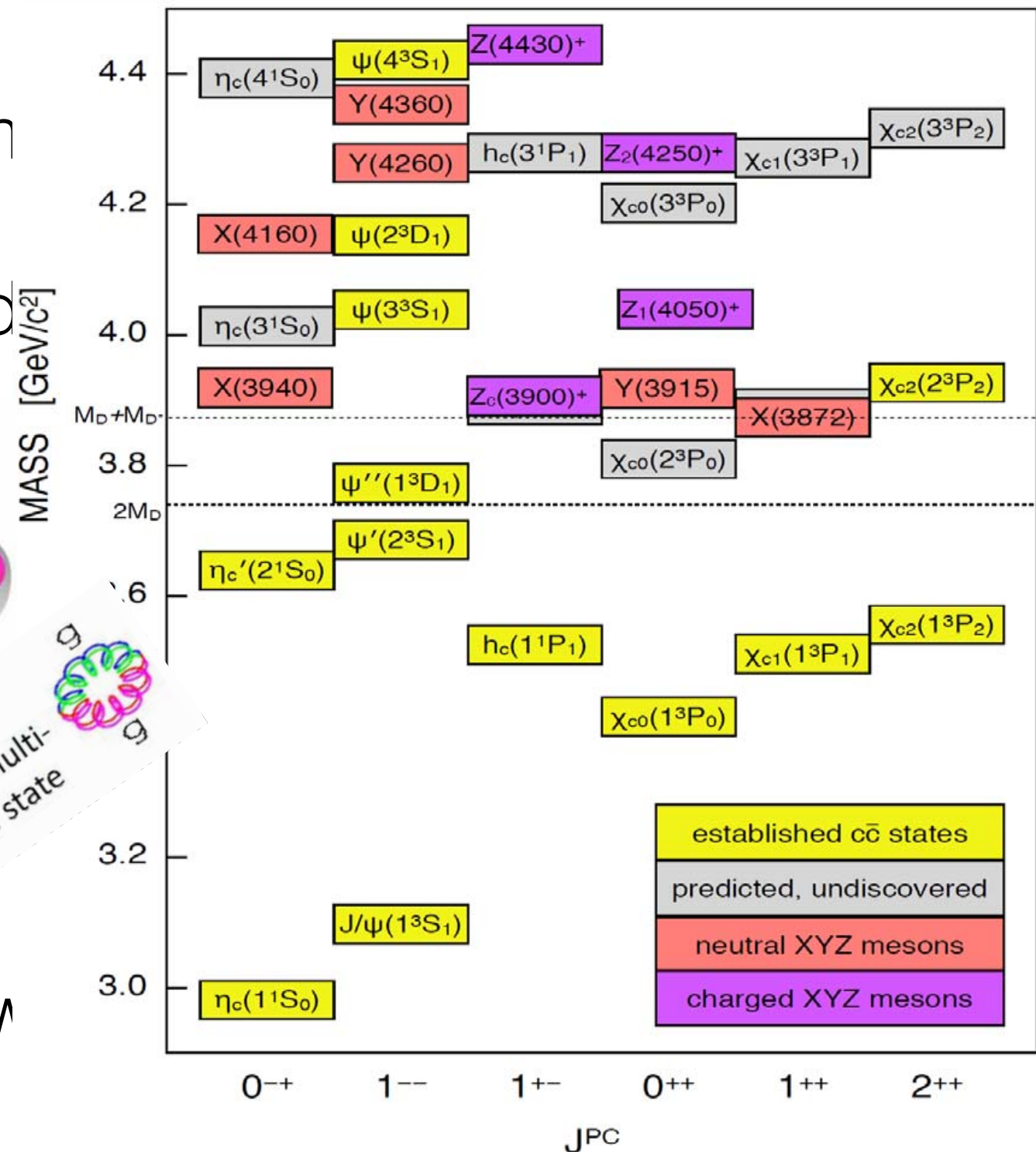
Diagrams and many results from Chengping Shen's talk at B2TIP Workshop, 28-31 October 2014, KEK

XYZ like states

- Plenty of new charm
- What are they? (And



- and how/where do v



Diagrams and many results from Chengping Shen's talk at B2TIP Workshop, 28-31 October 2014, KEK

XYZ like states

XYZ papers published in 2013 and 2014 (incomplete list)

X(3872)

LHCb: PRL 110, 222001 (2013)

BES III: Phys. Rev. Lett. 112, 092001 (2014)

BELLE: Phys. Rev. Lett. 110 252002 (2013)

Y(4008, 4260, 4360, 4660)

BES III Phys. Rev. Lett. 110, 252001 (2013)

Z(3900, 4020, 4200, 4430)

BELLE: Phys. Rev. Lett. 110 252002 (2013)

BELLE: Phys. Rev. D 89, 072015 (2014)

LHCb (2014): Phys.Rev.Lett. 112 (2014) 222002

BELLE (2014): Phys.Rev. D88 (2013) 074026

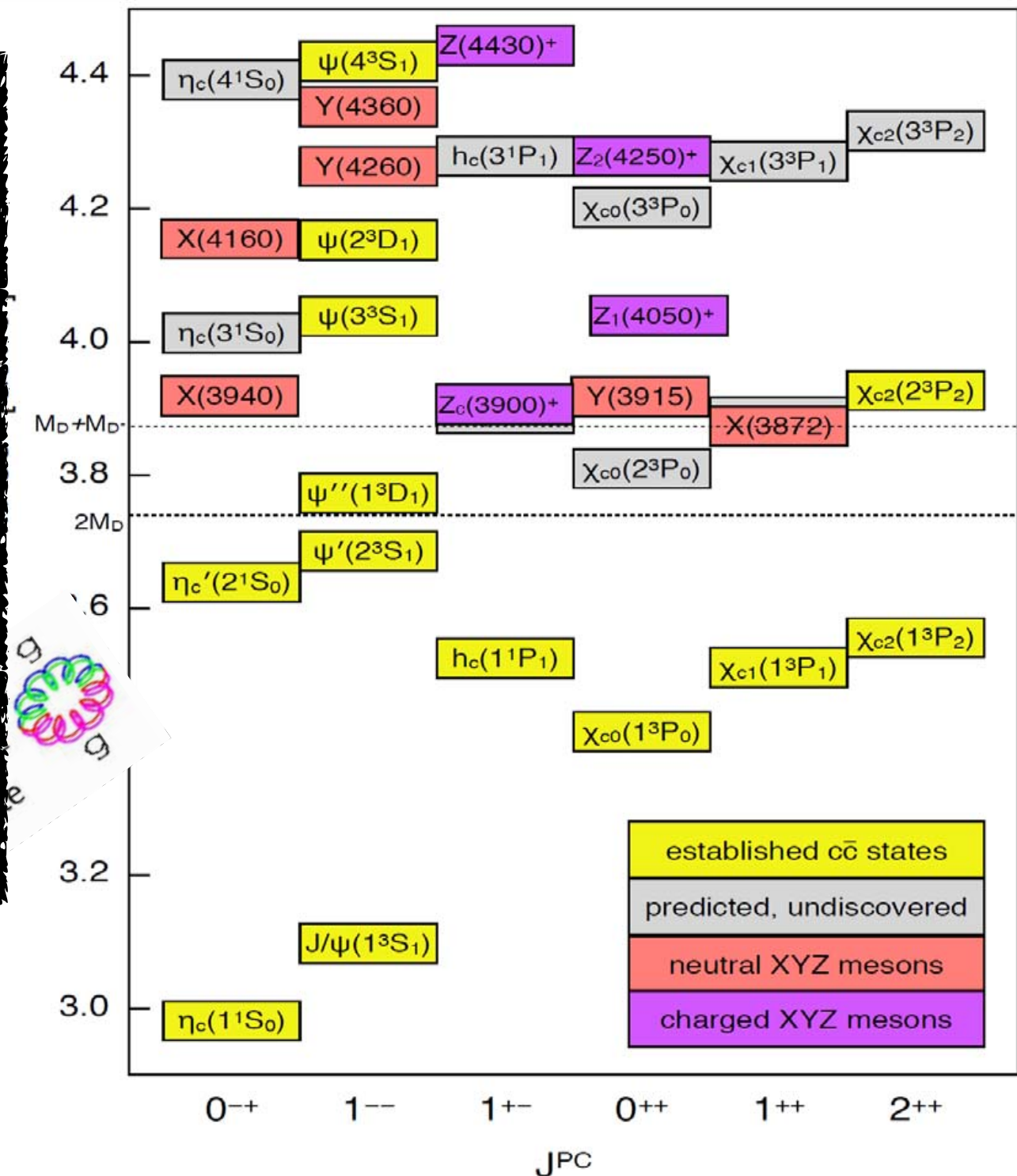
(no) Zcs

BES III Phys. Rev. Lett. 111, 242001 (2013)

BaBar: PRD 89, 111103(R) (2014)

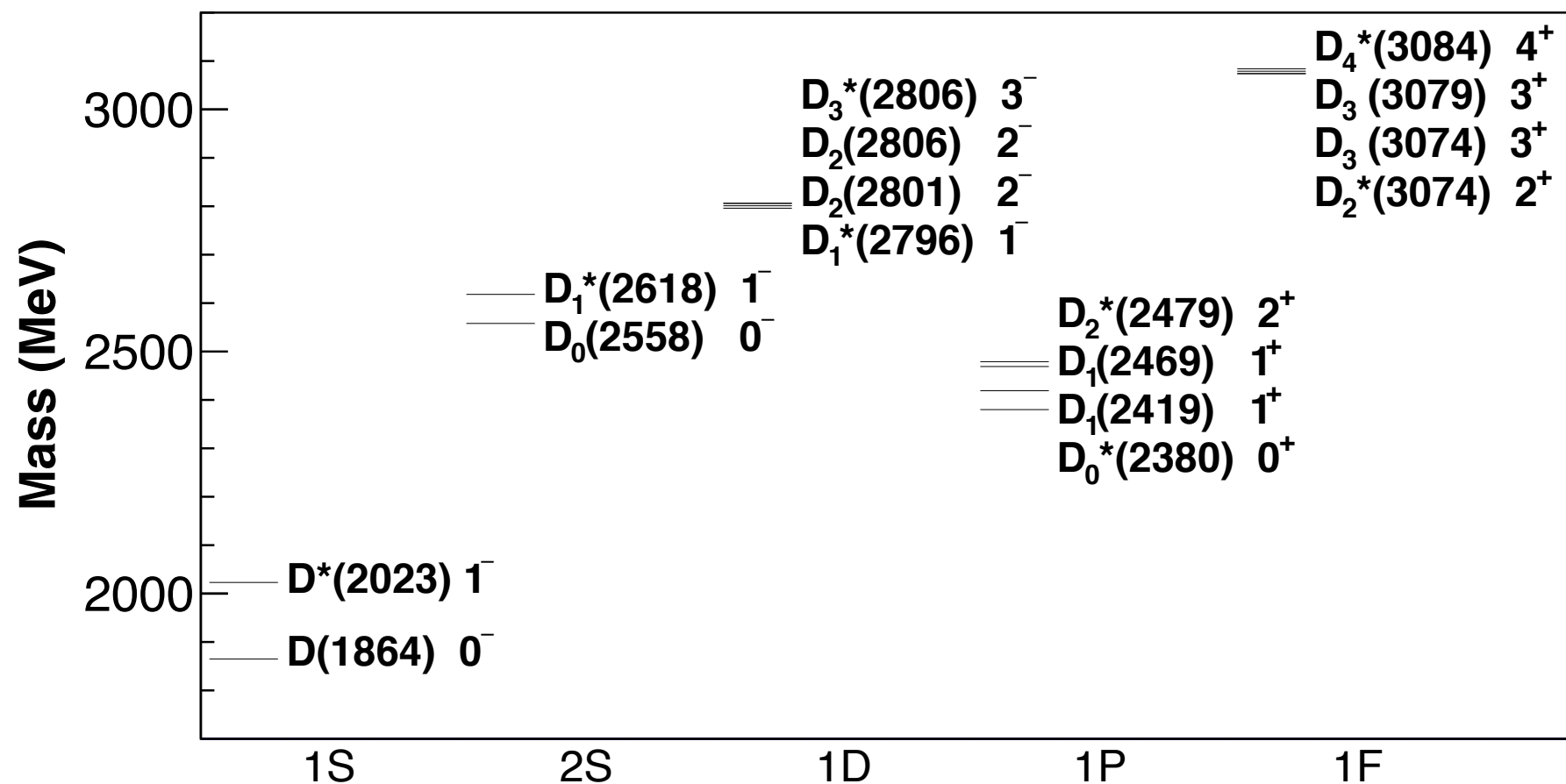
BES III: PRL 111, 032001 (2013) X(3823)

- and how/where do v



Diagrams and many results from Chengping Shen's talk at B2TIP Workshop, 28-31 October 2014, KEK

Spectroscopy

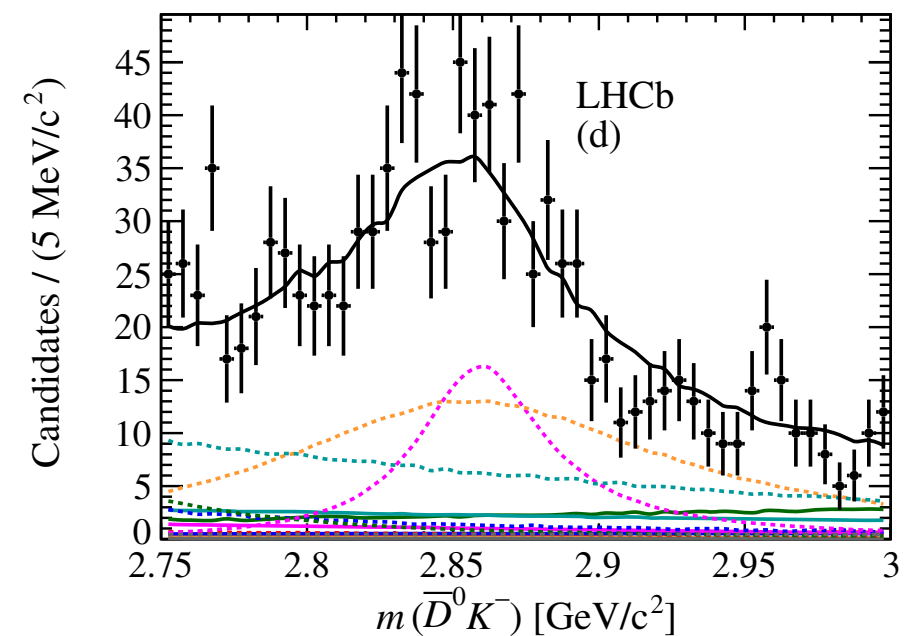
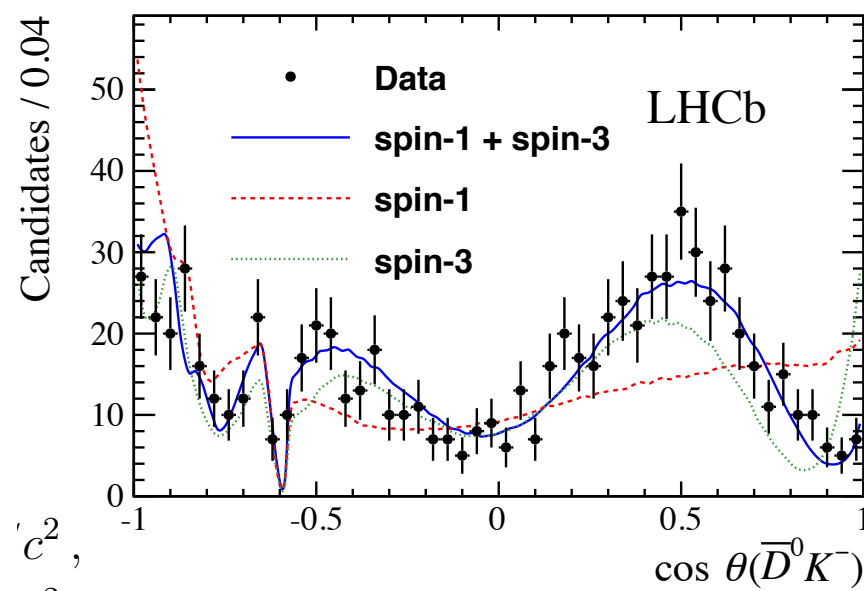
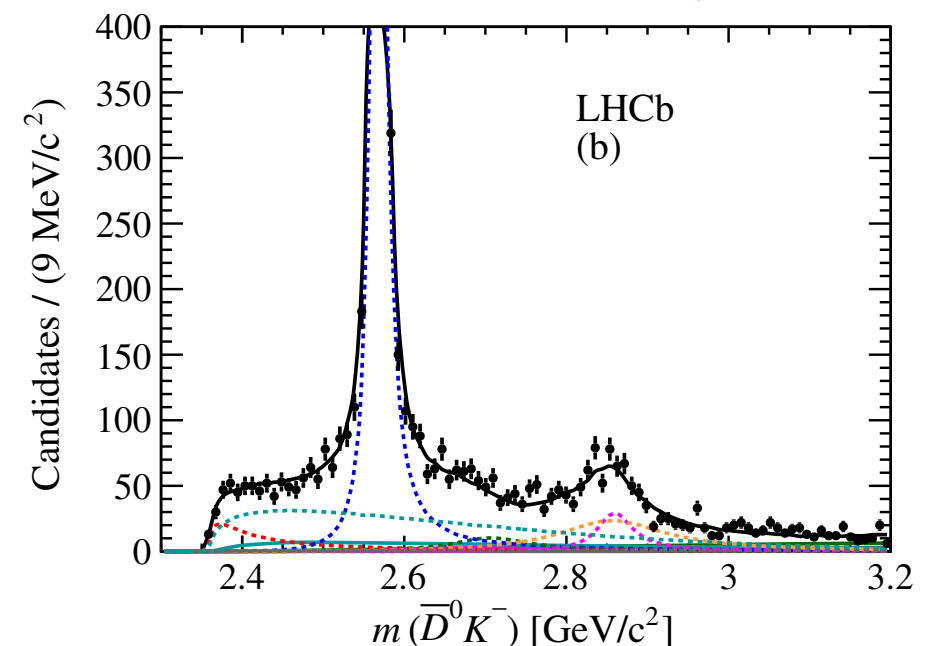


$$B_s \rightarrow \bar{D} K^- \pi^+$$

- Amongst many new results: The $D_{sJ}^*(2860)$ does exist - not only once, but twice:

$B \rightarrow \bar{D} K^- \pi^+$ Dalitz plot analysis finds two particles in the same mass region, one with spin 1, one with spin 3.

DK spectra in $B \rightarrow \bar{D} K^- \pi^+$ at LHCb (Phys.Rev. D90 (2014) 072003)



$B_s \rightarrow J/\psi \pi \pi$ CP content

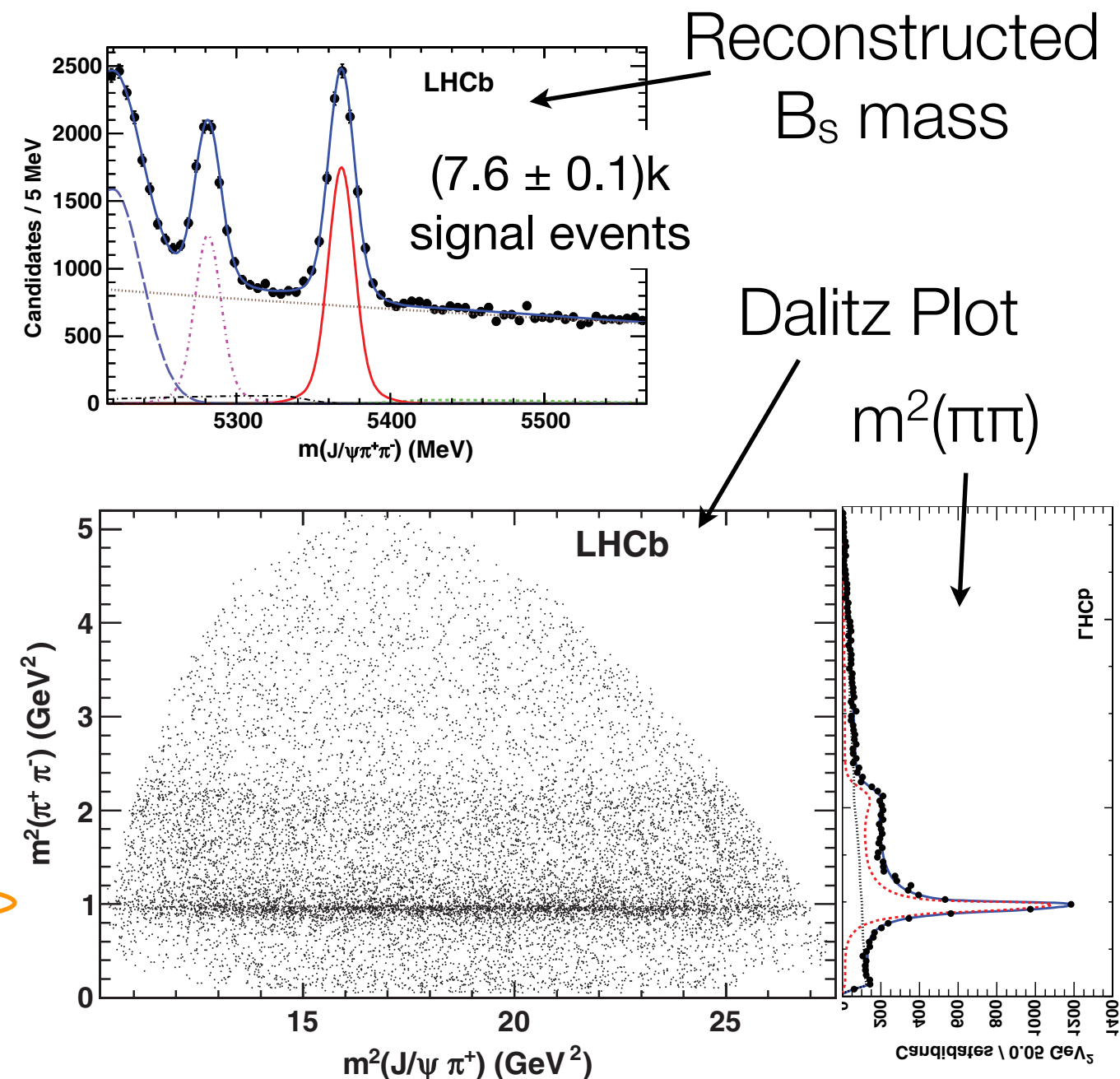
PRD 86, 052006 (2012)

- Amplitude analysis to evaluate the CP content of $B_s \rightarrow J/\psi \pi \pi$
- 4-dimensional analysis: 2 masses, 2 helicity angles.

Result:

Resonance	Normalized fraction (%)
$f_0(980)$	69.7 ± 2.3
$f_0(1370)$	21.2 ± 2.7
non-resonant $\pi^+ \pi^-$	8.4 ± 1.5
$f_2(1270), \Lambda = 0$	0.49 ± 0.16
$f_2(1270), \Lambda = 1$	0.21 ± 0.65

- Nearly all (>97.7% at 95 C.L.) CP-odd
- \Rightarrow No need for angular analysis to extract ϕ_s !



(see also arXiv:1302.1213 for an amplitude analysis of $B_s \rightarrow J/\psi KK$)

Model-independent check

PRD 86, 052006 (2012)

- Decay rate can be expressed in terms of spherical harmonics

$$\frac{d\Gamma}{d(\cos \theta)} = a_l^m Y_l^m(\cos \theta)$$

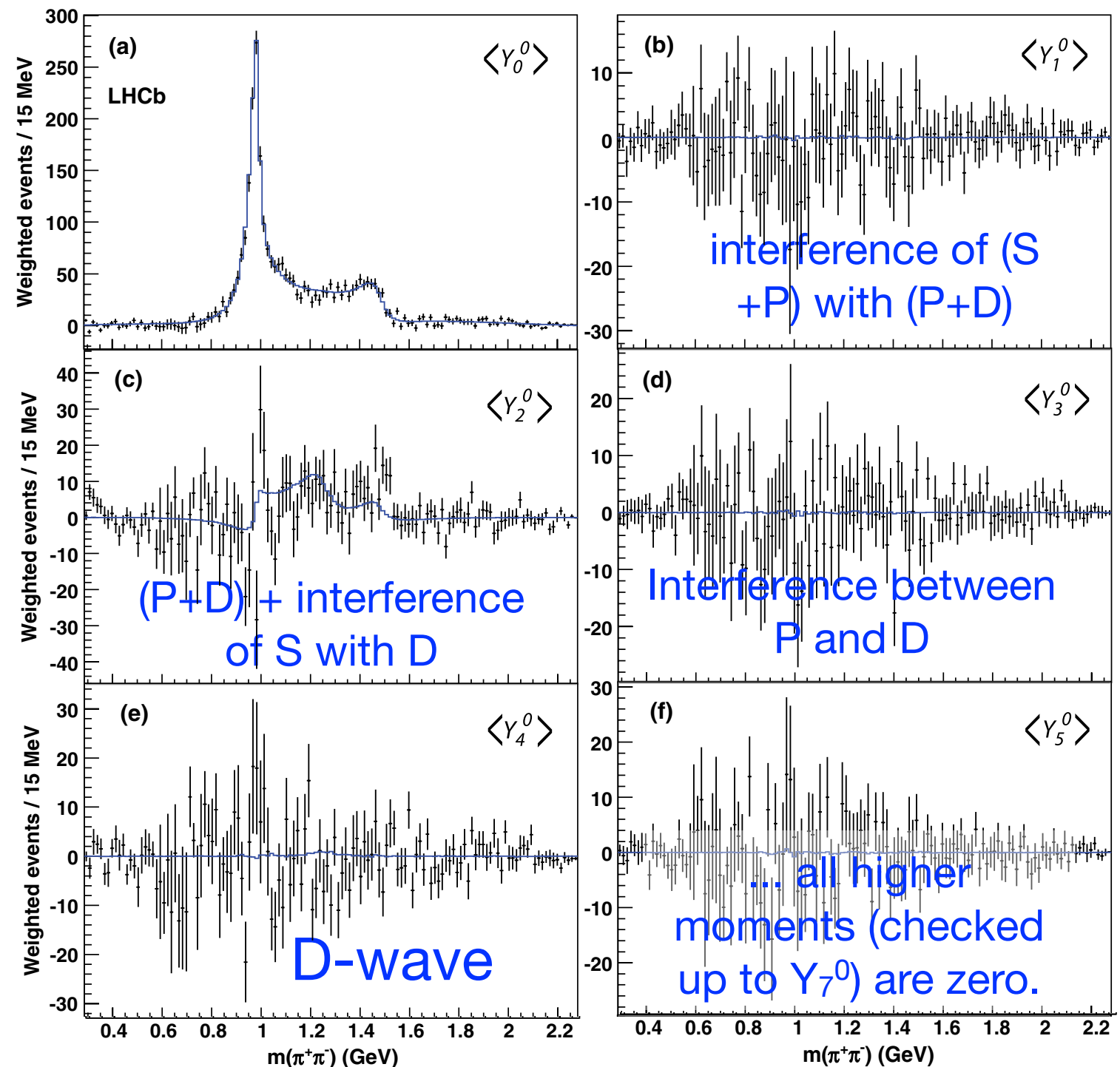
- These can be related to different S, P, D amplitude components.
- To project out a given component:

$$a_l^m = \int Y_l^m(\cos \theta) \frac{d\Gamma}{d(\cos \theta)} d(\cos \theta)$$

