2. Constructing Effective Field Theories (EFTs) from String Theory

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2.1. Basic concepts and notions

2.1.1. What is String Theory about?

- See also talks by Oskar & Fabrizio!
  More details: e.g. [BBS, 2006]
  or (very introductory!): B. Zwiebach: A first course in String Th.
- The subsequent summary should give you a coarse overview of what we need to understand how the String Landscape arises.
- In QFT we quantise e.g. point particles.
  String Th. can be considered as a QFT of strings:

open string   closed string

- Consistency conditions (e.g. Lorentz invariance) fix the numbers of spacetime dimensions D:

  D = 26 for bosonic string

  D = 10 for superstring ← to be considered
- Thus, Superstring Theory is a theory of ten spacetime dimensions. ⇒ predicts six extradimensions
  ⇒ to avoid contradiction w/ observations, these extra-dimensions need to be made small.
  → "<u>String-Compactification</u>"
  → More on this later!
- Nice "by-product" of ST: gravity is automatically included!
  (Graviton ≙ closed string excitation)

$\boxed{P.8}$

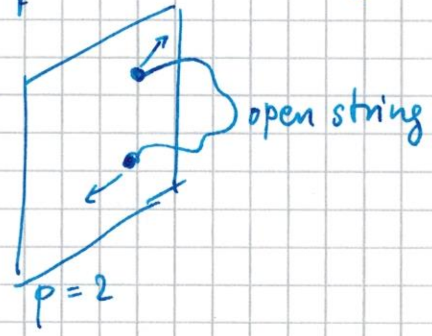
- ST not only contains strings but also higher dimensional objects (e.g. membranes, ...).

  $\to$ <u>Branes</u>

  Intuitive "understanding":

  Case 1: Ends of open string can move freely in the whole space.

  Case 2: Ends of open string can move only on a $\overset{(p+1)}{\cancel{p}}$-dim. submanifold $\mathcal{M} \subset \mathbb{R}^{1,9}$:



  $\to$ call this object a Dp-brane

  "Dirichlet"

  Notice: $(p+1)$ Neumann boundary cond. & $(D-p-1)$-Dirichlet b.c. give rise to a Dp-brane.

- Facts: - Strings carry charges under a 2-Form field B (Kalb-Ramond field): $B = \frac{1}{2} B_{\mu\nu}\, dx^\mu \wedge dx^\nu$
  Corresponding field strength tensor $H = dB$.
  - Similarly, Dp-branes carry charges under (p+1)-form fields $C_{\mu_1 \ldots \mu_p \mu_{p+1}}$.

- Don't worry: in the following it's enough to be aware of the fact that D-branes carry charges.

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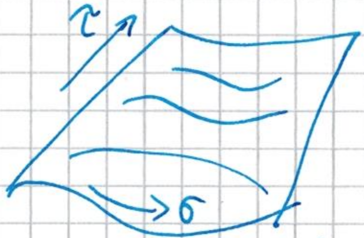
- Here, I briefly sketch how one can understand string- & D-brane charges:
  - Recall Electrodynamics of a point particle.
    τ parametrizes worldline in Minkowski space.
    $\Rightarrow$ Tangent vector of particle trajectory: $\frac{dx^\mu(\tau)}{d\tau}$

    Get a Lorentz-scalar by contraction w/ gauge field A_μ:

$$\underbrace{q}_{\text{charge}} \int A_\mu(x(\tau)) \frac{dx^\mu(\tau)}{d\tau} d\tau \subset \underbrace{S}_{\text{action of ~~pa~~ charged particle}}$$

  - Same logic for strings:

    In Minkowski space, Strings cut out a worldsheet!



Two tangent vectors: $\frac{\partial X^\mu}{\partial \tau}$ and $\frac{\partial X^\mu}{\partial \sigma}$.

