Measuring $|V_{ub}|$ at LHCb

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RTG Students Lecture

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Outline

1. Lecture
   - CKM Mechanism
   - How to measure CKM matrix elements in general?
2. Lecture Today: How to measure CKM matrix elements in B-decays?
   - Differences between B-factories and Hadron colliders
   - $|V_{cb}|$
   - $|V_{ub}|$
3. Lecture: Specific LHCb measurements
   - $\Lambda_b \to p\mu\nu$
   - $B_s^0 \to K^- \mu^+ \nu_\mu$

Differences between B-factories and Hadron colliders

B-factories

- **b-production:** $B\bar{B}$ events from $\Upsilon(4S)$ decays without additional background tracks $\Upsilon(4S) \rightarrow B^0\bar{B}^0$ 50% and $\Upsilon(4S) \rightarrow B^+B^-$ 50%, cross section is 1.1nb
- known kinematics and beam energy: $e^+e^- \rightarrow \Upsilon(4S)$ at 10.58 GeV/c
- **asymmetric beam energies** to get boost $\beta\gamma \sim 0.4$ for B meson to get better spatial separation of the two b-meson decay vertices $\rightarrow$ for time-dependent measurements
- **fully hermetic detector**: full reconstruction of all particles except neutrino, also other B for flavour tagging: tagging power $\sim 30\%$
- able to perform **inclusive measurements** $B \rightarrow Xl\nu$:
  X not explicitly reconstructed, sum over all possible resonant and non-resonant hadronic final states

B-factories

- Neutrino reconstruction:
  - from missing energy and momentum in event

$$P_\nu = (E_\nu, \vec{p}_\nu) = (E_{miss}, \vec{p}_{miss}) = (E_{\Upsilon(4S)}, \vec{p}_{\Upsilon(4S)}) - \left(\sum_i E_i, \sum_i \vec{p}_i\right)$$

  $M^2_{miss} = P^2_\nu \sim 0$ because of experimental resolution has long tails

  - better resolution achieved if second B meson in event is fully reconstructed $\rightarrow$ B-tagging:
    - in general: all charged particles are assigned to one of two B candidates & small remaining energy required

$$P_B = P_{\Upsilon(4S)} - P_{B_{tag}}, \quad P_\nu = P_B - P_l - P_X$$

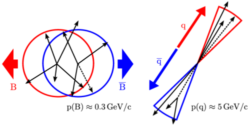
  resulting in narrow peak around zero for correctly reconstructed signal decays

B-factories

Background suppression & experimental techniques

$e^+e^- \to q\bar{q}$	Cross section (nb)
$b\bar{b}$	1.05
$c\bar{c}$	1.30
$s\bar{s}$	0.35
$d\bar{d}$	0.35
$u\bar{u}$	1.39
$\tau^+\tau^-$	0.94
$\mu^+\mu^-$	1.16
e^+e^-	~40

- e^+e^- cross section at $\Upsilon(4S)$ high contribution from non-$B\bar{B}$ events
- main background for B-decays:
  - *Continuum bkg*: $e^+e^- \to q\bar{q}$ $(q = u, d, s, c)$, $e^+e^- \to l^+l^-$ $(l = e, \mu, \tau)$, estimated from off-resonance data
    - fewer tracks then signal decay
    - more directional: use event shape variables
  - *Combinatorial bkg*: $B\bar{B}$ events where one/more particles wrongly assigned to signal B decay (from other B)
    - suppressed by testing kinematic concistency with B meson:



$$\Delta E = E_B - E_{beam}, \quad M_{bc} = \sqrt{E_{beam}^2 - p_B^2}$$ beam constrained mass, Babar m_{ES}

Hadron colliders

Advantages

- **large production cross section** of beauty quarks: $\sigma(pp \to b\bar{b}X) = 284 \pm 20 \pm 49\mu b$ at 7 TeV
- Millions of B candidates available, **all b-hadrons produced**: $B^0, B^+, B_s, B_c, \Lambda_b, \ldots$
- Excellent vertex separation, tracking and PID systems

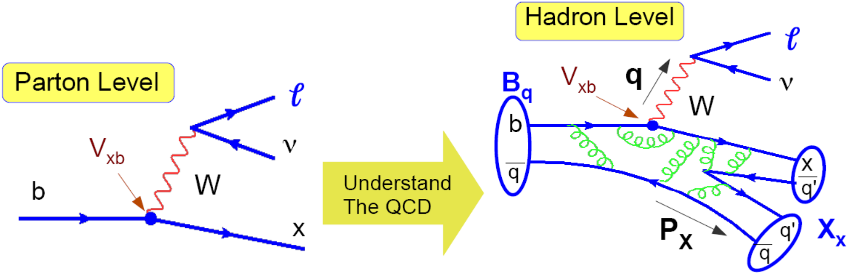
Disadvantages

- but **dirty environment**: many other particles produced in pp collisions
  $\to$ No possibility to use beam energy constraints
- No kinematic constraints from other (tagging) B, also b-hadron production fractions poorly known
- **unknown initial state** which makes reconstruction of neutrino challenging
- must trigger on **specific exclusive decay modes** and typically charged hadrons in final state
  $\to$ no inclusive measurements possible, hard to reconstruct neutrals

Theory Overview

- make use of fact that m_b is large compared to hadronic scale Λ_{QCD}
  $\rightarrow$ *heavy quark methods* used in B-physics
- expansions in powers of Λ_{QCD}/m_Q:
  Operator Production Expansion
  leading term corresponds to infinite mass limit (static heavy quark)
  $\rightarrow$ QCD exhibits new, additional symmetry:
  heavy quark symmetry (HQS): heavy quarks (b- and c-quarks) moving with same velocity

Measuring $|V_{xb}|$



- $|V_{xb}|$ measured using semileptonic decays
- 2 different strategies:
  - **exclusive** decays: $\bar{B}^0 \to \pi^+ l^- \bar{\nu}$ or $\bar{B}^0 \to D^{(*)+} l^- \bar{\nu}$
  - **inclusive** decays: $B \to X_c l^- \bar{\nu}$ or $B \to X_u l^- \bar{\nu}$

Why semileptonic decays?