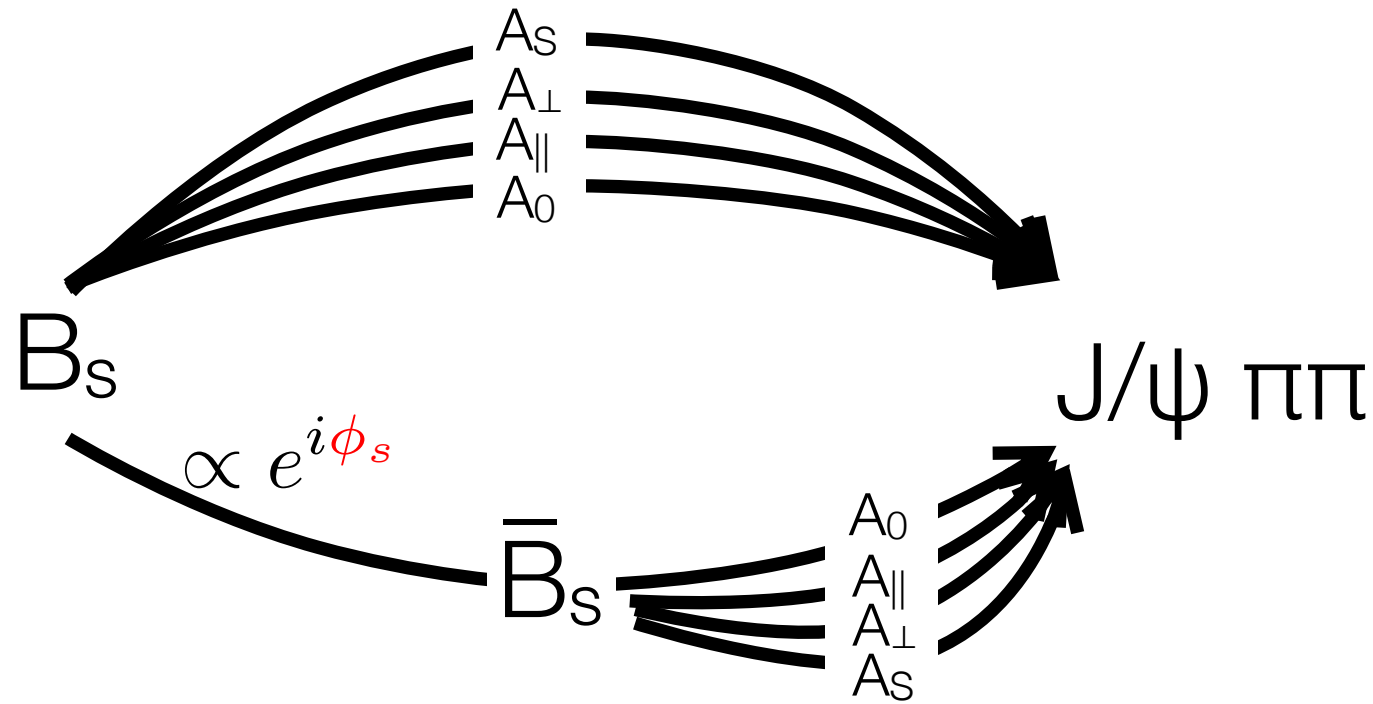
$$\approx \sum_{\text{events}} Y_l^m(\cos \theta_i)$$

= sum of weighted events

projection of weighted events onto $m(\pi\pi)$

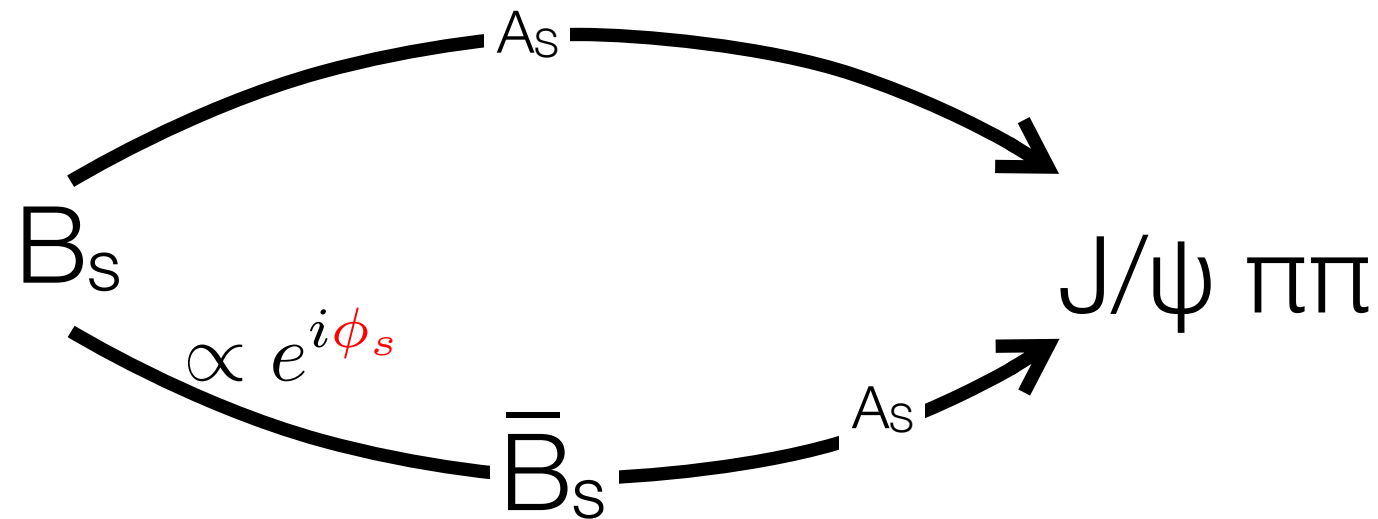


$B_s \rightarrow J/\psi \pi\pi$ for ϕ_s



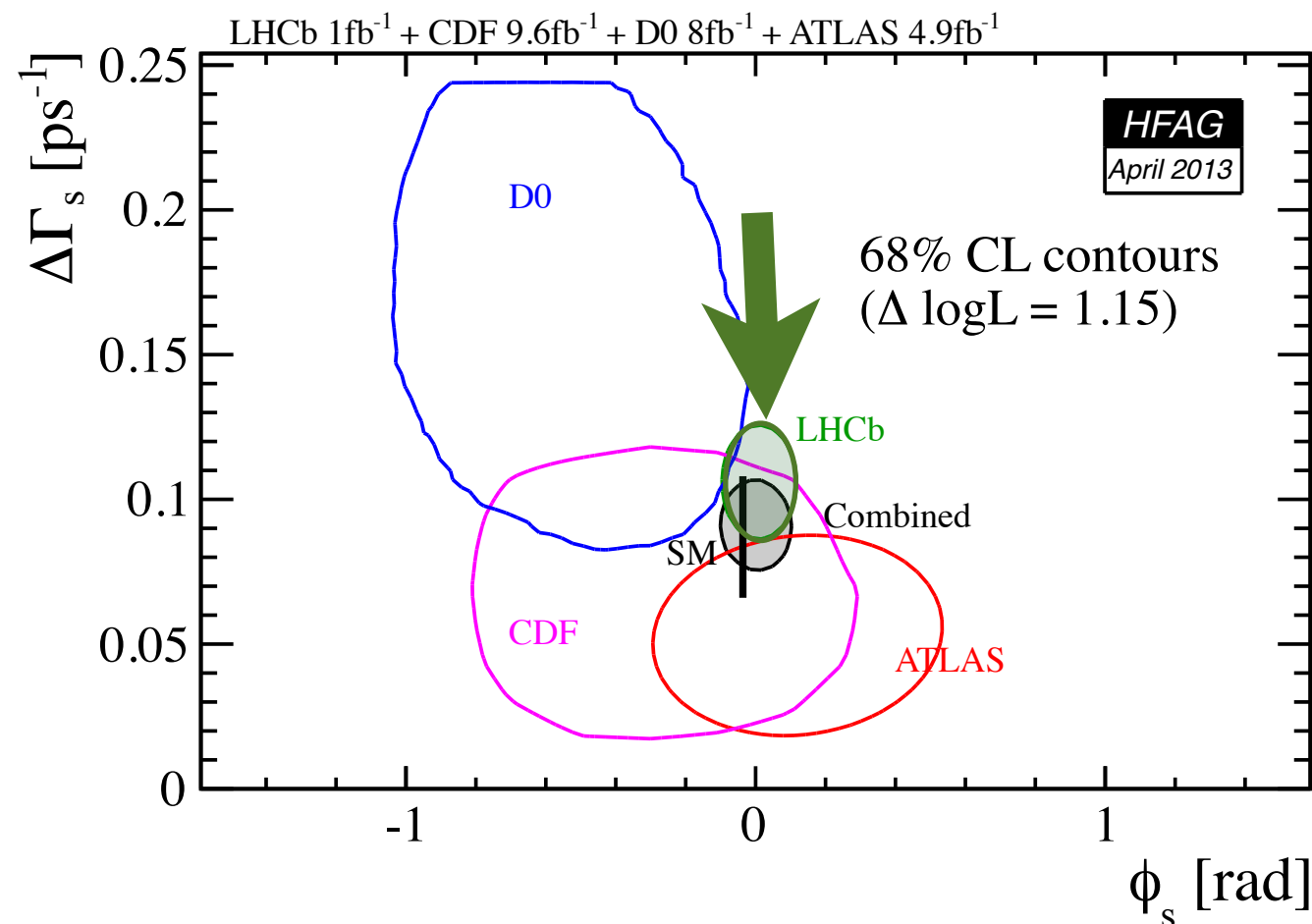
k	$f_k(\theta_\mu, \theta_K, \varphi_h)$	N_k	a_k	b_k	c_k	d_k
1	$2 \cos^2 \theta_K \sin^2 \theta_\mu$	$ A_0 ^2$	1	D	C	$-S$
2	$\sin^2 \theta_K (1 - \sin^2 \theta_\mu \cos^2 \varphi_h)$	$ A_\parallel ^2$	1	D	C	$-S$
3	$\sin^2 \theta_K (1 - \sin^2 \theta_\mu \sin^2 \varphi_h)$	$ A_\perp ^2$	1	$-D$	C	S
4	$\sin^2 \theta_K \sin^2 \theta_\mu \sin 2\varphi_h$	$ A_\parallel A_\perp $	$C \sin(\delta_\perp - \delta_\parallel)$	$S \cos(\delta_\perp - \delta_\parallel)$	$\sin(\delta_\perp - \delta_\parallel)$	$D \cos(\delta_\perp - \delta_\parallel)$
5	$\frac{1}{2} \sqrt{2} \sin 2\theta_K \sin 2\theta_\mu \cos \varphi_h$	$ A_0 A_\parallel $	$\cos(\delta_\parallel - \delta_0)$	$D \cos(\delta_\parallel - \delta_0)$	$C \cos(\delta_\parallel - \delta_0)$	$-S \cos(\delta_\parallel - \delta_0)$
6	$-\frac{1}{2} \sqrt{2} \sin 2\theta_K \sin 2\theta_\mu \sin \varphi_h$	$ A_0 A_\perp $	$C \sin(\delta_\perp - \delta_0)$	$S \cos(\delta_\perp - \delta_0)$	$\sin(\delta_\perp - \delta_0)$	$D \cos(\delta_\perp - \delta_0)$
7	$\frac{2}{3} \sin^2 \theta_\mu$	$ A_S ^2$	1	$-D$	C	S
8	$\frac{1}{3} \sqrt{6} \sin \theta_K \sin 2\theta_\mu \cos \varphi_h$	$ A_S A_\parallel $	$C \cos(\delta_\parallel - \delta_S)$	$S \sin(\delta_\parallel - \delta_S)$	$\cos(\delta_\parallel - \delta_S)$	$D \sin(\delta_\parallel - \delta_S)$
9	$-\frac{1}{3} \sqrt{6} \sin \theta_K \sin 2\theta_\mu \sin \varphi_h$	$ A_S A_\perp $	$\sin(\delta_\perp - \delta_S)$	$-D \sin(\delta_\perp - \delta_S)$	$C \sin(\delta_\perp - \delta_S)$	$S \sin(\delta_\perp - \delta_S)$
10	$\frac{4}{3} \sqrt{3} \cos \theta_K \sin^2 \theta_\mu$	$ A_S A_0 $	$C \cos(\delta_0 - \delta_S)$	$S \sin(\delta_0 - \delta_S)$	$\cos(\delta_0 - \delta_S)$	$D \sin(\delta_0 - \delta_S)$

$B_s \rightarrow J/\psi \pi\pi$ for ϕ_s



k	$f_k(\theta_\mu, \theta_K, \varphi_h)$	N_k	a_k	b_k	c_k	d_k
1	$2 \cos^2 \theta_K \sin^2 \theta_\mu$	$ A_0 ^2$	1	D	C	$-S$
2	$\sin^2 \theta_K (1 - \sin^2 \theta_\mu \cos^2 \varphi_h)$	$ A_{\parallel} ^2$	1	D	C	$-S$
3	$\sin^2 \theta_K (1 - \sin^2 \theta_\mu \sin^2 \varphi_h)$	$ A_{\perp} ^2$	1	D	C	S
4	$\sin^2 \theta_K \sin^2 \theta_\mu \sin 2\varphi_h$	$ A_{\parallel} A_{\perp} $	$\sin(\delta_{\perp} - \delta_{\parallel})$	$S \cos(\delta_{\perp} - \delta_{\parallel})$	$\sin(\delta_{\perp} - \delta_{\parallel})$	$D \cos(\delta_{\perp} - \delta_{\parallel})$
5	$\frac{1}{2} \sqrt{2} \sin 2\theta_K \sin 2\theta_\mu \cos \varphi_h$	$ A_0 A_{\parallel} $	$\cos(\delta_{\parallel} - \delta_0)$	$D \cos(\delta_{\parallel} - \delta_0)$	$C \cos(\delta_{\parallel} - \delta_0)$	$-S \cos(\delta_{\parallel} - \delta_0)$
6	$-\frac{1}{2} \sqrt{2} \sin 2\theta_K \sin 2\theta_\mu \sin \varphi_h$	$ A_0 A_{\perp} $	$C \sin(\delta_{\perp} - \delta_0)$	$S \cos(\delta_{\perp} - \delta_0)$	$\sin(\delta_{\perp} - \delta_0)$	$D \cos(\delta_{\perp} - \delta_0)$
7	$\frac{2}{3} \sin^2 \theta_\mu$	$ A_S ^2$	1	$-D$	C	S
8	$\frac{1}{3} \sqrt{6} \sin \theta_K \sin 2\theta_\mu \cos \varphi_h$	$ A_S A_{\parallel} $	$C \cos(\delta_{\parallel} - \delta_S)$	$S \sin(\delta_{\parallel} - \delta_S)$	$\cos(\delta_{\parallel} - \delta_S)$	$D \sin(\delta_{\parallel} - \delta_S)$
9	$-\frac{1}{3} \sqrt{6} \sin \theta_K \sin 2\theta_\mu \sin \varphi_h$	$ A_S A_{\perp} $	$\sin(\delta_{\perp} - \delta_S)$	$-D \sin(\delta_{\perp} - \delta_S)$	$C \sin(\delta_{\perp} - \delta_S)$	$S \sin(\delta_{\perp} - \delta_S)$
10	$\frac{4}{3} \sqrt{3} \cos \theta_K \sin^2 \theta_\mu$	$ A_S A_0 ^2$	$C \cos(\delta_0 - \delta_S)$	$S \sin(\delta_0 - \delta_S)$	$\cos(\delta_0 - \delta_S)$	$D \sin(\delta_0 - \delta_S)$

Combined $B_s \rightarrow J/\psi \text{ KK}$ and $B_s \rightarrow J/\psi \pi\pi$ for ϕ_s



arXiv:1304.2600 (2013)

supersedes previous results:
Phys. Rev. Lett. 108 (2012) 101803
Physics Letters B 713 (2012) 378

ϕ_s very sensitive to NP. But
no NP effects seen, yet...

$\Delta \Gamma_s$ less sensitive to NP
($\propto \cos(\phi^{\text{new}})$), but impressive
validation of HQE
calculation.

$$\text{SM: } \phi_s^{\text{SM}} = -0.036 \pm 0.002 \text{ rad}$$

$$\text{LHCb: } \phi_s = 0.07 \pm 0.09 \text{ (stat)} \pm 0.01 \text{ (syst)} \text{ rad,}$$

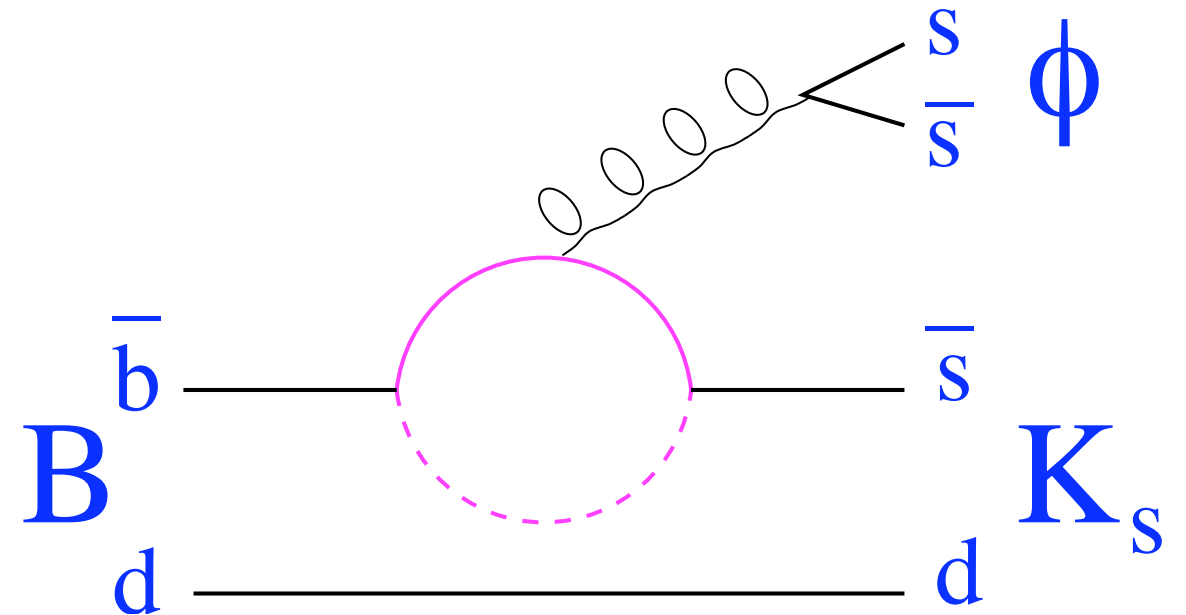
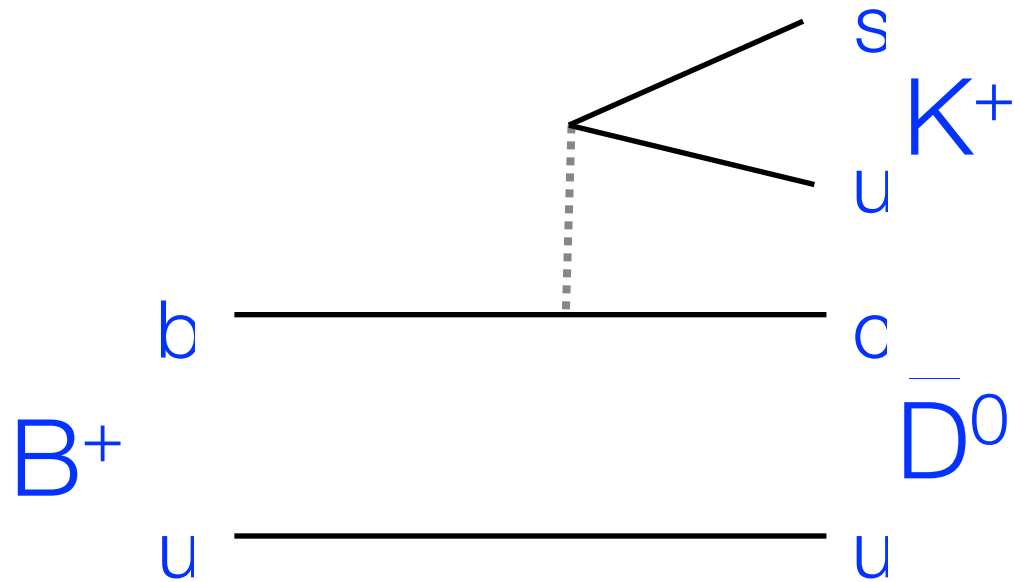
$$\Gamma_s \equiv (\Gamma_L + \Gamma_H)/2 = 0.663 \pm 0.005 \text{ (stat)} \pm 0.006 \text{ (syst)} \text{ ps}^{-1}$$

$$\Delta \Gamma_s \equiv \Gamma_L - \Gamma_H = 0.100 \pm 0.016 \text{ (stat)} \pm 0.003 \text{ (syst)} \text{ ps}^{-1}$$

Loops vs Trees

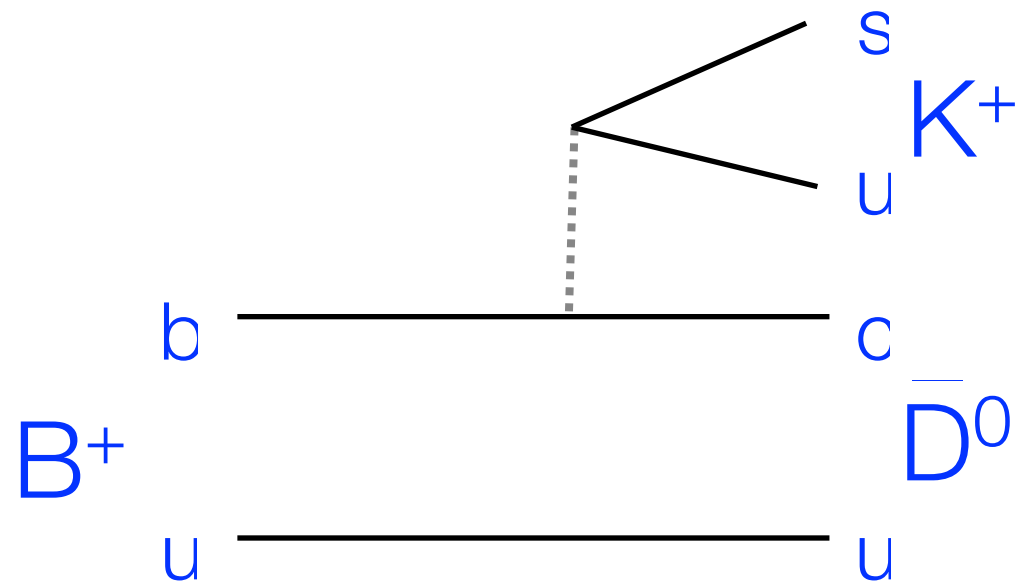
- Expect no New Physics in Trees

- New Physics in loops?

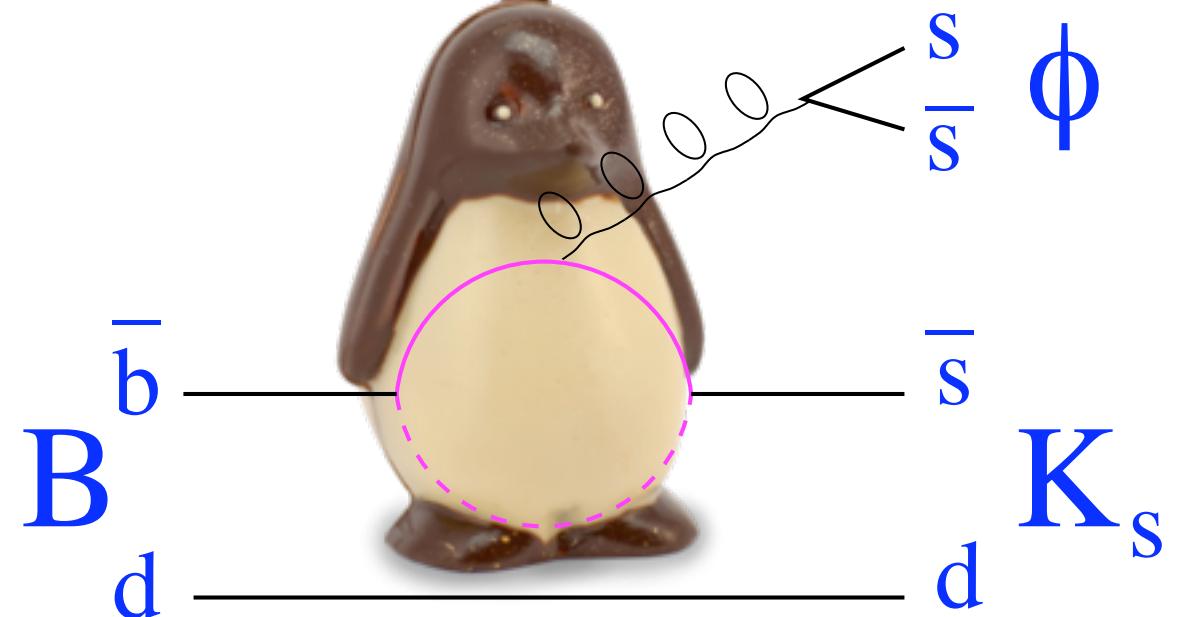


Loops vs Trees

- Expect no New Physics in Trees



- New Physics in loops?



Can penguins be bad?



<http://youtu.be/5IjmOSFtoJc>

Can penguins be bad?

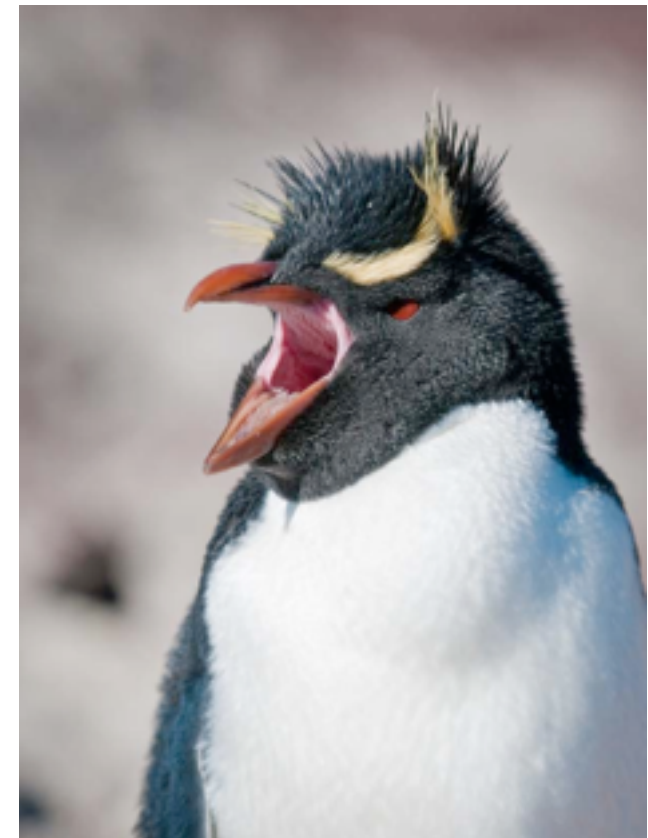


<http://youtu.be/5IjmOSFtoJc>

Can penguins be bad?