Obtain a Lorentz-scalar as follows:

$$-\int d\tau d\sigma \frac{\partial X^\mu}{\partial \tau} \frac{\partial X^\nu}{\partial \sigma} B_{\mu\nu}(X(\tau,\sigma)) \subset S$$

$B_{\mu\nu}$: gauge-field w/ two indices

  - Same logic for Dp-branes: Position of Dp-brane described by $X^\mu(\tau, \sigma^1, \dots, \sigma^p)$.

    Hence:

$$-\int d\tau d\sigma^1 \dots d\sigma^p \frac{\partial X^\mu}{\partial \tau} \frac{\partial X^{\mu_1}}{\partial \sigma^1} \cdots \frac{\partial X^{\mu_p}}{\partial \sigma^p} C_{\mu\mu_1\dots\mu_p}(X(\tau,\vec{\sigma}))$$

$C_{\mu\mu_1\dots\mu_p}$: gauge field w/ p+1 indices

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• Notice: There are actually *five* versions of superstring theories!
They are related (indirectly) to each other by dualities & M-Theory.
(No time to explain this further.
See e.g. [Den, 2008], [BBS, 2006], [IU, 2012], ...)

↳ We will focus on the socalled $\boxed{\text{Type IIB String Th.}}$

Reason: Best understood so far.

Notice: If we are more precise, we will only explore the Type IIB String Landscape later (and the F-Theory landscape).
Other superstring theories give rise to other parts of the full string landscape.

2.1.2. From 10 to 4 Dimensions

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- Obviously, we need to make the six extradimensions sufficiently small to have a chance to reproduce the correct physics.
- Interestingly, the geometry and the topology of the manifold that describes the six extradimensions determine the physics that comes out of ST!
  ↳ Unfortunately, the details are fairly technical, so 'explanations' only in pictures.
- After compactifying the 6 dimensions we obtain a 4D EFT.

  It contains now further (massless) fields, which we call <u>moduli</u>. The business of giving them a mass is called <u>moduli stabilisation</u>. → See 2.2.

- Let's look at an example (see [BM, 2014]) how geometric parameters can turn into fields/moduli:

  - Line element of 10D theory:

$$G_{MN}\, dX^M dX^N = e^{-6u(x)} g_{\mu\nu} dx^\mu dx^\nu + e^{2u(x)} \hat{g}_{mn} dy^m dy^n \tag{2.1}$$

  - $\hat{g}_{mn}$ is the metric of the 6D manifold, $g_{\mu\nu}$ the metric of our spacetime.

    Define volume of 6D manifold X_6:

$$\mathcal{V} = \int_{X_6} d^6y \sqrt{\hat{g}} \,. \tag{2.2}$$

  - $e^{u(x)}$ describes variations in size of the extra dimensions.

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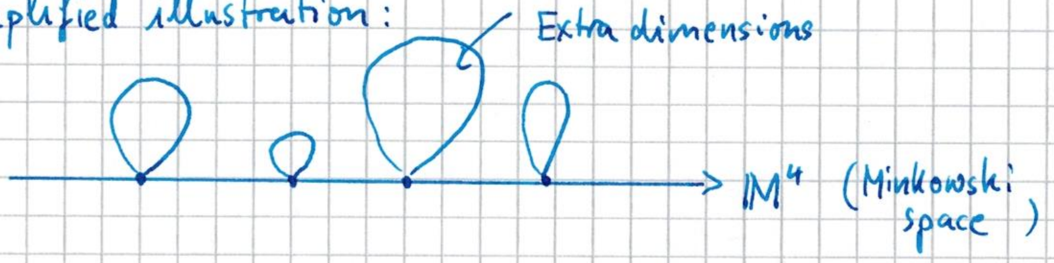
Simplified illustration:



- From 10D to 4D:
  * 10D Einstein-Hilbert action
  $$S_{EH}^{(10)} = \frac{1}{2\kappa^2}\int d^{10}X\sqrt{-G}\;R_{10} \qquad (2.3)$$
  * Computation with methods of GR class gives:
  $$S_{EH}^{(10)} = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}\int_{X_6} d^6y\,\sqrt{\hat{g}}\left(R_4 + e^{-8u}\hat{R}_6 + 12\,\partial_\mu u\,\partial^\mu u\right) \qquad (2.4)$$
  * By comparison with $S_{EH}^{(4)} = \frac{M_{PL}^2}{2}\int d^4x\,\sqrt{-g}\;R_4$
  we read off
  $$M_{PL}^2 = \frac{\mathcal{V}}{\kappa^2} \qquad (2.5)$$
- We can see now the following:
  * u enters 4D EFT as a scalar field.
  * In this case u is not stabilised:
  the potential for u, $V(u) \sim -e^{-8u}\hat{R}_6$
  has a run-away behaviour!
    If $\hat{R}_6 > 0$: $u \to -\infty \Rightarrow$ volume $\to 0$
    If $\hat{R}_6 < 0$: $u \to +\infty \Rightarrow$ volume $\to \infty$
  $\Rightarrow$ need a mechanism s.t. $V(u)$ allows for a minimum of u and thus stabilises the size of the extra dimensions
  $\rightarrow$ "Moduli stabilisation"

2.1.3. Towards moduli stabilisation: Some facts on geometry.

$\boxed{\text{P.13}}$

- Here we just want to state some facts about the geometric properties about the compactification of extra-dimensions.
  More e.g. in [Den, 2008], [BBS, 2006], ...

- We want to go from 10D to 4D via Type IIB String Th.
  When performing compactification one would like to have left some supersymmetry (e.g. for model building reasons).

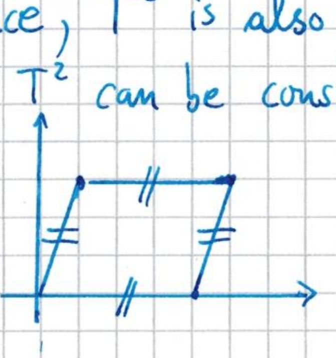
- This translates into the requirement that the 6D compactification manifold has to be a (compact) <u>Calabi-Yau manifold</u> (of complex dimension 3).

- <u>Definition</u>: A 2n-dimensional manifold w/ $SU(n)$ holonomy is called a Calabi-Yau n-fold (CY_n).

- Compactification of Type IIB on CY_3 leaves a 4D EFT w/ $\mathcal{N}=2$.

- ~~By including~~ Go to $\mathcal{N}=1$ susy on so called <u>Calabi-Yau orientifolds.</u>
  (→ p-form fluxes, D-branes, orientifold planes (→ to compensate the positive tenson of D-branes))

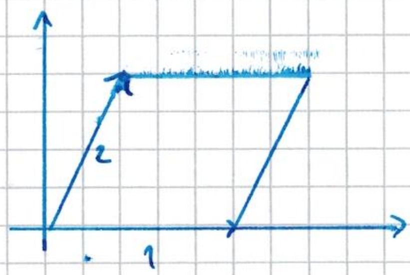
- Notice: $\mathbb{C}$ is CY_1.
  Hence, T^2 is also CY_1:
  T^2 can be constructed in the complex plane:



Identify opposite edges!

$\boxed{\text{P.14}}$

- From CY-compactifications we gain two particular moduli:
  - Kähler moduli
  - Complex structure moduli
- They correspond to deformations of the metric of the CY s.t. the deformed mfd is still CY.
- <u>Example:</u> Torus T^2