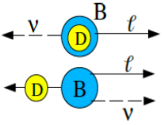
- advantages:
  1. very large event yields: $\sim 10\%$ of all B-decays
  2. theoretically clean:
     - much simpler to calculate then fully hadronic processes
     - leptons don't interact strongly $\rightarrow$ factorize strong (hadronic) and weak (leptonic) parts
     - hadronic matrix element parametrized by scalar functions of q^2 (momentum transfer to leptons) $\rightarrow$ so-called Form factors
       $\rightarrow$ absorb all non-pertubative effects into FF
- disadvantages:
  experimentally challenging since neutrino can't be directly reconstructed $\rightarrow$ partial reconstruction techniques needed

1. Part: $|V_{cb}|$

Exclusive $|V_{cb}|$

Theory Input

- QCD correction parametrized in the Form Factors:
- use velocities instead of momenta:
  - $v_B = \frac{p_B}{m_B}$, $v_D^{(*)} = \frac{p_D^{(*)}}{m_D^{(*)}}$, $w = v_B v_D^{(*)}$
  - w=1 corresponds to maximum momentum transfer to leptons $q^2_{max} = (m_B - m_D^{(*)})^2 \rightarrow$ Lattice-QCD



  - w_{max} corresponds to $q^2 = 0 \rightarrow$ LCSR
- interpolation between both regions needed to extract $|V_{cb}|$
- 2 FF parametrizations available using analyticity and unitarity bounds: BGL & CLN

Exclusive $|V_{cb}|$

Experimentally: $\bar{B} \to Dl\bar{\nu}$

$$\frac{d\Gamma(\bar{B} \to Dl\bar{\nu})}{dw} = \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B + m_D)^2 m_D^3 (w^2 - 1)^{3/2} |\eta_{EW}\mathcal{G}(w)|^2$$

- analysis performed by Belle & Babar with hadronic B-tagging:
  - reconstruct second B meson through hadronic decay
    $\to$ improve kinematic resolution and reduce combinatorial backgrounds
- use both $B \to D^0 l\nu$ and $B \to D^+ l\nu$ decays
- signal extracted from fits to M^2_{miss} in 10 bins of w, to measure the w dependence of the form factor $\mathcal{G}(w)$
- fit FF parametrization to $d\Gamma/dw$ to extract $|V_{cb}|$
- Largest background from $B \to D^* l\nu$

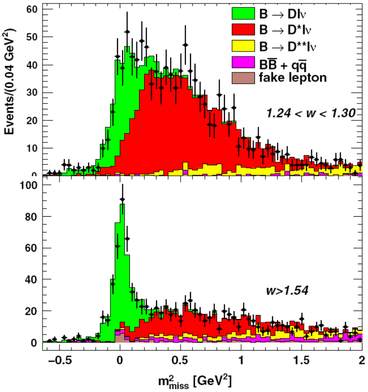
Exclusive $|V_{cb}|$ - $B \to Dl\nu$

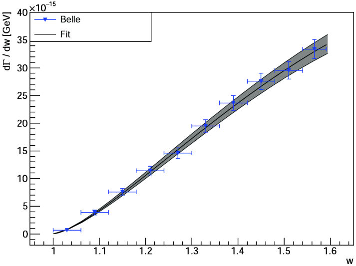
Babar (PRL **104,**011802 (2010))

- Babar used 460M $B\bar{B}$
- Fit $\sim$ 3200 signal events

Belle (Phys. Rev. D 93, 032006 (2016)))

- used 771M $B\bar{B}$
- Improved Hadronic B Tag based on NeuroBayes
- Fit $\sim$ 17000 signal events





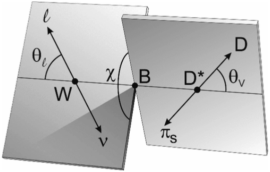
$\to$ FF parametrization dependent results
CLN: $|V_{cb}| = (39.86 \pm 1.33)x10^{-3}$
BGL: $|V_{cb}| = (40.83 \pm 1.13)x10^{-3}$
$\Rightarrow$ but both consistent

Exclusive $|V_{cb}|$ - $B \to D^* l\nu$

$$\frac{d\Gamma(B \to D^* l\nu)}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2(w^2-1)^{1/2}P(w)|\eta_{EW}\mathcal{F}(w)|^2$$

Babar (Phys.Rev.D77:032002,2008)

- based on $79 fb^{-1}$
- Fit 52.8K signal events

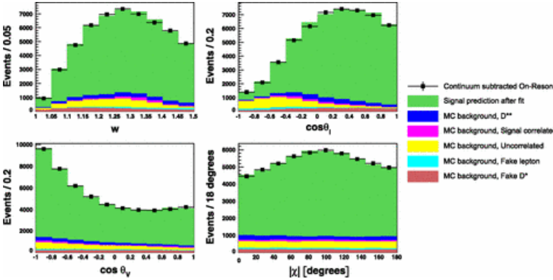


Belle (Phys.Rev.D82,112007(2010)))

