Amplitude analysis of $D^0 \to \pi^+\pi^-\pi^+\pi^-$ decays

RTG Lecture 3

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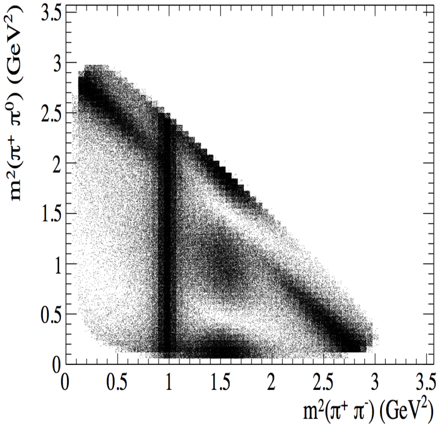
11.06.2017

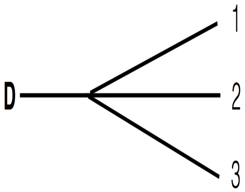
Lecture 1+2:

Phase-space, resonances and angular distributions



($D^0 \to \pi^+\pi^-\pi^0$ Toy Simulation)

Reminder: Kinematic of Multibody Decays



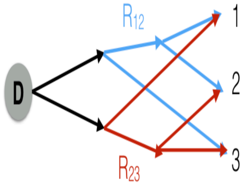
Four-momenta	**12**
Meson masses ($p_i^2 = m_i^2$)	**-3**
E, p conservation	**-4**
Arbitrary orientation (spinless particles)	**-3**
Independent variables	**2**

Fermi's Golden Rule for $D \to 123$

- **Differential decay rate:** $\mathrm{d}\Gamma \propto |A_{D\to123}|^2\,\mathrm{d}\phi_3$
- Convenient (but not unique) choice of variables: $m_{ij}^2 = (p_i^\mu + p_j^\mu)^2$
  $\mathrm{d}\Gamma \propto |A_{D\to123}|^2 \mathrm{d}m_{12}^2\,\mathrm{d}m_{23}^2$
- **Dalitz plot** = Scatter plot of m_{12}^2 vs m_{23}^2
  Deviation from flat distribution $\Rightarrow$ Information on $A_{D\to123}$!

Reminder: Interference of amplitudes

- Define phase-space point: $x = (m_{12}^2, m_{23}^2)$
- Amplitude for process 1:
  $A_1(x) = BW(m_{12}^2)P_L(\theta_{12}) = a_1(x)e^{i\phi_1(x)}$
- Amplitude for process 2:
  $A_2(x) = BW(m_{23}^2)P_L(\theta_{23}) = a_2(x)e^{i\phi_2(x)}$



- Total amplitude: $A(x) = A_1(x) + re^{i\delta}A_2(x)$
- $|A(x)|^2 = |a_1(x)|^2 + |r\,a_2(x)|^2 + 2|a_1(x)||r\,a_2(x)|\cos(\underbrace{\phi_1(x) - \phi_2(x)}_{\Delta\phi(x)} - \delta)$

Lecture 3:

Amplitude analysis of $D^0 \to \pi^+\pi^-\pi^+\pi^-$ decays

[JHEP 05 (2017) 143]

Why multibody charm decays ?

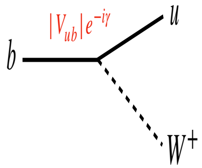
Spectroscopy

- Light hadron spectrum
- Measurement of basic properties:
  Mass, width, branching fractions, quantum numbers
- Provides insights into hadron dynamics

Matter vs. Antimatter

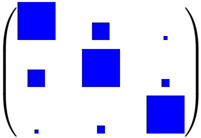
- *CPV* in charm decays not yet observed
  (SM prediction $\mathcal{O}(10^{-3})$)
- Observation of sizable *CPV* $\Rightarrow$ New Physics
- Measurement of CKM angle γ

