Lecture 2: "Gauge theories from String theory"

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05.05.14

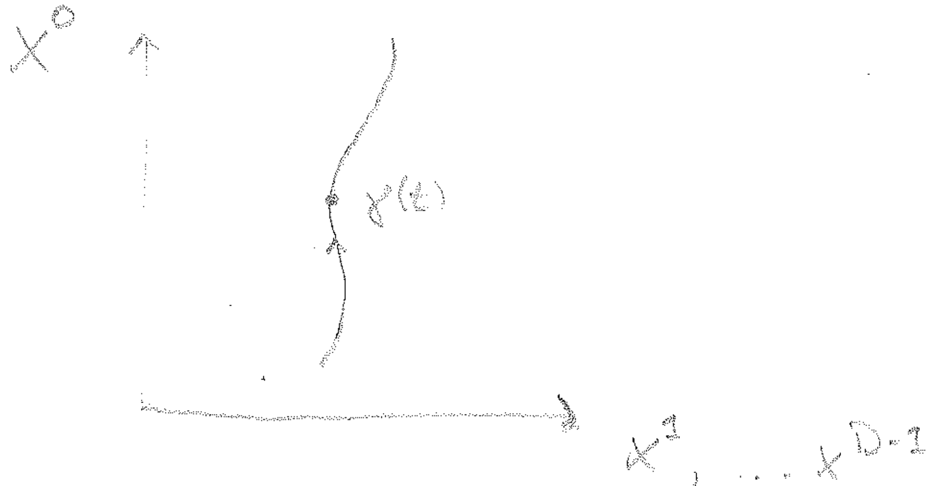
Outline:
- The classical string
- Quantization and spectrum
- Gauge theories and branes

Note: This is one approach to gauge theories from string theory, relevant for the type II theories.

Gauge theories from type I, heterotic or M-theory are realized quite differently.

The Relativistic Point Particle

- A point particle traces out a world-line in space-time



- Minkowski space-time: $ds^2 = -(dX^0)^2 + (dX^1)^2 + \dots + (dX^{D-1})^2$

  $ds^2 = -\eta_{\mu\nu} dX^\mu dX^\nu$ for $\eta_{\mu\nu} = \text{diag}(-1, 1, \dots, 1)$

- Path of particle parametrized by τ.

  $\gamma(t) : X^\mu(\tau)$

- Line element: $ds = \sqrt{-\eta_{\mu\nu} dX^\mu(\tau) dX^\nu(\tau)}$

  $= \sqrt{-\eta_{\mu\nu} \partial_\tau X^\mu(\tau) \partial_\tau X^\nu(\tau)}\, d\tau$

- Action: $S = -m \int ds = -m \int \sqrt{\eta_{\mu\nu} \partial_\tau X^\mu \partial_\tau X^\nu}\, d\tau$

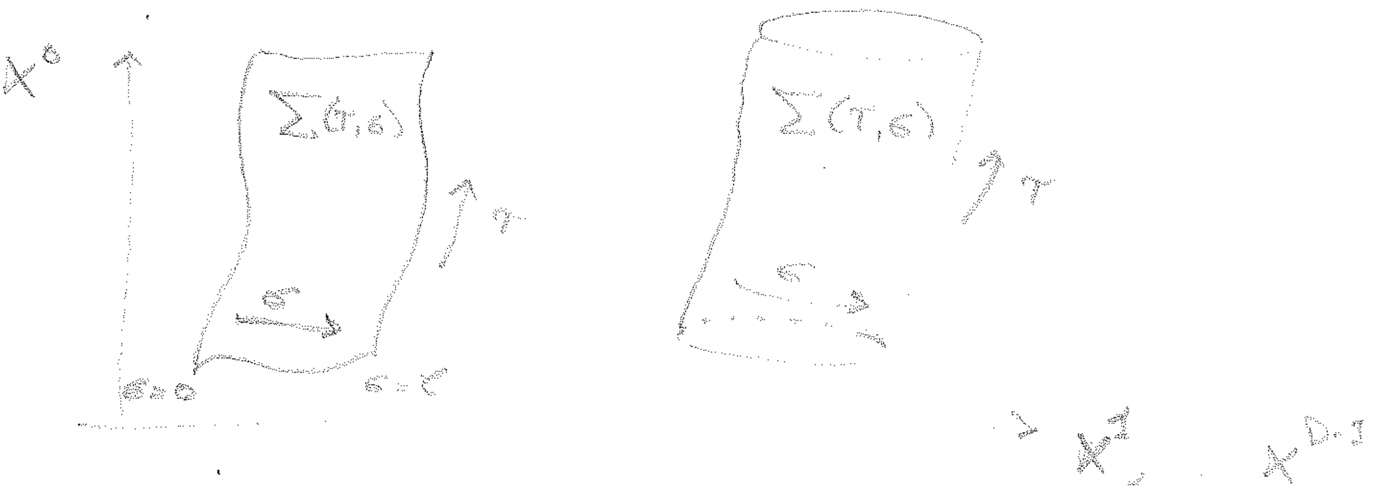
- Eq. of motion (minimizing S): $\partial_\tau^2 X^\mu(\tau) = 0$ (a straight line param. by τ)