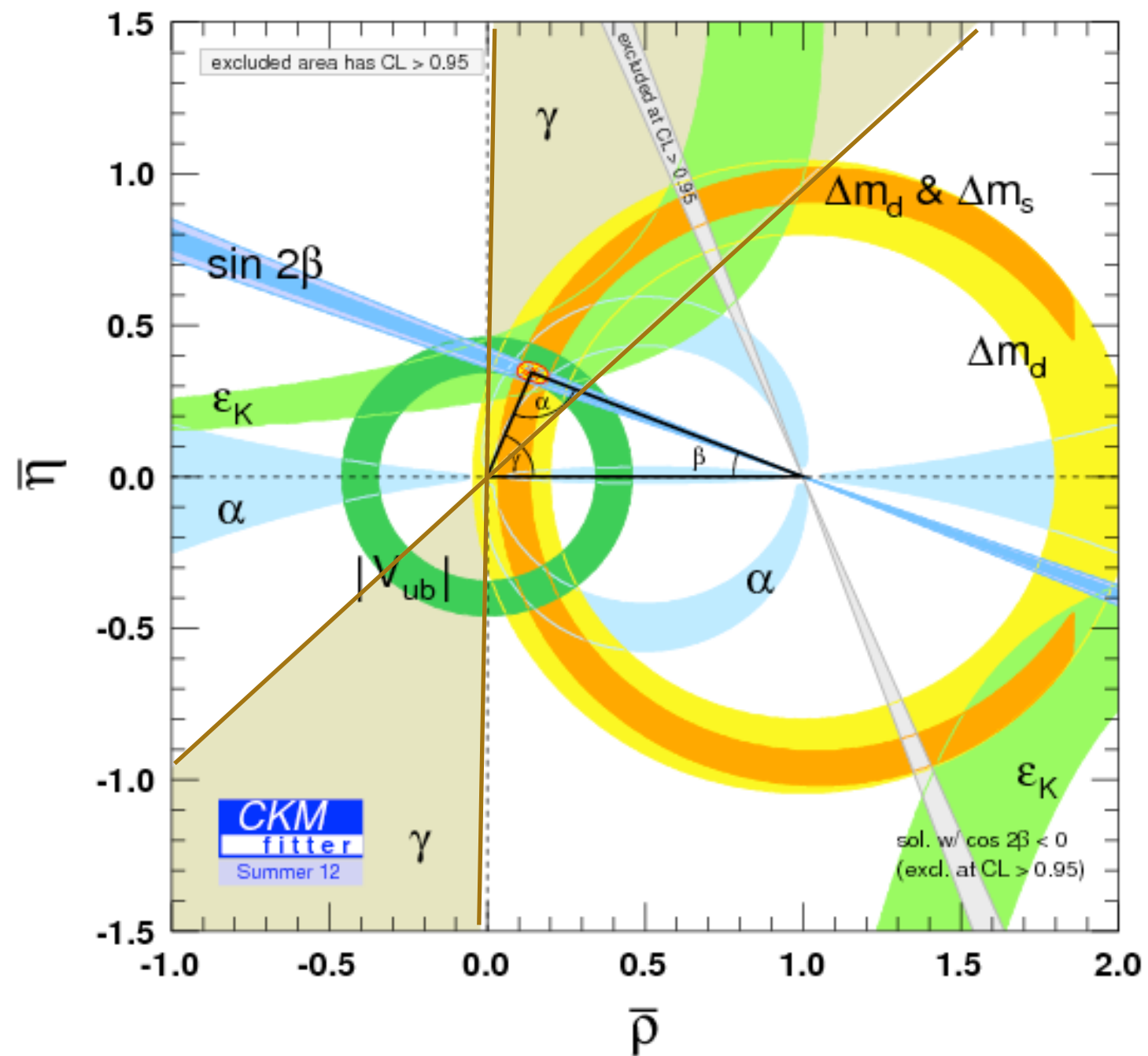


<http://youtu.be/5IjmOSFtoJc>

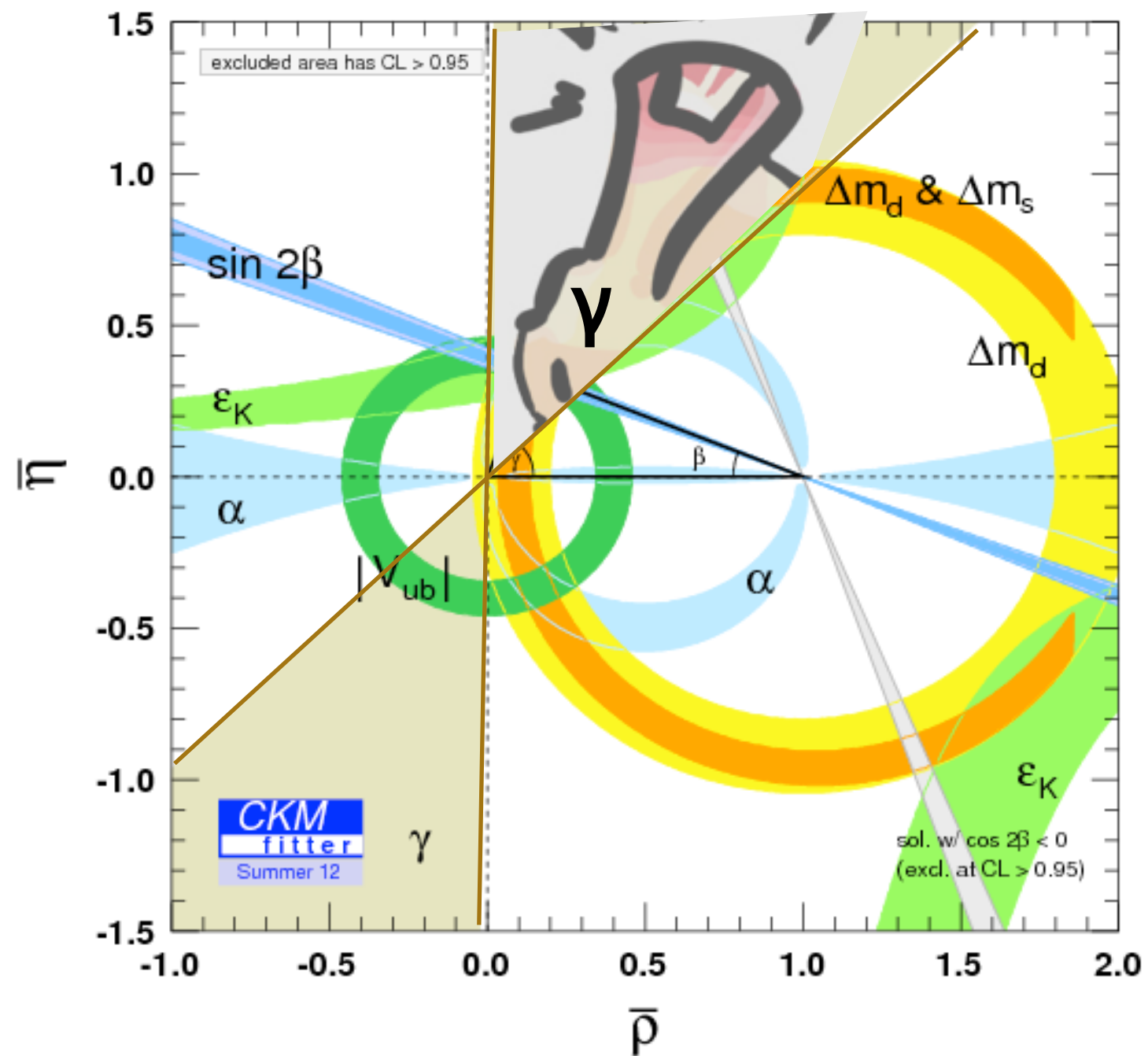


They can.

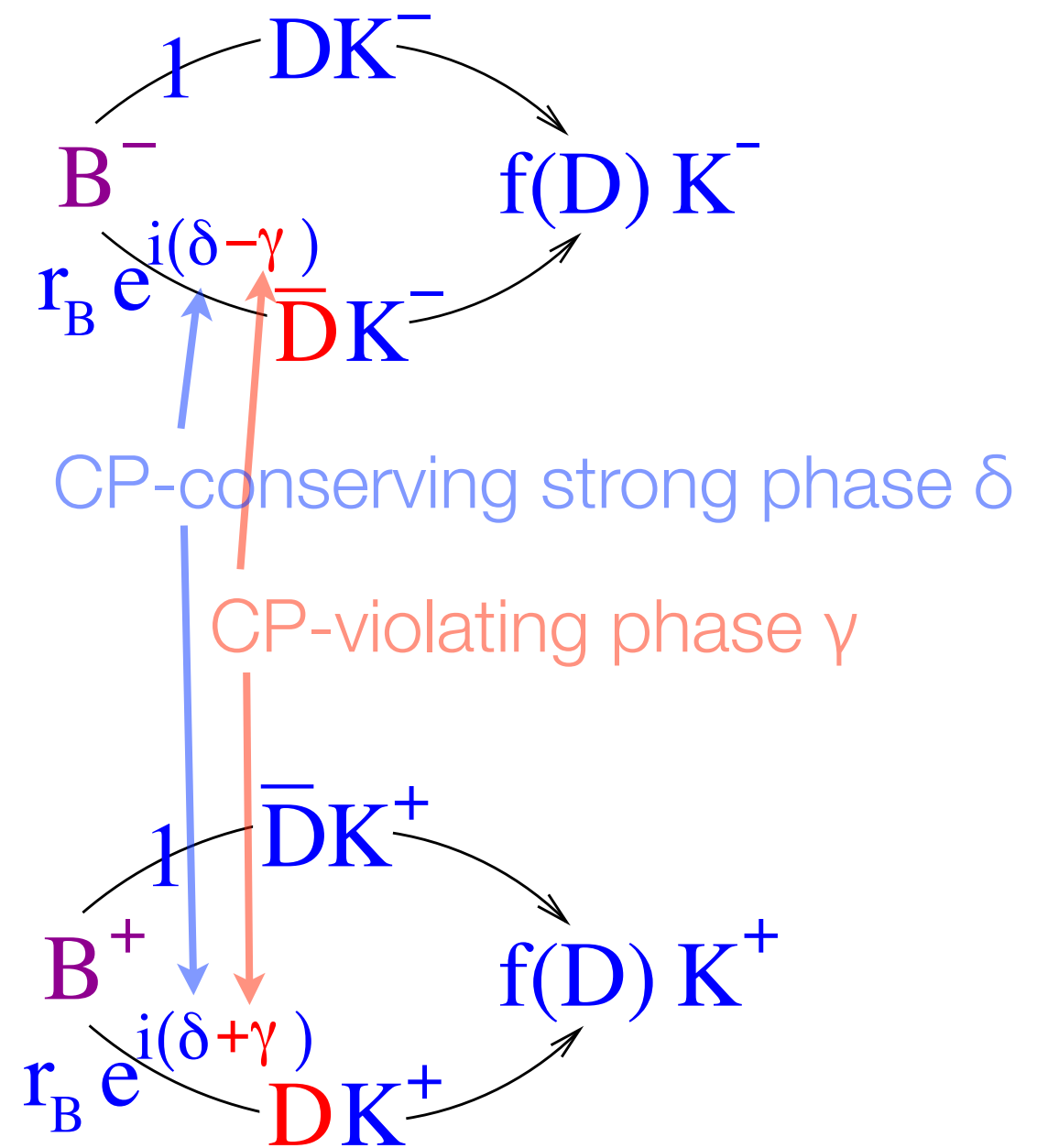
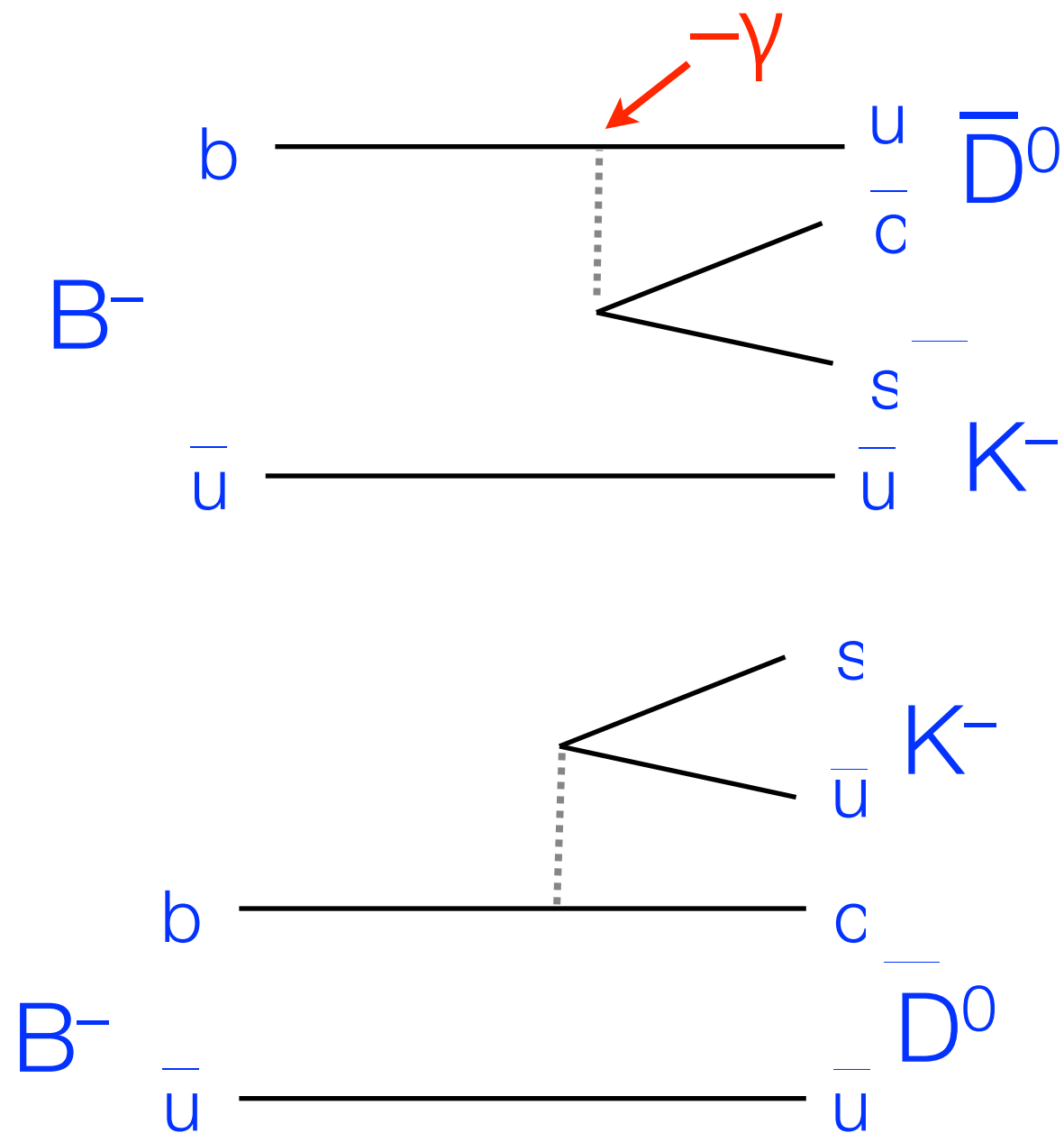
Measuring γ



Measuring γ

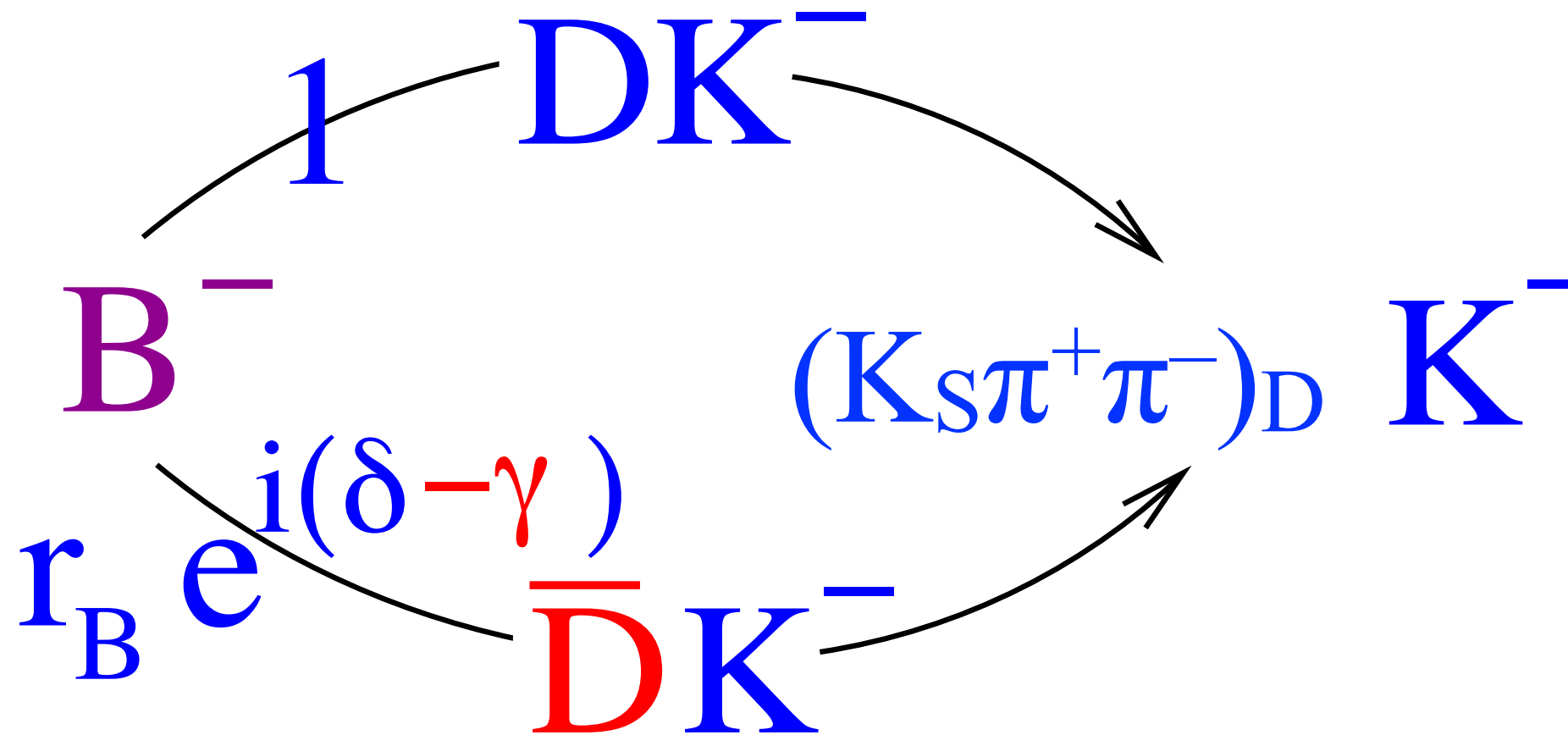


$$B^{\pm} \rightarrow DK^{\pm}$$



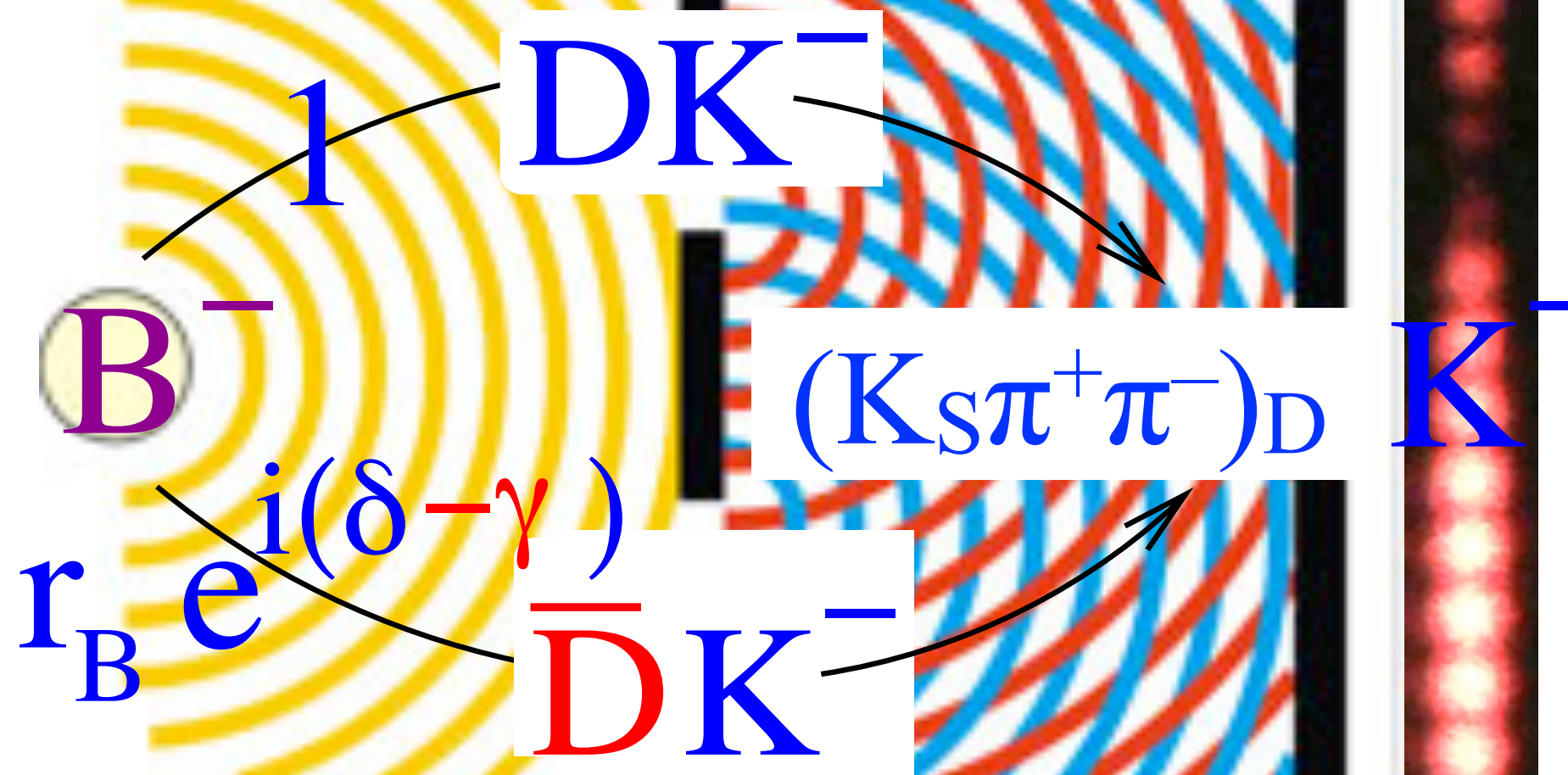
Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau, London Phys.Lett.B253:483-488,1991 (GLW) Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018 Belle Collaboration Phys.Rev. D70 (2004) 072003

CP violation is an interference effect



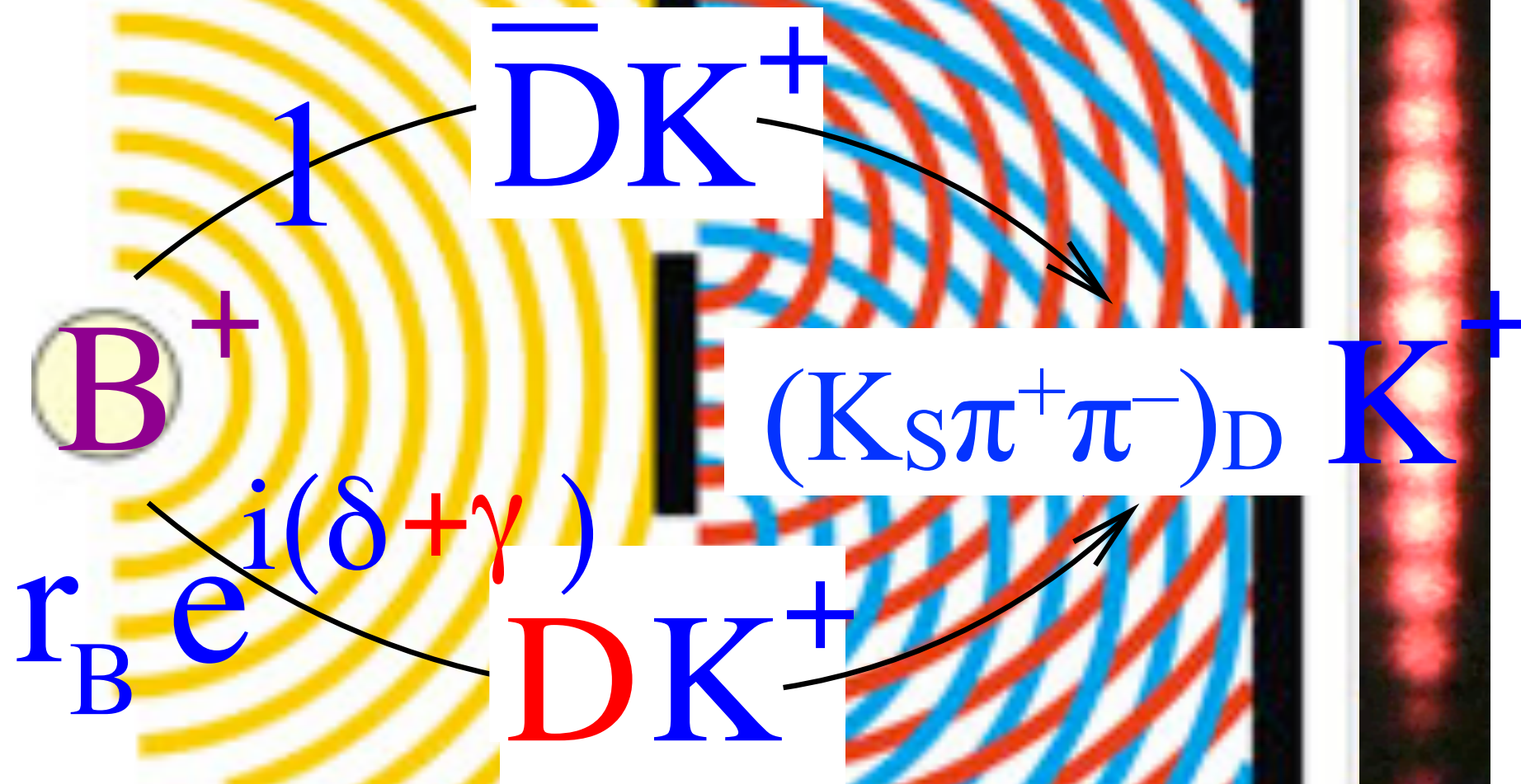
Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau, London Phys.Lett.B253:483-488,1991 (GLW) Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018 Belle Collaboration Phys.Rev. D70 (2004) 072003

CP violation is an interference effect



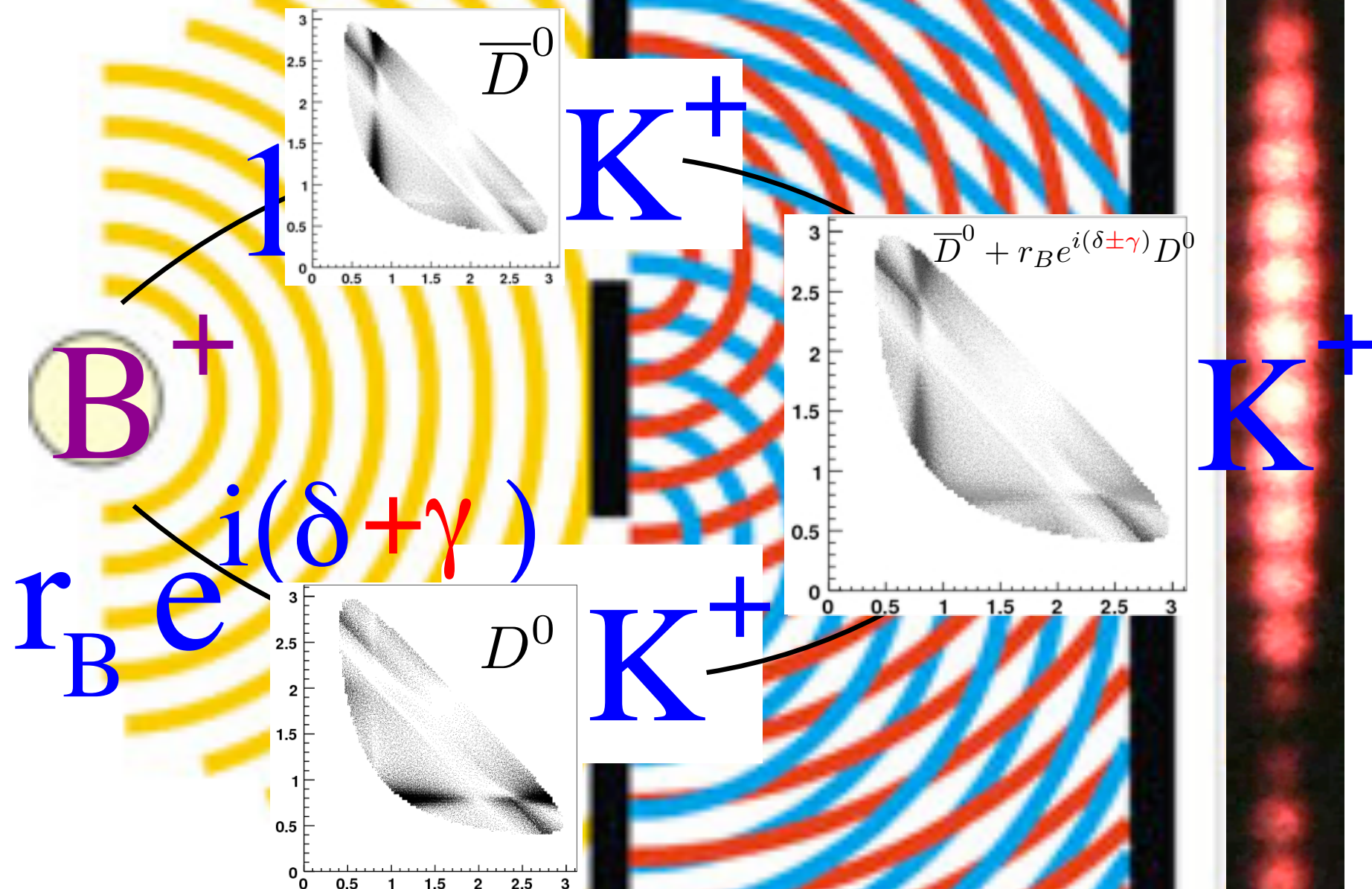
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CP violation is an interference effect



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CP violation is an interference effect



- For $D \rightarrow 3$ -body decays, the interference takes place in an abstract 2-D space (**Dalitz plot**)
- Analysing the Dalitz plot of the D decay, in D's that come from B^\pm 's, gives access to γ

Gronau, Wyler Phys.Lett.B265:172-176,1991, (GLW), Gronau, London Phys.Lett.B253:483-488,1991 (GLW) Atwood, Dunietz and Soni Phys.Rev.Lett. 78 (1997) 3257-3260 (ADS) Giri, Grossman, Soffer and Zupan Phys.Rev. D68 (2003) 054018 Belle Collaboration Phys.Rev. D70 (2004) 072003

Multi-Generational Flavour Physics



Edward V. Brewer (1883 – 1971)

Multi-Generational Flavour Physics



Edward V. Brewer (1883 – 1971)

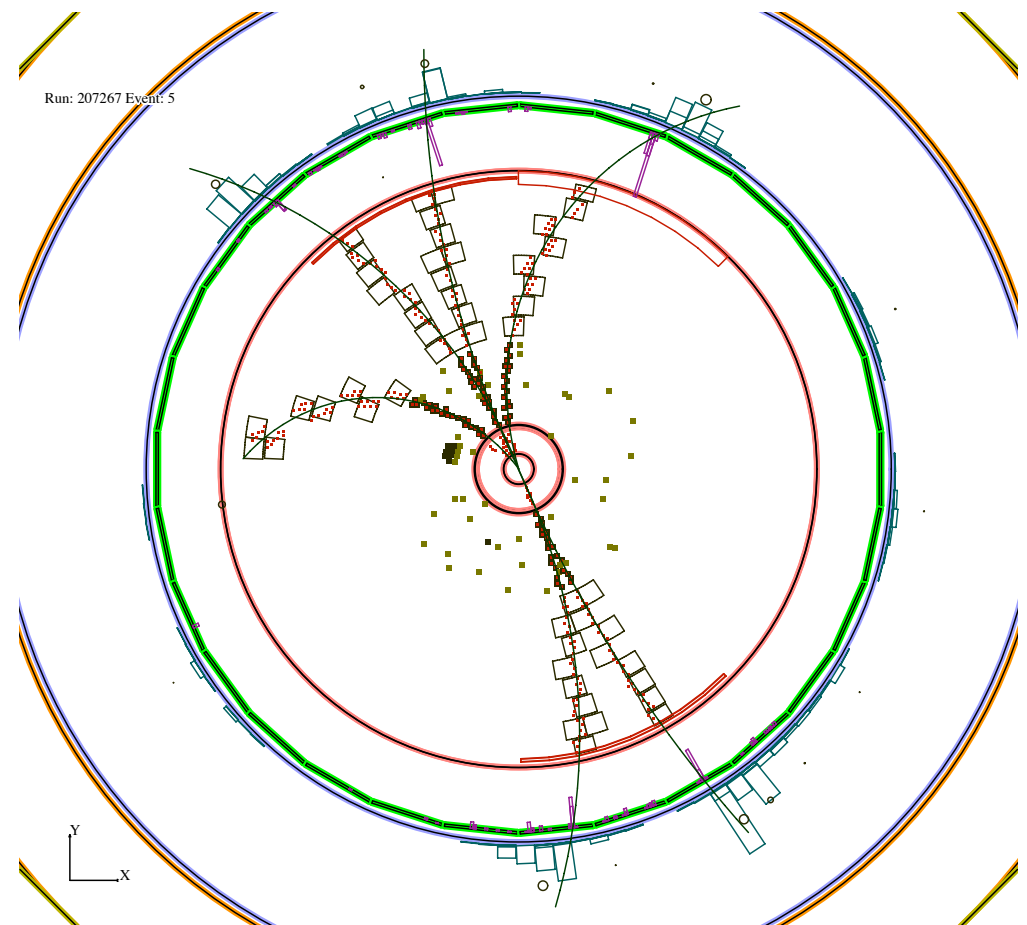
Regrettably, CLEO recently deceased - but her data live on.