Reminder: CKM Matrix



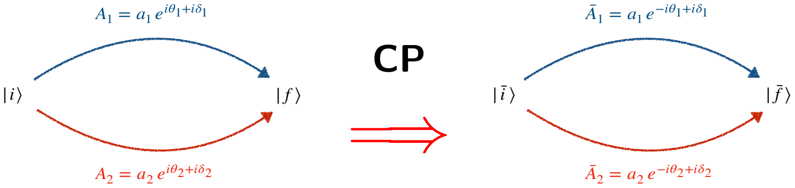
- Quark transitions are described by CKM matrix
- The matrix elements determine the transition probability
- Complex elements are **only** source of CPV in SM
- **Unitarity:** Only 3 real parameters and 1 phase are independent
- **Key test** of the SM: Verify unitarity

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{i\gamma} \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{i\beta} & |V_{ts}| & |V_{tb}| \end{pmatrix} =$$



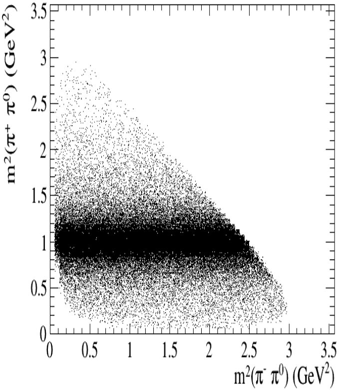
[See lectures by Dominik and Svende for more derails]

How to measure CP Violation ?

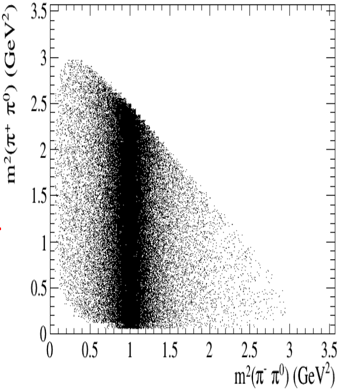


- Global phase is not observable:
  $A \to Ae^{i\theta}$, $|A|^2 \to |A|^2$
- Need at least two interfering processes with different:
  Weak phase : $CP\theta = -\theta$
  Strong phase: $CP\delta = +\delta$
- Asymmetry: $A_{CP} = \frac{\Gamma(i\to f) - \Gamma(\bar{i}\to\bar{f})}{\Gamma(i\to f) + \Gamma(\bar{i}\to\bar{f})} \propto 2\frac{a_2}{a_1} sin(\Delta\theta) sin(\Delta\delta)$

$D^0 \to [R \to \pi^+\pi^0]\pi^-$ $\xrightarrow{CP}$ $\bar{D}^0 \to [\bar{R} \to \pi^-\pi^0]\pi^+$

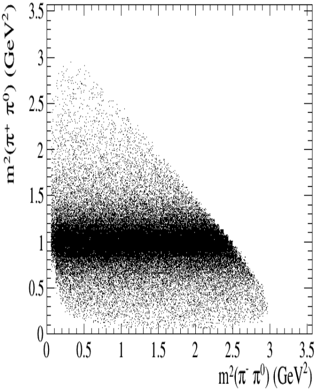


CP $\Longrightarrow$

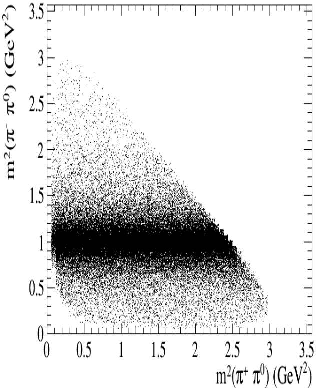


CP Violation in Charm

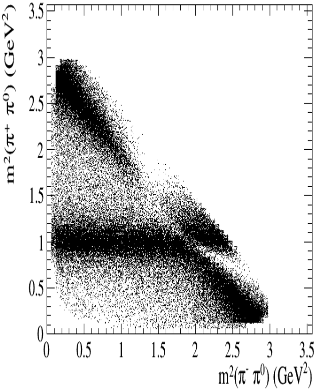
$A_{D^0}(m_{12}^2, m_{23}^2) \quad \overset{CP}{\longrightarrow} \quad A_{\overline{D}^0}(m_{12}^2, m_{23}^2) = A_{D^0}(m_{23}^2, m_{12}^2)$