- based on $711 fb^{-1}$
- Fit 120K signal events

both based on CLN parametrization
perform 4-D fit to w, cos θ_l, cos θ_v, χ

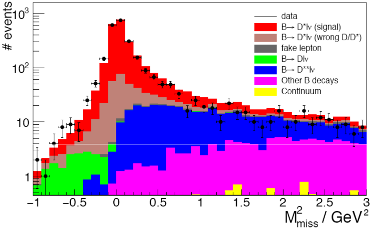
$|V_{cb}| = (38.71 \pm 0.47_{exp} \pm 0.59_{th})x10^{-3}$

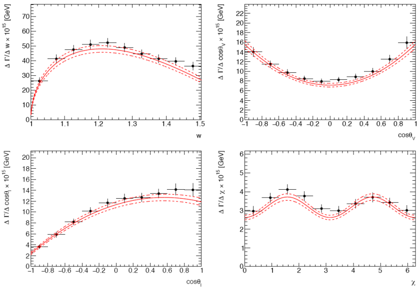


Exclusive $|V_{cb}|$ - $B \to D^* l\nu$

New Belle analysis (ArXiv:1702.01521v2)

- use hadronic tag
- Signal extracted from unbinned maximum likelihood fit to missing mass $\to \sim$ 2400 signal events
- Yields extracted in 4x10 bins of w and 3 angular variables
- gives $|V_{cb}|_{CLN} = (37.4 \pm 1.3)x10^{-3}$ consistent with world average of $|V_{cb}|_{WA} = (39.2 \pm 0.7)x10^{-3}$
- also published unfolded 4-D projections and full correlation matrix
  $\to$ can be fitted with different FF parametrizations: BGL





Exclusive $|V_{cb}|$

Final discussion:

- $|V_{cb}|$ averages from $B \to D^* l\nu$ and $B \to Dl\nu$ decays are consistent using the CLN parametrisation:

$$\eta_{EW}\mathcal{G}(1)|V_{cb}| = (41.57 \pm 1.00)x10^{-3} \quad (B \to Dl\nu, LQCD, CLN)$$
$$\eta_{EW}\mathcal{F}(1)|V_{cb}| = (35.61 \pm 0.43)x10^{-3} \quad (B \to D^* l\nu, LQCD, CLN)$$

  with $\mathcal{F}(1) = 0.906 \pm 0.013$ and $\mathcal{G}(1) = 1.054 \pm 0.004 \pm 0.008$

- BGL parametrization more general:

$$\eta_{EW}\mathcal{F}(1)|V_{cb}| = (38.2^{+1.7}_{-1.6})x10^{-3} \quad (B \to D^* l\nu, LQCD, BGL)$$

  10% shift to higher $|V_{cb}|$ value, well beyond experimental precision

$\Rightarrow$ large discussion ongoing at the moment

Inclusive $|V_{cb}|$

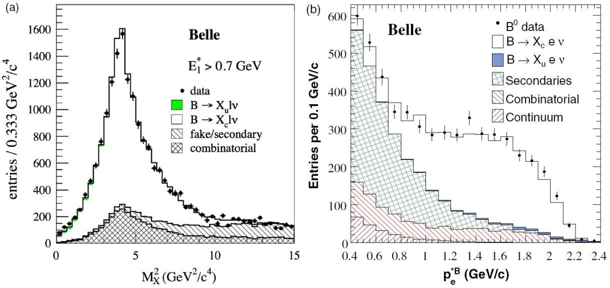
- Measurement of total semileptonic branching decay rate in inclusive semileptonic $b \to c$ transitions $\Rightarrow$ extract inclusive $|V_{cb}|$
- Theoretical foundation for calculation of the total semileptonic rate is the Operator Product Expansion (OPE) which yields the Heavy Quark Expansion (HQE)
- shapes of kinematic distributions, such as charged lepton energy hadronic invariant mass spectra and hadronic energy, of $B \to X_c l \nu$ decays are sensitive to the HQE parameters
- since rates and spectra depend strongly on m_b, fit is done using different mass definitions: kinetic or 1S scheme
- simultaneous fit to kinematic distributions is performed to determine $|V_{cb}|$ together with parameters of the HQE and quark masses

Inclusive $|V_{cb}|$

$$\langle E_\ell^n \rangle = \frac{1}{\Gamma_{E_\ell > E_{\rm cut}}} \int_{E_\ell > E_{\rm cut}} E_\ell^n \, \frac{d\Gamma}{dE_\ell} \, dE_\ell \; ,$$

$$\langle m_X^{2n} \rangle = \frac{1}{\Gamma_{E_\ell > E_{\rm cut}}} \int_{E_\ell > E_{\rm cut}} m_X^{2n} \, \frac{d\Gamma}{dm_X^2} \, dm_X^2$$

- several measurements of lepton energy $\langle E_l^n \rangle$ and hadronic mass $\langle m_X^{2n} \rangle$ performed by Belle, Babar, CLEO and DELPHI

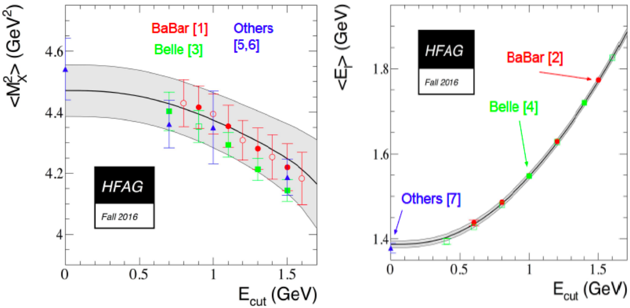


Phys. Rev. D 75, 032005

Phys. Rev. D 75, 032001

Inclusive $|V_{cb}|$

- most precise results are obtained from gloabl fits that include the moments from all experiments



$\rightarrow$ latest HFLAV average gives $|V_{cb}|_{incl.} = (42.19 \pm 0.78)x10^{-3}$ together with $\mathcal{B}(B \rightarrow X_c l\nu) = 10.65 \pm 0.16\%$ and $m_b^{kin} = 4.554 \pm 0.018$ GeV