- Momenta: $p^\mu = \dfrac{\delta L}{\delta(\partial_\tau X^\mu)}$

  Satisfies $p^\mu p_\mu + m^2 = 0 \Leftrightarrow E^2 = |\vec{p}|^2 + m^2$

The Classical Relativistic String

- Consider instead a string, moving in space-time

- 

- By analogy to the point-particle, take action proportional to world-sheet area

- $$S = -T\int_\Sigma dA$$
- Can be written as

$$S = -\frac{T}{2}\int d\tau d\sigma \sqrt{-\det h}\, h^{\alpha\beta}\, \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

for $X^\mu(\tau,\sigma)$ embedding string into space-time and $h_{\alpha\beta}$ metric on world-sheet.

- T is the string tension, $[T] = \frac{\text{mass}}{\text{length}}$
$\Leftrightarrow [T] = (M_{\text{string}})^2$

- Equations of motion: $\Box X^\mu(\tau,\sigma) = \left(\frac{\partial^2}{\partial\tau^2} - \frac{\partial^2}{\partial\sigma^2}\right) X^\mu(\tau,\sigma) = 0$

and $\frac{\delta S}{\delta h_{\alpha\beta}} = 0 \Leftrightarrow T_{\alpha\beta} = 0$

stress-energy tensor vanishes

Classical Solutions

- Boundary conditions gives two sectors:

  Closed strings: $X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \ell)$
  periodic b.c

$\boxed{\longrightarrow \text{Associated with gravity.}}$

  Open strings: Have to set b.c at end points.

  Neumann b.c: $\partial_\sigma X^\mu \Big|_{\substack{\sigma=0 \\ \sigma=\ell}} = 0$

  Dirichlet b.c

$$\left.\begin{aligned} X^\mu\Big|_{\sigma=0} &= X^\mu_0 \\ X^\mu\Big|_{\sigma=\ell} &= X^\mu_\ell \end{aligned}\right\} \text{const. w.r.t } \tau, \sigma$$

$\boxed{\longrightarrow \text{Associated with gauge fields}}$

- We only consider open strings from now on.

Open string with Neumann b.c

- General solution:

$$X^\mu(\tau,\sigma) = x^\mu + \ell_s^2 p^\mu \tau + i\ell_s \sum_{m\neq 0} \frac{1}{m} \alpha_m^\mu e^{-im\tau} \cos(m\sigma)$$

for $\frac{1}{2}\ell_s^2 = \alpha' = \frac{1}{2\pi T}$

- x^μ is the center-of-mass position of the string
- p^μ is the c.o.m momentum.
- α_m^μ are the oscillator modes of the string

Quantization and String States.

- Mechanics of the string:
  - Conjugate momenta: $P^\mu(\tau,\sigma) = \frac{\delta S}{\delta(\partial_\tau X^\mu(\tau,\sigma))}$
  - Poisson brackets: $\{P^\mu(\tau,\sigma), P^\nu(\tau,\sigma')\} = 0$

$$\{X^\mu(\tau,\sigma), X^\nu(\tau,\sigma')\} = 0$$

$$\{P^\mu(\tau,\sigma), X^\nu(\tau,\sigma')\} = \eta^{\mu\nu}\delta(\sigma-\sigma')$$

- Quantization: $\{\ ,\ \} \to -i[\ ,\ ]$, promoting fields to operators.