CLEO-c

$$e^+e^- \rightarrow \psi(3770) \rightarrow D\bar{D}$$

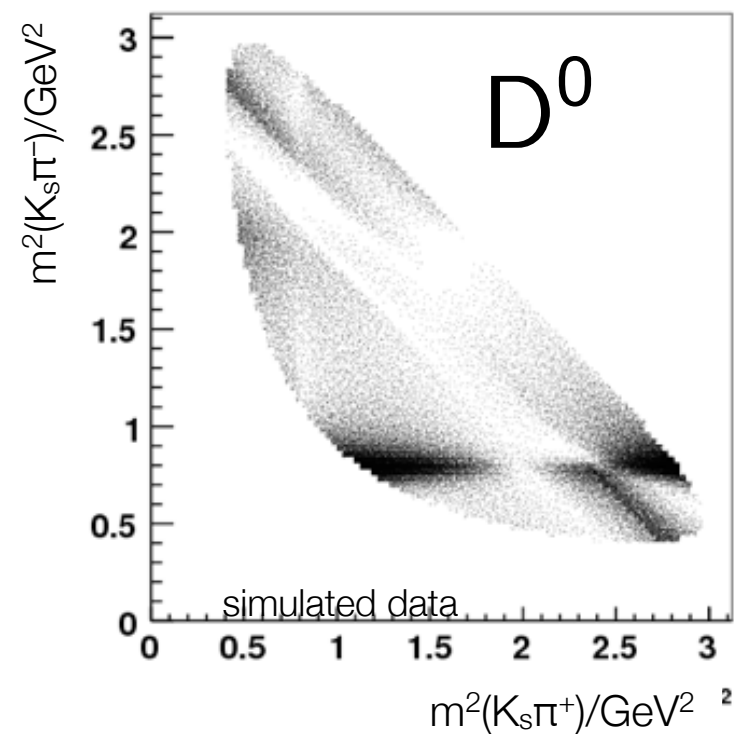
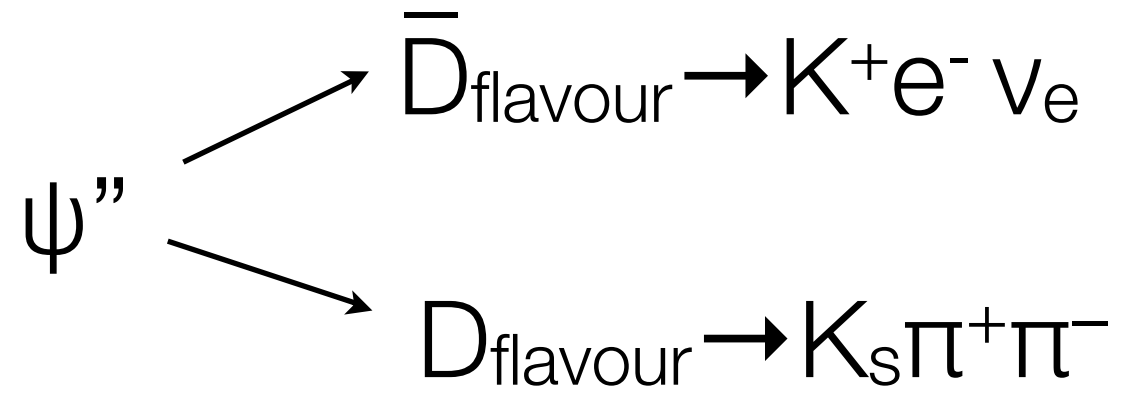
- Threshold production of correlated $D\bar{D}$.
- Final state must be CP-even with $L=1$: D mesons must have opposite intrinsic CP.
- Final state is also flavour-neutral.
- That gives us access to both amplitude and phase across the Dalitz plot.

CLEAN-c

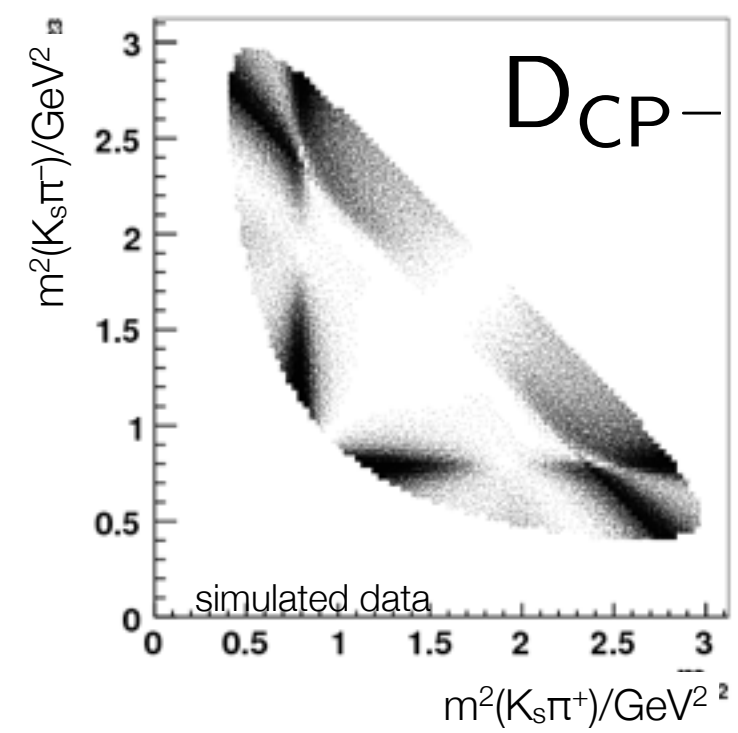
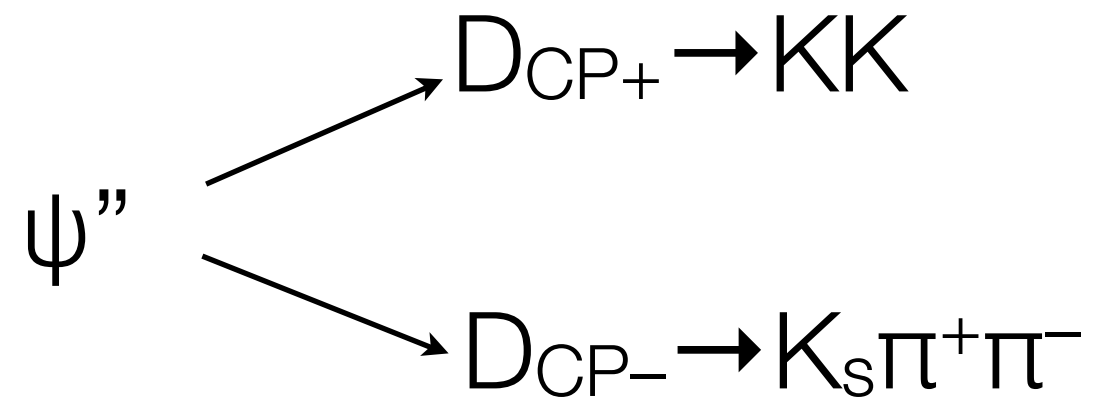
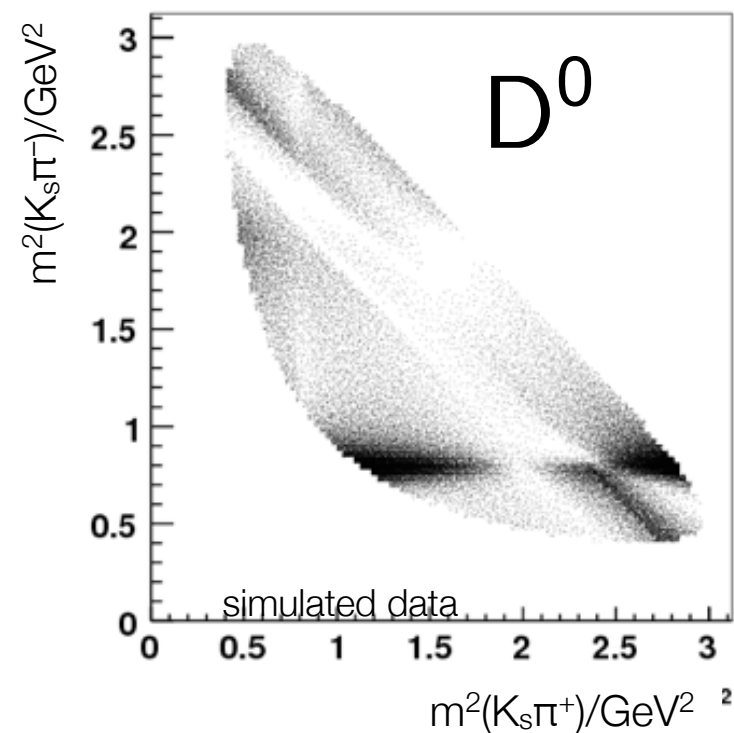
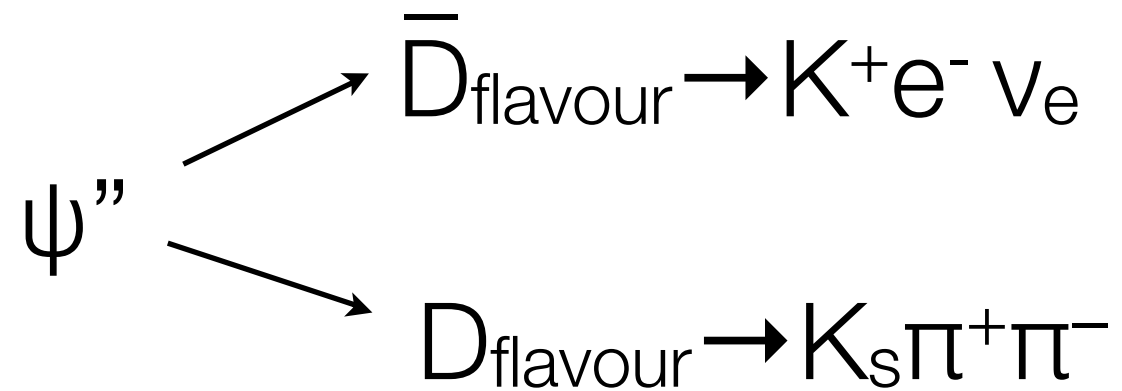


$$\psi(3770) \rightarrow D^0(K_S \pi^+ \pi^-) \bar{D}^0(K^+ \pi^-)$$

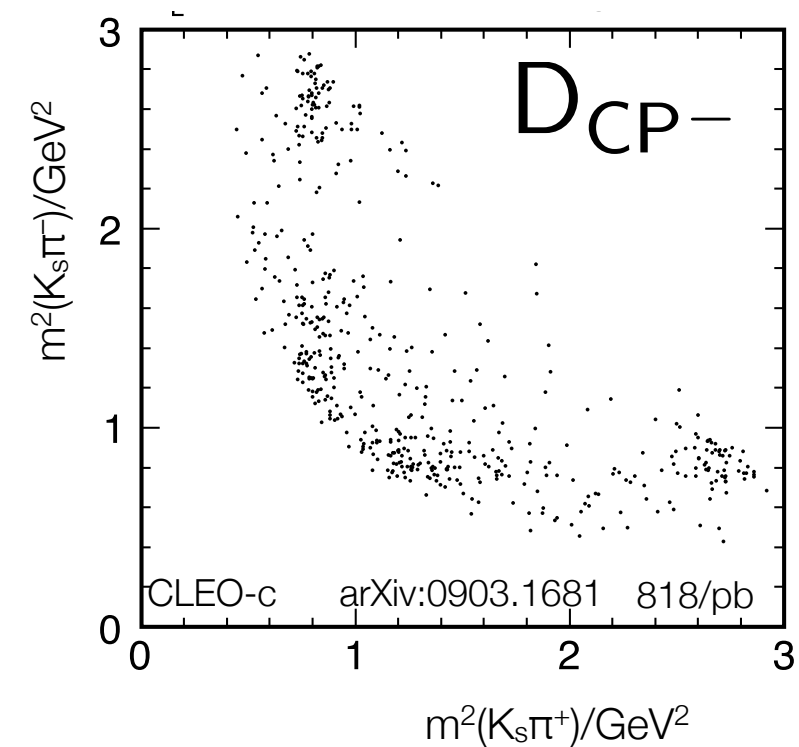
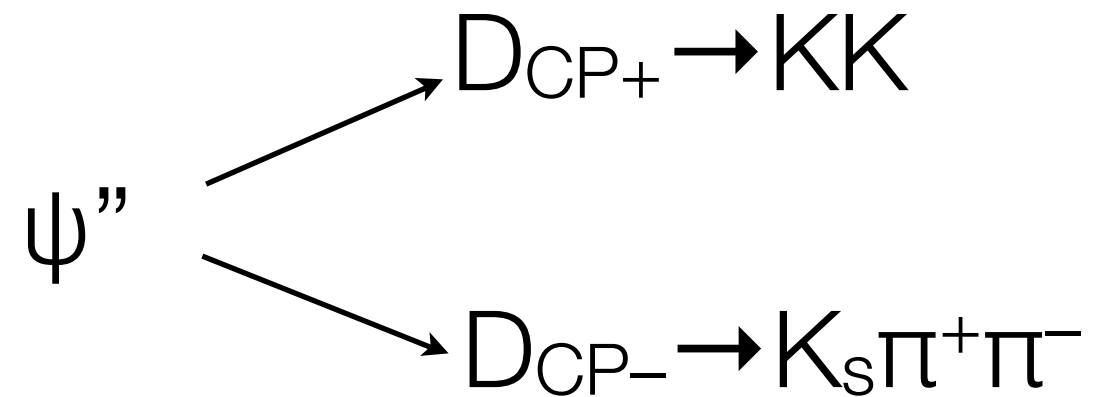
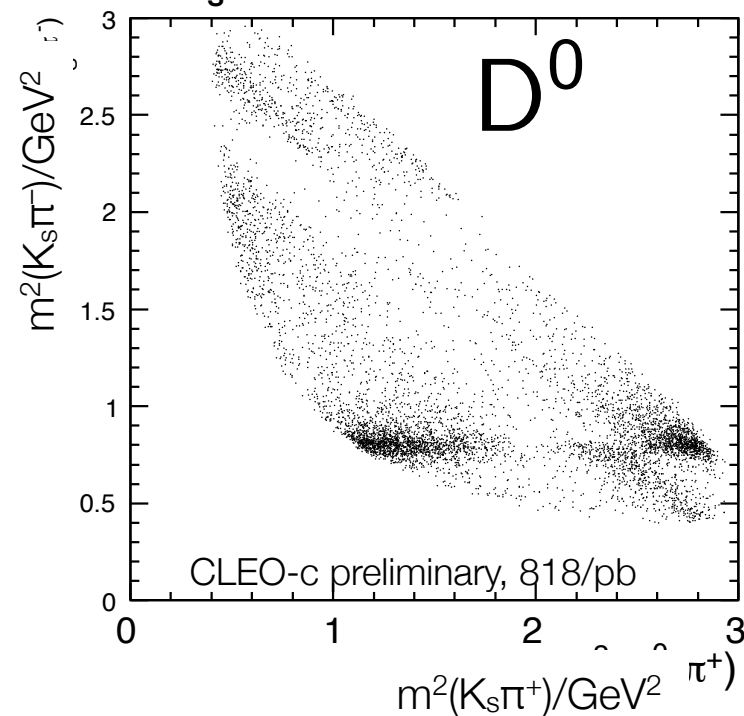
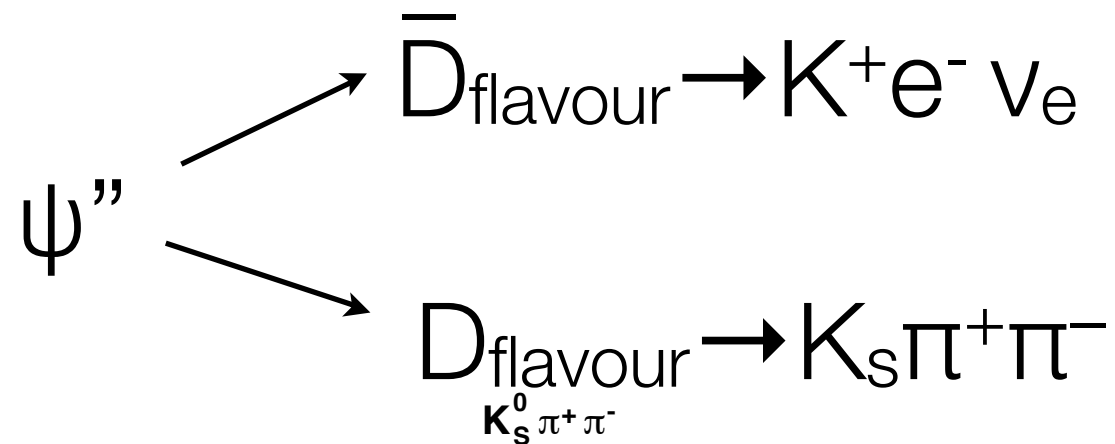
CP and flavour tagged D^0



CP and flavour tagged D^0



CP and flavour tagged D^0 at CLEO



Model independent γ fit

Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

- Binned decay rate:

$$\Gamma(B^\pm \rightarrow D(K_s \pi^+ \pi^-) K^\pm)_i =$$

$$\mathcal{T}_i + r_B^2 \mathcal{T}_{-i} + 2r_B \sqrt{\mathcal{T}_i \mathcal{T}_{-i}} \{c_i \cos(\delta \pm \gamma) + s_i \sin(\delta \pm \gamma)\}$$

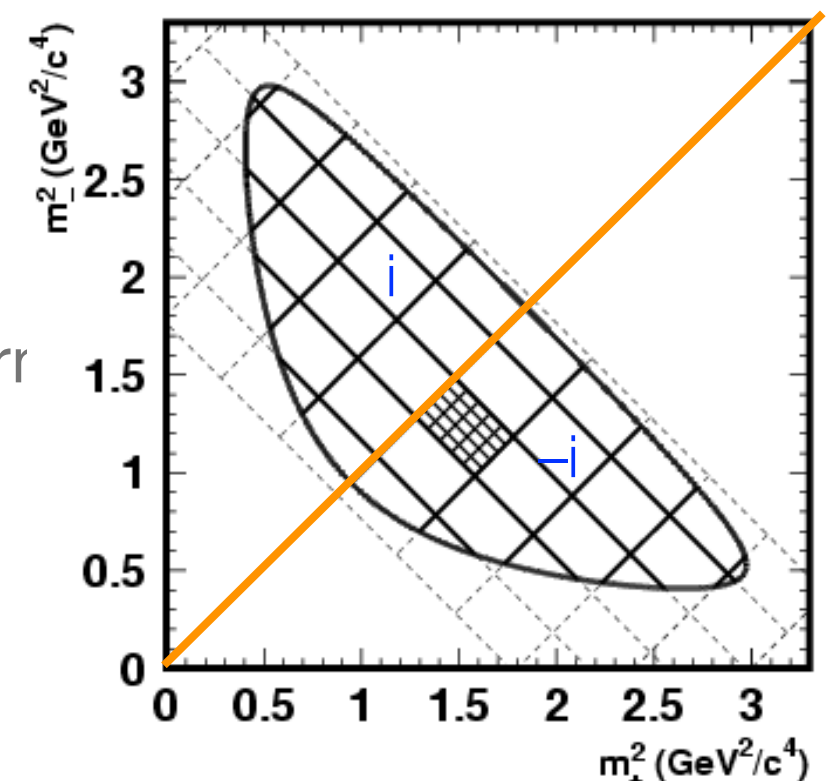
\mathcal{T}_i known from flavour-specific D decays (e.g. D^*)

(weighted) average of $\cos(\delta_D)$ and $\sin(\delta_D)$ over bin i , where δ_D = phase difference between $D \rightarrow K_s \pi \pi$ and $D^{\text{bar}} \rightarrow K_s \pi \pi$

- Binning such that such that $c_i = c_{-i}$, $s_i = -s_{-i}$

- Distribution sensitive to c_i , s_i , r_B , δ and γ .

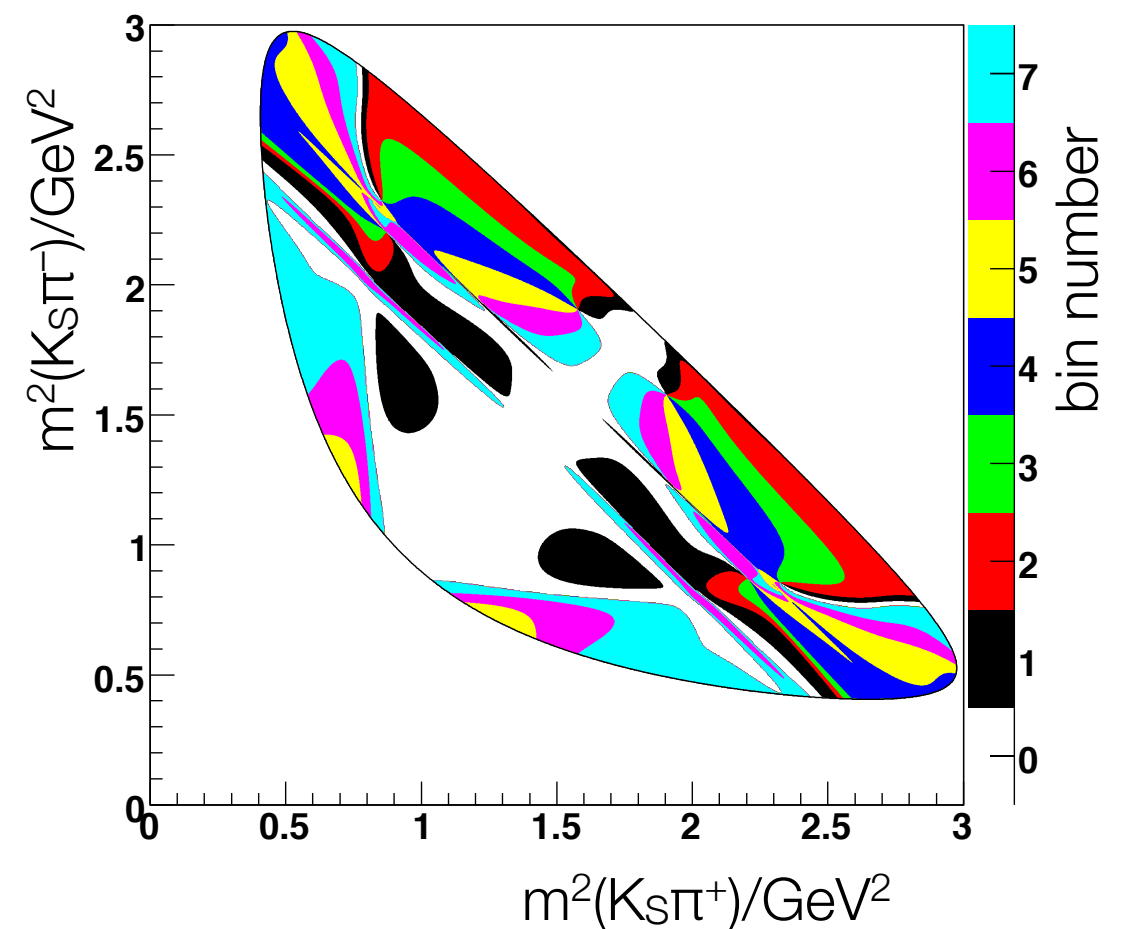
- To extract γ from realistic numbers of B events need external input from CLEO's quantum-correlated $D\bar{D}$ pairs.



Optimal binning

- Best γ sensitivity if phase difference δ_D is as constant as possible over each bin^[1].
- Plot shows CLEO-c's 8 bins, uniform in δ_D , (based on BaBar isobar model*).
- Choice of model will not bias result. (At worst a bad model would reduce the statistical precision of the result.)

