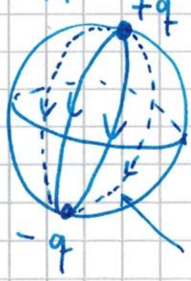
3. Counting the string vacua

In 2.2. we learned about the basic tools how to <u>construct</u> string vacua.
We would like to see now <u>how many</u> these are.

3.1. Constraints from tadpoles

- In 2.2.1, we fixed S and z_i by turning on fluxes (F_3, H_3), i.e. by choosing nonzero flux vectors N_F, N_H.
- Since every choice of flux vectors gives rise to other vacua, one might think that there should be infinitely many vacua, because there are infinitely many integers and thus infinitely many flux choices.
- But this is not true!
  The so called <u>tadpole-cancellation condition</u> for D3-branes imposes a constraint on the flux vectors s.t. only a finite ~~choice of fluxes~~ number of flux choices is possible.

- Simple example of a tadpole-cancellation condition:
  Electrostatics on a sphere:
  You can't put just one charged particle on a sphere, because the electric field lines intersect.
  Solution: Put another charge with opposite charge at the opposite point:



General:
$$\sum_{i=1}^{N} q_i = 0$$

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- The requirement of charge cancellation in electrodynamics on S^2 is a consequence of the fact that S^2 is <u>compact</u>.
- Our manifold X_6 is also compact and D3-branes carry charges (under C_4), see Sect. 2.1.1.

  $\Rightarrow$ there must be a cancellation of the D3-brane charges!

  $\rightarrow$ "tadpole-cancellation constraint"
- Indeed, from the equations of motion of the 10D Type IIB supergravity action one can show that

$$\int_{X_6} H_3 \wedge F_3 = Q_3^{loc} \qquad (3.1)$$

  (up to normalisation factors).
- Q_3^{loc} counts the charges of ~~D~~ D3-branes, O3-planes (localised objects)
- Interestingly, Q_3^{loc} is related to topological quantities that arise from X_6.

  [<u>Advanced Comment</u>:

  One can calculate Q_3^{loc} from M-Theory on a <u>4-fold</u> Y_8 and one finds $Q_3^{loc} = \frac{\chi(Y_8)}{24}$.

  $\chi(Y_8)$ is the Euler characteristic of Y_8. Recall that χ is directly related to the number of holes (genus g) by $\chi = 2g-2$. (E.g. sphere: $g=0 \Rightarrow \chi=-2$, torus $g=1 \Rightarrow \chi = 0, \dots$)

  In F-theory one is interested in elliptic fibrations Y_8 over a base space X_6. Knowing some topological data of X_6 (e.g. Chern classes, ...) and specifying the fibration, one can then compute $\chi(Y_8)$ and hence Q_3^{loc}.]

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- The LHS of (3.1) gives rise to a sum of products of the flux numbers.
  So we can roughly write

$$n_I A^{IJ} n_J \sim Q_3^{loc} \qquad (3.2)$$

where we denote by n_I the flux numbers that arise from the flux vectors N_F, N_H, introduced in (2.15). A^{IJ} is a matrix defined by evaluating the LHS of (3.1). A is <u>positive definite</u> for susy vacua! The indices I, J run from $1, \dots, 4(h^{2,1}(X_6)+1)$.

3.2. Counting the vacua

- We are now ready to use (3.2) to count the numbes of flux vacua.
- We start very <u>naively</u> as follows:
  - Typical CY_3 geometries X_6 have
    $Q_3 \sim 1000$
    and $h^{2,1}(X_6) \sim 100$
  - (ab)using (3.2) we can estimate the "maximal" size that most flux numbers have for most of the vacua:

$$400 \cdot n_{max}^2 \sim 1000$$

$$\Rightarrow \quad |n_{max}| \leq 2 \qquad \Rightarrow \quad n \in \{-2, -1, 0, +1, +2\}$$

mostly

  - Then, the number of solutions to (3.2) is at least
    ~~roughly~~ $5^{400} \sim 10^{280}$
  - At most, $|n_{max}| \leq \sqrt{1000} \sim 30$, so there are no more ~~that~~ than $60^{400} \sim 10^{700}$ vacua

- So there are between 10^{280} and 10^{700} type IIB flux vacua!