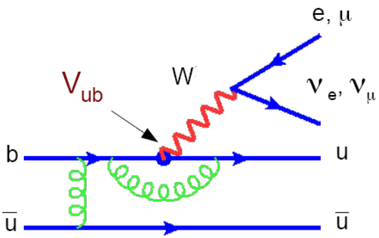
Comparison Inclusive vs. Exclusive $|V_{cb}|$

- inclusive $|V_{cb}| = (42.19 \pm 0.78)x10^{-3}$
- exclusive $|V_{cb}|_{WA} = (39.2 \pm 0.7)x10^{-3}$
  $\rightarrow$ gives $\sim 3\sigma$ tension between inclusive and exclusive determination of $|V_{cb}|$
  $\rightarrow$ long standing discrepancy

  BUT: exclusive $|V_{cb}|$ depends on FF parametrization, using more general BGL parametrization from $B \rightarrow D^* l\nu$ gives:
  $|V_{cb}|_{excl.} = (41.9^{+2.0}_{-1.9})x10^{-3}$
  $\rightarrow$ moves much closer to inclusive value
  $\rightarrow$ tension resolved by that???
  $\rightarrow$ large discussions ongoing

2. Part: $|V_{ub}|$

Exclusive $|V_{ub}|$



for massless leptons only one FF:

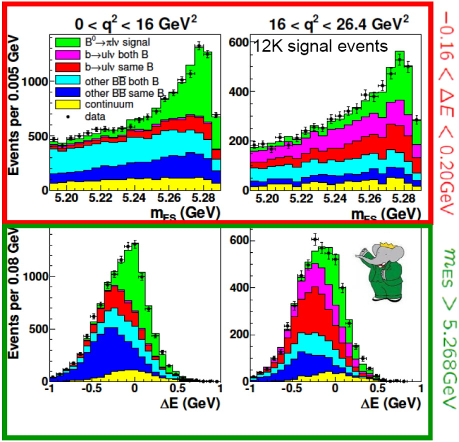
$$\frac{d\mathcal{B}(\bar{B}^0 \to \pi^+ l^- \bar{\nu})}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |p_\pi|^3 |f^+(q^2)|^2$$

Theory Input

- Lattice QCD (UKQCD, FNAL, HPQCD, . . .)
  - Works at high q2
  - Unquenched calculations (2+1, 2+1+1)
  - Other mesons (ρ, ω,...) difficult on lattice
- Light Cone Sum Rules
  - Reliable at low q2
  - Works for both pseudo-scalars and vector decays

Exclusive $|V_{ub}|$ - Untagged $\bar{B}^0 \to \pi^+ l^- \bar{\nu}$ decays

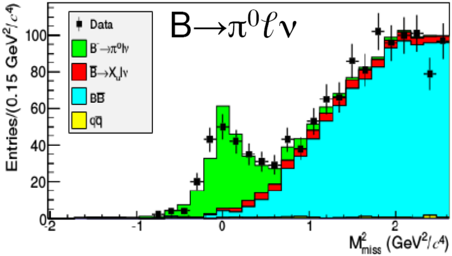
- performed by CLEO, Belle & Babar
- higher bkg (S/B<1)) and more restrictive kinematic cuts wrt. tagged analysis, but better precision on q^2 dependence on FF
- perform fit to m_{ES} and ΔE in bins of q^2 to extract signal

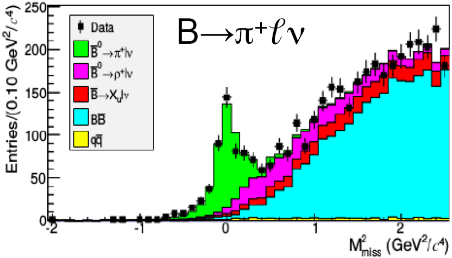


12.5K signal events (Phys.Rev.D86(2012) 092004)

Exclusive $|V_{ub}|$ - Tagged $\bar{B}^0 \to \pi l \bar{\nu}$ decays

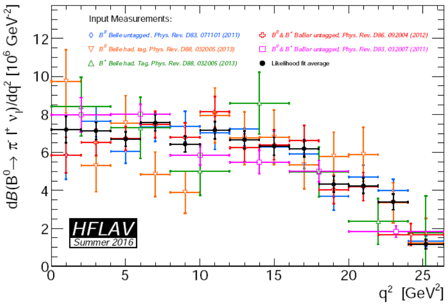
- second B is fully reconstructed in either hadronic or semileptonic mode
- have high and uniform acceptance, S/B$\sim$10 but low statistical power
- Belle measurement using 711 $fb^{-}1$ (Phys.Rev.D88,032005) using hadronic tag gives $\sim$200 $\bar{B} \to \pi^0 l \bar{\nu}$ and $\sim$500 $\bar{B}^0 \to \pi^+ l^- \bar{\nu}$ events