Binning at CLEO-c based on BaBar model*



[1] Bondar, Poluektov hep-ph/0703267v1 (2007)

*model = BaBar PRL 95 (2005) 121802

LHCb model-independent γ from $B^\pm \rightarrow (K_S \pi \pi)_D K$ and $B^\pm \rightarrow (K_S K K)_D K$

LHCb-CONF-2013-004

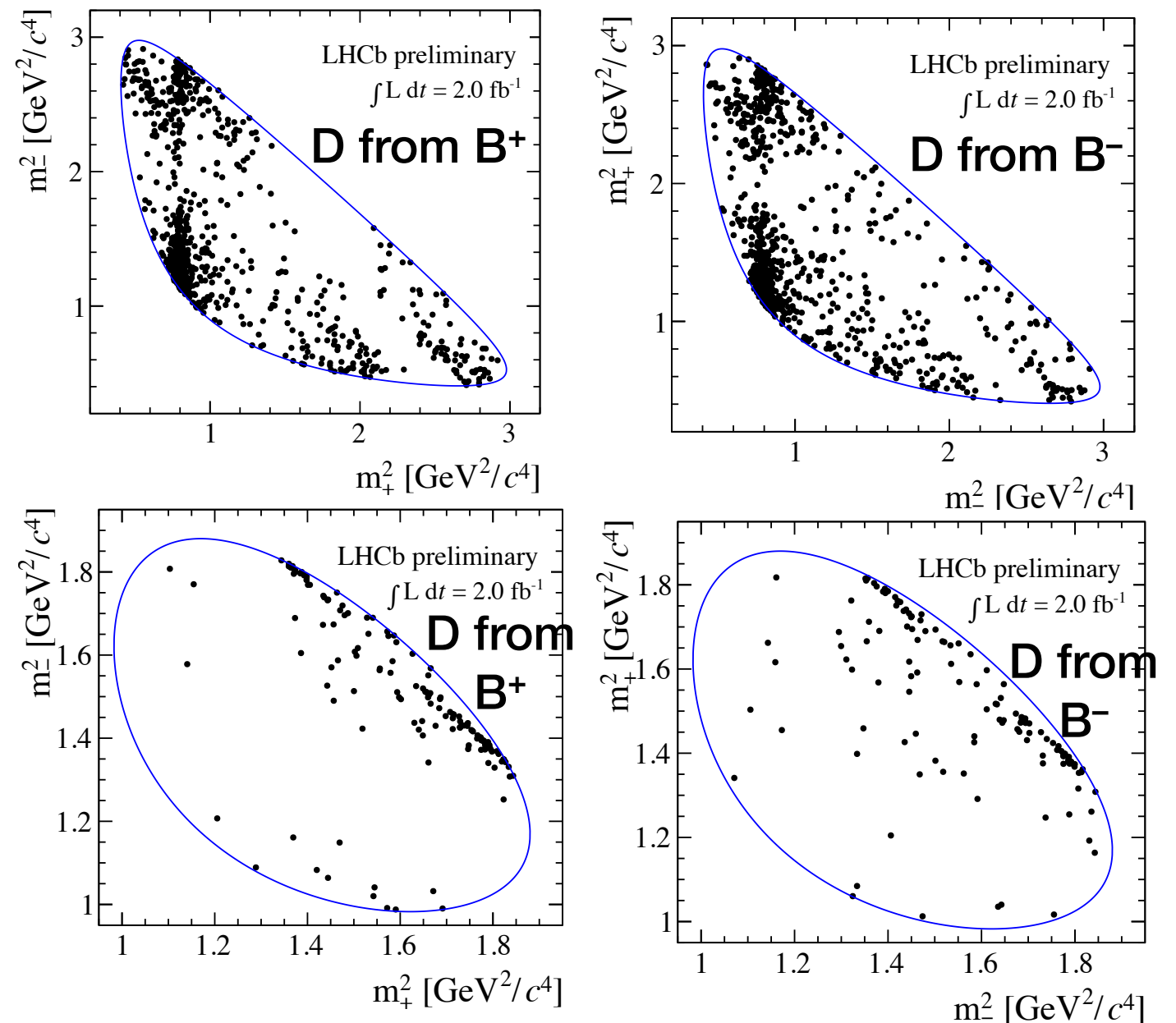
LHCb 2011 Result: Phys. Lett. B718 (2012) 43

- Binned, model-independent analysis using CLEO-c input.
Phys. Rev. D 82 112006.
- Plots show LHCb 2012 data
- Result of combined analysis (2011 & 2012 data, $K_S \pi \pi$ & $K_S K K$):

$$\gamma = (57 \pm 16)^\circ$$

$$\delta_B = (124^{+15}_{-17})^\circ$$

$$r_B = (8.8^{+2.3}_{-2.4}) \times 10^{-2}$$



CLEO-c input: Phys. Rev. D 82 112006.

Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

Optimal binning: Bondar, Poluektov hep-ph/0703267v1 (2007)

BELLE's first model-independent γ measurement: PRD 85 (2012) 112014

LHCb's γ combination

- LHCb combines inputs from

$$B^\pm \rightarrow (hh')_D K^\pm$$

$$B^\pm \rightarrow (K_S \pi \pi)_D K^\pm$$

$$B^\pm \rightarrow (K_S K K)_D K^\pm$$

$$B^\pm \rightarrow (K \pi \pi \pi)_D K^\pm$$

- Result:

$$\gamma = (67.2 \pm 12)^\circ$$

- More channels available, including $B^\pm \rightarrow D \pi^\pm$, $B^0 \rightarrow DK^*$.

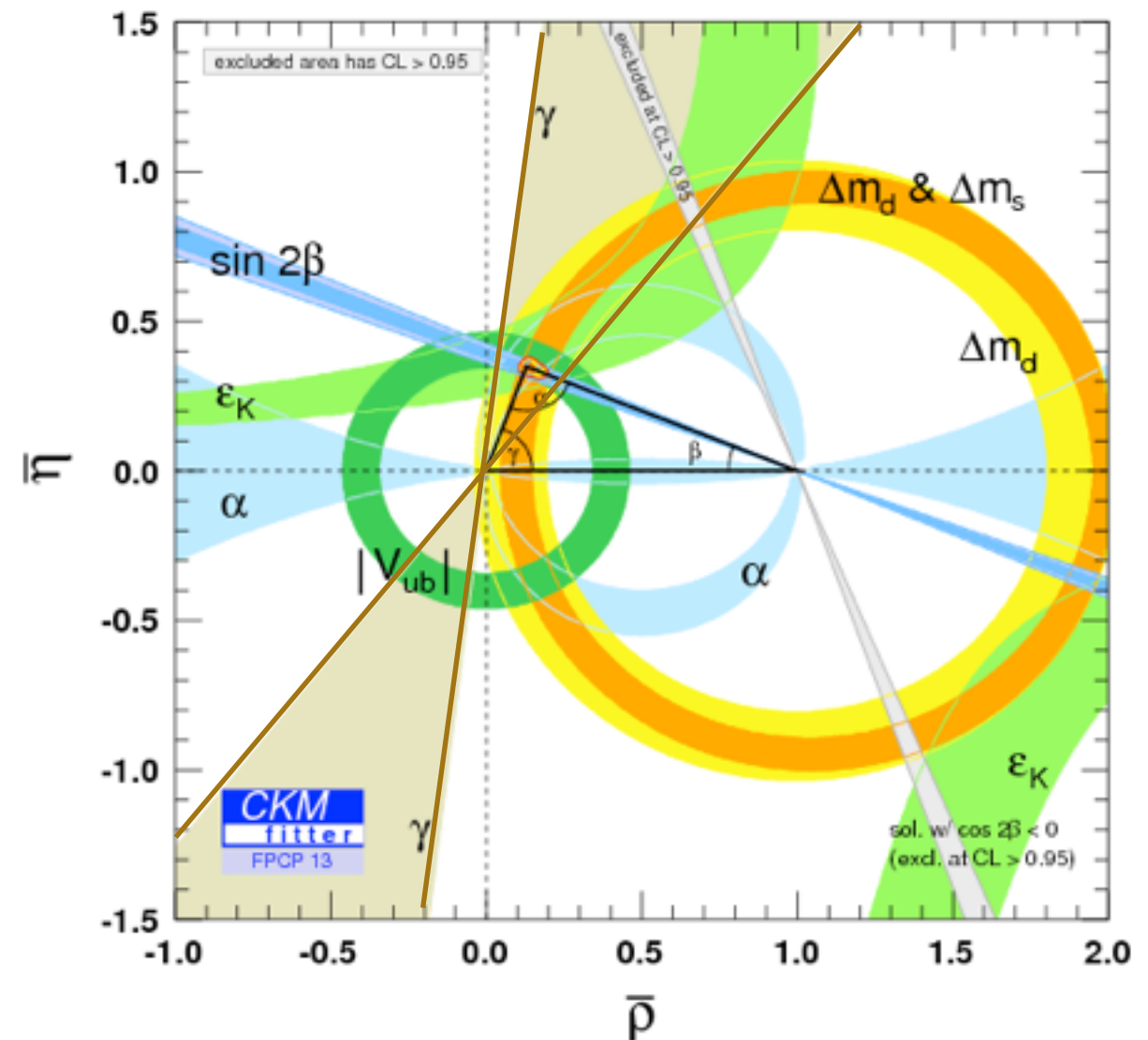
- Most recent addition: $B^\pm \rightarrow (K_S K \pi)_D K^\pm$
(see [arXiv:1402.2982](https://arxiv.org/abs/1402.2982), 2014)

previous world average
(Moriond 2012): $\gamma = 68^\circ \pm 12^\circ$

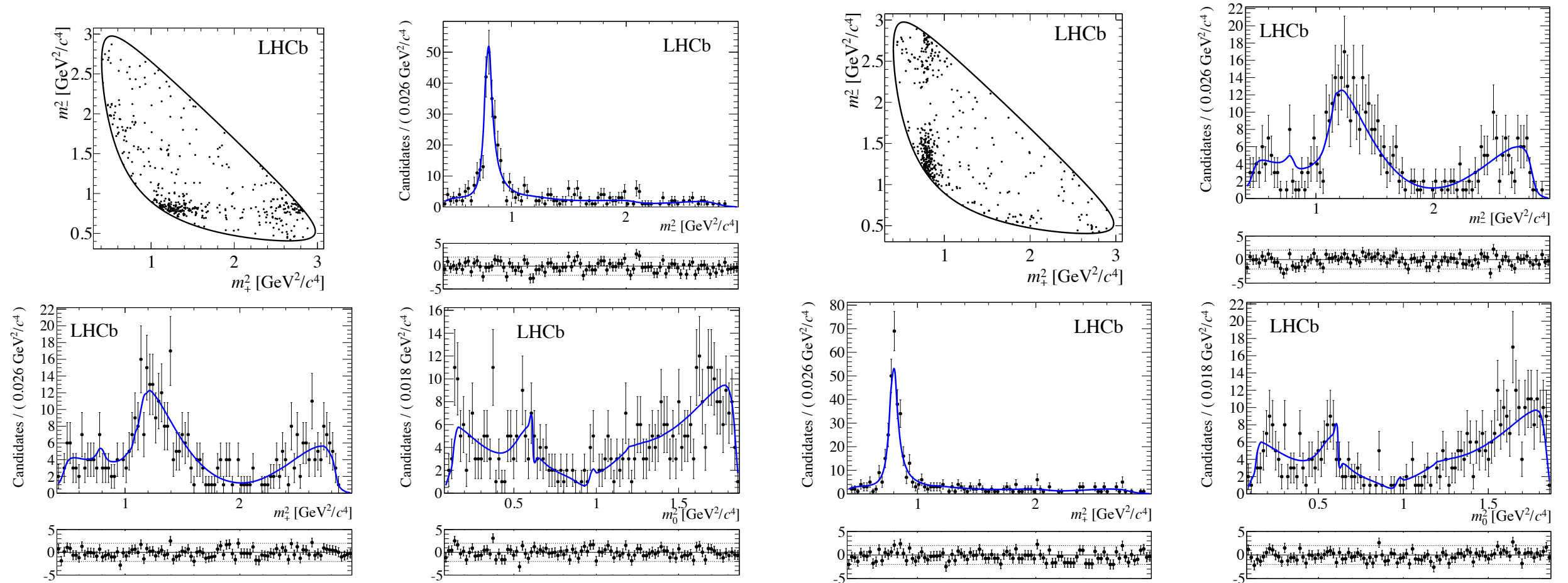
LHCb: $\gamma = 67.2^\circ \pm 12^\circ$

$$\gamma = 68^\circ +8.0^\circ -8.5^\circ$$

World averages by CKM Fitter



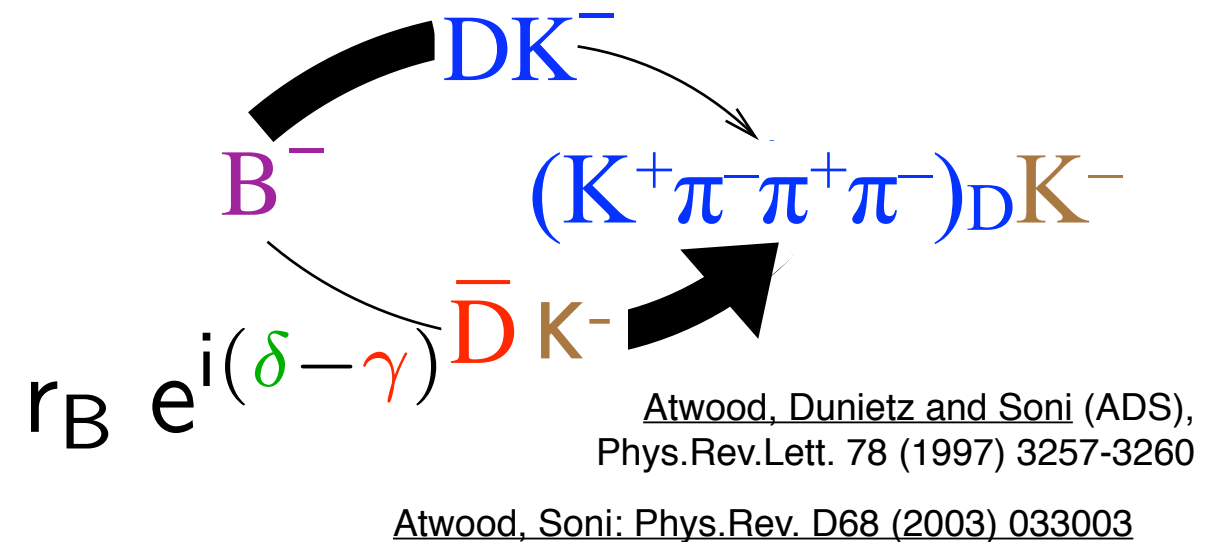
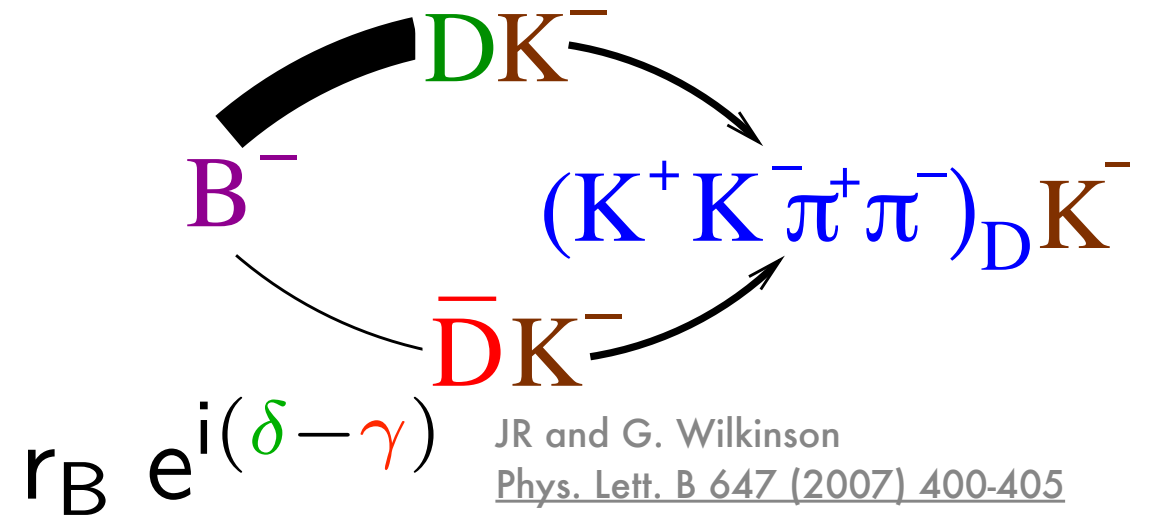
LHCb model-dependent γ from $B^\pm \rightarrow (K_S \pi \pi)_D K$



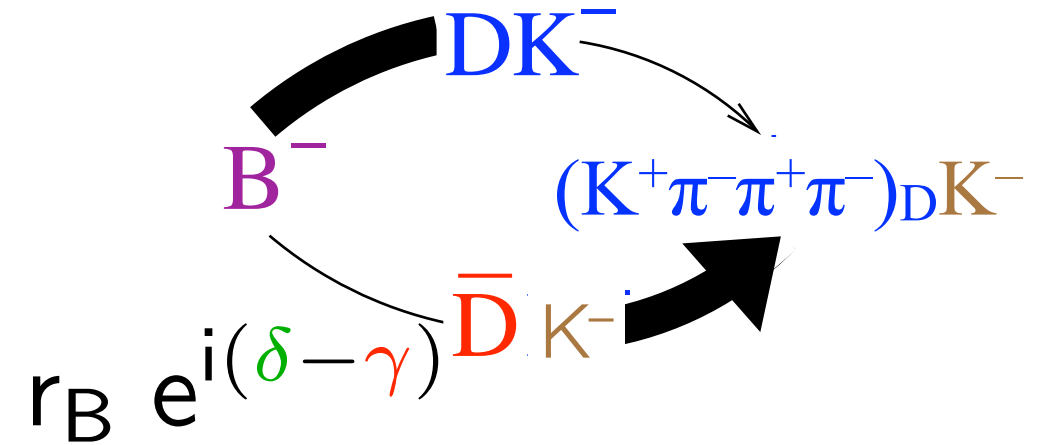
$$\gamma = (84_{-42}^{+49})^\circ$$

Why stop here

- Why stop at 3-body decays?
- 4-body amplitude analyses very promising for γ measurement at LHCb.
- Tricky... “Dalitz Plot” becomes 5-dimensional, phase space not flat, spin factors more complicated...



Coherence Factor Analysis of



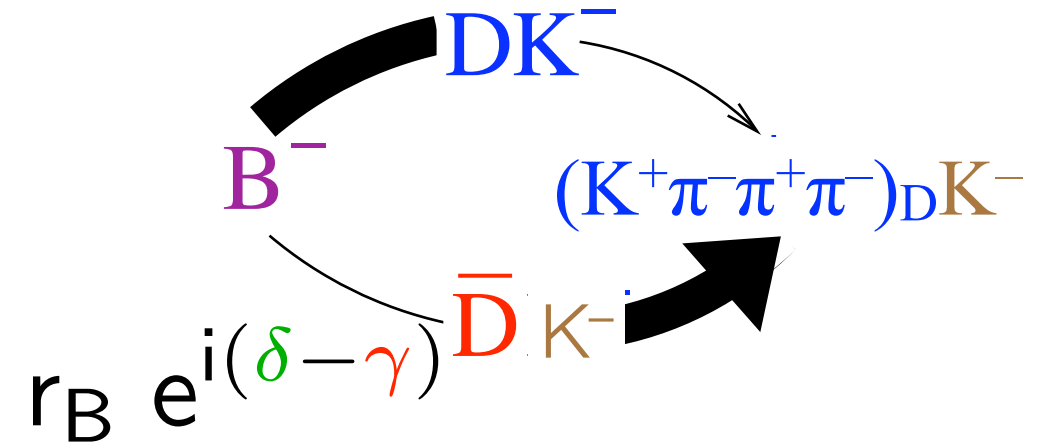
- Treat $K3\pi$ like two-body decay with single effective strong phase δ_D .
- Complex coherence parameter $Z = c + i s = R e^{i\delta}$ with coherent factor $R < 1$.

$$\Gamma(B^- \rightarrow (K^+ 3\pi)_D K^-) \propto r_B^2 + (r_D^{K3\pi})^2 + 2R_{K3\pi} r_B r_D^{K3\pi} \cdot \cos(\delta_B + \delta_D^{K3\pi} - \gamma)$$

$$r_B = \left| \frac{A(B^- \rightarrow \bar{D}^0 K^-)}{A(B^- \rightarrow D^0 K^-)} \right|$$

$$r_D = \left| \frac{A(D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-)}{A(\bar{D}^0 \rightarrow K^+ \pi^- \pi^+ \pi^-)} \right|$$

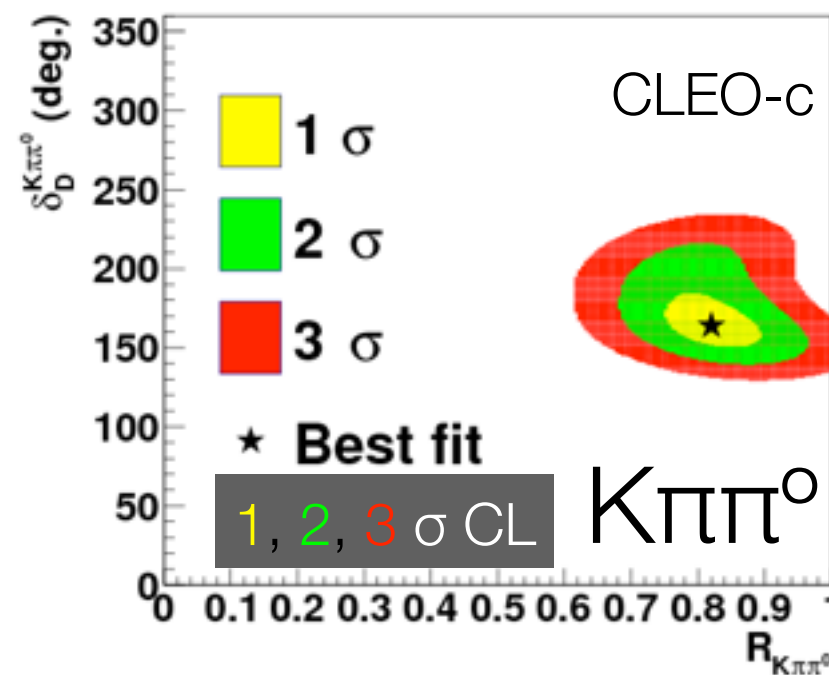
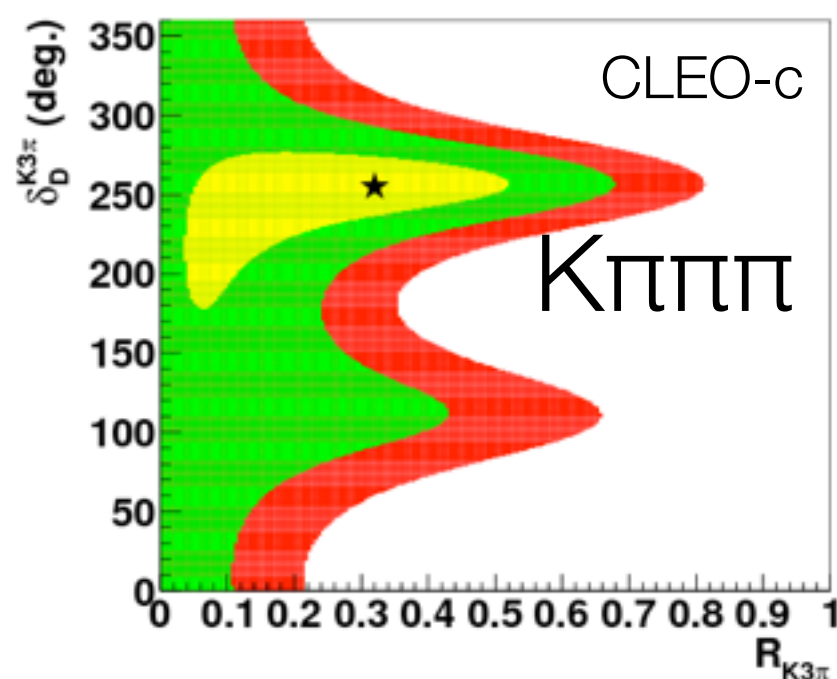
Coherence Factor Analysis of



- Treat $K3\pi$ like two-body decay with single effective strong phase δ_D .
- Complex coherence parameter $Z = c + i s = R e^{i\delta}$ with coherent factor $R < 1$.

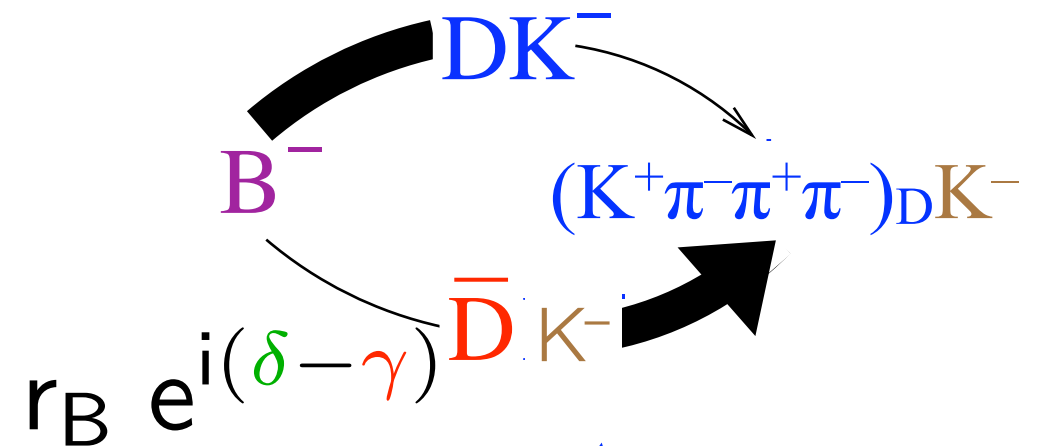
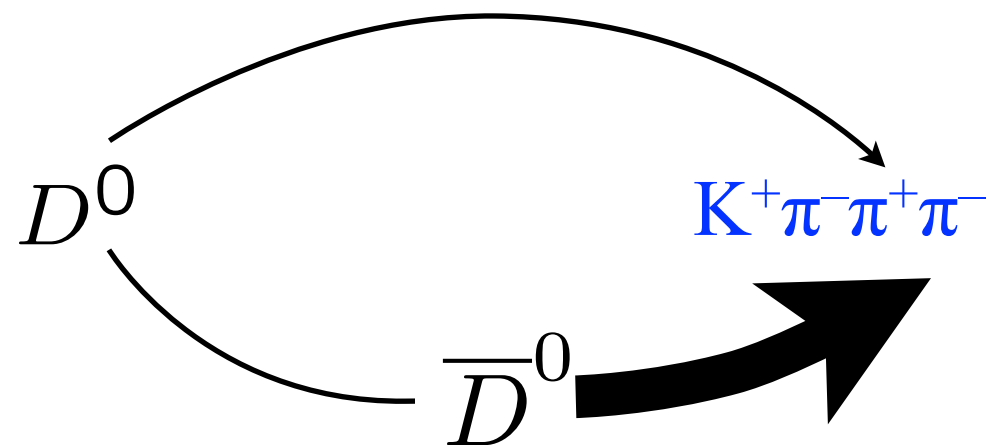
$$\Gamma(B^- \rightarrow (K^+ 3\pi)_D K^-) \propto r_B^2 + (r_D^{K3\pi})^2 + 2R_{K3\pi} r_B r_D^{K3\pi} \cdot \cos(\delta_B + \delta_D^{K3\pi} - \gamma)$$

- CLEO-c used coherent $\psi(3770) \rightarrow DD$ events to measure R , δ_D for $K\pi\pi\pi$ and $K\pi\pi^0$.



Theory:
 Atwood, Soni: Phys.Rev. D68
 (2003) 033003
 CLEO-c input:
 Phys.Rev.D80:031105,2009
 Phys.Lett. B731 (2014) 197-203
 LHCb CPV result:
 Physics Letters B 723 (2013), 44

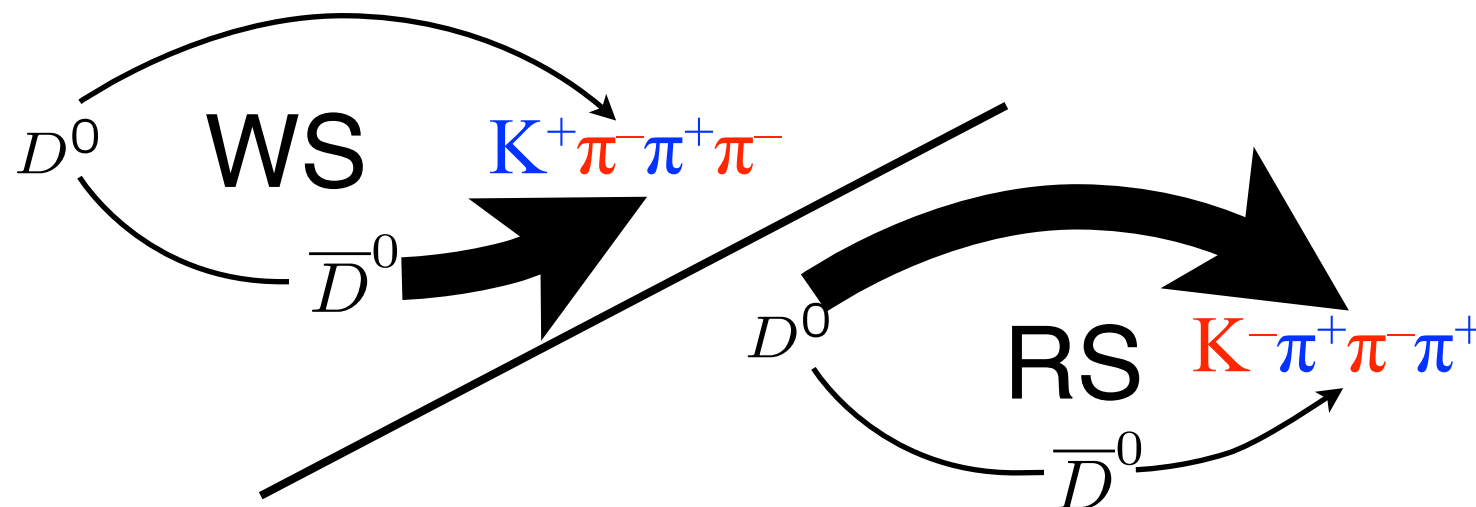
D⁰ Mixing as input to γ from $B^\pm \rightarrow DK^\pm$



This process is sensitive to the same D-D⁻ interference effects that pollute **this** measurement.

