LECTURE II

Repetition

$$\mathcal{L} = g^Z_{u_L} \bar{u}_L \gamma_\mu Z^\mu u_L + g^Z_{u_R} \bar{u}_R \gamma_\mu Z^\mu u_R$$
$$+ g^Z_{d_L} \bar{d}_L \gamma_\mu Z^\mu d_R + \ldots$$
$$+ g^A_{u_L} \bar{u}_L \gamma_\mu A^\mu u_L + \ldots$$
$$+ g^W \bar{u}_L \gamma_\mu W^+_\mu d_L + \ldots$$
$$+ Y_u \bar{u}_L \tilde{H} u_R + Y_d \bar{d}_L H d_R + \ldots$$

$$u_L^i = (u_L, c_L, t_L, \ldots)$$
$$d_L^i = (d_L, s_L, b_L, \ldots)$$

Y_u, Y_d are general complex $n \times n$ matrices. They can be diagonalized by a bi-unitary transformation

$$Y_u^{diag} = \frac{v}{\sqrt{2}} (m_u, m_c, m_t) = V_u^\dagger Y_u W_u$$
$$Y_d^{diag} = \frac{v}{\sqrt{2}} (m_d, m_s, m_b) = V_d^\dagger Y_d W_d$$

This corresponds to a change of basis

$$u_L^i = V_u^{ij} u_L'^j \qquad d_L^i = V_d^{ij} d_L'^j$$
$$u_R^i = W_u^{ij} u_R'^j \qquad d_R^i = W_d^{ij} d_R'^j$$

Therefore,

- $\bar{u}_L Y_u u_R H \to \bar{u}_L^i \underbrace{V_u^\dagger Y_u W_u}_{Y_u^{diag}} u_R^i H$, with $H \to v + h$ (mass → v, higgs couplings → h)

- $g_{uL}^Z \bar{u}_L \gamma_\mu \underset{A^\mu}{Z^\mu} u_L \to g_{uL}^Z \bar{u}_L^i V_u^\dagger V_u \gamma_\mu \underset{A^\mu}{Z^\mu} u_L^i$

$\Rightarrow$ No FCNC's

- $g^W \bar{u}_L \gamma_\mu W_+^\mu d_L \to g^W \bar{u}_L^i \underbrace{V_u^\dagger V_d}_{V_{CKM}} \gamma_\mu W_+^\mu d_L$

How many free parameters?

	2	3	N (Real, Im)
N×N matrix	(4, 4)	(9, 9)	(N^2, N^2)
Unitarity	(−3, −1)	(−6, −3)	$\left(-\frac{N(N+1)}{2}, -\frac{N(N-1)}{2}\right)$
Quark phases	(−, −3)	(−, −5)	$(-, -(2N-1))$
	(1, 0)	(3, 1)	$\left(\frac{N(N-1)}{2}, \frac{N(N-1)+2}{2}\right)$

For ≥3 Generations there is a phase in the CKM matrix.

Explicit example:

$$|K_L\rangle = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1+|\epsilon|^2}} \Big[(1+\epsilon) \underset{d\bar{s}}{|K_0\rangle} + (1-\epsilon) \underset{s\bar{d}}{|\bar{K}^0\rangle} \Big]$$

$$K_0 \to \pi^+ e^- \bar{\nu}_e \qquad \bar{K}_0 \to \pi^- e^+ \nu_e$$

For $\epsilon = 0$, these processes occur with the exact same rates, but

$$\Gamma(K_L \to \pi^+ e^- \bar{\nu}_e) \propto |\langle \bar{K}^0 | K_L \rangle|^2 \propto |1-\epsilon^2|$$
$$\Gamma(K_L \to \pi^- e^+ \nu_e) \propto |\langle K^0 | K_L \rangle|^2 \propto |1+\epsilon^2|$$

~ 0.8 % more often

$$\epsilon = 2 \times 10^{-3}$$

For each 1000 electrons, there are 1008 positrons.

Q: You are the international panel on Matter-Antimatter and an Alien spaceship is coming to visit us. What one question would you ask to make sure the aliens are made of matter?

BRUNNEN

W

Why is this phase important?

Assume there is a process

$$P \rightarrow a + b$$
$$\bar{P} \rightarrow \bar{a} + \bar{b}$$

with two different amplitudes each

$$M_1 (P \rightarrow a + b) \quad , \quad M_2 (P \rightarrow a + b)$$
$$\bar{M}_1 (\bar{P} \rightarrow \bar{a} + \bar{b}) \quad , \quad \bar{M}_2 (\bar{P} \rightarrow \bar{a} + \bar{b})$$

$$M_j = |M_j| \, e^{i\theta_j} \, e^{i\varphi_j}$$
$$\bar{M}_j = |M_j| \, e^{i\theta_j} \, e^{-i\varphi_j}$$

without CP violation these two terms must be the exact same complex number. With CP-violation, $\varphi \neq 0$ they are not

