Dalitz-Plot Analyses

RTG Lecture 2: Resonances and angular distributions

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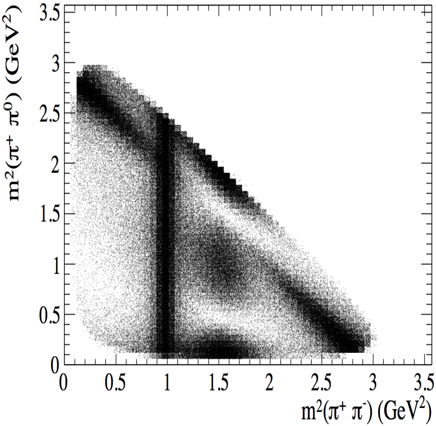
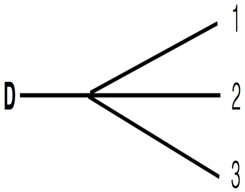
Lecture 2:
Resonances and angular distributions



($D^0 \to \pi^+\pi^-\pi^0$ Toy Simulation)

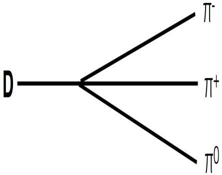
Reminder: Kinematic of Multibody Decays

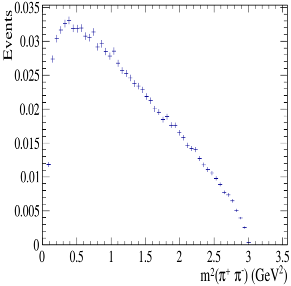


Four-momenta	**12**
Meson masses ($p_i^2 = m_i^2$)	**-3**
E, p conservation	**-4**
Arbitrary orientation (spinless particles)	**-3**
Independent variables	**2**

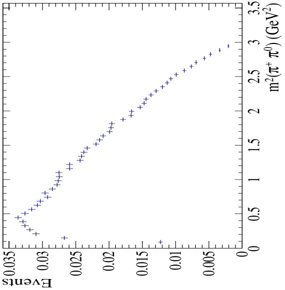
Fermi's Golden Rule for $D \to 123$

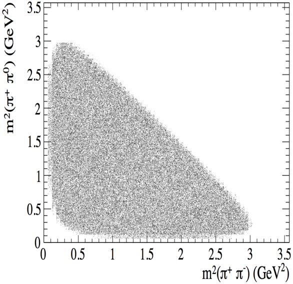
- **Differential decay rate:** $\mathrm{d}\Gamma \propto |A_{D\to123}|^2 \, \mathrm{d}\phi_3$
- Convenient (but not unique) choice of variables: $m_{ij}^2 = (p_i^\mu + p_j^\mu)^2$
  $\mathrm{d}\Gamma \propto |A_{D\to123}|^2 \mathrm{d}m_{12}^2 \, \mathrm{d}m_{23}^2$
- **Dalitz plot** = Scatter plot of m_{12}^2 vs m_{23}^2
  Deviation from flat distribution $\Rightarrow$ Information on $A_{D\to123}$!





$$A_{D\to 3\pi} = const.$$

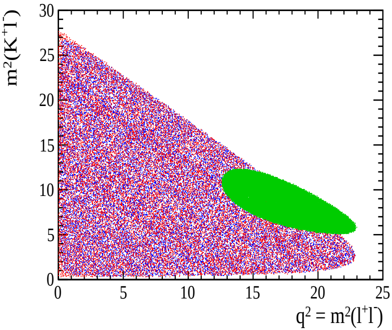


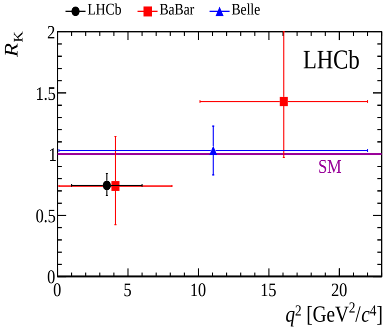


$$\Gamma(M \to 123) \overbrace{=}^{A\approx 1} \frac{1}{M^3} \int \mathrm{d}m_{12}^2 \mathrm{d}m_{23}^2 \propto V_{phsp}$$

Search for lepton flavor violation in $B^+ \to K^+ l^+ l^-$

$\frac{\Gamma(B^+ \to K^+ \mu\mu)}{\Gamma(B^+ \to K^+ ee)} \approx 0.99$ $\frac{\Gamma(B^+ \to K^+ \mu\mu)}{\Gamma(B^+ \to K^+ \tau\tau)} \approx 9$





[Phys. Rev. Lett. 113, 151601]