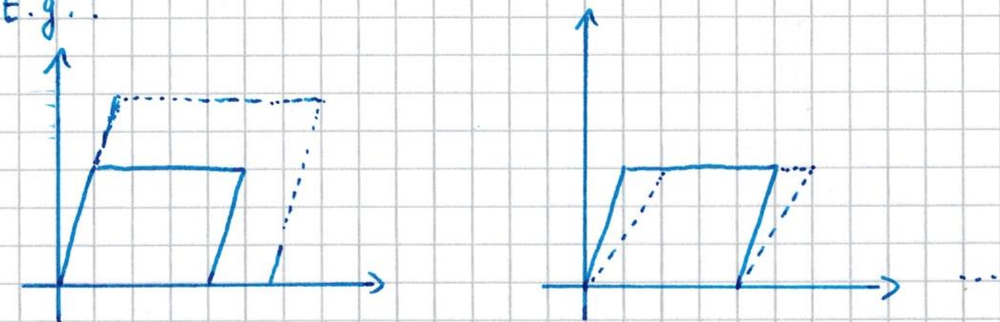


- Kähler modulus ρ $\hat{=}$ area of T^2
- Complex structure modulus τ $\hat{=}$ shape of T^2

For rectangular torus: $\rho = i R_1 R_2$
$\tau = i \frac{R_2}{R_1}$ radii of T^2

Deformations of Kähler-structure or complex-structure ~~leave~~ deform the torus, but the object remains a torus.

E.g.:



...

2.2 Moduli stabilisation: Constructing string vacua

P.15 · We can now understand many features of Type IIB compactifications in terms of field theory.

· ~~Ind~~ Indeed, there is a prescription of how to find 4D Lagrangians from type IIB string theory:

① Choose a geometry (i.e. a particular CY_3-orientifold)
→ this determines how and how many Kähler- and complex-structure moduli enter $\mathcal{L}$.

② To the field strength tensors H_3 & F_3 we can associate flux numbers (integers). Those can be used to obtain different Lagrangians.
(this is why there is a Landscape!)
Also, D-branes can be inserted.

③ Now, one can just calculate the Lagrangian ($\mathcal{N}=1$):
$$\mathcal{L} = - K_{i\bar{j}}\, \partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{j}} - V_F(\phi, \bar{\phi}) \qquad (2.6)$$

· How to read (2.6)?
- The Kähler metric $K_{i\bar{j}}$ is obtained from the Kähler potential $\mathcal{K}$ as follows:
$$K_{i\bar{j}} = \frac{\partial^2 \mathcal{K}}{\partial \phi^i \partial \bar{\phi}^{\bar{j}}} \qquad (2.7)$$
ϕ^i are the complex (super-)fields, e.g. Kähler moduli, ...
- Furthermore, a (holomorphic!) superpotential W enters $\mathcal{L}$ (via V_F). Since W is holomorphic, it can only depend on ϕ^i but not on $\bar{\phi}^{\bar{j}}$.

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- Notice that the Kähler-potential $\mathcal{K}$ is fully determined by the chosen geometry for the extra dimensions.
- W is also determined by the geometry, but also by the chosen flux numbers.
- V_F is the F-term scalar potential given by:

$$V_F = e^{\mathcal{K}} \left[K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2 \right], \qquad (2.8)$$

where the (Kähler-) covariant derivatives $D_i W$ are given by:

$$D_i W = \partial_i W + \mathcal{K}_i W \qquad (2.9)$$

- Thus, if we know how to calculate $\mathcal{K}$ and W from string theory, $\mathcal{L}$ can be computed.

• But some more words on SUSY and SUSY-breaking before (see also [KQS, 2.10]):
  - F-term potential in <u>global</u> SUSY:

$$V_F = K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} \qquad (2.10)$$

  SUSY broken $\Longleftrightarrow$ $DW \neq 0$

  $\Longleftrightarrow$ $\langle V_F \rangle > 0$

  - In local SUSY, i.e. supergravity:
    * SUSY preserved if $D_i W = 0$
    * ~~SUSY~~ doesn't imply any longer that $\langle V_F \rangle > 0$. (obvious from (2.8)).