$\Rightarrow$ $[x^\mu, p_\nu] = i\delta^\mu_\nu$

$[\alpha_m^\mu, \alpha_n^\nu] = m\eta^{\mu\nu}\delta_{m+n,0}$ $\quad (\delta_{i,j} = 1 \text{ iff } i=j)$

- Harmonic oscillators:

By rescaling $a^\mu_m = \frac{1}{\sqrt{m}} \alpha^\mu_m$

$(a^\mu_m)^\dagger = \frac{1}{\sqrt{m}} \alpha^\mu_{-m}$, $m > 0$

$\Rightarrow [a^\mu_m, (a^\nu_n)^\dagger] = \eta^{\mu\nu} \delta_{m,n}$

- Ground state: $|0\rangle$ defined by

$a^\mu_m |0\rangle = 0 \quad \forall \, m > 0$

- A general state, with momentum k^μ

$|\phi\rangle = (a^{\mu_1}_{m_1})^\dagger \ldots (a^{\mu_k}_{m_k})^\dagger |0; k\rangle$

with $p^\mu |\phi\rangle = k^\mu |\phi\rangle$

- Note: any state constructed by the $a^\dagger$'s carries space-time indices μ, ν ...

$\longrightarrow$ Oscillations of the string appears as particles in space-time!

- Constraint: The stress-energy tensor $T_{\alpha\beta} = \frac{\delta S}{\delta h_{\alpha\beta}}$ must vanish

- $T_{\alpha\beta}$ is of the form $\sim \partial_\alpha X^\mu \partial_\beta X_\mu$

...

- For open strings: $T_{10} = T_{01} = 0$ automatically and by inserting the open-string solution

$$T_{\alpha\alpha} = 2\ell_s^2 \sum L_m e^{-2im(\tau \pm \sigma)}$$

$\pm$ for $\alpha = 0,1$

with $L_m = \frac{1}{2} \sum_{n=-\infty}^{\infty} \alpha^\mu_{m-n} \alpha^\nu_n \eta_{\mu\nu}$ classically

and $L_{-m} = L_m^\dagger$, and $\alpha_0^\mu = \ell_s p^\mu$

- Classically $T_{\alpha\beta} = 0 \Rightarrow L_m = 0 \ \forall m$

- In the quantum theory we instead have $\langle \phi | L_m | \phi \rangle = 0 \ \forall m$

$\Rightarrow L_m |\phi\rangle = 0 \ \forall m > 0$, equivalent to

$\langle \phi | L_m^\dagger = \langle \phi | L_{-m} = 0 \ \forall m > 0$

- The operators $L_m = \frac{1}{2} \sum \alpha^\mu_{m-n} \alpha^\nu_n \eta_{\mu\nu}$ are well defined for all $m \neq 0$, since $[\alpha_{m-n}, \alpha_n] = 0$ for $m \neq 0$

- For L_0 however, there is an ambiguity due to normal ordering of the α's:

$$L_0 \equiv \frac{1}{2} \sum :\alpha^\mu_{-n} \alpha^\nu_n: \eta_{\mu\nu}$$

- Since there might be contributions from the commutation - relations we insist that $(L_0 - a)|\phi\rangle = 0$ for some constant a.

- This is analogous to the 'zero - energy' in QFT :

$$H = \int \frac{d^3p}{(2\pi)^3} E_p a_p^\dagger a_p + \underbrace{\Delta \delta(0)}_{\text{from } [a_p, a_p^\dagger]}$$

- In string theory, absence of negative norm states $\Rightarrow a = 1$, and gives $D = 26$ in this case (bosonic string theory). With fermions on the world-sheet, $D = 10$ (superstring theory)

Mass Formula

- $$L_0 - a = \frac{1}{2}\sum_{m=-\infty}^{\infty} : \alpha_{-m}^\mu \alpha_m^\nu : \eta_{\mu\nu} - 1$$

$$= \frac{1}{2}\alpha_0^\mu \alpha_0^\nu \eta_{\mu\nu} + \sum_{m>0} : \alpha_{-m}^\mu \alpha_m^\nu : \eta_{\mu\nu} - 1$$

and using $\alpha_0^\mu = l_s p^\mu$

$$\Rightarrow L_0 - 1 = \frac{1}{2} l_s^2 p^\mu p_\mu + \sum_{m>0} : \alpha_{-m}^\mu \alpha_m^\nu : \eta_{\mu\nu} - 1$$