Particle decay

- Total decay rate: $\Gamma = \sum_f \Gamma(i \to f)$
- Branching fraction: $B_f = \frac{\Gamma(i \to f)}{\Gamma}$
- Number of particles remaining after time t:
  $N(t) = N(0)e^{-\Gamma t} \Rightarrow$ Average lifetime: $\tau = \frac{1}{\Gamma}$
- Heisenberg Uncertainty: $E\tau \sim \hbar$
  Finite lifetime $\Rightarrow$ uncertain energy

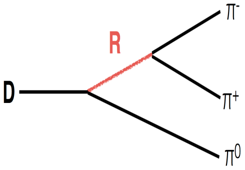
QM description of decaying states

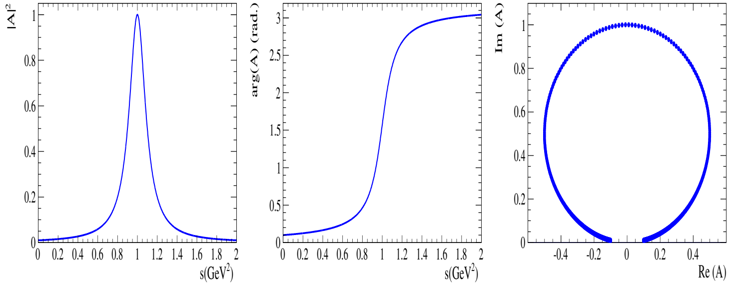
- **Free stable particle:**
  $\psi(t) = \psi(0) e^{-iE_0 t} \Rightarrow |\psi(t)|^2 = |\psi(0)|^2$
- **Unstable particle:**
  $\psi(t) = \psi(0) e^{-iE_0 t} e^{-\Gamma/2 t} \Rightarrow |\psi(t)|^2 = |\psi(0)|^2 e^{-\Gamma t}$
- **Fourier transform:**
  $\phi(E) = \int \psi(t) e^{iEt} dt = \psi(0) \int e^{i[(E-E_0)+i\Gamma/2]t} dt$

$$\phi(E) = \psi(0) \underbrace{\frac{1}{(E-E_0)+i\Gamma/2}}_{\text{Breit-Wigner}=BW(E)}$$

Resonances

- Decay of D via intermediate hadron state
- Short lived resonance:
  $\tau \approx \mathcal{O}(10^{-23}\mathrm{s}) \Rightarrow \Gamma_0 = \frac{1}{\tau} \approx \mathcal{O}(100\,\mathrm{MeV})$
- Peak in scattering amplitude:
  $BW(s) = \frac{1}{m_0^2 - s - i m_0 \Gamma_0}$





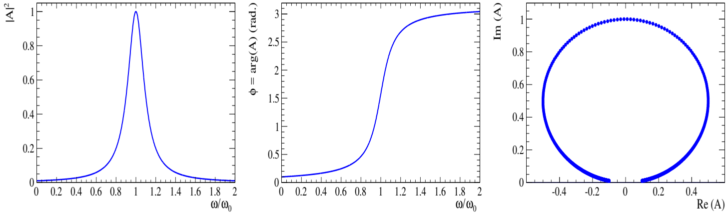
Mechanical Resonance

Forced harmonic oscillator

- $\ddot{x}(t) = -\underbrace{\omega_0^2 x(t)}_{\text{spring}} - \underbrace{\gamma \dot{x}(t)}_{\text{friction}} + \underbrace{F_0 cos(\omega t)}_{\text{external force}}$
- $x(t) \xrightarrow[t\to\infty]{} A\, cos(\omega t + \phi)$

  $A^2 = \frac{F^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2},\; tan\phi = -\frac{\gamma\omega}{\omega_0^2 - \omega^2}$
- **Resonance**:

  $\omega = \omega_0 \Rightarrow A = \frac{F}{\gamma\omega_0}, \phi = 90^\circ$



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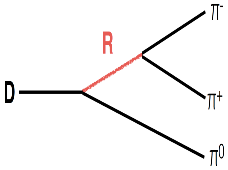
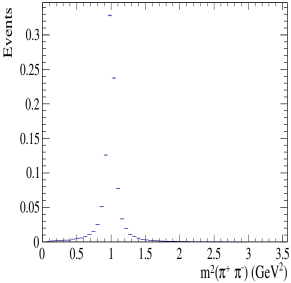
  $A^2 = \frac{F^2}{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2},\ tan\phi = -\frac{\gamma\omega}{\omega_0^2-\omega^2}$
- **Resonance**:

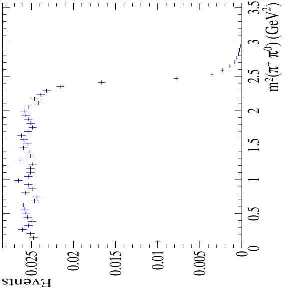
  $\omega = \omega_0 \Rightarrow A = \frac{F}{\gamma\omega_0}, \phi = 90^\circ$