CP

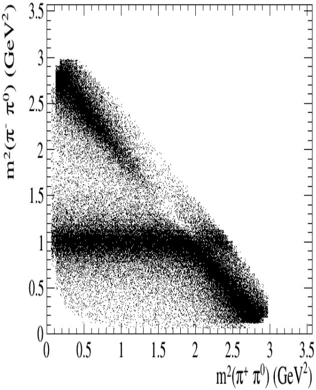


$A_{D^0}(m_{12}^2, m_{23}^2)$ $\xrightarrow{CPV}$ $A_{\overline{D}^0}(m_{12}^2, m_{23}^2) \neq A_{D^0}(m_{23}^2, m_{12}^2)$

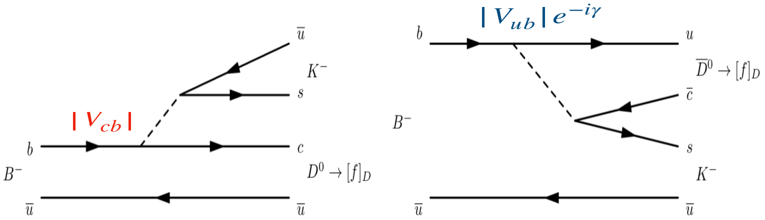


CPV?

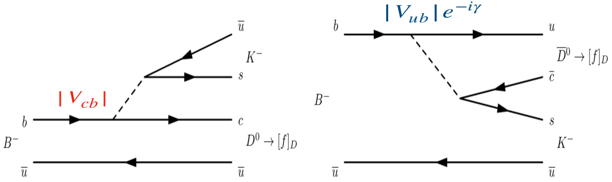
$\Longrightarrow$



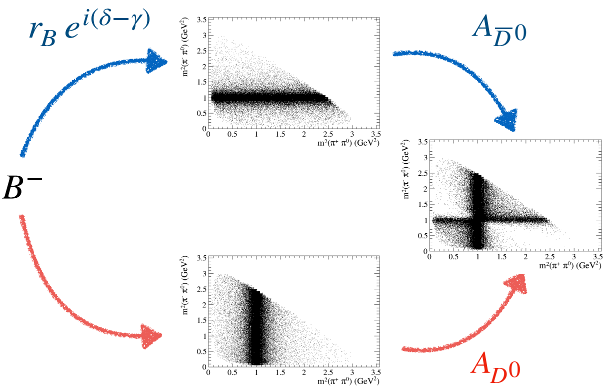
CP Violation in B: Measurement of CKM angle γ



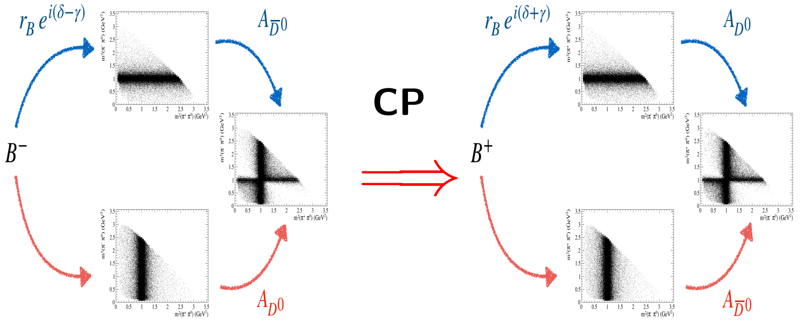
- Close sensitivity gap:
  World average: $\gamma = (73.5^{+4.3}_{-5.7})^\circ$
  Indirect measurement: $\gamma = (65.3^{+1.0}_{-2.5})^\circ$
- Exploit interference between $b \to c$ and $b \to u$ transitions
- Promising example: $B^- \to D^0 K^-$ and $B^- \to \bar{D}^0 K^-$
- D and $\bar{D}$ need to have common final state to achieve interference



$|V_{cb}|$
b
B^-
$\bar{u}$
$\bar{u}$
K^-
s
c
$D^0 \to [f]_D$
$\bar{u}$
$|V_{ub}|e^{-i\gamma}$
b
u
$\overline{D}^0 \to [f]_D$
B^-
$\bar{c}$
s
K^-
$\bar{u}$
$\bar{u}$