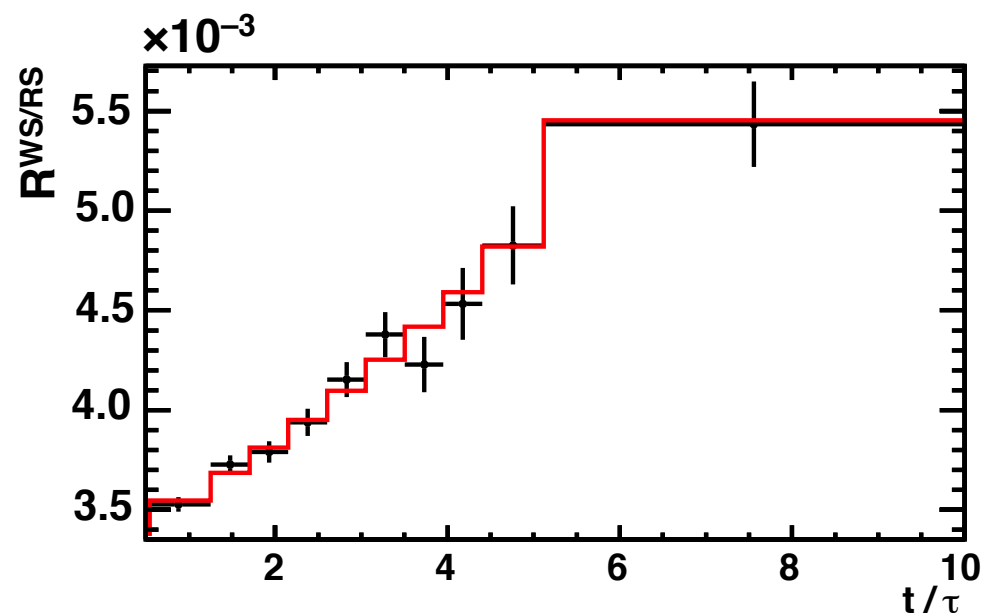
Phys.Lett. B728 (2014) 296-302

D⁰ Mixing as input to γ from $B^\pm \rightarrow DK^\pm$

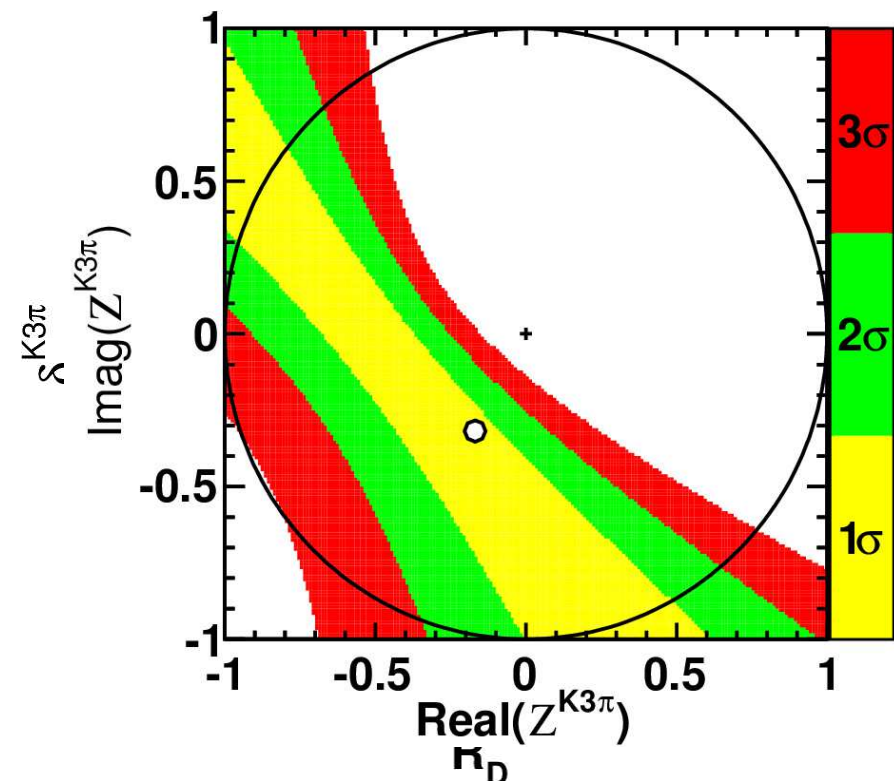


D \bar{D} tagged with $D^* \rightarrow D\pi$

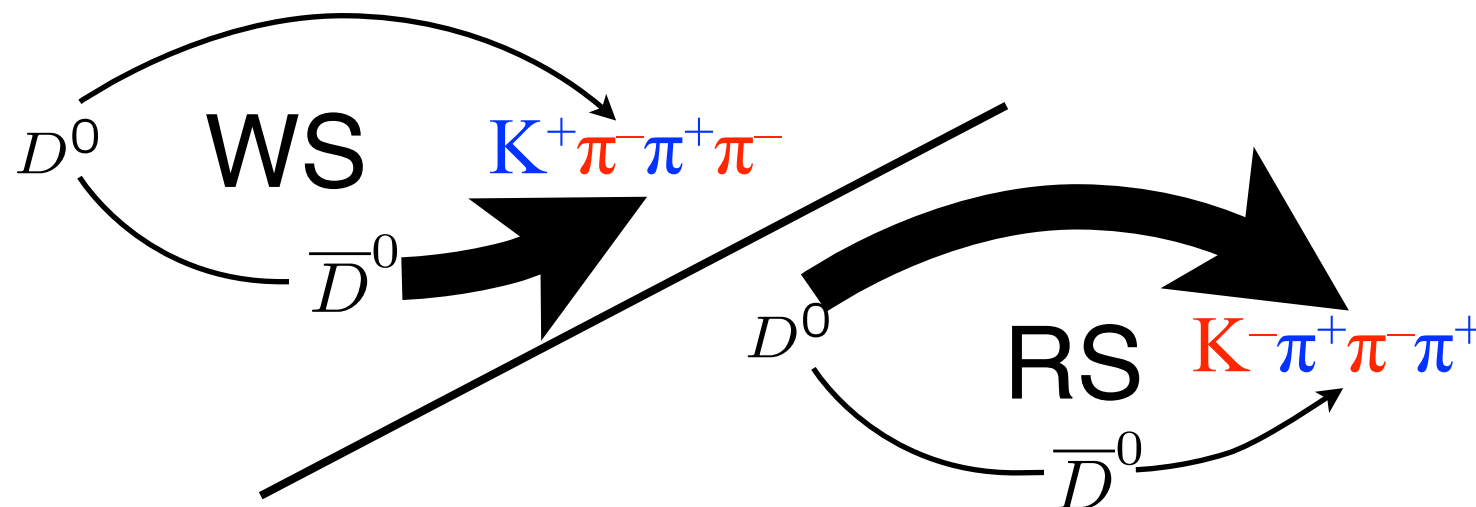
$$\frac{\Gamma(D^0 \rightarrow K^+ 3\pi)}{\Gamma(D^0 \rightarrow K^- 3\pi)}(t) = r_D^{K3\pi^2} + r_D^{K3\pi} (y \text{Re} Z^{K3\pi} + x \text{Im} Z^{K3\pi}) \Gamma t + \frac{x^2 + y^2}{4} (\Gamma t)^2$$



simulated data

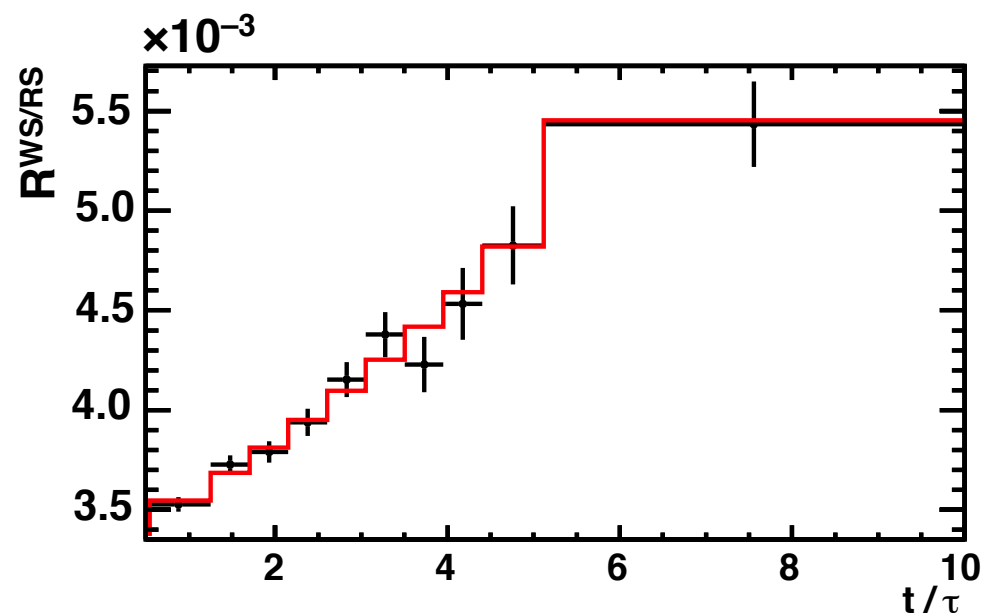


D⁰ Mixing as input to γ from $B^\pm \rightarrow DK^\pm$

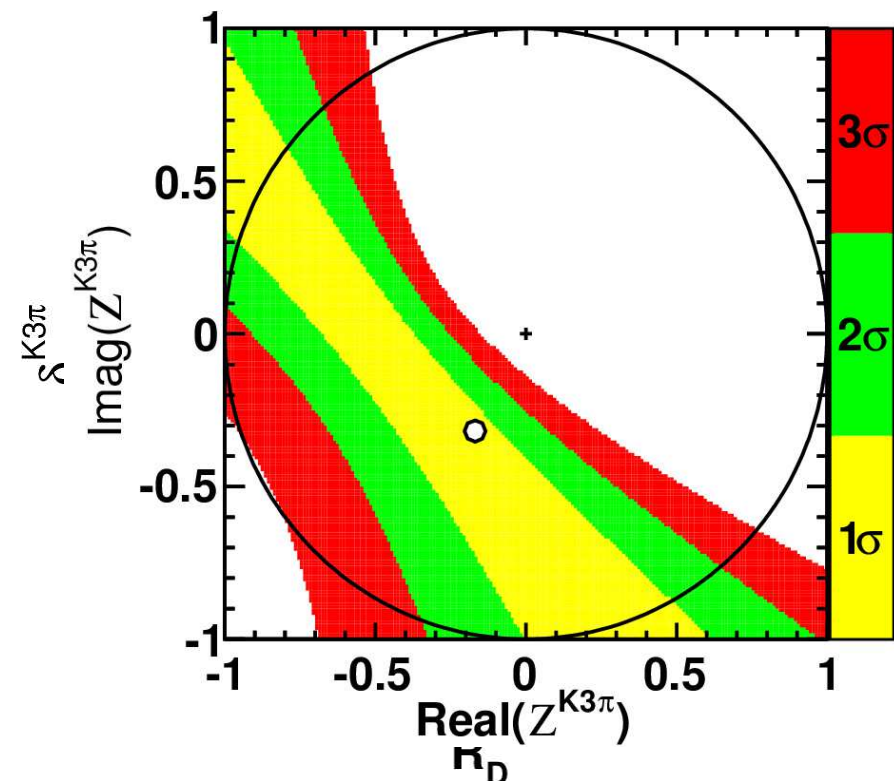


D \bar{D} tagged with $D^* \rightarrow D\pi$

$$\frac{\Gamma(D^0 \rightarrow K^+ 3\pi)}{\Gamma(D^0 \rightarrow K^- 3\pi)}(t) = r_D^{K3\pi^2} + r_D^{K3\pi} (y \text{Re} Z^{K3\pi} + x \text{Im} Z^{K3\pi}) \Gamma t + \frac{x^2 + y^2}{4} (\Gamma t)^2$$



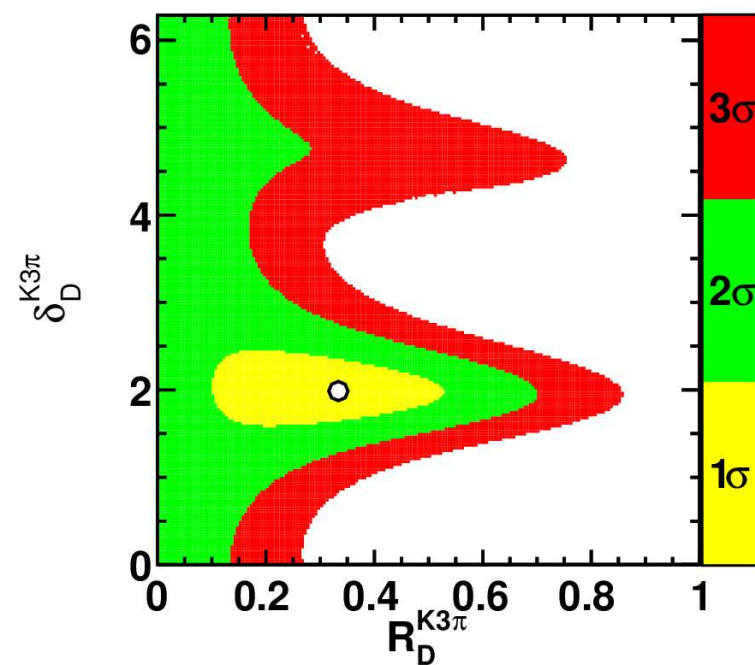
simulated data



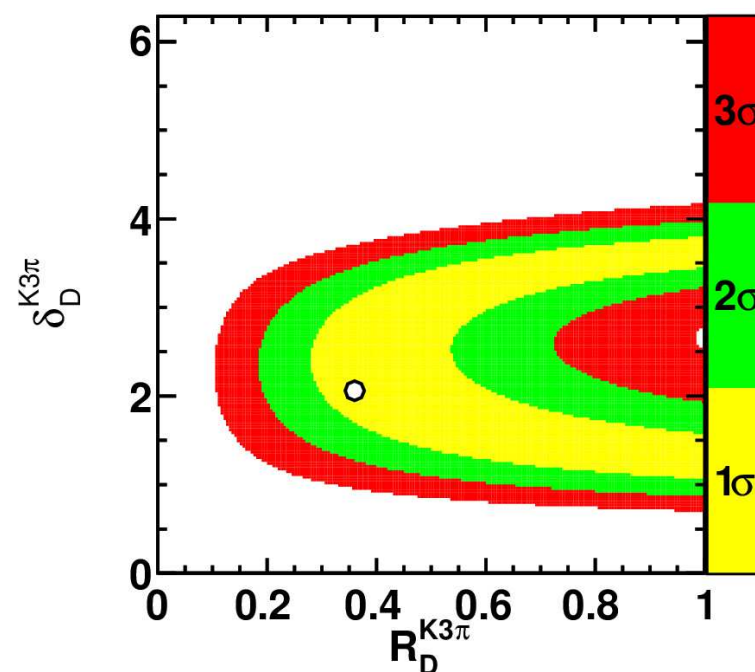
Unpublished, unofficial preview:

$D \rightarrow K^- \pi^+ \pi^- \pi^+$ coherence factor from mixing at LHCb

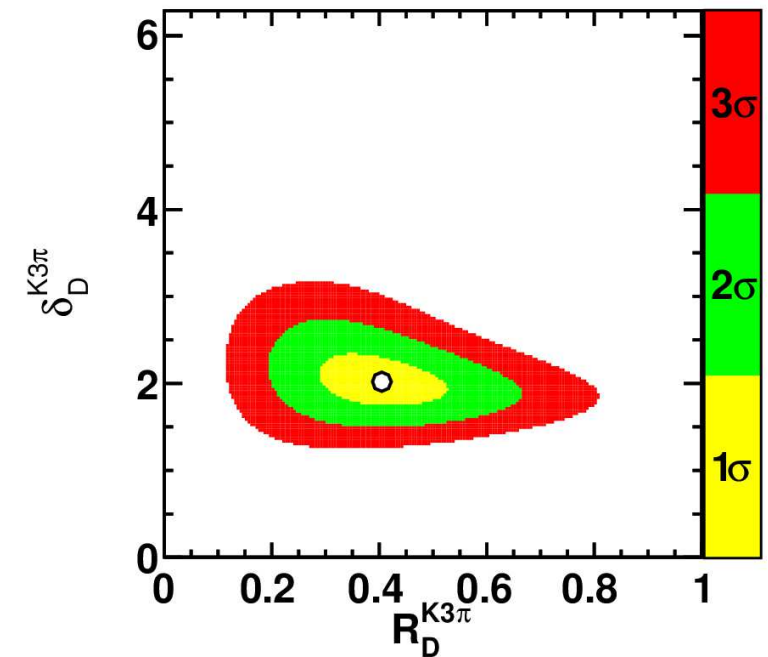
CLEO-c
(previous result)



Simulated
LHCb 1/fb



Both

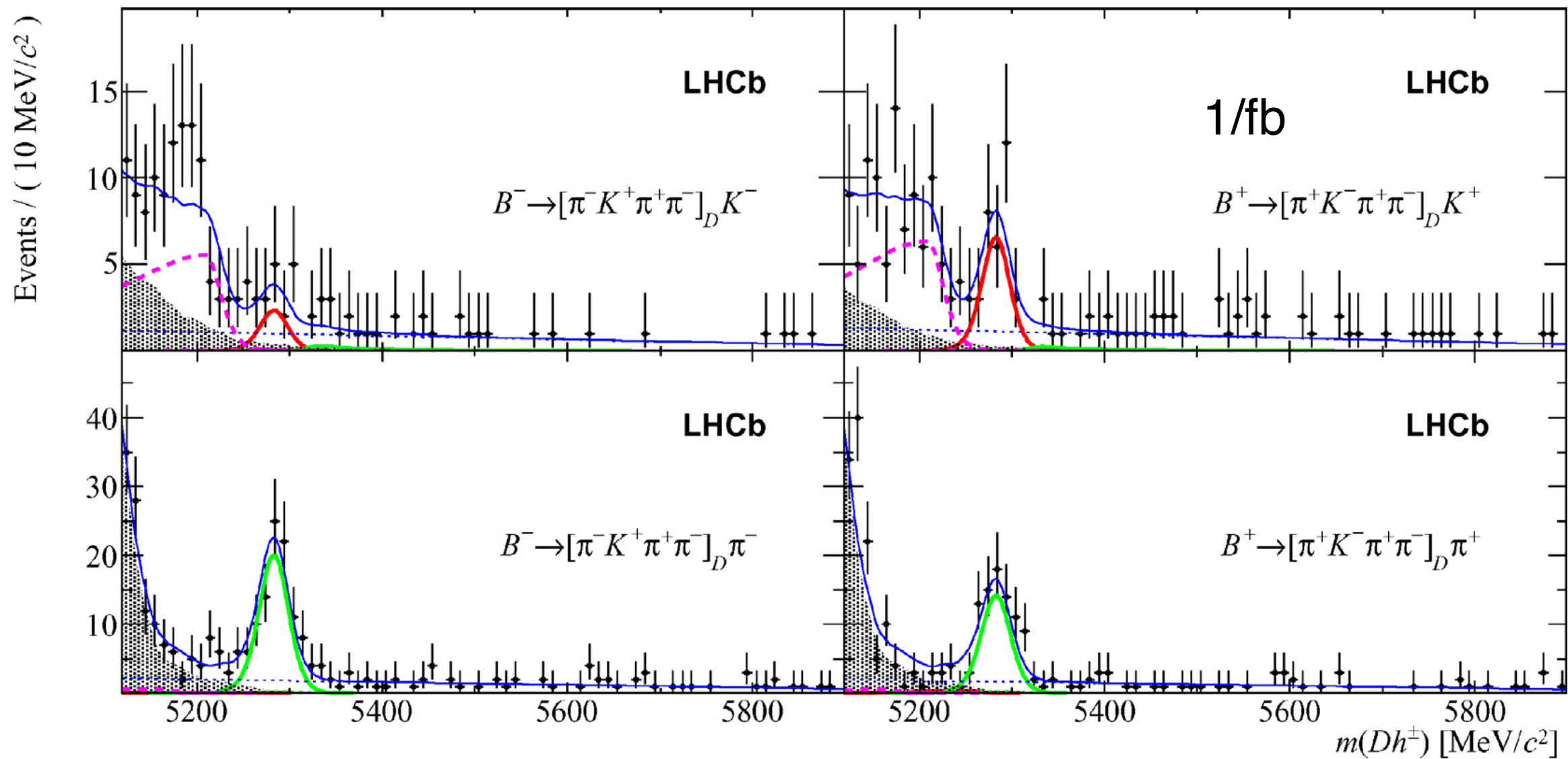


CLEO-c input: [Phys.Rev.D80:031105,2009](#)
[Phys.Lett. B731 \(2014\) 197-203](#)

D-mixing input: [Phys.Lett. B728 \(2014\) 296-302](#)

LHCb $B^\pm \rightarrow D(K\pi\pi\pi)K^\pm$

Physics Letters B 723 (2013), pp. 44-53



A closer look at Z

$$\frac{\Gamma(B^- \rightarrow DK^-, D \rightarrow f)_\Omega}{\Gamma(B^- \rightarrow DK^-, D \rightarrow \bar{f})_{\bar{\Omega}}} = r_{D,\Omega}^2 + r_B^2 + r_{D,\Omega} r_B \left| \mathcal{Z}_\Omega^f \right| \cos(\delta_B - \delta_\Omega^f - \gamma)$$

$$\mathcal{A}_\Omega \equiv \sqrt{\int_\Omega |\langle f_{\mathbf{p}} | \hat{H} | D^0 \rangle|^2 \left| \frac{\partial^n \phi}{\partial(p_1 \dots p_n)} \right| d^n p}, \quad \mathcal{B}_\Omega \equiv \sqrt{\int_\Omega |\langle f_{\mathbf{p}} | \hat{H} | \bar{D}^0 \rangle|^2 \left| \frac{\partial^n \phi}{\partial(p_1 \dots p_n)} \right| d^n p},$$

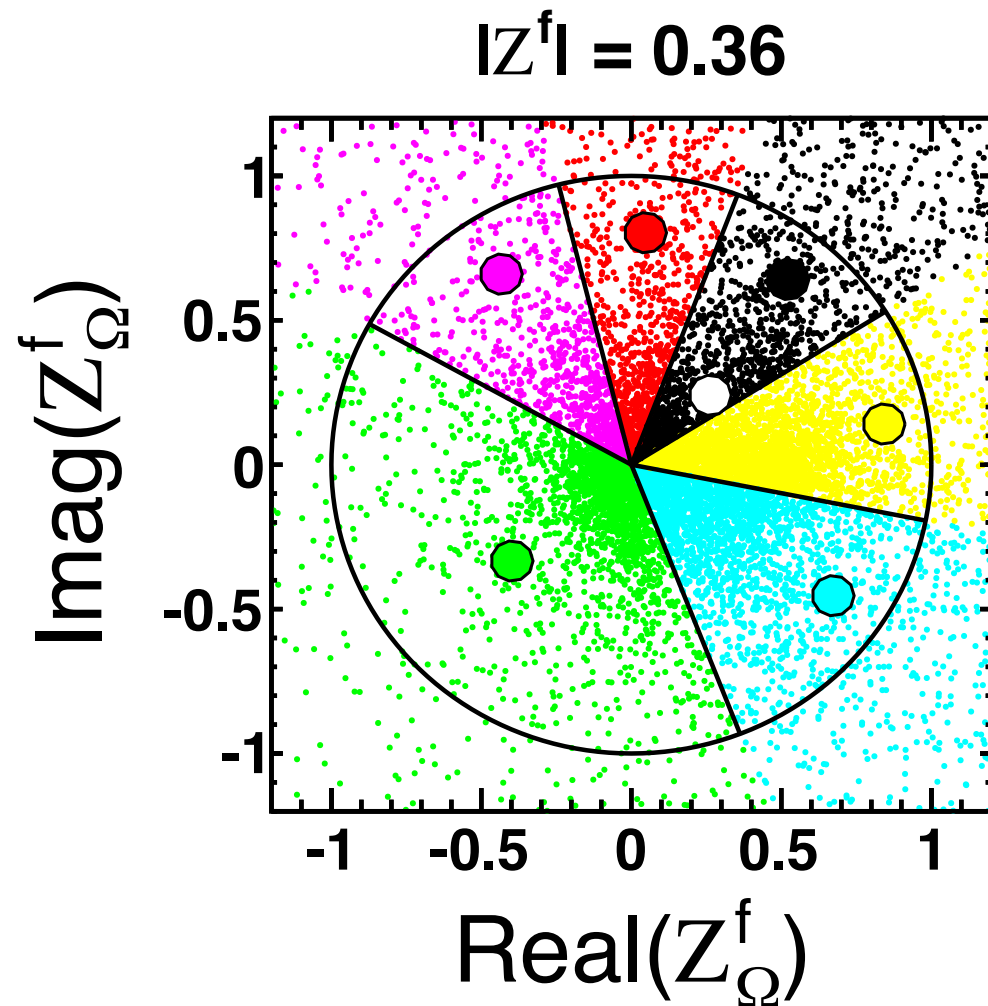
$$\mathcal{Z}_\Omega^f \equiv \frac{1}{\mathcal{A}_\Omega \mathcal{B}_\Omega} \int_\Omega \langle f_{\mathbf{p}} | \hat{H} | D^0 \rangle \langle f_{\mathbf{p}} | \hat{H} | \bar{D}^0 \rangle^* \left| \frac{\partial^n \phi}{\partial(p_1 \dots p_n)} \right| d^n p.$$

amplitude of D (\bar{D}) going to 5-D phase space point p

Binning is good for you

arXiv:1412.7254

$$\frac{\Gamma(B^- \rightarrow DK^-, D \rightarrow f)_\Omega}{\Gamma(B^- \rightarrow DK^-, D \rightarrow \bar{f})_{\bar{\Omega}}} = r_{D,\Omega}^2 + r_B^2 + r_{D,\Omega} r_B |Z_\Omega^f| \cos(\delta_B - \delta_\Omega^f - \gamma)$$

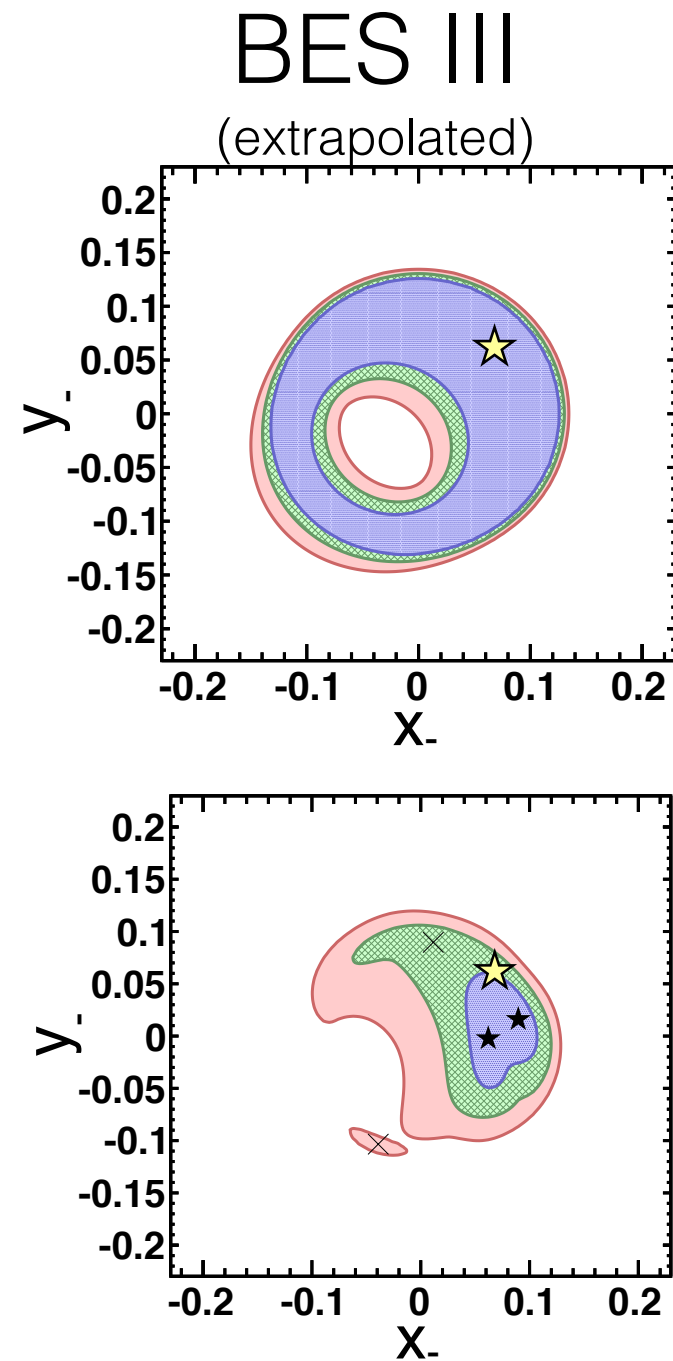


$$Z_\Omega^f \equiv \frac{1}{\mathcal{A}_\Omega \mathcal{B}_\Omega} \int_\Omega \langle f_{\mathbf{p}} | \hat{H} | D^0 \rangle \langle f_{\mathbf{p}} | \hat{H} | \bar{D}^0 \rangle^* \left| \frac{\partial^n \phi}{\partial(p_1 \dots p_n)} \right| d^n p.$$

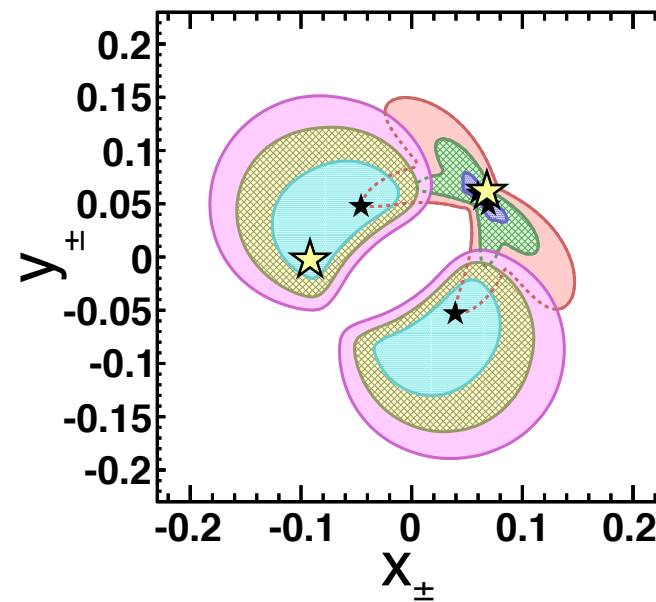
Mean $|Z|$ increases if you bin in terms of the phase difference between D and \bar{D} amplitudes.