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- Now, we want to state how $\mathcal{K}$ and W can be obtained from string theory
- Let's choose a CY-orientifold X_6.
  The field content is then precisely determined:
  - there are $h_+^{1,1}(X_6)$ Kähler moduli T_i
  - there are $h_-^{2,1}(X_6)$ complex structure moduli z_i
  - there is one axio-dilaton $S = C_0 + \frac{i}{g_s}$

  C_0: universal axion; g_s: string coupling

  [For those who are interested in mathematics:
  What are $h^{1,1}$ and $h^{2,1}$?
  Recall: On a complex manifold X you can define differential forms ω, e.g. $\omega_{1,1} = dz^1 \wedge d\bar{z}^1 + dz^2 \wedge d\bar{z}^3 \ldots$
  In particular, there are closed forms, i.e. forms ω s.t. $d\omega = 0$. There are also exact forms ω, which can then be written as $\omega = d\alpha$, where α is another differential form on X. Since $d^2=0$, any exact ω is (trivially) closed! (The converse isn't always true.)
  More precisely, one defines Dolbeault operators $\partial, \bar{\partial}$ s.t. $d\omega = \partial\omega + \bar{\partial}\omega$ and $\partial^2 = \bar{\partial}^2 = \partial\bar{\partial} + \bar{\partial}\partial = 0$.
  One can now construct the so called (r,s)-th Dolbeault Cohomology Group $H^{(r,s)}(X,\mathbb{C})$ from the equivalence relation $\omega_{r,s} \sim \omega_{r,s} + \bar{\partial}\alpha_{r,s-1}$:

$$H^{(r,s)}(X,\mathbb{C}) = \frac{\{\omega_{r,s} \mid \bar{\partial}\omega_{r,s} = 0\}}{\{\alpha_{r,s} \mid \alpha_{r,s} = \bar{\partial}\beta_{r,s-1}\}} \quad \text{(Vector space!)}$$

$$h^{r,s}(X) \equiv \dim H^{(r,s)}.$$

Note that the Kähler- and complex str. moduli are associated to cycles, i.e. objects w/o boundaries: a cycle $\Rightarrow$ $\partial a = 0$

Attention: NOT a Dolbeault op.

$\boxed{P.18}$

Then, there are cycles a that can be expressed as boundaries of other cycles b (one dimension higher): $a = \partial b$. Then $\partial a = 0$ follows trivially, because boundaries don't have boundaries (by definition).

By Stokes theorem, one can construct isomorphisms between cycles and differential forms. This justifies to count e.g. the number of Kähler moduli as $h^{1,1}$.

$\pm$ denotes whether even$^{(+)}$ or odd $(-)$ parts of the Dolbeault Cohomology are counted under geometrical orientifold evolution.

More on this: E.g. [Den, 2008], [BBS, 2006] Nakahara, ...

]

- The Kähler potential $\mathcal{K}$ can be computed from geometrical and topological data:
  - ~~Käh~~ Complex structure moduli:

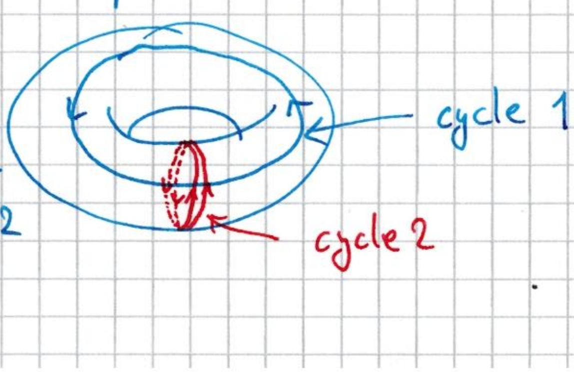
$$\mathcal{K}_{cs} = -\ln\left(\Pi_i \, \eta^{i\bar{j}} \, \overline{\Pi}_{\bar{j}} \right) \qquad (2.11)$$

symplectic form)

contains the z_i and their <u>intersection numbers</u>

in pictures:

cycle 2 intersects cycle 1 twice
$\Rightarrow$ intersection number ~~K~~ $\mathcal{K}_{12} = +2$