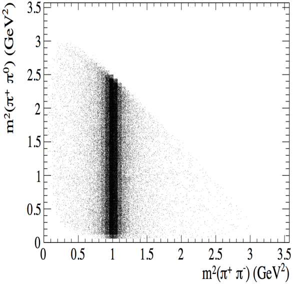
- Driving force F $\sim$ matrix element $R \to ab$
- Driving frequency ω $\sim$ invariant mass m(ab)
- Characteristic frequency ω_0 $\sim$ mass of resonance
- Friction γ $\sim$ decay width Γ

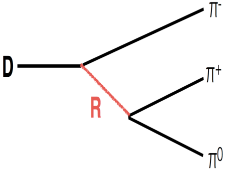


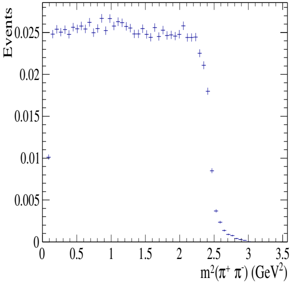


$$A_{D\to 3\pi} = BW(m^2_{\pi^+\pi^-})$$

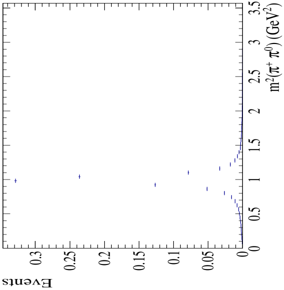


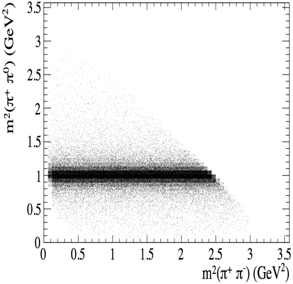


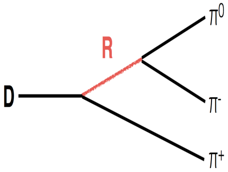


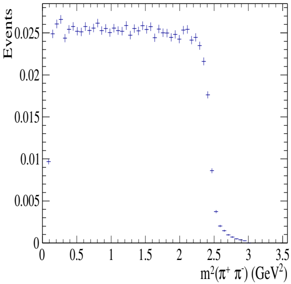


$$A_{D\to 3\pi} = BW(m^2_{\pi^+\pi^0})$$



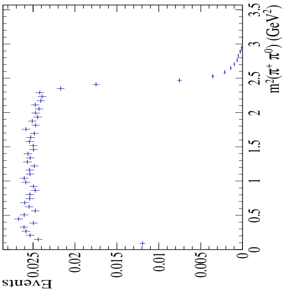


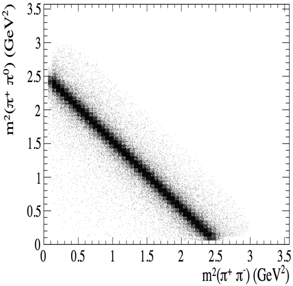




$$A_{D\to 3\pi} = BW(m^2_{\pi^0\pi^-})$$

$$m_D^2 + m_1^2 + m_2^2 + m_3^2 = m_{12}^2 + m_{13}^2 + m_{23}^2$$

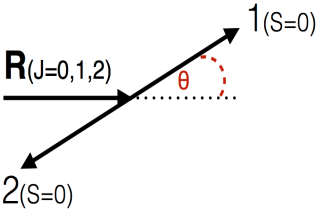


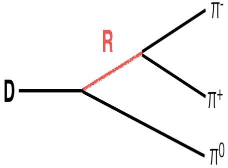


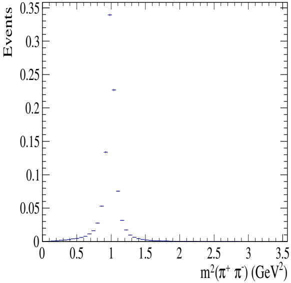
Angular distribution

- Non-isotropic distribution of decay products if **R** has **spin**
- Angular distribution given by spherical harmonics
  (In rest-frame of **R**)

$$
\begin{aligned}
J &= 0 : A \propto 1 \\
J &= 1 : A \propto cos\theta \\
J &= 2 : A \propto (cos\theta^2 - \tfrac{1}{3}) \\
J &\quad\; : A \propto P_J(\theta)
\end{aligned}
$$