Turns out: if you have sufficiently many bins, you can extract γ model-independently, even w/o input from the charm threshold.

Gets even better if we divide the 5-D space into bins



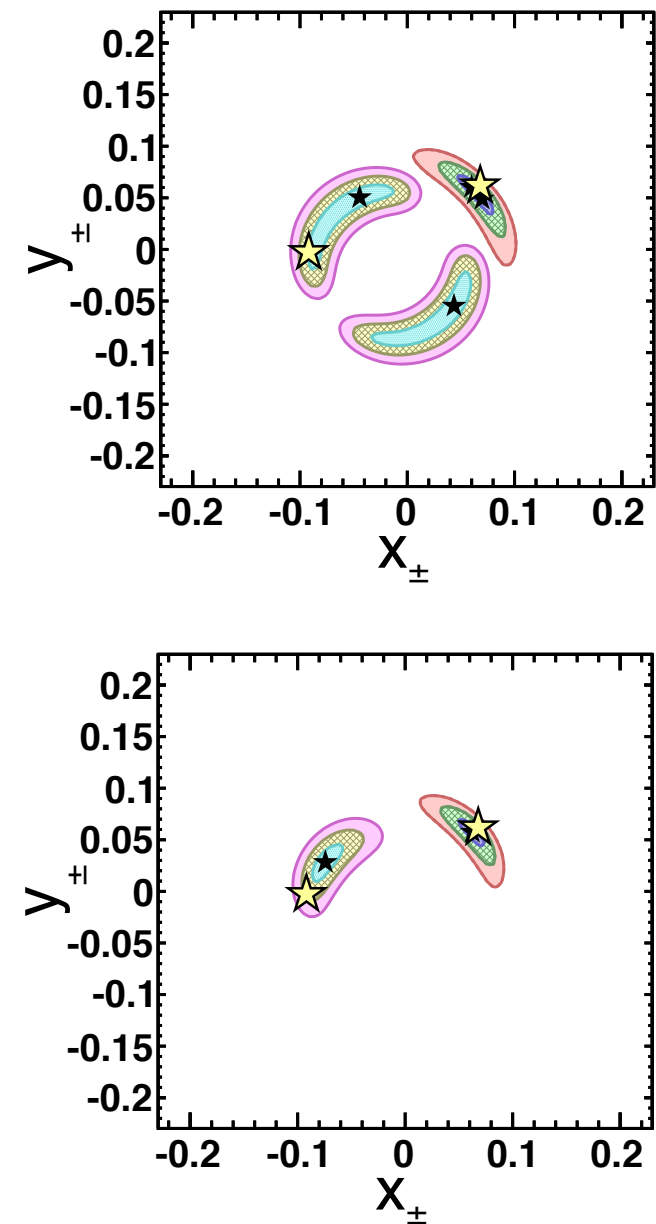
D-mixing
(binned, LHCb
run II statistics)



$$x_{\pm} = r_B \cos(\delta_B \pm \gamma)$$

$$y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$

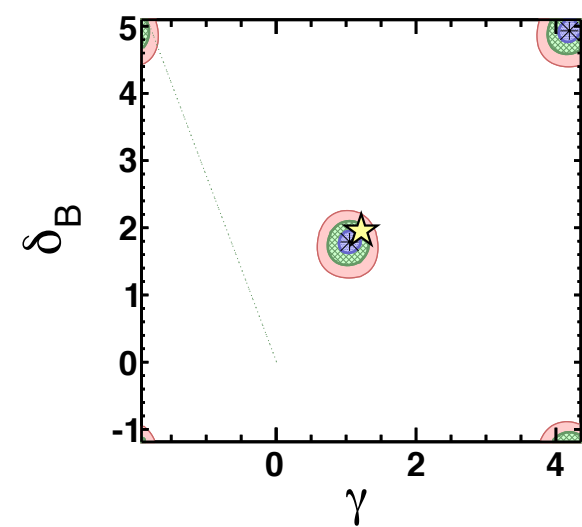
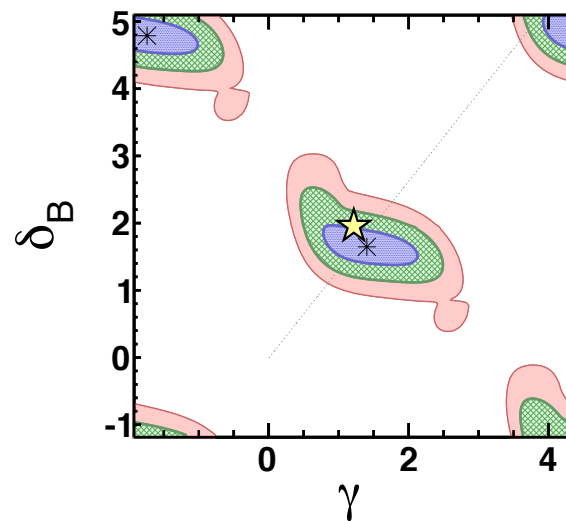
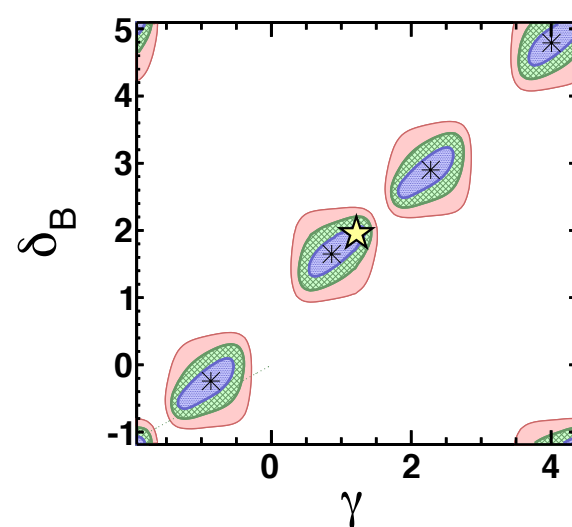
Combination



[arXiv:1412.7254](https://arxiv.org/abs/1412.7254) (accepted by JHEP)

Gets even better if we divide the 5-D space into bins

D-mixing
BES III - binned (binned, LHCb
run II statistics) Combination



(all simulated data)

[arXiv:1412.7254](#) (accepted by JHEP)

Searches for CPV by comparing binned Dalitz plots

PhysRevD.84.112008

- Compare yields in CP-conjugate bins

$$S_{CP} = \frac{N_i - \alpha \bar{N}_i}{\sigma(N_i - \alpha \bar{N}_i)}$$

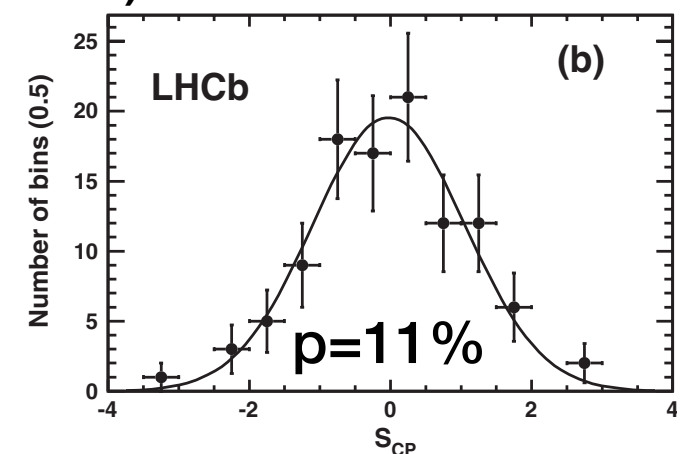
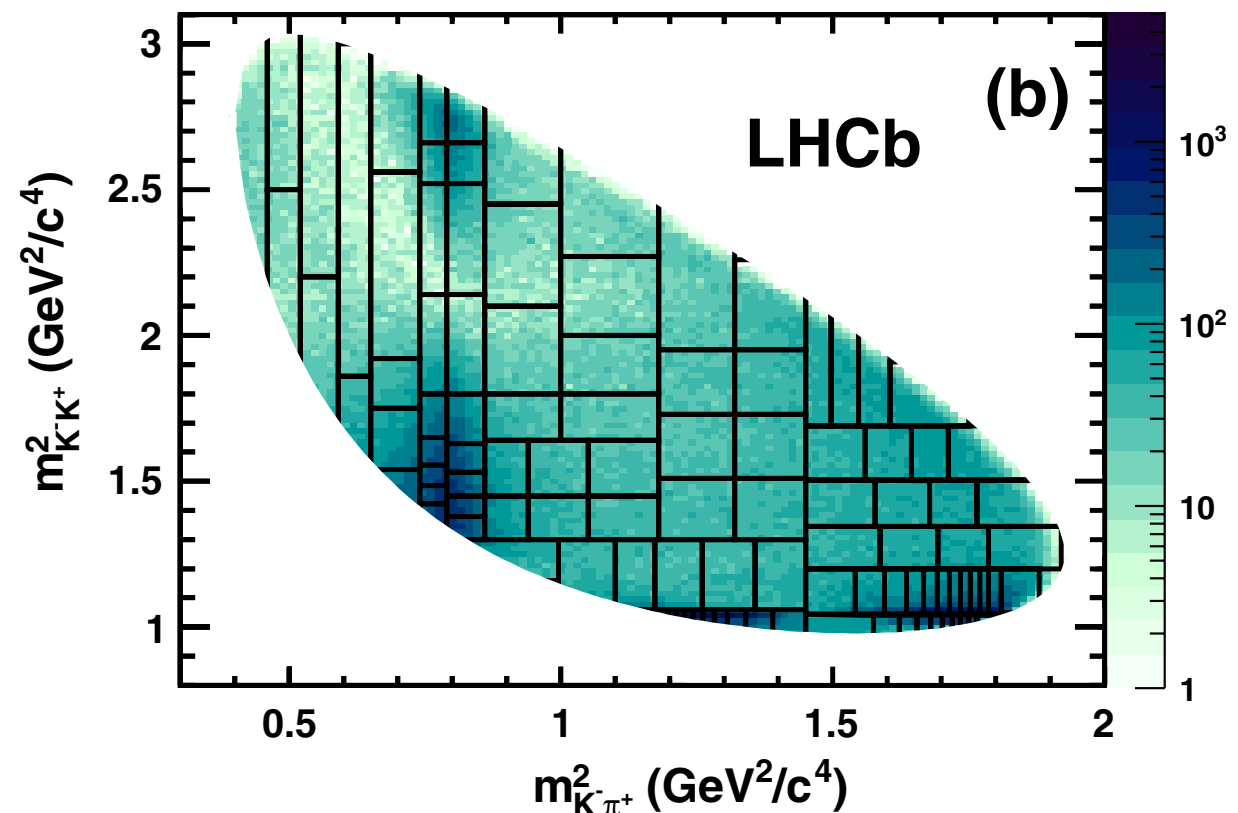
$$\alpha = \frac{N_{\text{total}}}{\bar{N}_{\text{total}}}$$

- Calculate p-value for no-CPV hypothesis based on

$$\chi^2 = \sum_i (S_{CP}^i)^2$$

- Model independent. Many production and detection effects cancel.

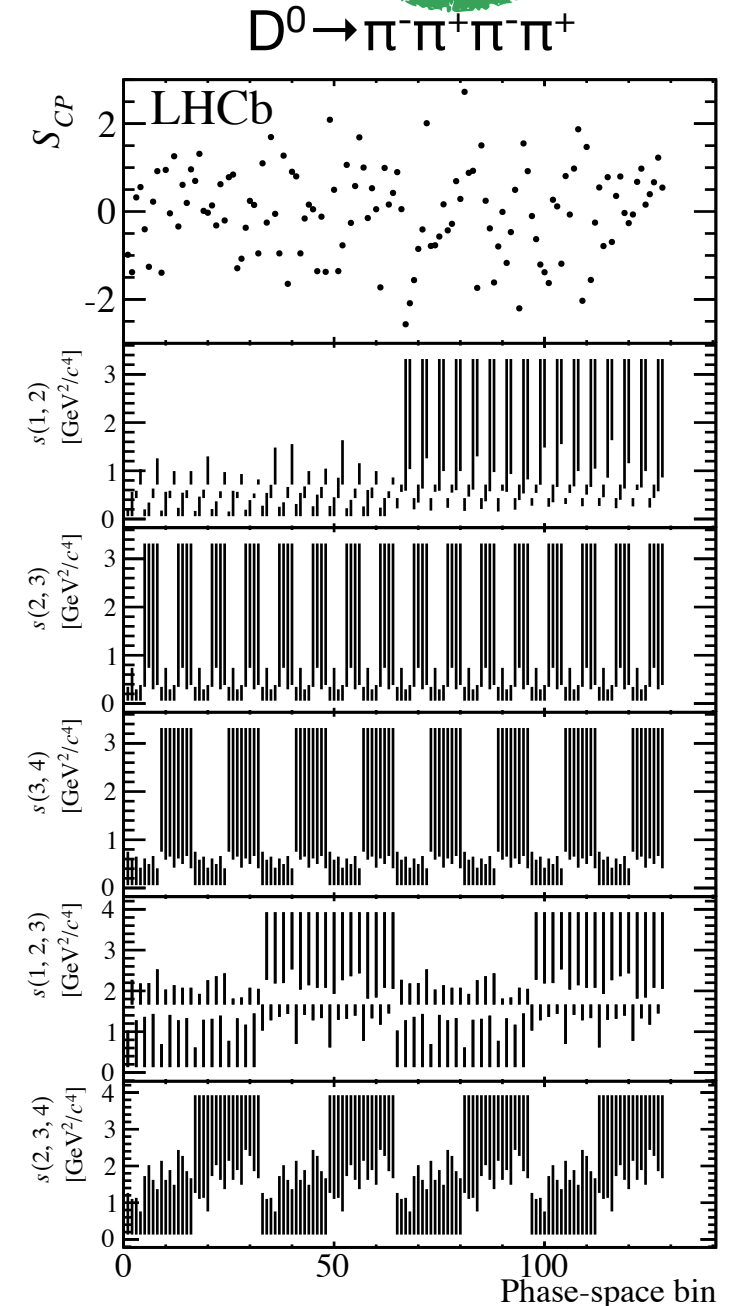
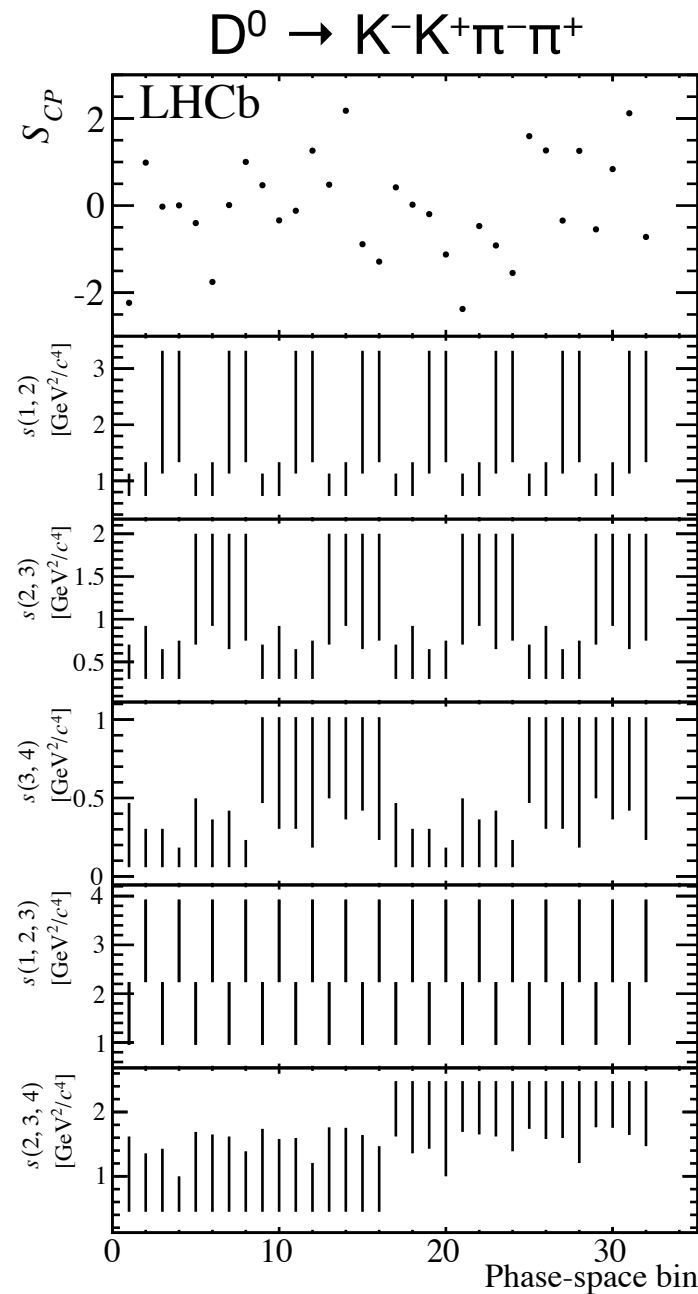
330k $D^+ \rightarrow K^- K^+ \pi^+$ in 35/pb



5-D binned analysis in $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$, $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$

LHCb 1fb⁻¹ Phys.Lett. B726 (2013) 623-633

- Binning in 5-dimensional hypercuboids.
- Adaptive binning to ensure similar number of entries per bin.
- Plots show for each bin the range in invariant mass squared and S_{CP} value in that bin.

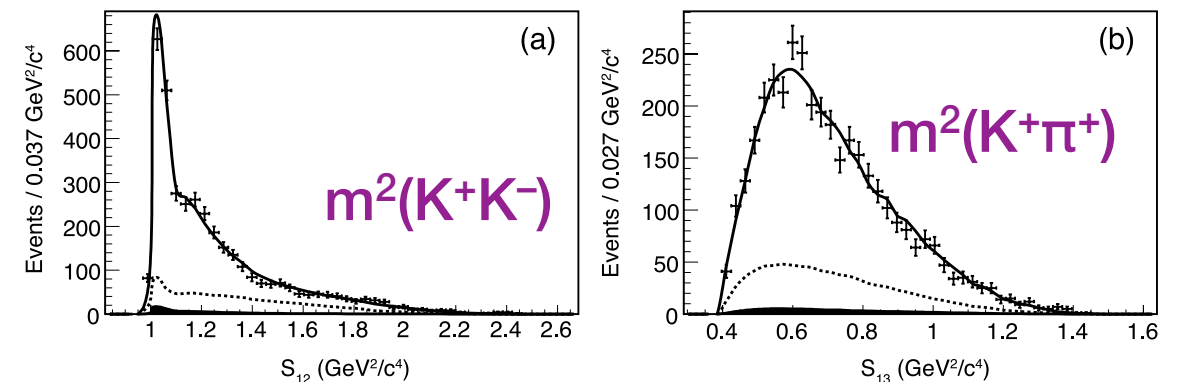


Model-dependent CPV search in $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$

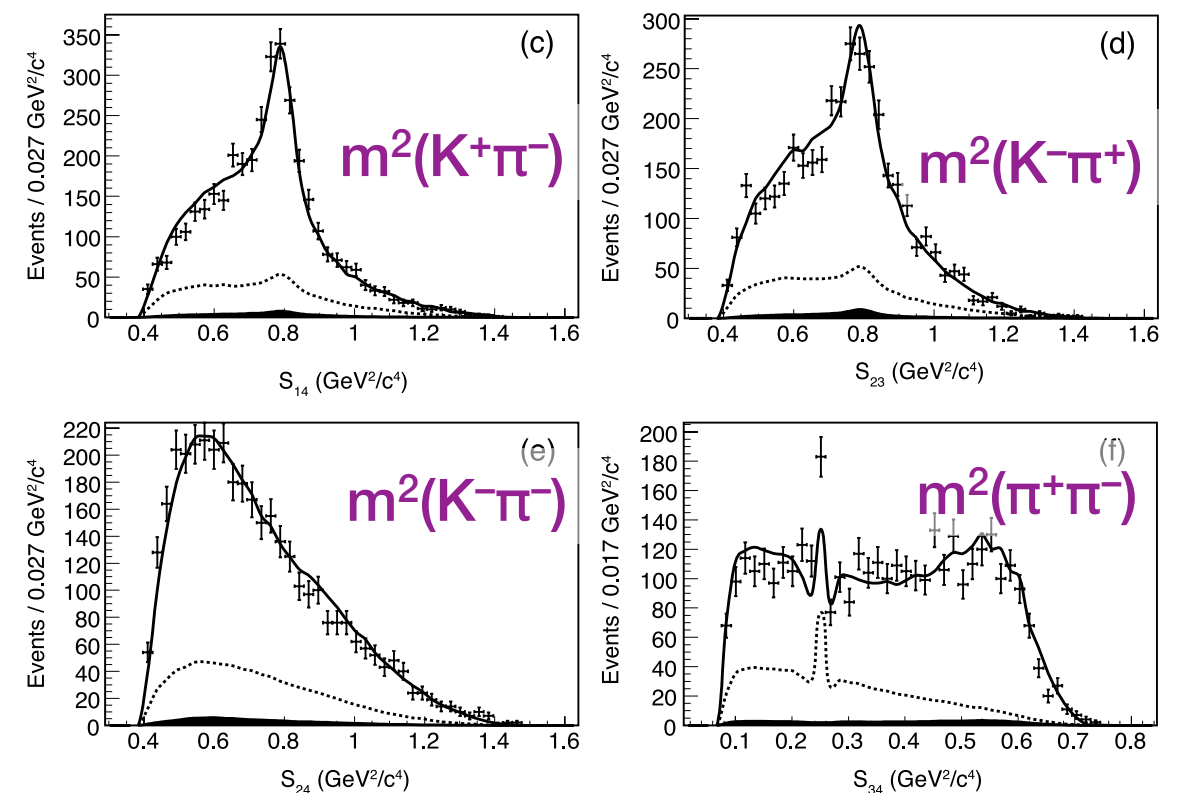
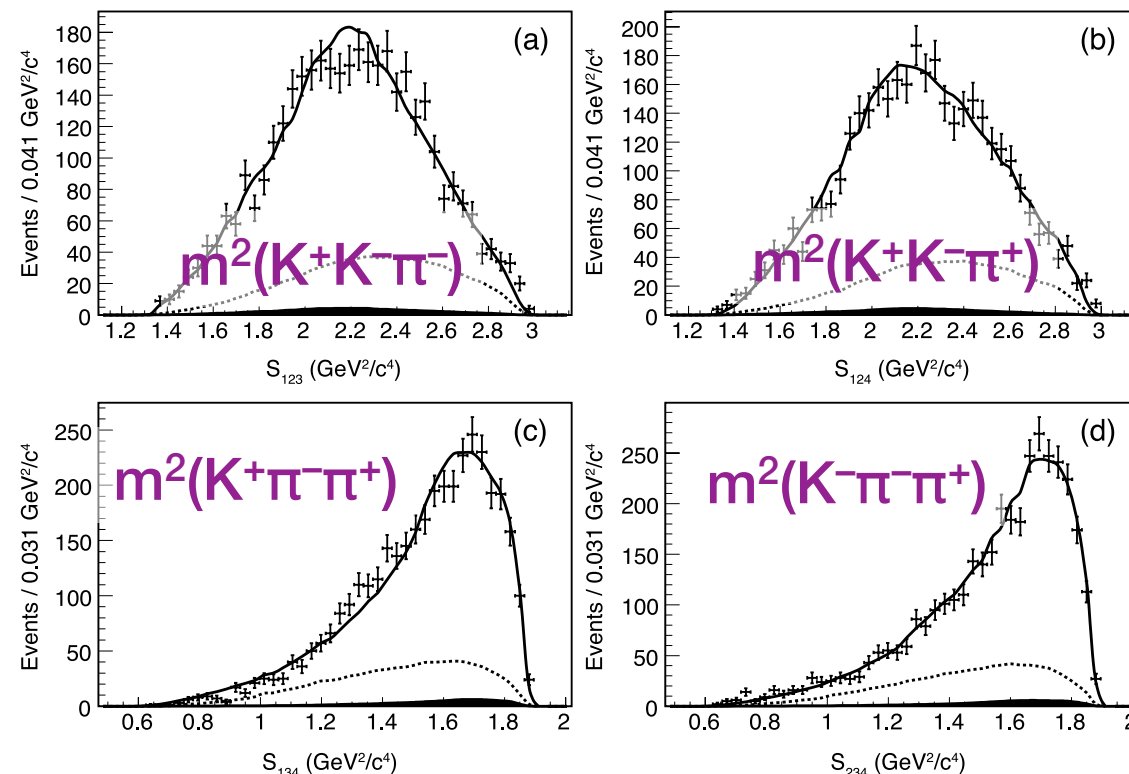
1-D projections of 5-D
amplitude fit
(D^0 , D^0 bar combined, charge
assignments in $m^2(K^+\pi^-)$ etc are for D^0
and are reversed for D^0 bar)

CLEO: Phys.Rev. D85 122002 (2012)