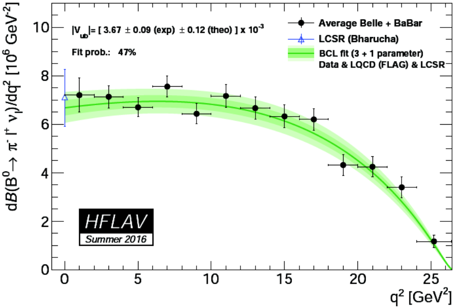




Exclusive $|V_{ub}|$ - HFLAV average

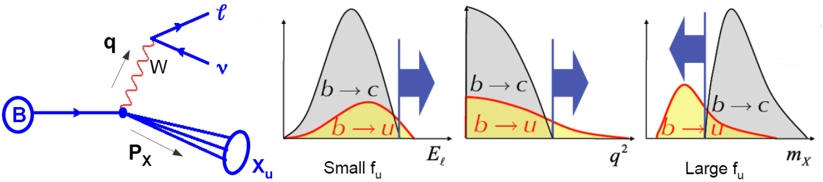
- $|V_{ub}|$ can be extracted from the average $\bar{B}^0 \to \pi l \bar{\nu}$ branching fraction together with the measured q^2 spectrum
- most sensitive method to do simultaneous fit to measured experimental partial rates and theory predictions versus q^2 to determine $|V_{ub}|$ and the first few coefficients of the BCL FF parametrization: $f+(q^2,\vec{b}) = \frac{1}{1-q^2/m_B^2} \sum_{k=0}^{K} b_k(t_0) z(q^2)^k$





gives $|V_{ub}| = (3.67 \pm 0.09_{exp} \pm 0.12_{theo}) \times 10^{-3}$

Inclusive $|V_{ub}|$



Theory Input

- inclusive $\bar{B} \to X_u l^- \bar{\nu}$ decays described with Heavy Quark Expansion
- hard to measure due to large bkg from CKM-favoured $\bar{B} \to X_c l^- \bar{\nu}$ transitions $\to$ need to go to phase space where those are kinematically suppressed, apply cut f_u
- there OPE breaks down $\to$ requires introduction of non-pertubative distribution function: 'shape function', in general unknown
- becomes important near the endpoint of $\bar{B} \to X_u l^- \bar{\nu}$ lepton spectrum $\to$ different kinematic regions give different sensitivity to them

Inclusive $|V_{ub}|$

shape functions

- at leading order single shape function which is universal for all heavy-to-light transitions, measured in $\bar{B} \to X_s\gamma$ decays
- at subleading order in $1/m_b$, several shape functions appear
- relations between shape functions and HQE parameters:

$$f(w) = \delta(w) + \frac{\mu_\pi^2}{6m_b^2}\delta''(w) - \frac{\rho_D^3}{18m_b^3}\delta'''(w) + ...$$

  $\to$ measurements of HQE parameters from global fits to $\bar{B} \to X_c l^- \bar{\nu}$ and $\bar{B} \to X_s\gamma$ moments can be used to constrain the SF moments
- HFLAV performs fits on the basis of several approaches, with varying degrees of model dependence and expansion orders in $1/m_b$ and α_s: BLNP, GGOU, DGE

Inclusive $|V_{ub}|$ - Measurements

1. Inclusive electron momentum:

   (Phys. Rev. Lett. 88, 231803 (2002),Phys. Lett. B621, 28 (2005), Phys. Rev. D73, 012006 (2006))

   - reconstruct a single electron to determine $\bar{B} \to X_u e^- \bar{\nu}$ near the kinematic endpoint
   - large selection efficiency but also large bkgs
   - decay rate can be extracted for $E_e > 2.3$GeV, cuts deep in the SF region, where theoretical uncertainties are large

2. untagged "neutrino reconstruction":(Phys. Rev. Lett. 97, 019903 (2006))

   - uses combination of a high-energy electron with missing momentum vector
   - large S/B$\sim$0.7 for $E_e > 2.0$GeV with small selection efficiency, but smaller accepted phase space and uncertainties associated with the determination of the missing momentum

3. tagged B, other decaying semileptonically:

   (Phys. Rev. D86, 032004 (2012), Phys. Rev. Lett. 95, 241801 (2005), Phys. Rev. Lett. 104, 021801 (2010), Phys. Rev. D95, 072001 (2017), Phys. Rev. Lett. 96, 221801 (2006))

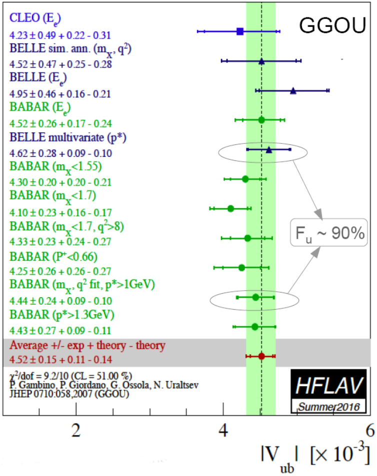
   - fully reconstruct a "tag" B candidate in about 0.5% (0.3%) of B^+B^- ($B^0\bar{B}^0$) events
   - electron or muon with momentum above 1.0 GeV required
   - full set of kinematic properties (E_l, m_X, q^2, etc.) are available
   - high selection efficiency $\sim$90%, but $\bar{B} \to X_c l^- \bar{\nu}$ remain important source of uncertainty

Inclusive $|V_{ub}|$ - Measurements

- Consistency between difference acceptance regions
- measured partial $\bar{B} \to X_u l^- \bar{\nu}$ rates and theoretical calculations from BLNP, GGOU and DGE are used to determine $|V_{ub}|$

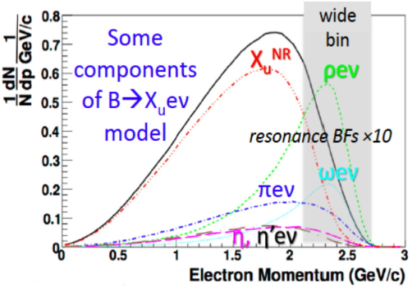
Framework	$\|V_{ub}\|[10^{-3}]$
BLNP	$4.44 \pm 0.15^{+0.21}_{-0.22}$
DGE	$4.52 \pm 0.16^{+0.15}_{-0.16}$
GGOU	$4.52 \pm 0.15^{+0.11}_{-0.14}$