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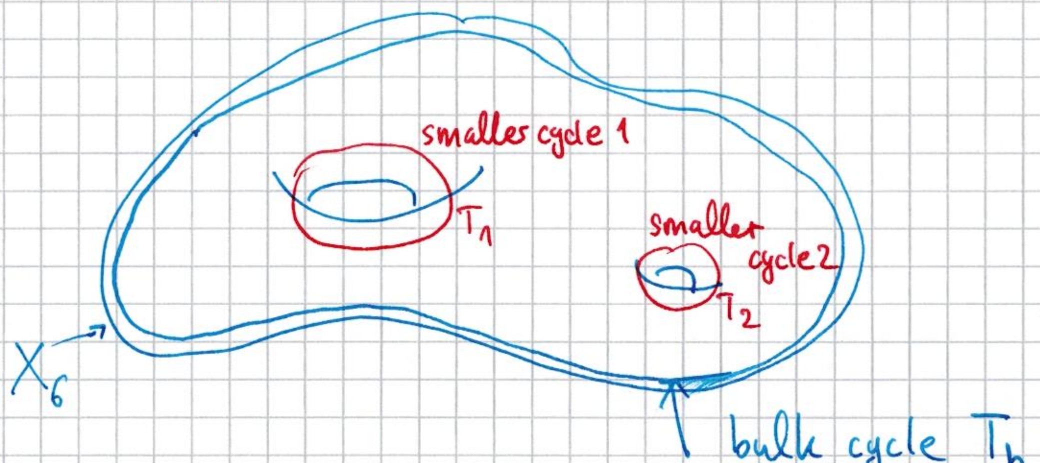
- Kähler moduli sector:

Suppose:



Then: $\mathcal{K}_K = -2\ln\left((T_b+\bar{T}_b)^{3/2} - (T_1+\bar{T}_1)^{3/2} - (T_2+\bar{T}_2)^{3/2}\right)$ (2.12)

Volume of X_6: $\mathcal{V} \simeq (T_b+\bar{T}_b)^{3/2}$

- Axio-dilaton S: ~~$\mathcal{K}_D = -\ln(S\bar{S})$~~

$\mathcal{K}_D = -\ln\left(i(S-\bar{S})\right)$ (2.13)

$\Rightarrow$ Kähler potential:

$\mathcal{K} = \mathcal{K}_D + \mathcal{K}_K + \mathcal{K}_{CS}$ (2.14)

- Superpotential W:
  - There is a part W_0 coming from the fluxes F_3 and H_3:

$W_0 = (N_F - S N_H)_\alpha \, \Pi^\alpha(z)$ (2.15)

$(2h^{2,1}_- + 2)$-dimensional flux vectors consisting of ~~flux~~ integers = (flux numbers)

- There are instantonic corrections W_{inst} :

P.20

e.g. $W_{inst} = A(z)\, e^{-2\pi a T}$ (2.16)

Notice: the bigger the cycle T, i.e. the bigger $\mathrm{Re}(T)$, the less important the corresp. instanton corrections.

$\Rightarrow$ Superpotential

$$W = W_0 + W_{inst} \qquad (2.17)$$

Mathematical comment on <u>Flux Quantisation</u>:

<u>Proposition</u>: d-dimensional theory, $F_q = dC_{q-1} \Rightarrow dF_q = 0$. $(q-2)$-branes charged under C_{q-1} w/ minimal charge Q. Then: $Q \int_{\Sigma_q} F_q = 2\pi \mathbb{Z}$.

$\Sigma_q \leftarrow$ non-trivial q-cycle

<u>Proof</u>: Write $\Sigma_q = \Sigma_+ - \Sigma_-$, where trivial $(q-1)$-cycle Π_{q-1} separates Σ_+ and Σ_-. E.g.

Then:

$$\exp\left(iQ \int_{\Pi_{q-1}} C_{q-1}\right) = \exp\left(iQ \int_{\partial\Sigma_\pm} C_{q-1}\right) =$$
$$= \exp\left(iQ \int_{\Sigma_\pm} F_q\right) \quad \text{by Stokes thm.}$$

However, $Q\left(\int_{\Sigma_+} F_q - \int_{\Sigma_-} F_q\right) = Q \int_{\Sigma_q} F_q$.