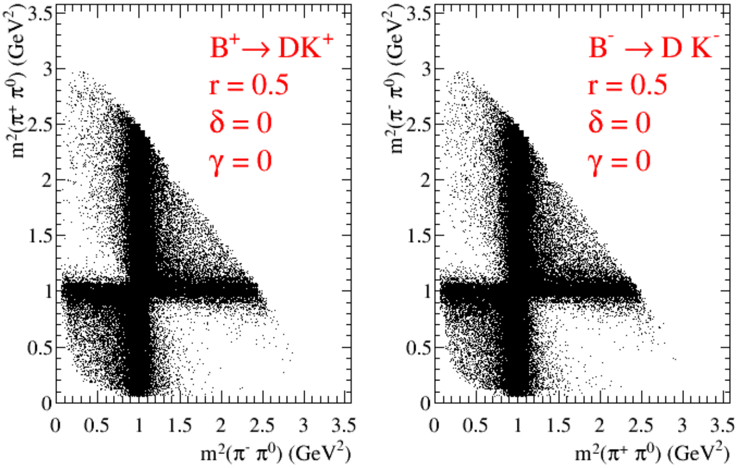
$r_B\, e^{i(\delta-\gamma)}$
$A_{\overline{D}0}$
B^-
A_{D0}
$m^2(\pi^- \pi^0)$ (GeV^2)
$m^2(\pi^+ \pi^0)$ (GeV^2)



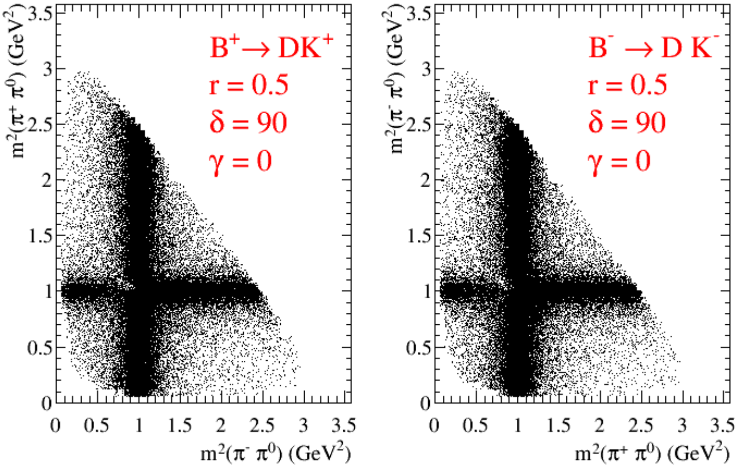
- Small rates: $\mathcal{B}(B^- \to D^0 K^-) \approx O(10^{-7})$
- Small interference: $r_B \approx 0.1$ (driven by CKM elements and color suppression)
- Combine a lot of final states: $f = K_s\pi^+\pi^-, \pi^+\pi^-\pi^0, \underline{\pi^+\pi^-\pi^+\pi^-}, \ldots$
- Need charm input:

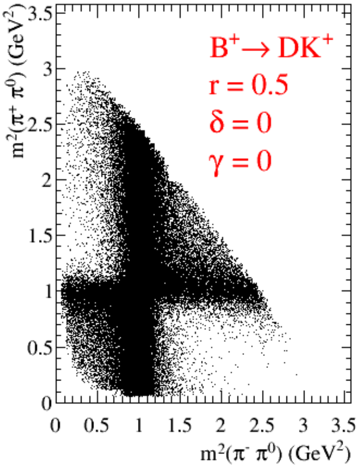
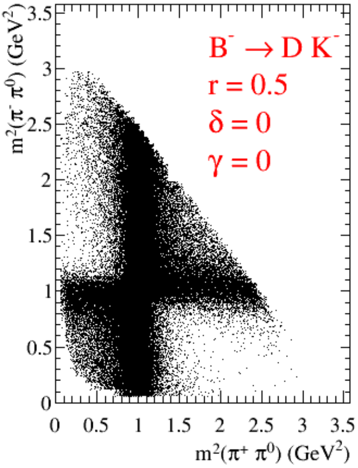
  Fit amplitude model to flavor tagged $D^0, \bar{D}^0$ decays

