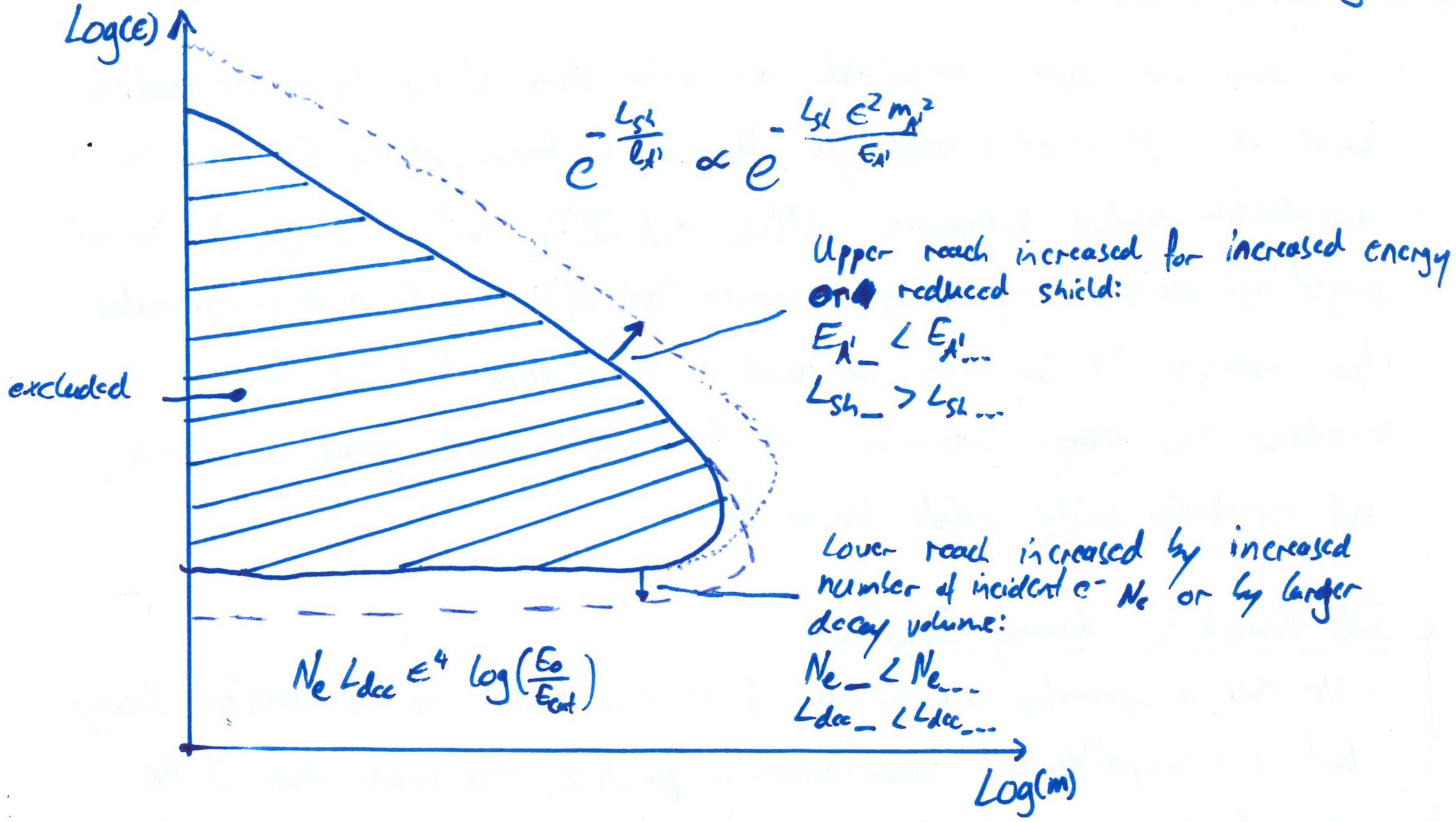
In the $m_{A'} - \epsilon$ plane the typical beam dump reach looks as following



Upper reach:

The upper reach of the beam dump limit is mainly controlled by the length of the shield L_{sh} compared to the typical A' decay length $l_{A'}$. As soon as $l_{A'} \ll L_{sh}$ practically all produced A' decay within the shield and don't reach the detector.

Lower reach:

The lower reach is depending on the ratio of the A' decay length to the total length of the experimental setup $L_{tot} = L_{sh} + L_{dec}$. Once $l_{A'} \gg L_{sh}, L_{dec}$ we can approximate

$$P = e^{-\frac{L_{sh}}{l_{A'}}}\left(1 - e^{-\frac{L_{dec}}{l_{A'}}}\right) \simeq \frac{L_{dec}}{l_{A'}} \propto L_{dec} \frac{m_{A'}^2 \epsilon^2}{E_{A'}}$$

In this regime the total number of events then scales as

$$N_{A'} \propto \int_{E_{cut}}^{E_0} dE_{A'} \frac{d\sigma}{dx_e} P \propto \int_{E_{cut}}^{E_0} dE_{A'} \frac{\alpha^3 Z^2 \epsilon^2}{m_{A'}^2} L_{dec} \frac{m_{A'}^2 \epsilon^2}{E_{A'}} \simeq \alpha^3 Z^2 \epsilon^4 L_{dec} \log\left(\frac{E_0}{E_{cut}}\right)$$

where E_{cut} denotes the experimental energy cut. The total number scales with ϵ^4 and therefore has a quite steep fall-off so that the lower reach is mainly statistics limited. Once we go to too small ϵ, too few A' are produced and many of them decay outside the detector.