$$M = |M_1| \, e^{i\theta_1} e^{i\varphi_1} + |M_2| \, e^{i\theta_2} e^{i\varphi_2}$$
$$\bar{M} = |M_1| \, e^{i\theta_1} e^{-i\varphi_1} + |M_2| \, e^{i\theta_2} e^{-i\varphi_2}$$

$$|M|^2 = |M_1|^2 \cancel{e^{2i\theta_1}} \cancel{e^{2i\varphi_1}} + |M_2|^2 + 2 |M_1||M_2| \, e^{i(\theta_1 - \theta_2)} e^{i(\varphi_1 - \varphi_2)}$$

$$|\bar{M}|^2 = |M_1|^2 + |M_2|^2 + 2 |M_1||M_2| \, e^{i(\theta_1 - \theta_2)} e^{i(\varphi_2 - \varphi_1)}$$

$$|M|^2 - |\bar{M}|^2 = 4 \, |M_1| \, |M_2| \, \sin(\theta_1 - \theta_2) \sin(\varphi_1 - \varphi_2)$$

Matter-Antimatter imbalance!

In the universe $\frac{n_b - n_{\bar{b}}}{n_\gamma} \approx 10^{-9}$ CKM: 10^{-20}

CKM matrix

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$\lambda \simeq 0.22$ Cabibbo angle!

$A \simeq 0.81$ $\rho = 0.135$ $\eta = 0.349$

$$|V_{CKM}|^2 \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

Assume the SM as an effective theory

$$\mathcal{L}_{SM} = \mu_H^2 H^2 + \lambda H^4 + \mathcal{L}_{gauge} + \mathcal{L}_{Yuk} + \frac{1}{\Lambda}(LH)^2 + \frac{1}{\Lambda^2}\mathcal{O}^6 + \dots$$

$\lambda = 0.13$

$m_H = \sqrt{2}\,\mu_H$

$g_3 \simeq 1$
$g \simeq 0.6$
$g' \simeq 0.3$

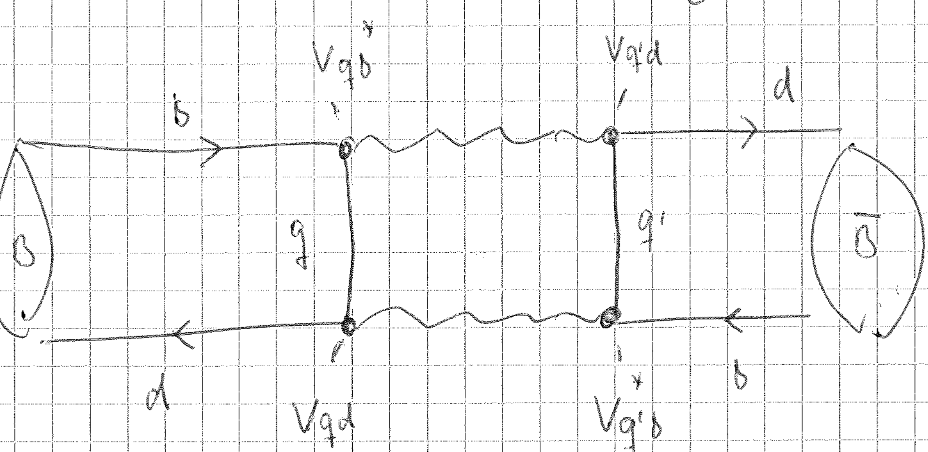
$$\mathcal{O}^6 = \frac{c}{\Lambda^2}(b\bar{d})(b\bar{d})$$

$m_t \sim \lambda^0 \frac{v}{\sqrt{2}}$ $m_s \sim \lambda^5 \frac{v}{\sqrt{2}}$

$m_b \sim \lambda^3 \frac{v}{\sqrt{2}}$ $m_d \sim m_u \sim \lambda^7 \frac{v}{\sqrt{2}}$

$m_c \sim \lambda^4 \frac{v}{\sqrt{2}}$ The SM flavor problem!

SM: $B_d - \bar{B}_d$ mixing



$$\mathcal{A} = \sum_{q q'} (V_{qb}^* V_{qd})(V_{q'b} V_{q'd}^*) A_{qq'}$$

$$V_{ub}^* V_{ud} + V_{tb}^* V_{td} + V_{cb}^* V_{cd} = 0 \qquad \text{Unitarity}$$

