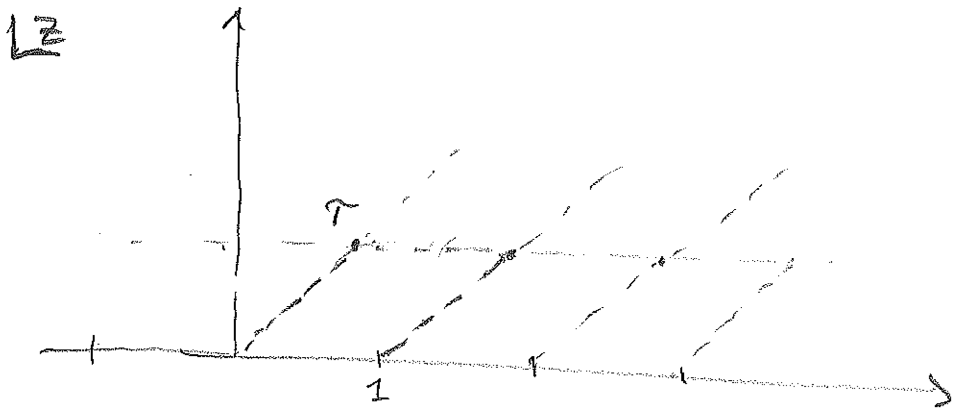
Lecture 3 : F-theory 12.05.2014

Outline:
- Mathematical background: Elliptic fibrations
- Type IIB string theory with D7-branes
- An F-theory GUT model

Background on Tori and Fibrations

- A flat torus T^2 is spanned by 2 periodic directions

- In the complex plane



Lattice Λ

- The quotient $\mathbb{C}/\Lambda = T^2 = \{z \mid z \sim z+1 \sim z+\tau\}$

identifies opposite sides of a cell in Λ

- The transformation $\tau \longrightarrow \dfrac{a\tau+b}{c\tau+d}$

$a,b,c,d \in \mathbb{Z}$, $ad-bc=1$ preserves Λ, and thus the torus T^2

- $SL(2,\mathbb{Z}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \;\middle|\; a,b,c,d \in \mathbb{Z},\ ad-bc=1 \right\}$

is the symmetry group of the torus.

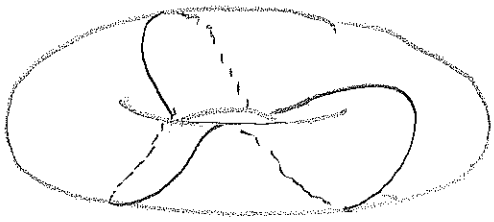
- A 1-cycle is a closed loop on the torus.

- A basis for all 1-cycles on T^2 is given by A and B.

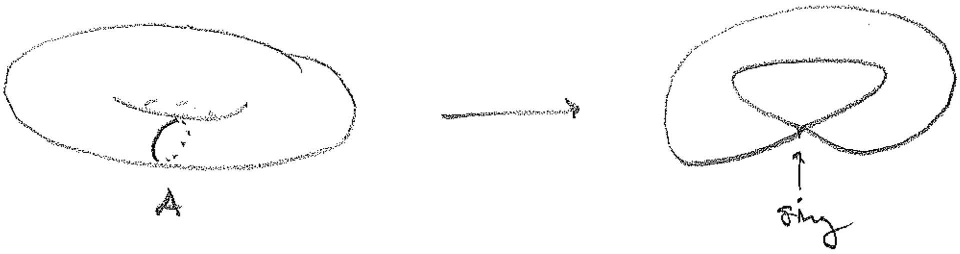
- Any 1-cycle is written as $pA + qB = (p,q)$ (up to deformation)

  i.e. p turns in the A-direction and q — " — B-direction

Ex. a $(3,1)$-cycle:

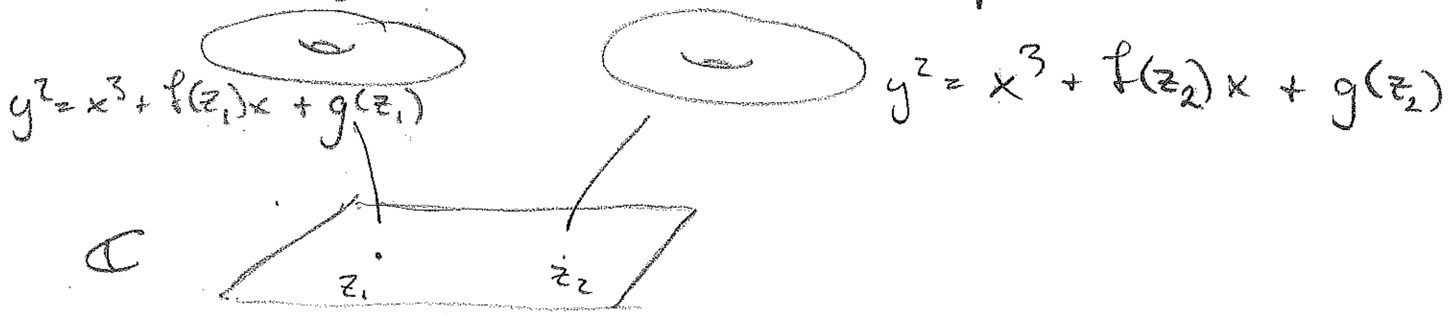


- If a 1-cycle B shrinked down to zero size $\longrightarrow$ singularity

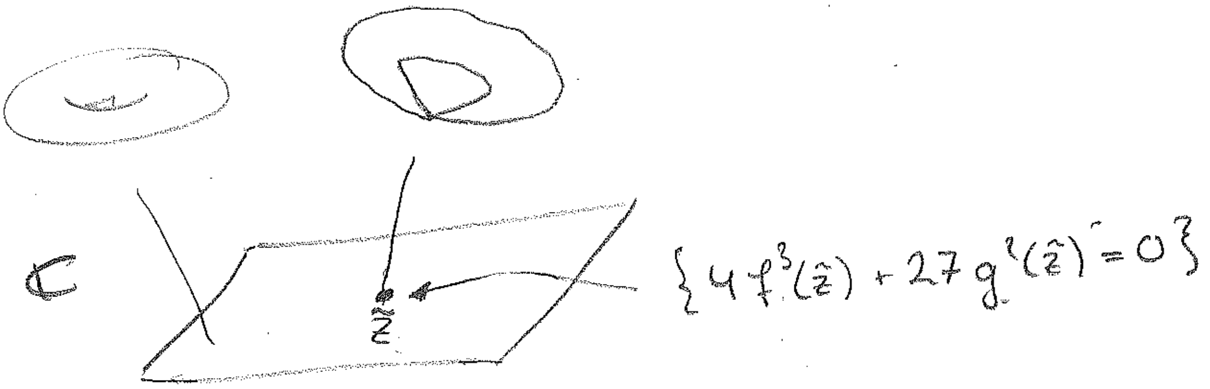


- A general singularity: a vanishing (p,q)-cycle.

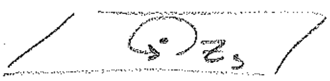
- A torus may also be described as the equation $y^2 = x^3 + f\,x + g$, $x, y \in \mathbb{C}$. (f, g determine τ)
- A fibration of tori: Take $f(z)$ and $g(z)$ to vary over the complex plane:



- T^2 becomes singular where $4f^3 + 27g^2 = 0$



- Monodromy: The singular point $\tilde{z} \in \mathbb{C}$ makes τ multivalued, transporting τ around z_s



gives $\tau \longrightarrow \dfrac{a\tau + b}{c\tau + d}$

- The $SL(2,\mathbb{Z})$ symmetry makes the fibration well-defined despite the monodromies

- The monodromy action by $M \in SL(2,\mathbb{Z})$ classifies the possible singularities of T^2.