4280112-013

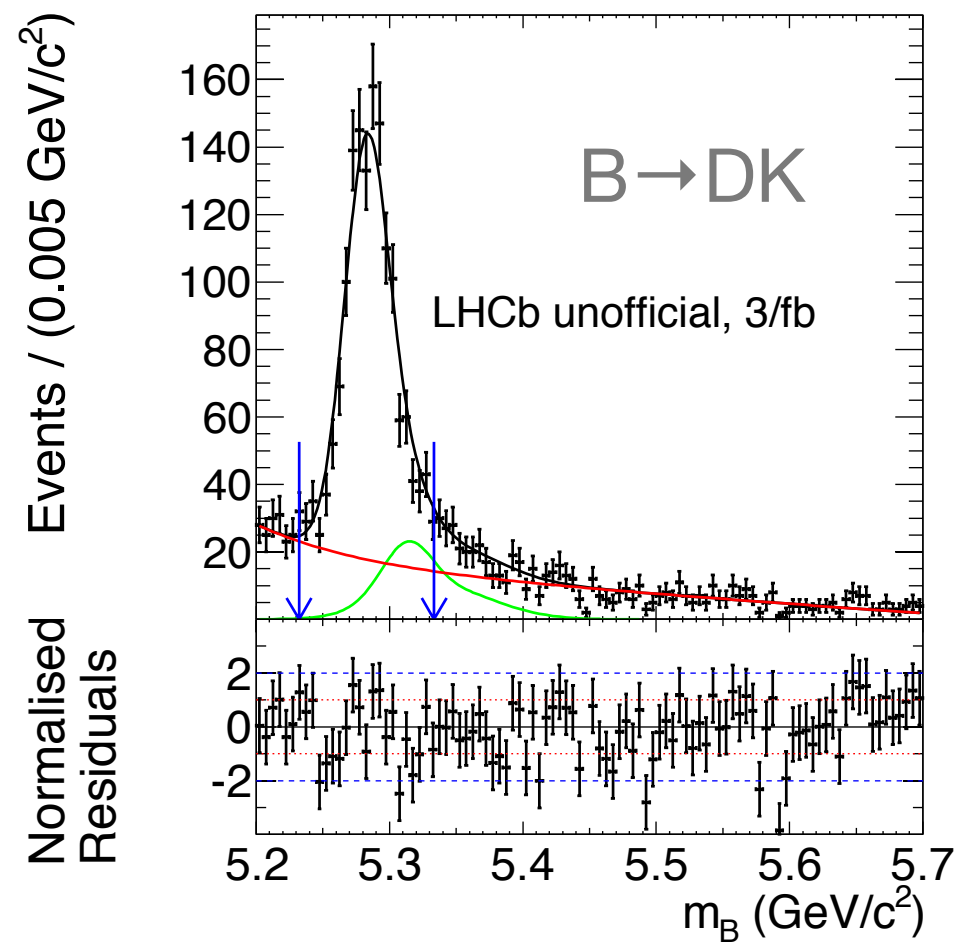


4280112-014

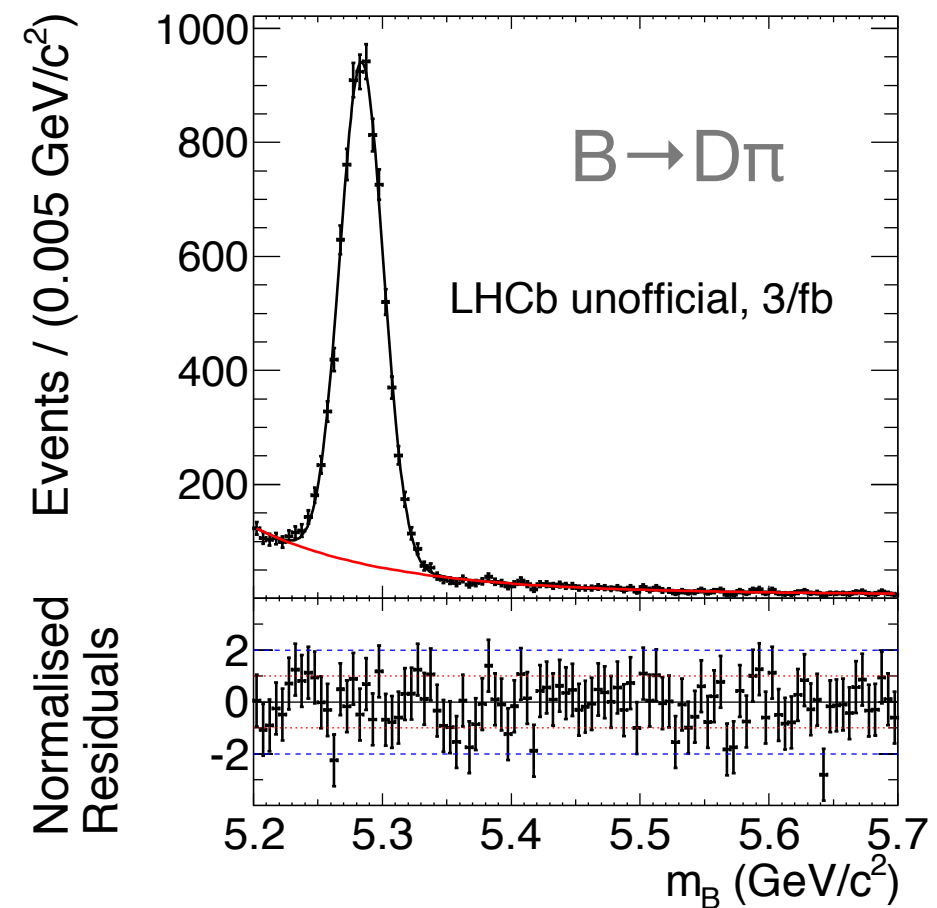


Towards γ with $B^\pm \rightarrow D(KK\pi\pi)K^\pm$

Signal



Control Channel



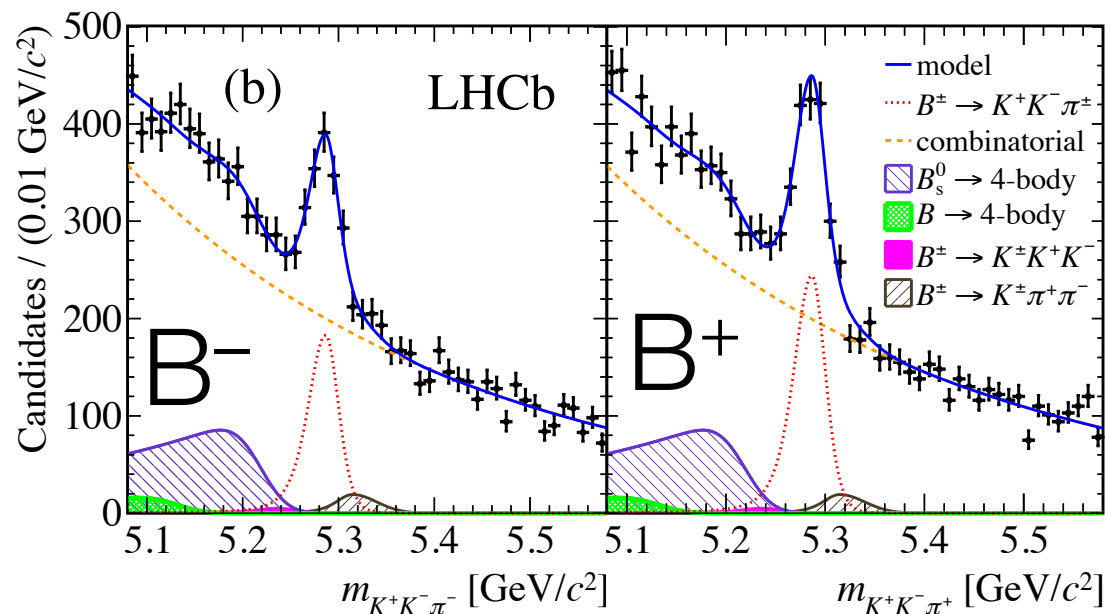
Nicole Skidmore & Jeremy Dalseno (Bristol)

CPV in $B^\pm \rightarrow \pi^\pm K^+ K^-$

Phys. Rev. Lett. 112, 011801 (2014)

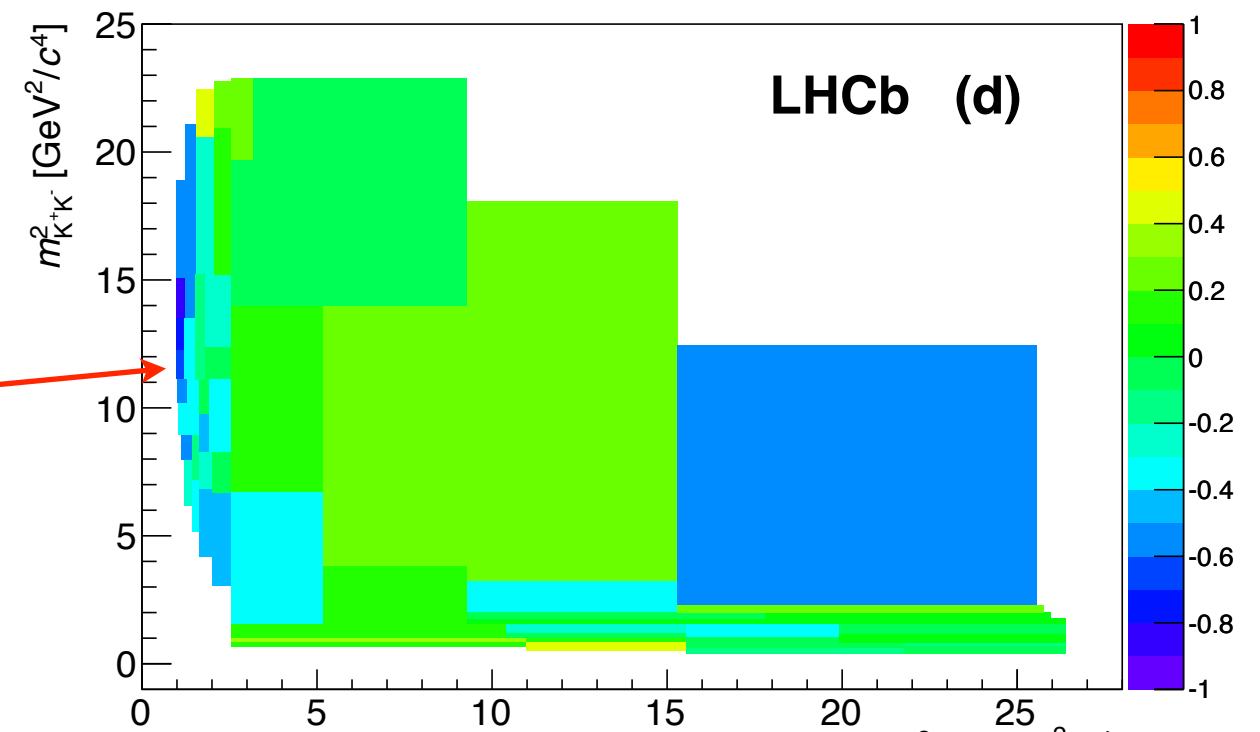
Phys. Rev. Lett. 111, 101801 (2013)

3/fb update: arXiv:1501.06777 (2014)



local:

$$A_{CP \text{ bin}} = \frac{N_{\text{bin}}(B^-) - N_{\text{bin}}(B^+)}{N_{\text{bin}}(B^-) + N_{\text{bin}}(B^+)}$$



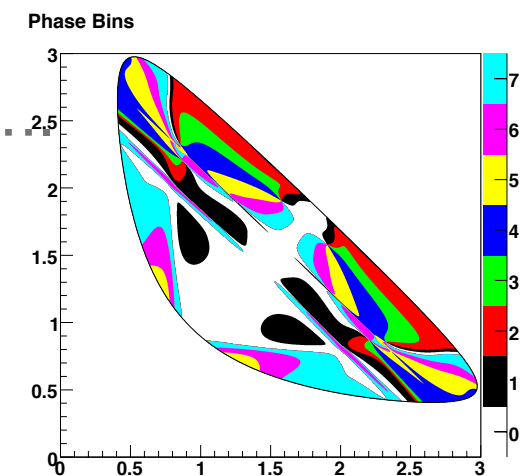
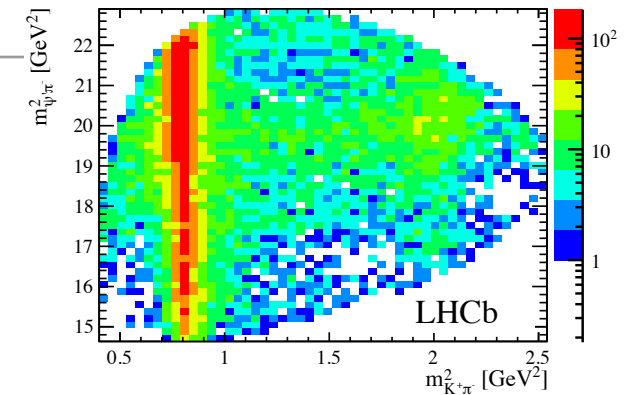
Large
local A_{CP} at low $m(KK)^2$, not
associated to a resonance

Also found large local CPV in low mass regions w/o clear
association to known resonances in other $B^\pm \rightarrow hhh$ modes:

$B^\pm \rightarrow K^\pm \pi^+ \pi^-$, $B^\pm \rightarrow K^\pm K^+ K^-$, $B^\pm \rightarrow \pi^\pm \pi^+ \pi^-$, $B^\pm \rightarrow \pi^\pm K^+ K^-$

Conclusions

- Amplitude Analyses are a very powerful tool used at LHCb and elsewhere for wide variety of measurements, including
 - searching for new resonances and characterising them
 - precision CP violation and mixing measurements in charm and beauty
- They are not “just” Dalitz plots. Vectors in final state, 4 body analyses,...
- Most remarkable strength: unique sensitivity to phases.
- Most annoying weakness: theoretically not well understood. This is increasingly problematic with increasingly ginormous data samples.
- Theorists are making tangible progress on theoretically sound models.
- Future: improved models, model independent methods, pragmatic compromises.



Credits

Special thanks to Antimo Palano and Marco Pagapallo, from whose excellent talks I lifted a particularly large number of plots.

Backup

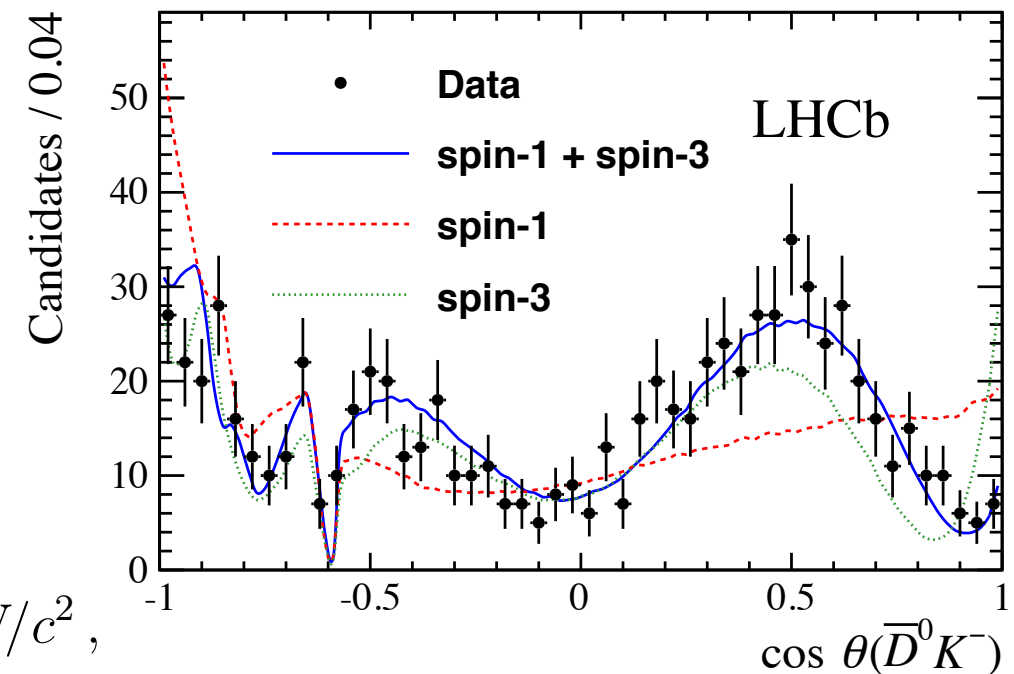
$B_s \rightarrow DK\pi$ at LHCb

(Phys.Rev. D90 (2014) 072003)

- Resolved the $D_{sJ}^*(2860)$ state into spin 1 and spin 3 states
 - Now part of a renaissance in D(s) spectroscopy (15 citations so far)

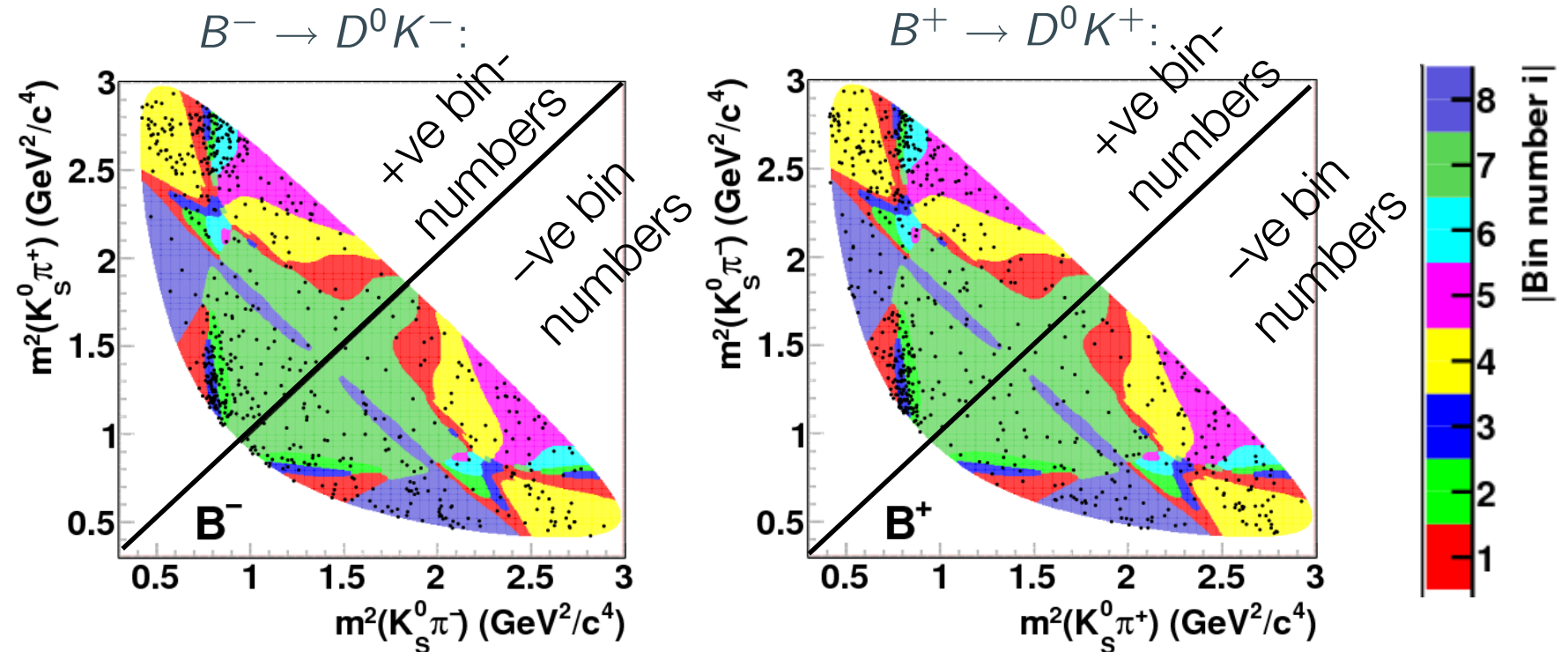
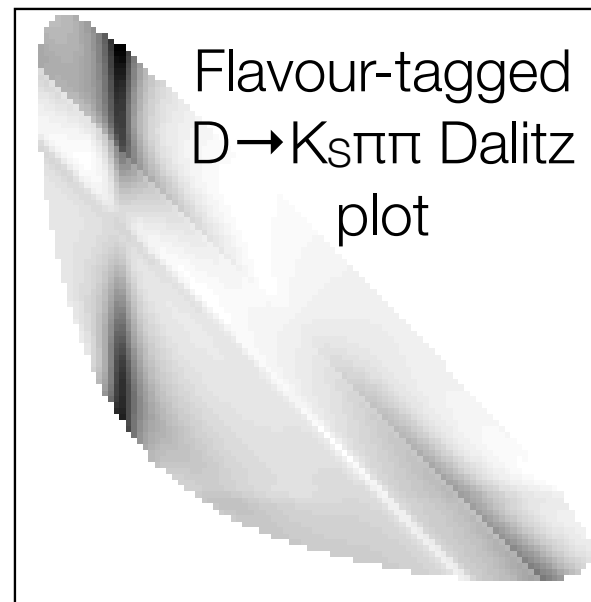
- Other results

- Mass, width and spin of D_{s2}^*
- Fit fractions
- Branching fractions
- Complex amplitudes



$$\begin{aligned}
 m(D_{s1}^*(2860)^-) &= 2859 \pm 12 \pm 6 \pm 23 \text{ MeV}/c^2, \\
 \Gamma(D_{s1}^*(2860)^-) &= 159 \pm 23 \pm 27 \pm 72 \text{ MeV}/c^2, \\
 m(D_{s3}^*(2860)^-) &= 2860.5 \pm 2.6 \pm 2.5 \pm 6.0 \text{ MeV}/c^2, \\
 \Gamma(D_{s3}^*(2860)^-) &= 53 \pm 7 \pm 4 \pm 6 \text{ MeV}/c^2,
 \end{aligned}$$

First model-independent γ measurement (BELLE)

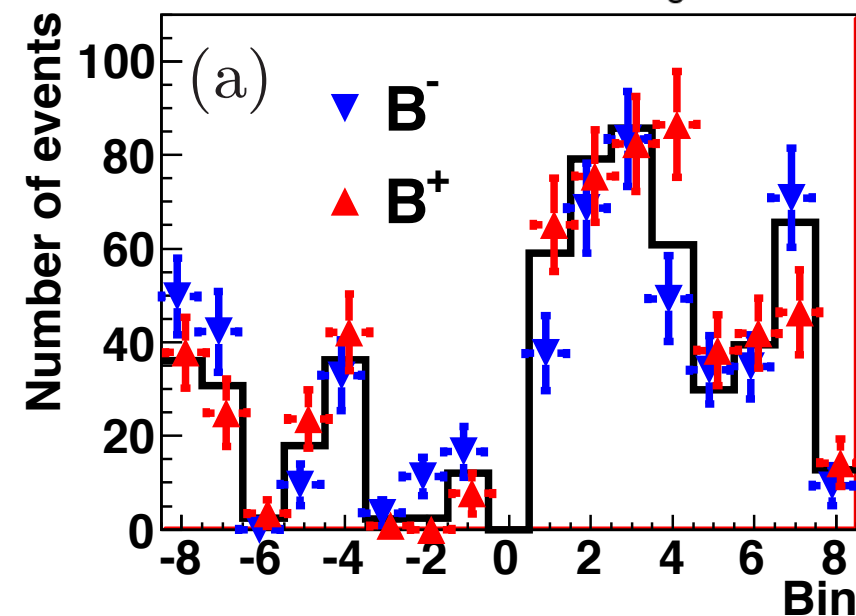


$$\gamma = (77.3^{+15.1}_{-14.9} \pm 4.2 \pm 4.3)^\circ$$

$$r_B = 0.145 \pm 0.030 \pm 0.011 \pm 0.011$$

$$\delta_B = (129.9 \pm 15.0 \pm 3.9 \pm 4.7)^\circ,$$

where the last uncertainty on γ of 4.3° the former model uncertainty of 8.9°



BELLE: [arXiv:1106.4046](https://arxiv.org/abs/1106.4046). See also Anton Poluektov's talk at Moriond EW 2011 (from which I lifted several of the plots shown here): <http://belle.kek.jp/belle/talks/moriondEW11/poluektov.pdf>
CLEO-c input: [Phys.Rev.D82:112006,2010](https://arxiv.org/abs/1106.4046).

LHCb model-independent γ from $B^\pm \rightarrow (K_S \pi \pi)_D K$ and $B^\pm \rightarrow (K_S K K)_D K$

LHCb-CONF-2013-004

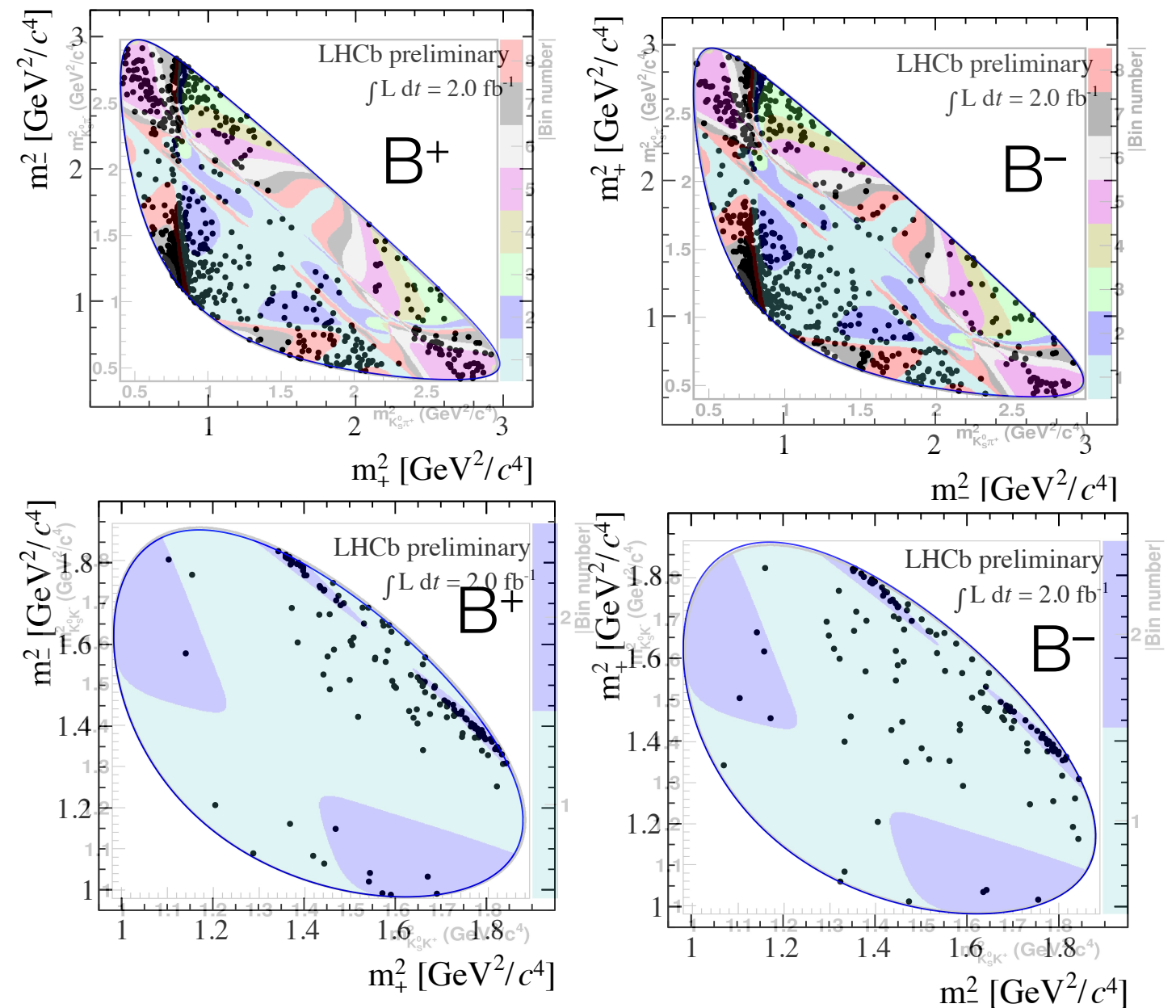
LHCb 2011 Result: Phys. Lett. B718 (2012) 43

- Binned, model-independent analysis using CLEO-c input.
Phys. Rev. D 82 112006.
- Plots show LHCb 2012 data - the colours represent the bins, shaped to optimise sensitivity.
- Result of combined analysis (2011 & 2012 data, $K_S \pi \pi$ & $K_S K K$):

$$\gamma = (57 \pm 16)^\circ$$

$$\delta_B = (124^{+15}_{-17})^\circ$$

$$r_B = (8.8^{+2.3}_{-2.4}) \times 10^{-2}$$



CLEO-c input: Phys. Rev. D 82 112006.

Model-independent method: Giri, Grossmann, Soffer, Zupan, Phys Rev D 68, 054018 (2003).

Optimal binning: Bondar, Poluektov hep-ph/0703267v1 (2007)

BELLE's first model-independent γ measurement: PRD 85 (2012) 112014

B->DK, D->3pi with BES & Mixing

$LHCb$ scenario	D^0 mix?	charm threshold?	$\sigma(\gamma)$ [°]	$\sigma(\delta_B)$ [°]	$\sigma(r_B)$ $\times 10^2$	$\sigma(x_+)$ $\times 10^2$	$\sigma(y_+)$ $\times 10^2$	$\sigma(x_-)$ $\times 10^2$	$\sigma(y_-)$ $\times 10^2$
run I	Y	none	26	47	1.6	8.7	9.1	8.8	8.2
run II			22	29	1.4	7.6	6.9	4.5	4.0
upgr			15	14	0.17	4.7	5.2	0.56	0.98
run I	Y	CLEO global	20	29	0.82	6.4	5.7	6.6	5.9
run II			15	19	0.62	5.4	3.9	2.5	2.7
upgr			11	10	0.16	3.8	2.8	0.44	0.50
run I	Y	BESIII global	19	25	0.78	6.4	5.5	6.5	5.8
run II			14	18	0.57	5.4	3.9	2.4	2.7
upgr			9.0	8.2	0.15	3.7	2.7	0.43	0.48
run I	N	CLEO binned	46	35	3.2	6.9	6.5	8.6	10
run II			50	34	3.3	6.9	6.7	8.9	11
upgr			52	35	3.3	7.6	6.7	8.9	11
run I	N	BESIII binned	40	24	2.6	4.1	5.0	5.7	6.2
run II			34	17	2.5	3.6	4.1	5.0	5.1
upgr			39	14	2.9	3.9	4.1	4.3	5.6
run I	Y	CLEO binned	16	18	0.78	2.1	3.5	2.6	3.1
run II			12	13	0.53	1.7	3.1	1.7	2.0
upgr			7.8	7.2	0.15	1.1	2.6	0.40	0.46
run I	Y	BESIII binned	12	14	0.68	1.6	2.6	2.0	2.5
run II			8.6	9.6	0.47	0.90	2.1	1.5	1.5
upgr			4.1	3.9	0.14	0.53	1.3	0.35	0.38

[arXiv:1412.7254](https://arxiv.org/abs/1412.7254)

(accepted by JHEP)