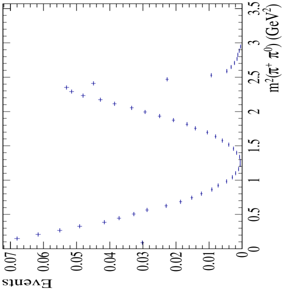


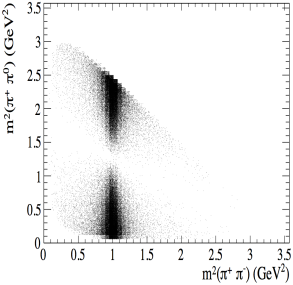


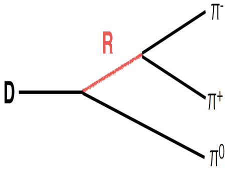


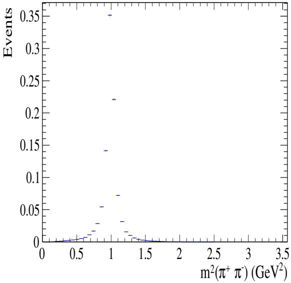
$$A_{D\to 3\pi} = BW(m^2_{\pi^+\pi^-})cos\theta$$

$$m^2_{\pi^+\pi^0} = (m^2_{\pi^+\pi^0})_{min}\frac{1+cos\theta}{2} + (m^2_{\pi^+\pi^0})_{max}\frac{1-cos\theta}{2}$$

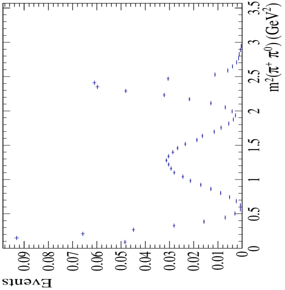


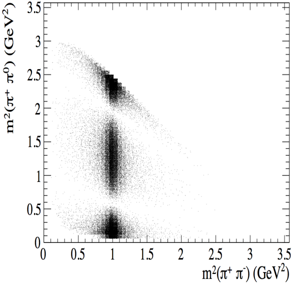


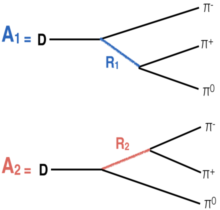




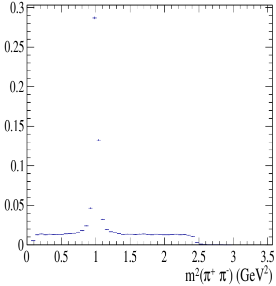
$$A_{D\to 3\pi} = BW(m^2_{\pi^+\pi^-})\,(cos\theta^2 - \frac{1}{3})$$

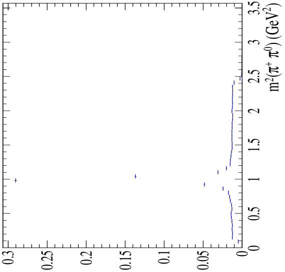


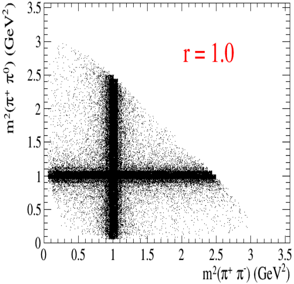


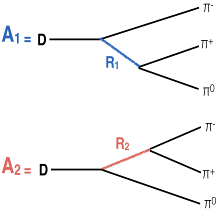


$$|A|^2 = |A_1|^2 + r^2 \cdot |A_2|^2$$

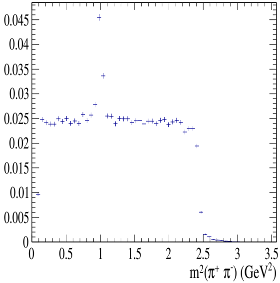


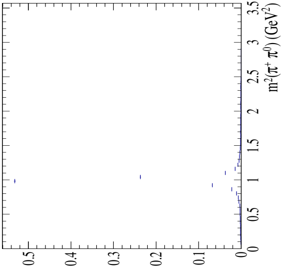


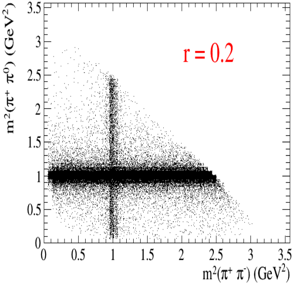




$$|A|^2 = |A_1|^2 + r^2 \cdot |A_2|^2$$

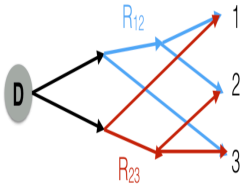


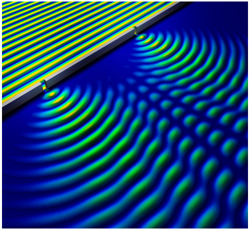




Interference Phenomenon

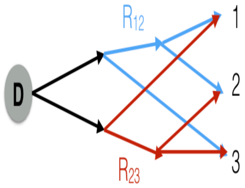
- Multiple resonances may appear as intermediate states
- Same initial state (D) and same final state $(1+2+3)$
  $\Rightarrow$ intermediate states interfere
- **Double slit experiment:**
  Wave-function: $\Psi = \Psi_1 + \Psi_2$
  Detector measures:
  $$|\Psi|^2 = \underbrace{|\Psi_1|^2 + |\Psi_2|^2}_{\text{coherent sum}} + \underbrace{2|\Psi_1||\Psi_2|\cos\delta}_{\text{interference}}$$