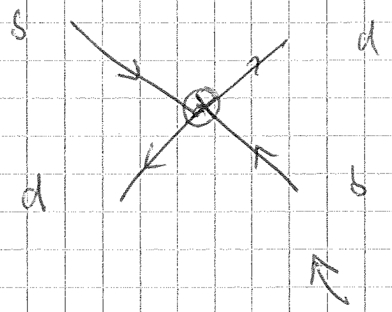
$$= \sum_{q} (V_{qb}^* V_{qd}) \left[\underbrace{V_{tb}^* V_{td}}_{\lambda^3} (A_{tq} - A_{uq}) + \underbrace{V_{cb}^* V_{cd}}_{\lambda^3} (A_{cq} - A_{uq}) \right]$$

$$A_{qq'} \sim \frac{g^4}{16\pi^2 m_W^2} \left[\text{Const.} + \frac{m_q m_{q'}}{m_W^2} + \dots \right] \langle \bar{B}_d | (\bar{s}_L \gamma_\mu d_L)^2 | B \rangle$$

$$= (V_{tb}^* V_{td})^2 \frac{g^4 m_t^2}{16\pi^2 m_W^4} + \dots$$

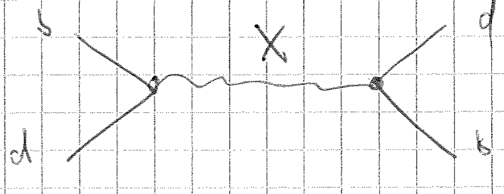
$$\approx 10^{-10} \frac{1}{\text{GeV}^2}$$

100 % correction



$$\mathcal{A}_{NP} \approx \frac{c}{\Lambda^2} \Rightarrow \Lambda \approx 10^5 \text{ GeV}$$

$$\approx M_X$$



Even stronger bounds in K-$\bar{K}$ mixing:

B_s : $V_{tb} V_{ts}^* \sim \lambda^2$

B_d : $V_{tb} V_{td}^* \sim \lambda^3$

K : $V_{ts} V_{td}^* \sim \lambda^5$

What kind of NP can induce such FCNCs?

Very simple example:

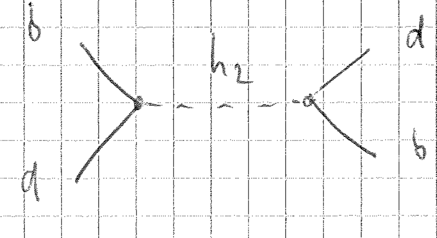
Additional scalar doublet h_2 (no vev)

$$\mathcal{L}_{Yuk} \to Y_1 \bar{d}_L h d_R + Y_2 \bar{d}_L h_2 d_R$$

$$\Big\downarrow \text{EWSB}$$

$$\bar{d}_L' \underbrace{V_d^\dagger Y_1 W_d}_{Y^{diag}} (v + h) d_R'$$

$$+ \bar{d}_L' \underbrace{V_d^\dagger Y_2 W_d}_{= Y_2' \text{ is not diagonal}} d_R'$$



$$\propto \frac{(Y_2'^*)_{13} (Y_2')_{31}}{m_{h_2}^2}$$

NP flavor problem!

Counter-example:

back to 3 quark model

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad s_L, \qquad u_R, \underbrace{d_R, s_R}$$

$$\sim (2, \tfrac{1}{6}) \qquad \sim (1, -\tfrac{1}{3}) \qquad (1, \tfrac{2}{3}) \qquad (1, -\tfrac{1}{3})$$

$$\mathcal{L}_A = g_u \left(\bar{u}_L \gamma_\mu A^\mu u_L + \bar{u}_R \gamma_\mu A^\mu u_R \right)$$

$$+ \left(\bar{d}_L \ \bar{s}_L \right) \gamma_\mu A^\mu \begin{pmatrix} e_d & \\ & e_d \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix} + \ldots$$

$$\mathcal{L}_Z = g^Z_{uL} \bar{u}_L \gamma_\mu Z^\mu u_L + g^Z_{uR} \bar{u}_R \gamma_\mu Z^\mu u_R$$

$$+ \left(\bar{d}_L \ \bar{s}_L \right) \gamma_\mu Z^\mu \begin{pmatrix} g_{dL} & \\ & g_{sL} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix}$$

$$+ \left(\bar{d}_R \ \bar{s}_R \right) \gamma_\mu Z^\mu \begin{pmatrix} g_{dR} & \\ & g_{sR} \end{pmatrix} \begin{pmatrix} d_R \\ s_R \end{pmatrix}$$

$g_{dR} = g_{sR} = g_{sL} \neq g_{dL}$

$$\mathcal{L}_{mass} = y_u \bar{u}_L h u_R$$

$$+ \left(y^d_1 \ y^d_2 \right) \bar{d}_L h \begin{pmatrix} d_R \\ s_R \end{pmatrix}$$

$$+ M \bar{s}_L s_R$$

$$\ni m_u \bar{u}_L u_R + \left(\bar{d}_L \ \bar{s}_L \right) \begin{pmatrix} v y^d_1 & v y^d_2 \\ v y^{d*}_2 & M \end{pmatrix} \begin{pmatrix} d_R \\ s_R \end{pmatrix}.$$

$$\uparrow \qquad \qquad \uparrow$$

$$V_d^{d\dagger} \qquad \qquad W_d$$

Leads to Z FCNCs at tree-level!