IV. Generalized Hidden Photons

- So far, we have considered an extra dark $U(1)_d$ symmetry under which the SM remains uncharged. However, we know that the SM has the accidental global symmetries $U(1)_B$ and $U(1)_{L_i}$ with $i \in \{e, \mu, \tau\}$. So we might ask ourselves, if we can promote (one of) those to a local symmetry. Upon gauging the symmetries we have to make sure that we do not introduce any gauge anomalies as this would break gauge invariance and eventually might violate unitarity!

Side Remark 3: Anomaly Cancellation

- We call a symmetry anomalous, if it is respected by the classical theory but is incompatible with quantization. In practice, this means that if the classical theory respects the conservation law

$$\partial_\mu J^\mu_{sym} = 0$$

we have to check whether in the quantum theory we still have

$$\partial_\mu \langle J^\mu_{sym} \rangle = 0 .$$

Chiral anomaly

- Probably the most famous example is the chiral anomaly of QED. The QED action is invariant (in the massless case) under axial rotations $\psi(x) \to e^{i\beta\gamma_5}\psi(x)$ with associated current $J^\mu_A = \bar{\psi}\gamma^\mu\gamma_5\psi$ that respects

$$\partial_\mu J^\mu_A = 2im\bar{\psi}\gamma_5\psi$$

- To study the quantum behavior we consider a Dirac fermion coupled to an external gauge field $A_\mu(x)$:

$$S_{int} = -e\int d^4x \; J^\mu_V(x) A_\mu(x)$$

with the usual vector current $J^\mu_V = \bar{\psi}\gamma^\mu\psi$.

Then

$$\langle J^\mu_A(x) \rangle_A = \frac{1}{Z}\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \; J^\mu_A(x) \, e^{i\int d^4y \left[\bar{\psi}(i\partial\!\!\!/ - m)\psi - e J^\mu_V A_\mu(y)\right]}$$

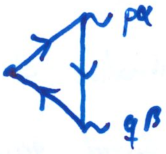
- To gain some insight one can expand this in the coupling constant e

$$\langle J_A^\mu(x)\rangle_{\mathcal{A}} = -ie\int d^4y\, \langle 0|T\{J_A^\mu(x) J_V^\alpha(y)\}|0\rangle \mathcal{A}_\alpha(y) - \frac{e^2}{2}\int d^4y_1\, d^4y_2\, \langle 0|T\{J_A^\mu(x) J_V^\alpha(y_1) J_V^\beta(y_2)\}|0\rangle \mathcal{A}_\alpha(y_1)\mathcal{A}_\beta(y_2) + \ldots$$

One finds that the first term vanishes. Going to momentum space, one finds for the second term

$$\partial_\mu \langle J_A^\mu(x)\rangle_{\mathcal{A}} = \frac{i}{2}\int d^4y_1\, d^4y_2\, \mathcal{A}^\alpha(y_1)\mathcal{A}^\beta(y_2)\int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}(p+q)_\mu\, i\Gamma^{\mu\alpha\beta}(p,q)\, e^{ip(y_1-x)+iq(y_2-x)}$$

where to lowest order

$$i\Gamma^{\mu\alpha\beta}(p,q) = (p+q)^\mu$$  $$+ \quad (p+q)^\mu$$ 

Calculating these triangle diagrams finally leads to

$$\boxed{\partial_\mu\langle J_A^\mu(x)\rangle = \frac{e}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}}$$

- An important theorem by Adler and Bardeen guarantees that this result does not receive any quantum corrections and is 1-loop exact!

Gauge anomalies

- For Abelian gauge theories one can calculate the anomaly analogously from noticing that

$$J_L^\mu = \frac{1}{2}(J_V^\mu - J_A^\mu) \qquad J_R^\mu = \frac{1}{2}(J_V^\mu + J_A^\mu)$$

and only considering the parity-odd combinations

$$\langle T\{J_A J_V J_V\}\rangle + \langle T\{J_V J_A J_V\}\rangle + \langle T\{J_V J_V J_A\}\rangle + \langle T\{J_A J_A J_A\}\rangle$$

For N_R right-handed and N_L left-handed fermions one finds

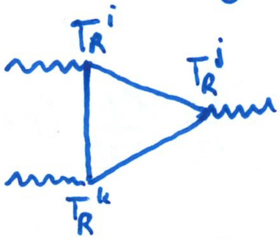
$$\boxed{\partial_\mu\langle J^\mu\rangle_{\mathcal{A}} = \frac{1}{96\pi^2}\left(\sum_{j=1}^{N_R}\tilde{Q}_j^3 - \sum_{k=1}^{N_L}Q_k^3\right)\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}}$$

- Thus the condition for anomaly cancellation is $\boxed{\sum_{j=1}^{N_R}\tilde{Q}_j^3 - \sum_{k=1}^{N_L}Q_k^3 = 0}$.