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- Of course, one can count the number of vacua more rigorously as follows:
  - Notice that (3.2) should actually be an inequality, i.e.
$$n_I A^{IJ} n_J \leq Q^{loc}_{3,max}, \qquad (3.3)$$
    because we can add $N_{D3} \geq 0$ D3-branes to the system and thus
$$Q_3^{loc} = Q^{loc}_{3,max} - N_{D3}$$
    as every D3-brane contributes -1 to the charge Q.
  - Then, notice that (3.3) is not completely different from
$$x_1^2 + x_2^2 + \ldots + x_d^2 \leq R^2 \qquad (3.4)$$
    which describes a d-dimensional sphere of radius R.
  - Then, the number of flux vacua should be proportional to the volume of a d-dimensional sphere of radius $\sqrt{Q^{loc}_{3,max}}$ with $d = 4(h^{2,1}(X_6) + 1)$.
  - Even more rigorous computations confirm this expectation! See [DD, 2004] and [Den, 2008].
  - The result is
$$N_{vac} \simeq \frac{(2\pi Q_{3,max})^{d/2}}{(d/2)! \sqrt{\det A}} \qquad (3.5)$$
    (up to a factor proportional to the integral over the density of flux vacua in the moduli space of the complex structure moduli).

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- When we plug in typical numbers for $Q_{3,max}$ and d, we obtain roughly

$$10^{500}$$

flux vacua!

[Numbers used: $Q_{3,max} = 972$, $h^{2,1} = 149$, i.e. $d = 600$.]

- Notice that this estimate ignores the degrees of freedom due to D7-brane motions.
  One can ~~st~~ take them into account in F-theory, where d will then become much larger.
  Then, one estimates the number of F-theory flux vacua to

$$10^{1700}$$

  based on (3.5).
  [However, (3.5) might no longer be a good estimate for the number of flux vacua; notice that the volume of a d-dim. sphere goes to zero as $d \to \infty$.
  In [Den, 2008] it was argued that there can be even 10^{3400} F-theory flux vacua.]

- Also notice that every compactification geometry has another number of flux vacua.

- This degeneracy of ~~of~~ flux vacua is what we call the
  String Theory Landscape.

3.3. How to deal with the Landscape?

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- The idea of reproducing the parameters of the Standard Model in a unique way is gone!
  However, General Relativity, for instance, also allows us to create different solar systems. In this case, the degeneracy is much worse than in String Theory, because there are continuous parameters (masses, ...) and thus there is a uncountable set of solutions to the problem in GR.
- The idea of the <u>multiverse</u> can put the <u>problem of fine-tuning</u> into a new context:
  - If there is a mechanism that populates all the possible string vacua (each of it is another "universe"), then one should automatically find a "universe" with parameters we observe!
  - In this context one might "understand", why our cosmological constant is observed to be extremely small. See Sect. 1.2.!
  - Indeed, there is such a mechanism (independently of String Theory!): <u>Eternal inflation</u>
    (Idea in brief: Inflation stops only locally
    $\Rightarrow$ creation of small "pocket" universes; like holes in swiss-cheese ...)
  - One could in principle find observational hints for the multiverse idea (bubble collisions, ...)
  - But: Difficult to make reasonable statistical statements on parameters and to make meaningful predictions: <u>Measure Problem</u>

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- Why can't we just use EFTs from the bottom-up approach instead of following the top-down approach (string compactifications)?
  - Answer: Any "consistent-looking" EFT (bottom-up) is most likely to be actually inconsistent with Quantum Gravity.
    I.e. if String Th. is a correct description of our universe, the EFTs from string compactifications are very special.

Swampland : → "Swampland" of inconsistent theories vs. Landscape.

See [Vaf, 2005].

  - Example: Any EFT in which gravity is not the weakest force, should sit in the swampland. ⇝ "Weak Gravity Conjecture" ⇝ see [AMNV, 2006]

⇒ Research area: Exploration of the boundaries of the Landscape!

- Landscape allows to constrain stringy models: If a model is fine-tuned to precision ε, then we expect only εN_{vac} appropriate vacua. If we have N tunings to precision ε and if $10^{500} \varepsilon^N < 1$, we conclude that this scenario isn't realised in our (string) vacuum.