Interference of amplitudes

- Define phase-space point: $x = (m_{12}^2, m_{23}^2)$
- Amplitude for process 1:
  $A_1(x) = BW(m_{12}^2)P_L(\theta_{12}) = a_1(x)e^{i\phi_1(x)}$
- Amplitude for process 2:
  $A_2(x) = BW(m_{23}^2)P_L(\theta_{23}) = a_2(x)e^{i\phi_2(x)}$

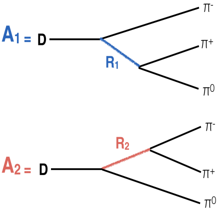


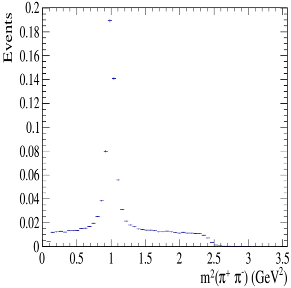
- Total amplitude: $A(x) = A_1(x) + re^{i\delta}A_2(x)$
- $|A(x)|^2 = |a_1(x)|^2 + |r\, a_2(x)|^2 + 2|a_1(x)||r\, a_2(x)|\cos(\underbrace{\phi_1(x) - \phi_2(x)}_{\Delta\phi(x)} - \delta)$

Fit fractions and interference fractions:

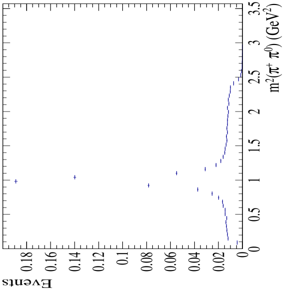
$$F_1 = \frac{\int |A_1(x)|^2 dx}{\int |A_1(x)+re^{i\delta}A_2(x)|^2 dx} \quad F_2 = \frac{\int |re^{i\delta}A_2(x)|^2 dx}{\int |A_1(x)+re^{i\delta}A_2(x)|^2 dx} \quad IF_{12} = \frac{\int 2\mathrm{Re}(A_1(x)re^{-i\delta}A_2^*(x)) dx}{\int |A_1(x)+re^{i\delta}A_2(x)|^2 dx}$$

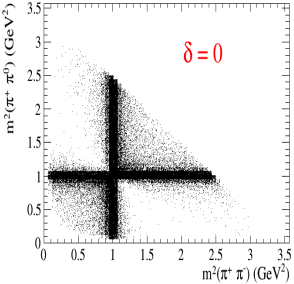
$$1 = \underbrace{F_1 + F_2}_{\neq 1} + IF_{12}$$

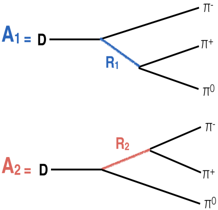


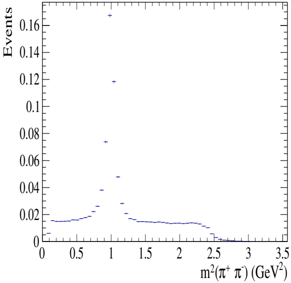


$$|A(x)|^2 = |a_1(x)|^2 + |r\, a_2(x)|^2 + 2|a_1(x)||r\, a_2(x)|\cos(\Delta\phi(x) - \delta)$$