- Ex. Simplest singularity, vanishing A-cycle:

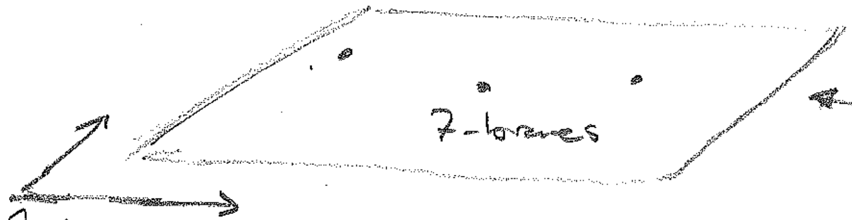
$$M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \Longrightarrow \tau \to \frac{\tau + 1}{1} = \tau + 1$$

- Hint: The classification agrees with the classification of simply laced Lie algebras: $SU(N)$
  $SO(2N)$
  E_6, E_7, E_8

Type IIB string Theory with 7-branes

- Theory of closed and open strings, with Neumann boundary conditions in 7+1 directions

- Space-time 9+1 dimensional



← 9 spatial dimensions

2 transverse directions to 7-brane ~ $\mathbb{C}$

- Low energy limit ↔ strings look pointlike

Fields (bosonic)

$\tau = C_0 + i e^{-\phi}$ complex scalar

$g_{\mu\nu}$ graviton

B_2, C_2 U(1) gauge fields (not from branes, but from including fermions)

- Action invariant under $SL(2,\mathbb{Z})$ transf.

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

$$\begin{pmatrix} B_2 \\ C_2 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} B_2 \\ C_2 \end{pmatrix}$$

- τ is called the axio-dilaton field

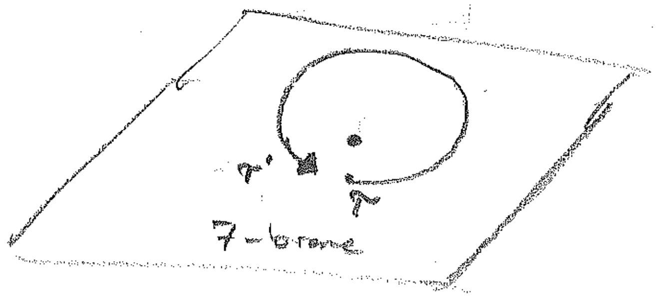
$\tau = C_0 + i e^{-\phi}$, $\langle e^{\phi} \rangle = g_s$, the string coupling

- $SL(2,\mathbb{Z})$ generated by

$$\begin{cases} \tau \to \tau + 1 & \text{, perturbative symmetry} \\ \tau \to -\frac{1}{\tau} & \text{, exchanges weak-strong coupling} \end{cases}$$

- Since τ may vary, both weakly and strongly regions may occur

- 7-branes are **massive** objects, which are charged under τ



$$\tau' = \frac{a\tau + b}{c\tau + d}$$

Analogy: Integrating the magnetic field around a wire $\int B \sim I$, the current in the wire

Ex. 1 D7 brane has charge 1 under C_0

$\Rightarrow C_0 \to C_0 + 1$ (or $\tau \to \tau + 1$) when encircling the brane.

$\tau \to \tau + 1$ is the element $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \in SL(2,\mathbb{Z})$

Ex n D7 branes : $\tau \to \tau + n$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \in SL(2,\mathbb{Z})$$

<u>Generalization:</u> The monodromy of τ around a (p,q)-brane is given by

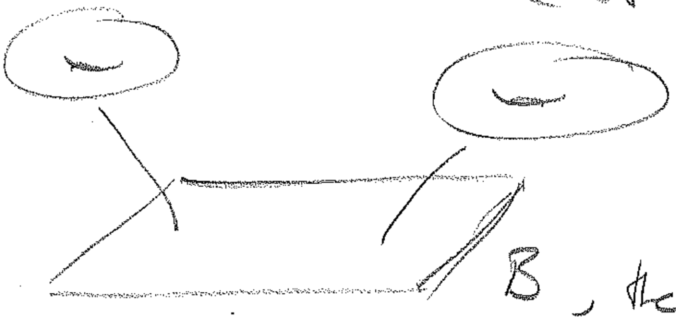
$$M = \begin{pmatrix} 1-pq & p^2 \\ q^2 & 1+pq \end{pmatrix} \in SL(2,\mathbb{Z})$$

<u>Note:</u> A Dirichlet D7-brane is of type $(1,0)$ which is a perturbative solution.