Thus:

$$\exp\left(iQ \int_{\Sigma_-} F_q\right) = \exp\left(iQ \int_{\Sigma_-} F_q + iQ \int_{\Sigma_q} F_q\right).$$

Ambiguity disappears iff $Q \int_{\Sigma_q} F_q = 2\pi \mathbb{Z}$. $\square$

<u>Application</u>: We have fluxes F_3, H_3 with $dF_3 = dH_3 = 0$ and X_6 contains non-trivial 3-cycles. Thus, the proposition applies and we define <u>flux numbers</u> by:

$$\int_{\Sigma^i} F_3 = 2\pi N_F^i \quad , \quad \int_{\Sigma^i} H_3 = 2\pi N_H^i$$

for all 3-cycles Σ^i.

2.2.1. Stabilisation of the axio-dilaton and the complex structure moduli

P.21

- In order to stabilise S and z_i we have to find a minimum for ~~this~~ these fields of the F-term scalar potential given in (2.8).
- For this purpose let's start with the following model:

$$\mathcal{K} = -\ln(-i(S-\bar{S})) - 3\ln(T+\bar{T}) - \mathcal{K}_{cs}(z,\bar{z}) \qquad (2.18)$$

and

$$W = W_0 = (N_F - S N_H)\cdot \Pi(z) \qquad (2.19)$$

(no instanton corrections for the moment)

- Exercise: Show that

$$K^{T\bar{T}} D_T W D_{\bar{T}} \bar{W} = 3|W|^2. \qquad (2.20)$$

This is the no-scale structure.

- From (2.20) it follows that (2.8) reduces to

$$V_F = e^{\mathcal{K}} K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} \qquad (2.21)$$

where $\alpha, \beta \in \{S, z_i\}$.

- Hence, T doesn't show up in V_F and thus is not stabilised!
- But it's obvious how to find the minimum for S and z_i:

$$D_z W = 0$$
$$D_S W = 0 \qquad (2.22)$$

~~This gives~~ For nonzero flux vectors these equations fix the z_i and S!

- This gives a Minkowski vacuum (because $V_F = 0$). It is non-supersymmetric because $D_T W \neq 0$.
- Details in [GKP, 2001]!

2.2.2. Stabilising Kähler moduli

P.22

- In [KKLT, 2003] it was suggested to break the no-scale structure of V_F by simply including instanton corrections (see (2.16)).
  $\Rightarrow$ V_F depends now on T
  $\Rightarrow$ $D_T W = 0$ stabilises T !
- The other moduli S and z_i are stabilised by $D_{z_i} W = 0$ and $D_S W = 0$ as explained previously.
- The obtained minimum is now, however, supersymmetric. (See explanations around eq. (2.10).)
  Moreover, $V_F < 0$ at the minimum! $\rightarrow$ SUSY AdS vacua!
  Clearly, at this stage, this is not a realistic string vacuum!
- Solution: Add anti-D3-branes ($\overline{D3}$) to the system.
  They have negative tension and therefore act repulsively.
  $\Rightarrow$ $V = V_F + V_{\overline{D3}} > 0$ possible.

  [Notice: This $\overline{D3}$-brane uplift is under discussion in the community.
  Problems: $D3/\overline{D3}$ annihilation, ...]
- This KKLT-sceanrio requires some tunings:
  - $|W_0| \ll 1$ (e.g. $|W_0| \sim 10^{-4}$)
  - One has to balance $V = V_F + V_{\overline{D3}}$ s.t. $V \sim 10^{-120} M_{Pl}^4$ for getting the right cosmological constant.
- There are other famous mechanisms for Kähler moduli stab., e.g. the Large Volume Scenario (LVS).
  ~~Prob~~ Problem: Dark Radiation !