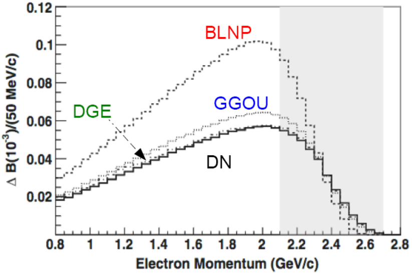
- All calculations yield compatible $|V_{ub}|$ values and similar error estimates
- HFLAV average: $|V_{ub}| = (4.52 \pm 0.15_{exp}{}^{+0.11}_{-0.14 theo}) \times 10^{-3}$
- BUT: $|V_{ub}|$ is calculated from partial rates measured with *only one signal model*



New Inclusive $|V_{ub}|$ Measurement

- recently new BABAR measurement based on inclusive electron spectrum for Ee > 0.8GeV *Phys.Rev.D 95,072001 (2017)*
- Highest sensitivity to $\bar{B} \to X_u e^- \bar{\nu}$ in the wide bin 2.1-2.7 GeV
- Models make different predictions for the fractional rate in this bin





→ $|V_{ub}| \times 10^3$ is $4.56 \pm 0.13^{+0.28}_{-0.26}$ (BLNP), $3.96 \pm 0.10 \pm 0.17$ (GGOU) and $3.85 \pm 0.11^{+0.08}_{-0.07}$ (DGE)

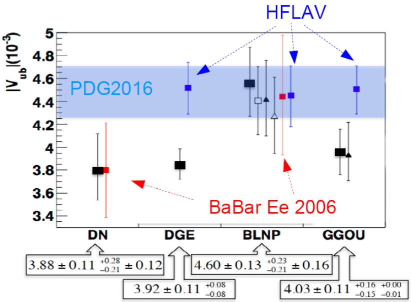
→ Results are lower than previous measurement

Comparison Inclusive vs. Exclusive $|V_{ub}|$

- exclusive $|V_{ub}| = (3.67 \pm 0.09_{exp} \pm 0.12_{theo}) \times 10^{-3}$
- inclusive $|V_{ub}| = (4.52 \pm 0.15_{exp}{}^{+0.11}_{-0.14 theo}) \times 10^{-3}$

  $\rightarrow$ gives $\sim 3.5\sigma$ tension between inclusive and exclusive determination of $|V_{ub}|$

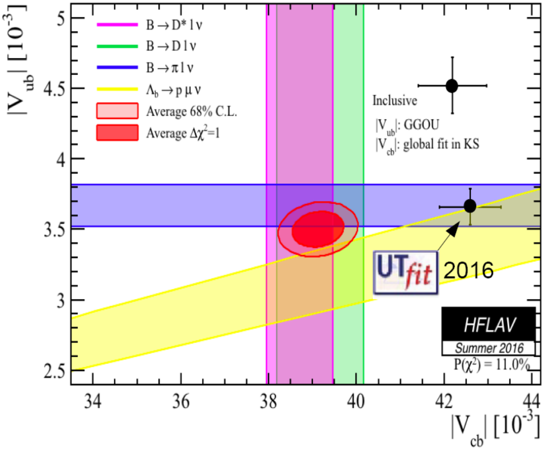
  $\rightarrow$ long standing puzzle



inclusive $|V_{ub}|$ depends on signal model, crucial to consider this and use same model for both signal extraction and $|V_{ub}|$

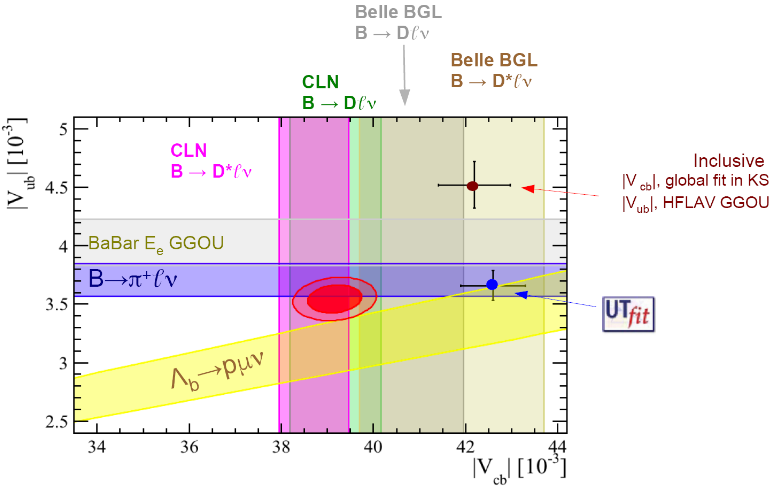
Putting it all together

Total uncertainties better than 2% for $|V_{cb}|$ and at about 5-6 % for $|V_{ub}|$

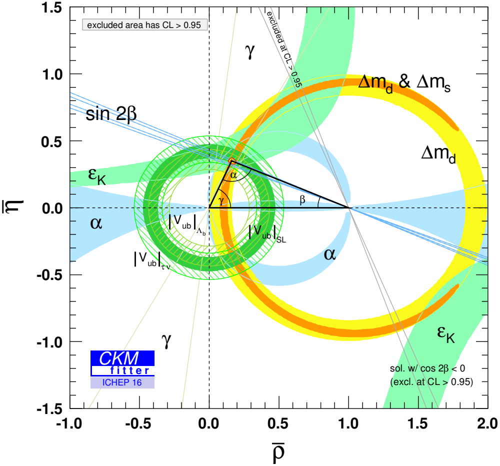


Indirect determinations from CKM fits prefer inclusive $|V_{cb}|$ and exclusive $|V_{ub}|$

New Global Picture?