- On a physical state $p^\mu p_\mu = -M^2$ and $(L_0 - 1)|\phi\rangle = 0 \Rightarrow$

$$\underbrace{\tfrac{1}{2}\ell_s^2}_{=\alpha'} M^2 = \sum_{m>0} :\alpha_m^\mu \alpha_m^\nu: \eta_{\mu\nu} - 1$$

$$\Rightarrow \quad M^2 = \frac{1}{\alpha'}\Big(\underbrace{\sum_{m>0} :(\alpha_m^\mu)^\dagger \alpha_m^\nu: \eta_{\mu\nu}}_{=\sum_{m>0} m :a_m^{i\dagger} a_m^i: \, = N} - 1\Big)$$

the numbers operator, whose eigenvalues count the numbers of excited modes

- $$\boxed{M^2 = \frac{1}{\alpha'}(N-1)}$$

- <u>Spectrum</u>:
  - $N=0$, the ground state $|0;k\rangle$
    $M^2 = -\frac{1}{\alpha'}$, a tachyon
  - $N=1$, $(\alpha_1^\mu)^\dagger|0;k\rangle$
    $M^2 = 0$, space-time vector
  - $N=2$, $\begin{cases}(\alpha_2^\mu)^\dagger|0;k\rangle \\ (\alpha_1^\mu)^\dagger(\alpha_1^\nu)^\dagger|0;k\rangle\end{cases}$
    $M^2 = \frac{1}{\alpha'} \sim (M_{string})^2$
    very heavy.

- <u>Conclusion</u>: Open string spectrum $\supset$ massless vector!

Gauge symmetries and Branes

- **Thm.** A massless vector boson is a $U(1)$ gauge field.
  (see Weinberg - 'The Quantum Theory of Fields')

- Lorentz symmetry $\Longrightarrow$

$$\begin{cases} A_\mu(x) \to A_\mu + \partial_\mu \Lambda(x) \\ \psi(x) \to e^{i\Lambda(x)}\psi \end{cases}$$

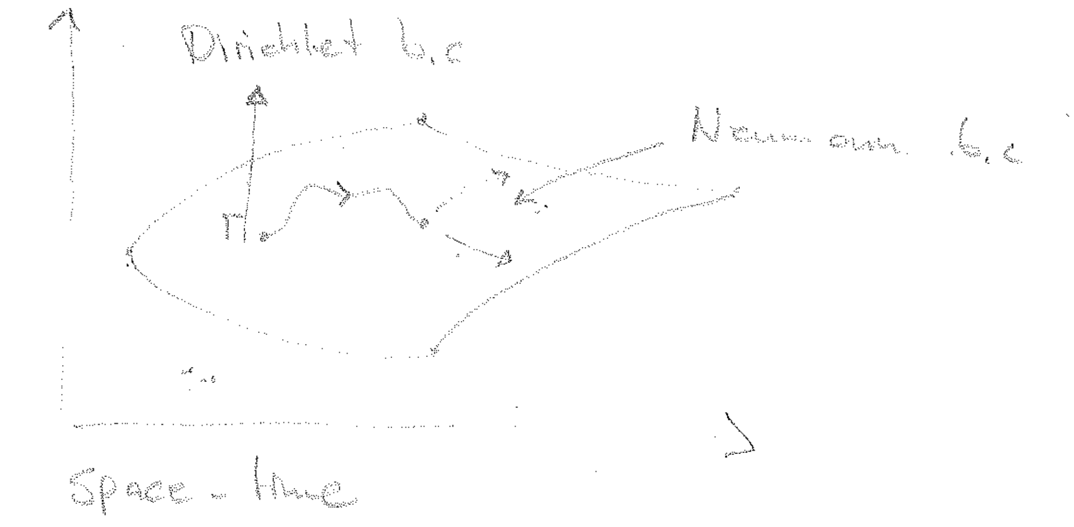
leaves theory invariant for any matter field that is charged under A_μ

- **Hence**: Open string $\to$ massless vector
  $\to U(1)$ gauge theory.

- **D-branes:**

Change b.c for open strings

$$X^\mu(\tau,\sigma) \longrightarrow \begin{cases} X^a(\tau,\sigma) & \text{Neumann b.c } a = 0,\dots,p \\ X^I(\tau,\sigma) & \text{Dirichlet b.c } I = p+1,\dots,D \end{cases}$$