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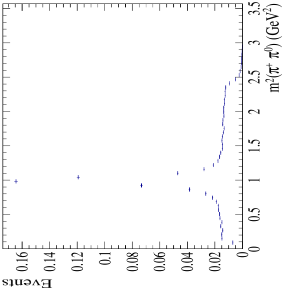
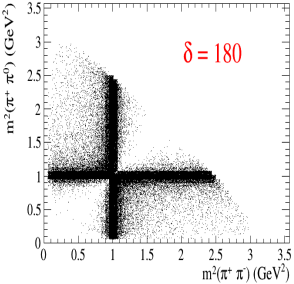
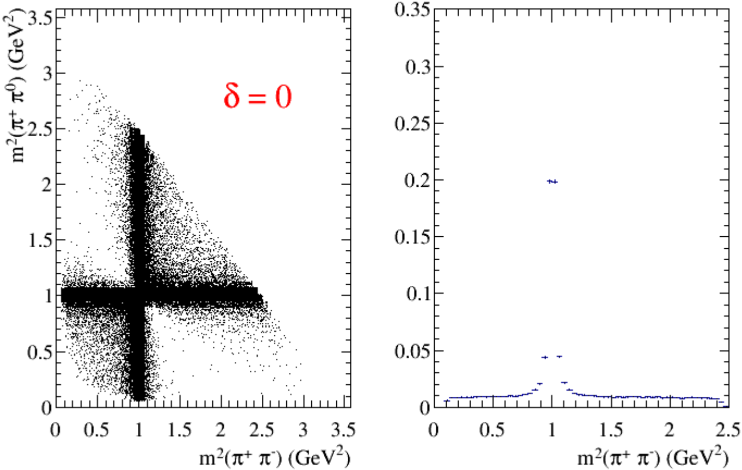
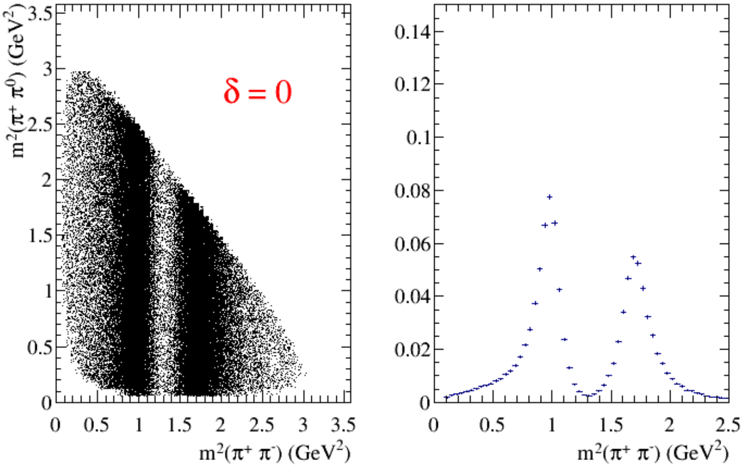


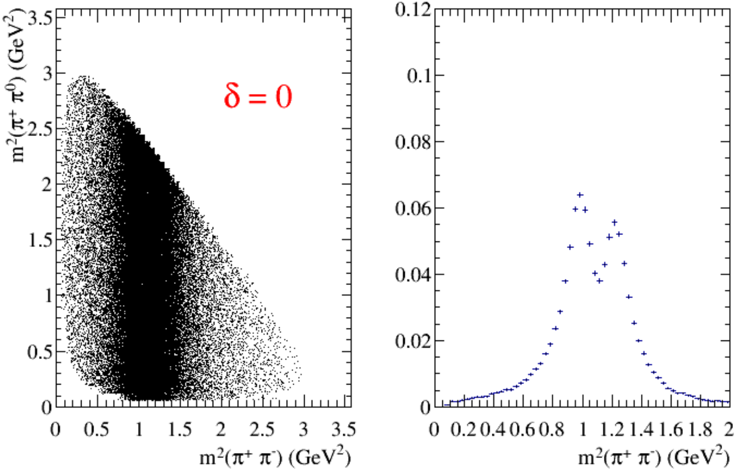




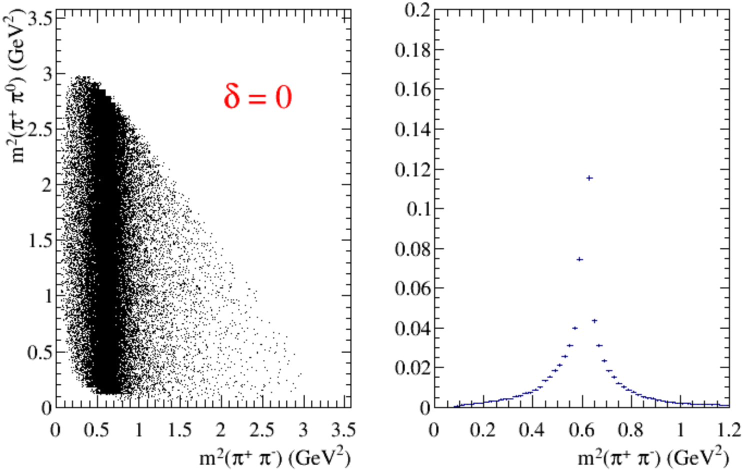
$\delta = 0$
$m^2(\pi^+ \pi^0)$ (GeV^2)
$m^2(\pi^+ \pi^-)$ (GeV^2)
0
0.5
1
1.5
2
2.5
3
3.5
0.05
0.1
0.15
0.2
0.25
0.3
0.35