Unitarity Triangle



$$|V_{CKM}| \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

with $\lambda \sim 0.22$

Conclusions

- Exclusive $|V_{cb}|$: General agreement to move to model independent FF parametrizations
- Inclusive $|V_{cb}|$: Everything consistent here
- Exclusive $|V_{ub}|$: so far all measurements very consistent
- Inclusive $|V_{ub}|$: internally consistent but above CKM fit and exclusive determination
  $\rightarrow$ very dependent on model predictions, Theory/parameters uncertainties dominate
- Inclusive – Exclusive puzzle cannot be considered solved in $|V_{cb}|$ ($\sim 3\sigma$ tension) nor $|V_{ub}|$ ($\sim 3.5\sigma$ tension)
- in general $|V_{ub}|$ tension is seen as more striking $\rightarrow$ need further independent measurements to check it
  $\rightarrow$ exclusive $|V_{ub}|$ can be also measured in LHCb
  $\rightarrow$ covered in next lecture

Sources

- Neckarzimmern B-Physics Workshop 2016 Looking for Semileptonic b-hadron decays at LHCb, C. Bozzi Semi-Leptonic Theory, T.Mannel & 2017 From hadron colliders to e+e-, flavour physics at Belle II, T. Kuhr
- Quark and Lepton Flavor Physics Lectures by Ulrich Uwer https://www.physi.uni-heidelberg.de/ uwer/lectures/Flavor/notes.html
- Review of Particle Physics, C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update, http://pdg.lbl.gov/2017/reviews/rpp2017-rev-vcb-vub.pdf
- Leptonic and semileptonic decays of B mesons, Jochen Dingfelder and Thomas Mannel, Rev. Mod. Phys. 88, 035008 – Published 21 September 2016
- Mini review on $|V_{ub}|$, $|V_{cb}|$ @LHCb and B-factories, Marcello Rotondo, https://cds.cern.ch/record/2301174/files/rotondo_sldecays.pdf
- Heavy Flavor Averaging Group (HFLAV), Eur. Phys. J. C (2017) 77:895
- CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005) [hep-ph/0406184]

Thanks for your attention!

Backup Slides

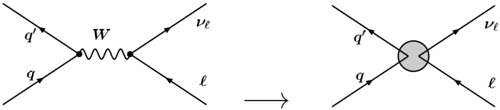
In general: weak decays of hadrons

- theoretically quarks are fundamental particles participating in Interaction
- experimentally one probes hadrons as asymptotic states $\rightarrow$ introduce parametrization to treat the problem
- factorization: physics at different scale decouples $\rightarrow$ factorize different physical effects in the transition amplitude
- form factors: describe shape corrections to the approximation that the scattering object is not point-like (e.g. non-relativistic Rutherford scattering) $\rightarrow$ encodes all non-pertubative QCD effects
- decay constant: absorbs the non-pertubative properties of meson decays

Approximations

Effective 4-fermion interaction:

- Since W is much heavier than the b quark, once can integrate out the W boson: $\langle 0| T[W_\mu(x)W^*_\nu(0)]|0\rangle \sim \frac{1}{M_W^2}\delta^4(x)$



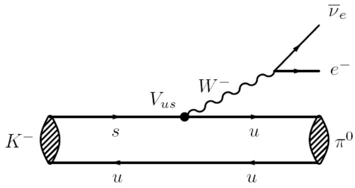
- semi-leptonic effective Hamiltonian:

$$H^{sl}_{eff} = \frac{4G_F}{\sqrt{2}}(\bar{u}_L\gamma_\mu V_{CKM} d_L)(\bar{e}_L\gamma_\mu\bar{\nu}_{e,L} + \bar{\mu}_L\gamma_\mu\bar{\nu}_{\mu,L} + \bar{\tau}_L\gamma_\mu\bar{\nu}_{\tau,L}) + h.c.$$

with $G_F = \frac{g^2}{4\sqrt{2}M_W^2}$ correct up to order m_b^2/M_W^2

Factorization

consider decay $K^- \to \pi^0 e^- \nu$



$$\begin{aligned} A &= \langle \pi^0 e^- \nu | \mathcal{O} | K^+ \rangle \\ &= \underbrace{\langle e^- \nu | \mathcal{O} | 0 \rangle}_{\text{leptonic part}} \frac{1}{M_W^2} \underbrace{\langle \pi^0 | \mathcal{O} | K^+ \rangle}_{\text{hadronic part}} \end{aligned}$$

hadronic part includes QCD binding of quarks, quite difficult to calculate
$\to$ in general parametrized by scalar functions of $q^2 = (p_K - p_\pi^0) \Rightarrow$ FF

Form Factor- exclusive Vub

exclusive semi-leptonic decays
for pseudoscalar final state $P(p_P)$:

$$\langle P(p_P)|\,\bar{q}\gamma^\mu b\,|B(p_B)\rangle = f_+(q^2)\left(p_B^\mu + p_P^\mu - \frac{m_B^2 - m_P^2}{q^2}q^\mu\right) + f_0(q^2)\frac{m_B^2 - m_P^2}{q^2}q^\mu$$

$$\langle P(p_P)|\,\bar{q}\gamma^\mu\gamma_5 b\,|B(p_B)\rangle = 0$$

$f_+(q^2)$ and $f_0(q^2)$ are form factors:

- Lattice QCD: at high $q^2 \sim (M - B - m_P)^2$
- QCD Sum rules: at low $q^2 \sim m_l^2$
- interpolation between both regions
- use FF bounds from analyticity and unitarity

Form Factor Parametrization - exclusive Vcb

need to extrapolate FF to zero-recoil point to extract CKM matrix element
$\rightarrow$ Parametrization use analyticity and unitarity constraints