- In general a (p,q)-brane is a non-perturbative IIB solution, which is hard to analyze in string theory.

<u>The F-theory Idea:</u>

- Exchange the axio-dilaton τ for a fibration of tori



B, the IIB background geometry

- Along loci in B, where the torus becomes singular, 7-branes are found in the IIB description.

- A vanishing (p,q)-cycle in T^2
  $\longleftrightarrow$ (p,q)-brane in the base B
- A 'non-perturbative' description of IIB string theory with branes.

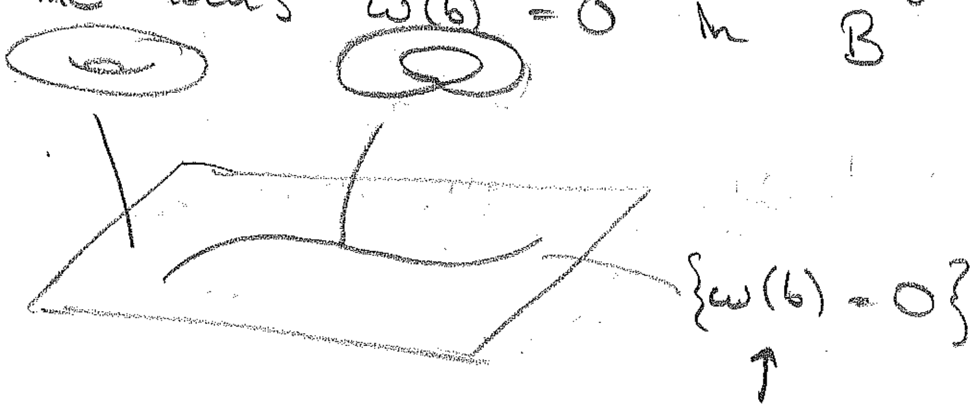
- Geometrization: Questions about fields and branes can be analyzed through the geometry of the fibration.

An F-theory model

- Arrange an fibration of tori

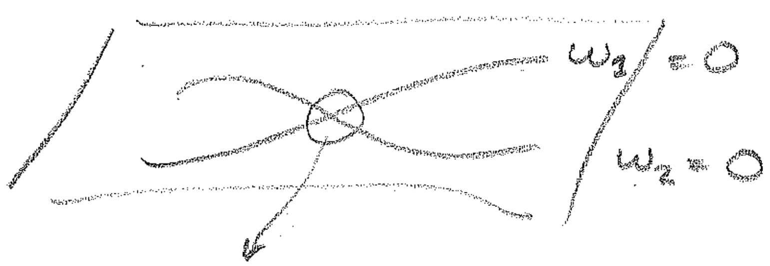
$$y^2 = x^3 + f(b)x + g(b)$$

s. th the singularity type is, say $SU(5)$ at some locus $\omega(b) = 0$ in B (codim 1)



- In type IIB this is a stack of 5 D7 branes at $\{\omega(b) = 0\}$

- Having multiple such loci $\{\omega_i(b) = 0\}$ gives multiple factors to the gauge group



- At intersection loci, the singularity type 'enhances', and the pattern of the enhancement determines the matter representations at the intersection (as in IIB ...)

Virtues:

- Non-perturbative in the coupling strength

  ⇒ The $10\ 10\ 5_H$ SU(5) coupling cannot be constructed perturbatively in IIB

- The geometric formulation is well suited for attacking problems with field theory GUTS : Doublet-triplet splitting

$$\updownarrow$$

Higgses 5_H, $\bar{5}_H$ on separate curves (intersection

: Proton decay (dim 4)

no $10\ \bar{5}\ \bar{5}$ coupling

$$\updownarrow$$

need $\bar{5}_m$ and $\bar{5}_H$ on different matter curves