- 

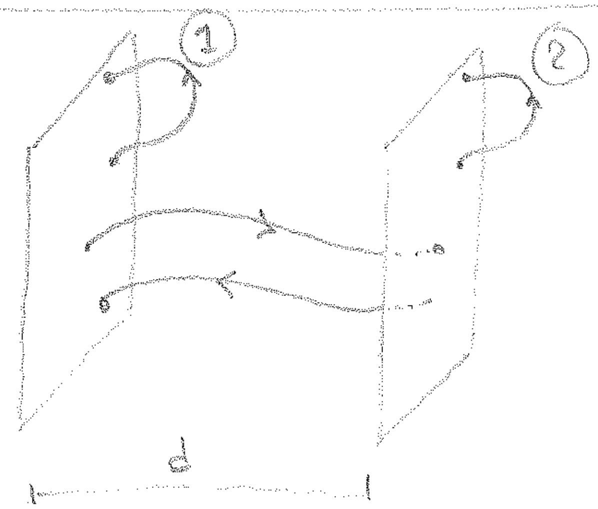
- Confines the string to a $p+1$ - dimensional submanifold of space-time "the world-volume" of a D_p - brane

- Mass formula now gives $(\alpha_1^a)^\dagger |0,k\rangle$ as a massless vector, free to move along the brane

$$\updownarrow$$

U(1) gauge theory on the brane

- More branes $\Leftrightarrow$ More symmetry



1 : U(1) gauge boson
2 : — " —
} $U(1)^2$ theory

3, 4 } Massive vectors (no U(1) symmetry) mass ~ d

- Limit $d \to 0$ "branes on top of each other"
3 and 4 becomes massless vectors

- 1 brane $\longleftrightarrow$ 1 massless state

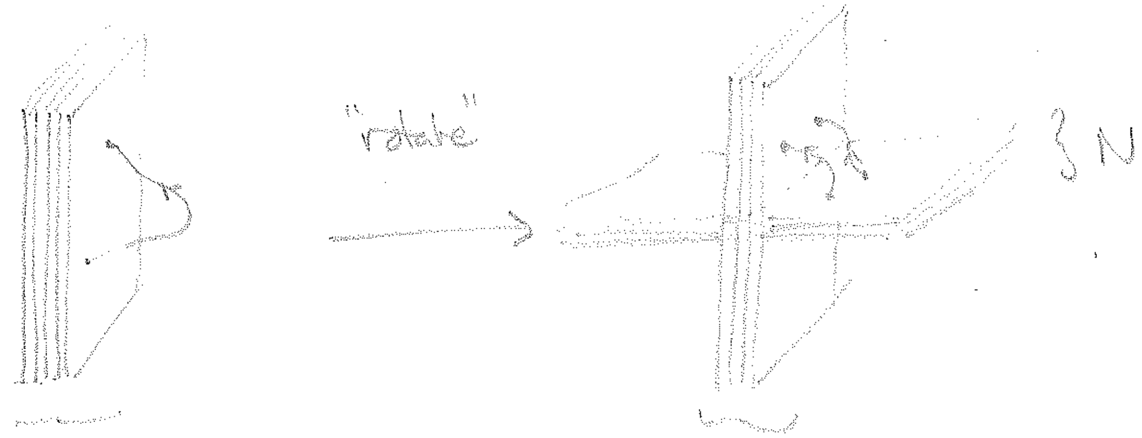
  2 branes $\longleftrightarrow$ 4 massless states

  $\vdots$

  N branes $\longleftrightarrow$ N^2 massless states

- $N^2 = \dim U(N)$, $U(N) = SU(N) \times U(1)$
- Point: Coincident branes $\longleftrightarrow$ non-abelian gauge symmetry !

- Consider M+N branes in a stack



U(M+N) - theory $\longrightarrow$ $U(M) \times U(N)$ - theory

adjoint rep $\qquad$ adj. reps

$$(M+N)^2 \longrightarrow \underbrace{(M^2,1) + (1,N^2)}_{\text{adj. reps}} + (M,N) + (\bar{M},\bar{N})$$

$U(M) \times U(N)$ theory, with bi-fundamental matter

- Final remark:

- String theory is very unified $\Leftrightarrow$ a priori gauge- and gravitational interactions are equally strong.
- What makes gravity so weak?

- Graviton $\Leftrightarrow$ closed string, not confined to branes

  Gauge fields $\Leftrightarrow$ open strings confined to submanifolds

$$\left(M^{4d}_{Planck}\right)^2 = \frac{1}{G_N} \sim \mathrm{Vol}\,(\text{comp. space})$$

$$\frac{1}{g_{GUT}} \sim \mathrm{Vol}\,(\text{brane})$$

- Localization of gauge d.o.f $\Rightarrow$ gauge coupling constant $\gg$ gravitational coupling