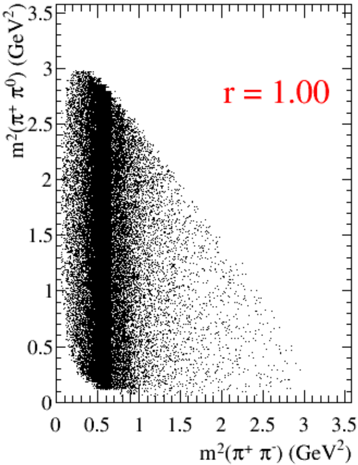
δ = 0
m²(π⁺ π⁰) (GeV²)
m²(π⁺ π⁻) (GeV²)
m²(π⁺ π⁻) (GeV²)

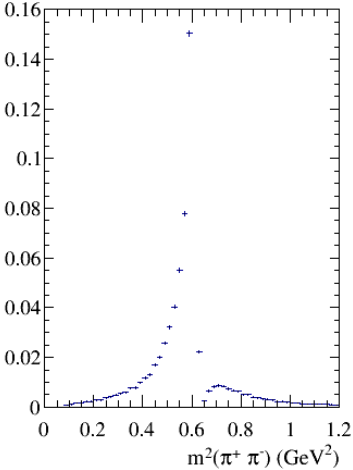


$\delta = 0$
$m^2(\pi^+ \pi^0)$ (GeV^2)
$m^2(\pi^+ \pi^-)$ (GeV^2)
$m^2(\pi^+ \pi^-)$ (GeV^2)



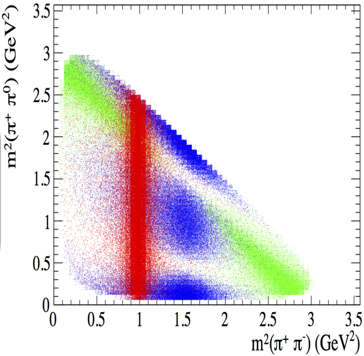
δ = 0
m²(π⁺ π⁰) (GeV²)
m²(π⁺ π⁻) (GeV²)
m²(π⁺ π⁻) (GeV²)


r = 1.00
m^2($\pi^+ \pi^0$) (GeV^2)
m^2($\pi^+ \pi^-$) (GeV^2)


m^2($\pi^+ \pi^-$) (GeV^2)

Toy Dalitz Plot for $D \to \pi^+\pi^-\pi^0$

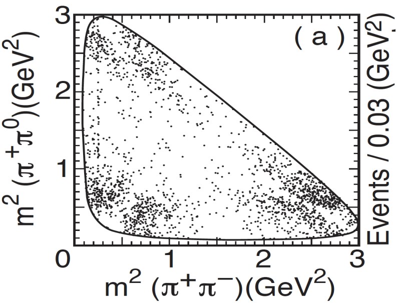
$f_0(980)^0 \to \pi^+\pi^-$ (Spin-0)
$\rho(770)^- \to \pi^-\pi^0$ (Spin-1)
$f_2(1270)^0 \to \pi^+\pi^-$ (Spin-2)



Real Life Dalitz Plot for $D \to \pi^+\pi^-\pi^0$

Consider 4 contributions:

- $A_1 = A(D \to \rho^+\pi^-)$
- $A_2 = A(D \to \rho^0\pi^0)$
- $A_3 = A(D \to \rho^-\pi^+)$
- $A_4 = A(D \to \pi^+\pi^-\pi^0)$



[Phys.Rev.D72:031102,2005]

Dalitz-Fit

- Total amplitude: $A = A_1 + r_2 e^{i\delta_2} A_2 + r_3 e^{i\delta_3} A_3 + r_4 e^{i\delta_4} A_4$
- Prob. density: $\mathcal{P}(x) = \frac{|A(x)|^2}{\int |A(x)|^2 dx}$
- Fit determines relative magnitudes and phases: r_i, δ_i