1. BGL expansion:
   expressed in terms of variable: $z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$

$$F(z) = \frac{1}{P_F(z)\phi_F(z)} \sum_{n=0}^{\infty} a_n z^n$$

2. CLN parametrization:

$$F(w) = F(1) - \rho^2(w-1) + c(w-1)^2 + ...$$

   with ρ is the slope of the FF, based on heavy quark limit

- LQCD FF calculations use HQS, unquenched: using realistic sea-quarks with 2+1 flavours, gives total uncertainty of 1-2%, main error from chiral extrapolation to realistic u,d quark masses and discretisation errors: $F(1) = 0.906 \pm 0.013$, sum rules give lower values

Exclusive $|V_{cb}|$

$\bar{B} \to Dl\bar{\nu}$ and $B \to D^* l\nu$ provide clean way to extract $|V_{cb}|$

$$\frac{d\Gamma(\bar{B} \to Dl\bar{\nu})}{dw} = \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B + m_D)^2 m_D^3(w^2 - 1)^{3/2}|\eta_{EW}\mathcal{G}(w)|^2$$

Form Factor: $\mathcal{G}(w) = h_+(w) - \frac{m_B - m_D}{m_B + m_D} h_-(w)$
η_{EW} represents electroweak corrections

- using HQS: $\mathcal{G}$ is normalized at w=1, $h_+(1) = 1$ and $h_-(1) = 0$
- fit FF parametrization to $d\Gamma/dw$ to extract $|V_{cb}|$

Inclusive $|V_{cb}|$- Theory Input

Heavy Quark Expansion (HQE)

- use optical theorem to relate decay rate to forward matrix element of a scattering amplitude

$$\Gamma = Im \int d^4x \langle B(p_b)| \, T[H_{eff}(x)H_{eff}(0)] \, |B(p_B)\rangle$$

- time-ordered product can be written as an operator product expansion (OPE):

$$\int d^4x T[H_{eff}(x)H_{eff}(0)] = \sum_{n,i} \frac{1}{m_Q^n} \underbrace{C_{n,i}}_{\text{Wilson coeff.}} \underbrace{\mathcal{O}_{n+3,i}}_{\text{operators of dimension n+3}}$$

$C_{n,i}$ are pertubatively calculable coefficients, $\mathcal{O}_{n+3,i}$ non-pertubative operators

- dimension of operators can be related to hadronic quantities:
  n=0, dim. 3: no unknown hadronic matrix element= partonic rate
  n=2, dim. 5:
  $2m_B\mu_\pi^2 = -\langle B(p_B)| \, \bar{b}_\nu (iD)^2 b_\nu \, |B(p_B)\rangle$, $2m_B\mu_G^2 = -\langle B(p_B)| \, \bar{b}_\nu \sigma_{\mu\nu}(iD^\mu)(iD^\nu) b_\nu \, |B(p_B)\rangle$
  n=3, dim. 6:
  $2m_B\rho_D^3 = -\langle B(p_B)| \, \bar{b}_\nu (iD_\mu)(i\nu D)(iD^\mu) b_\nu \, |B(p_B)\rangle$,
  $2m_B\rho_{L,S}^3 = -\langle B(p_B)| \, \bar{b}_\nu \sigma_{\mu\nu}(iD^\mu)(i\nu D)(iD^\nu) b_\nu \, |B(p_B)\rangle$

Inclusive $|V_{cb}|$

$$\begin{aligned}\Gamma = & |V_{cb}|^2 \frac{G_F^2 m_b^5(\mu)}{192\pi^3}(1+A_{\rm ew})\times \\ & \left[z_0^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_0^{(1)}(r) + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 z_0^{(2)}(r) + \cdots \right. \\ & + \frac{\mu_\pi^2}{m_b^2}\left(z_2^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_2^{(1)}(r) + \cdots \right) \\ & + \frac{\mu_G^2}{m_b^2}\left(y_2^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} y_2^{(1)}(r) + \cdots \right) \\ & + \frac{\rho_{\rm D}^3}{m_b^3}\left(z_3^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} z_3^{(1)}(r) + \cdots \right) \\ & \left. + \frac{\rho_{\rm LS}^3}{m_b^3}\left(y_3^{(0)}(r) + \frac{\alpha_s(\mu)}{\pi} y_3^{(1)}(r) + \cdots \right) + \ldots \right]\end{aligned}$$

- μ_π^2,μ_G^2, ρ_D^3, $\rho_{L,S}^3$ non-perturbative input into the heavy quark expansion
- in the same way HQE can be set up for the moments of distributions of charged-lepton energy, hadronic invariant mass and hadronic energy, e.g.:

$$\langle E_e^n \rangle_{E_e > E_{cut}} = \int_{E_{cut}}^{E_{max}} \frac{d\Gamma}{dE_e} E_e^n dE_e \Bigg/ \int_{E_{cut}}^{E_{max}} \frac{d\Gamma}{dE_e} dE_e$$

Inclusive $|V_{cb}|$

- shapes of kinematic distributions, such as charged lepton energy hadronic invariant mass spectra and hadronic energy , of $B \to X_c l\nu$ decays are sensitive to the HQE parameters $\to$ HQE parameters are determined by fitting the HQE to these moments
- moments of the distribution of an observable E_e:

$$\langle E_e^n \rangle_{E_e > E_{cut}} = \int_{E_{cut}}^{E_{max}} \frac{d\Gamma}{dE_e} E_e^n dE_e \bigg/ \int_{E_{cut}}^{E_{max}} \frac{d\Gamma}{dE_e} dE_e$$

- moments are measured as a function of the minimum lepton energy, as their dependence on E_{cut} contains information on the HQE parameters and thus provides additional sensitivity for their determination