For non-Abelian symmetries the condition is $\boxed{\mathrm{Tr}\left[T_R^a\{T_R^b, T_R^c\}\right] = 0}$ for fermions transforming in a representation R.

IV.1 Anomaly-free extensions of the SM

- For a gauge symmetry G the anomaly coefficient is determined from the trace over the gauge coefficients T_R^i of the representation R under which the fermions transform in the triangle diagrams



$$\propto \mathrm{Tr}\left[T_R^i \{ T_R^j, T_R^k \} \right]$$

- If we want to add a symmetry to the SM we have to determine these coefficients for all the possible combinations of gauge bosons. For an extra $U(1)_X$ in the SM there are the following mixed gauge anomalies:

Anomaly	Constraints
$U(1)_X^3$	$\sum_X Q_i^3 = 0$
$SU(3)^2\, U(1)_X$	$\sum_{\text{color}} Q_i = 0$
$SU(2)^2\, U(1)_X$	$\sum_{\text{left-handed}} Q_i = 0$
$\text{grav}^2\, U(1)_X$	$\sum_X Q_i = 0$
$U(1)_X^2\, U(1)_Y$	$\sum_{X,Y} Q_i^2 Y_i = 0$
$U(1)_X\, U(1)_Y^2$	$\sum_{X,Y} Q_i Y_i^2 = 0$

- Within the field content of the SM we can find 4 combinations of the global symmetries $U(1)_B$ and $U(1)_{L_i}$ that are anomaly-free*:

$$\boxed{U(1)_{B-L} \qquad U(1)_{L_\mu - L_e} \qquad U(1)_{L_e - L_\tau} \qquad U(1)_{L_\mu - L_\tau}}$$

* Note that for $U(1)_{B-L}$ to be anomaly-free requires the addition of 3 right-handed neutrinos.

IV.2 Phenomenology of anomaly-free Hidden Photons

- Finally we can consider the full interaction term for the Hidden Photon

$$\mathcal{L}_{int} = g_d\, j_d^{\mu} A'_{\mu} - \epsilon e\, j_{EM}^{\mu} A'_{\mu}$$

- The associated gauge currents for the anomaly-free symmetries $U(1)_{B-L}$ and $U(1)_{L_i-L_j}$ with $i,j \in \{e,\mu,\tau\}$ read

$$j^{\mu}_{B-L} = \frac{1}{3}\bar{Q}\gamma^{\mu}Q + \frac{1}{3}\bar{u}_R\gamma^{\mu}u_R + \frac{1}{3}\bar{d}_R\gamma^{\mu}d_R - \bar{L}\gamma^{\mu}L - \bar{l}_R\gamma^{\mu}l_R - \bar{\nu}_R\gamma^{\mu}\nu_R$$

$$j^{\mu}_{i-j} = \bar{L}_i\gamma^{\mu}L_i + \bar{l}_{R_i}\gamma^{\mu}l_{R_i} - \bar{L}_j\gamma^{\mu}L_j - \bar{l}_{R_j}\gamma^{\mu}l_{R_j}$$

- The lepton-family groups $U(1)_{L_i-L_j}$ are special in that they can be embedded into a larger group $G_{L_i-L_j}$ that breaks only to them $G_{L_i-L_j} \to U(1)_{L_i-L_j}$. This renders the loop-induced kinetic mixing finite:

$$\epsilon_{ij}(q^2) \simeq \frac{3\, e\, g_d}{4\pi^2}\int_0^1 dx\, x(x-1)\log\left(\frac{m_j^2 + q^2 x(x-1)}{m_i^2 + q^2 x(x-1)}\right)$$

$\Rightarrow$ This leads to a hierarchy in the couplings of the order

$$\frac{\epsilon_{ij}\, e}{g_d} \propto \underbrace{\frac{6\alpha}{\pi}}_{\sim 0.014} \underbrace{\log\left(\frac{m_j}{m_i}\right)}_{2\ldots3} \sim 0.03$$

Phenomenological consequences:

i) The presence of the term $\bar{L}\gamma^{\mu}L$ in the gauge currents leads to neutrino couplings of the gauge bosons. These are generically absent for minimal Hidden Photons! This leads to important constraints from neutrino scattering experiments, White Dwarf cooling or N_{eff} during BBN.

ii) The $U(1)_{L_i-L_j}$ only have gauge couplings to leptons and loop-suppressed kinetic mixing couplings to hadrons. This makes it very hard to produce these gauge bosons in hadron beam dumps and colliders. Therefore, constraints from hadronic processes lose a lot of sensitivity!