Live-Stream: Dalitz Fit

Assume resonances $\rho^{\pm}, \rho^{0}$ have spin $S = 0$

Iteration = 0

$r_2 = 1.000$

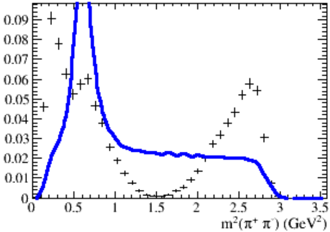
$\delta_2 = 0.000$

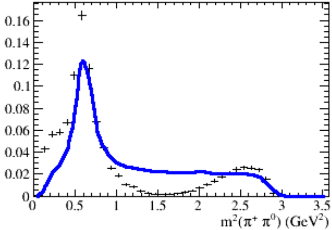
$r_3 = 1.000$

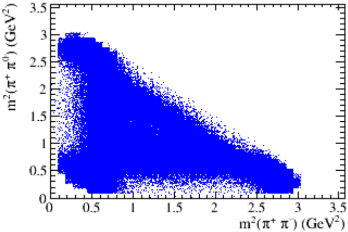
$\delta_3 = 0.000$

$r_4 = 0.100$

$\delta_4 = 0.000$







Assume resonances $\rho^{\pm}, \rho^0$ have spin $S = 1$

Iteration = 0

$r_2 = 1.000$

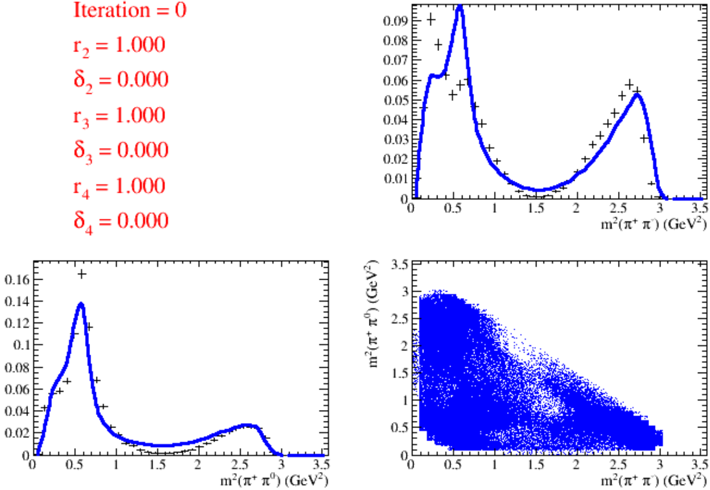
$\delta_2 = 0.000$

$r_3 = 1.000$

$\delta_3 = 0.000$

$r_4 = 1.000$

$\delta_4 = 0.000$



Assume resonances $\rho^{\pm}, \rho^0$ have spin $S = 1$, Fit m_0, Γ_0 of $\rho^{\pm}$

Iteration = 0

$r_2 = 1.000$

$\delta_2 = 0.000$

$r_3 = 1.000$

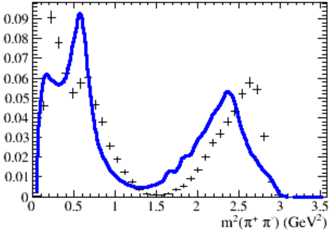
$\delta_3 = 0.000$

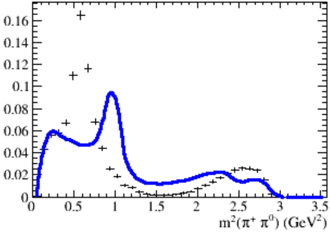
$r_4 = 1.000$

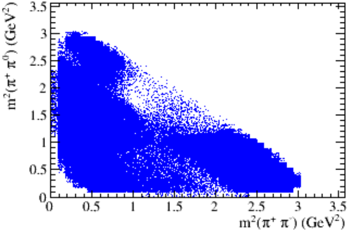
$\delta_4 = 0.000$

$m_0(\rho^+) = 1000$ MeV

$\Gamma_0(\rho^+) = 100$ MeV

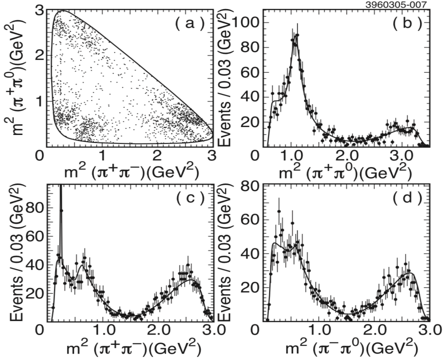






Real life Dalitz Plot for $D \to \pi^+\pi^-\pi^0$

Decay channel	F_i (%)
$D \to \rho^+\pi^-$	$76.6 \pm 1.8 \pm 2.5$
$D \to \rho^0\pi^0$	$23.9 \pm 1.8 \pm 2.1$
$D \to \rho^-\pi^+$	$32.3 \pm 2.1 \pm 1.3$
$D \to \pi^+\pi^-\pi^0$	$2.7 \pm 0.9 \pm 0.2$



[Phys.Rev.D72:031102,2005]

Summary

Dalitz plot

- Visualization of decay process
- Powerful tools for **Spectroscopy:**
  Determine mass, width, spin, parity of resonances
- **Interference** effects provide sensitivity to phases

Backup: Resonance Example

$e^+e^- \to \gamma/Z \to f\bar{f}$

- Resonance curve: $\sigma(s) \propto \frac{1}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}$
- Total decay width: $\Gamma_Z = \Gamma_{had} + 3\Gamma_{lep} + N_\nu\Gamma_\nu$
- Branching fractions: $\mathcal{B}(Z \to f\bar{f}) = \frac{\Gamma_f}{\Gamma_Z}$
- $N_\nu = 2.9840 \pm 0.0082$
  $\Rightarrow$ No room for new physics: Z $\to$ new