  Or measure **model-independent** at CLEO-c

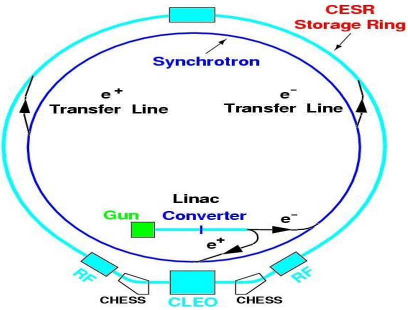


$B^+ \to DK^+$
r = 0.5
δ = 0
γ = 0
$B^- \to D K^-$
r = 0.5
δ = 0
γ = 0
$m^2(\pi^+ \pi^0)$ (GeV^2)
$m^2(\pi^- \pi^0)$ (GeV^2)
$m^2(\pi^- \pi^0)$ (GeV^2)
$m^2(\pi^+ \pi^0)$ (GeV^2)


B+→ DK+
r = 0.5
δ = 90
γ = 0
B- → D K-
r = 0.5
δ = 90
γ = 0
m2(π+ π0) (GeV2)
m2(π- π0) (GeV2)
m2(π- π0) (GeV2)
m2(π+ π0) (GeV2)
0
0.5
1
1.5
2
2.5
3
3.5

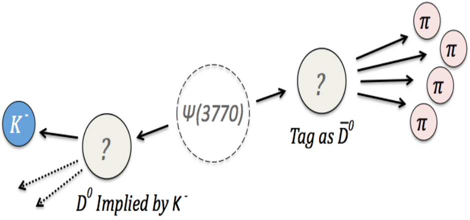

B+→ DK+
r = 0.5
δ = 0
γ = 0
m2(π+ π0) (GeV2)
m2(π- π0) (GeV2)
0
0.5
1
1.5
2
2.5
3
3.5

B- → D K-
r = 0.5
δ = 0
γ = 0
m2(π- π0) (GeV2)
m2(π+ π0) (GeV2)
0
0.5
1
1.5
2
2.5
3
3.5

Introduction to CLEO-c



- Detector at the Cornell Electron Storage Ring (CESR)
- Symmetric $e^+ e^-$ collider
- Operated at $\psi(3770)$ resonance
- $L_{int} = 818$ 1/pb collected

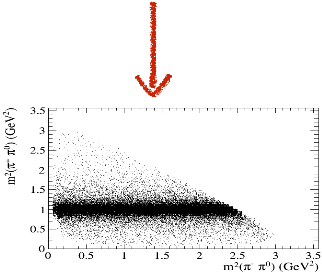
Data Set



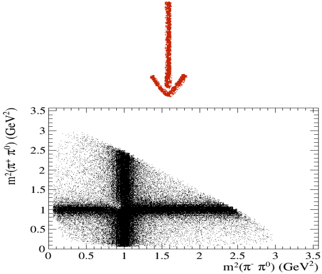
$e^+ e^- \to \psi(3770) \to D_a D_b$

- **Flavour tag:**
  $|c\bar{c}\rangle \to |\bar{c}u\rangle + |c\bar{u}\rangle$
  $D_b \to K^- e^+ \nu_e \Rightarrow D_a = \overline{D^0}$
- **CP tag:**
  $CP(\psi) = 1 \to CP(D_a)CP(D_b)(-1)^{L=1}$
  $D_b \to K^- K^+ (CP+) \Rightarrow D_a =$ CP-
  $\mathcal{A}_{CP\pm} \approx (\mathcal{A}_{D^0} \pm \mathcal{A}_{\overline{D^0}})$

$\psi(3770) \to D_a(D_b \to K^- e^+ \nu_e)$



$\psi(3770) \to D_a(D_b \to K^- K^+)$



Flavor tag:

$|A_{D^0}|^2$

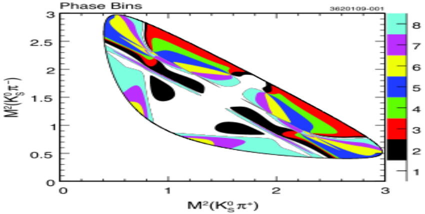
$|A_{\overline{D}^0}|^2$

CP tag:

$|A_{D^0}|^2 + |A_{\overline{D}^0}|^2 \pm 2|A_{D^0}||A_{\overline{D}^0}|cos(\delta_D)$

$\Rightarrow$ Measure phase by counting: $cos(\delta_D) \propto \frac{N_{CP+}}{N_{CP+} + N_{CP-}}$

Multigenerational flavor physics



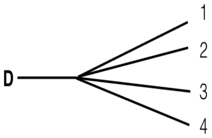
[Phys.Rev. D82 112006]

$D \to 4\pi$ at CLEO-c

- Determine amplitude model from flavor tagged data
- Use model to define optimal binning
- Measure phase from CP tagged data

$B \to (D \to 4\pi)K$ at LHCb

- Use charm input to measure γ

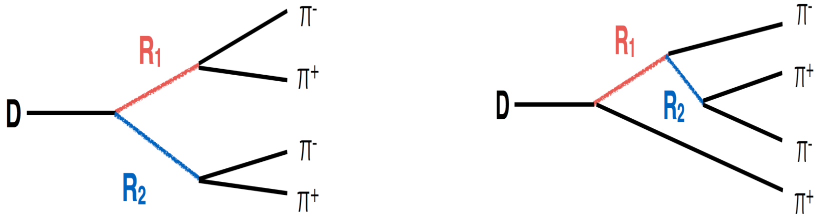


Four-momenta	**12** $\rightarrow$	**16**
Meson masses ($p_i^2 = m_i^2$)	**-3** $\rightarrow$	**-4**
E, p conservation		**-4**
Arbitrary orientation		**-3**
Independent variables	**2** $\rightarrow$	**5**

Decay rate

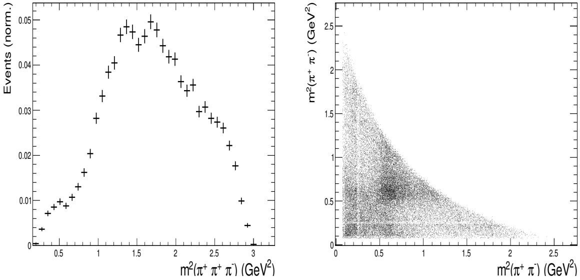
- $d\Gamma \approx |M_{fi}|^2 \, \Phi_4 \, dm_{12}^2 \, dm_{23}^2 \, dm_{34}^2 \, dm_{123}^2 \, dm_{234}^2$
- Phase space density function is not flat ($\Phi_4 \neq 1$)
- 5D phasespace $\Rightarrow$ cannot easily be visualized

Amplitude analysis



- Amplitude $A_i \approx \mathcal{B}W_{R_1} \cdot \mathcal{B}W_{R_2} \cdot S_f$
- $PDF \approx |\sum_i r_i e^{i\delta_i} A_i|^2$
- Amplitude fit determines relative magnitudes r_i and phases $e^{i\delta_i}$

$\approx 7k\ D \to 4\pi$ flavor-tagged signal candidates



Resonances ?

- $a_1(1260) \to \pi\,\pi\,\pi$
- $\rho(770) \to \pi\,\pi$

Decay channels

Plenty of possible decay channels:

- cascade decays
  $D \to \pi^- \left[a_1(1270)^+ \to \rho(770)\,\pi^+\right]$
  $D \to \pi^- \left[a_1(1270)^+ \to \sigma\,\pi^+\right]$
  $D \to \pi^+ \left[a_1(1270)^- \to \rho(770)\,\pi^-\right]$
  $D \to \pi^- \left[a_2(1320)^+ \to \rho(770)\,\pi^+\right]$
- quasi-two-body
  $D \to \sigma\,\sigma$
  $D[S,P,D] \to \rho(770)\,\rho(770)$
- single resonance
  $D \to \sigma\,(\pi\pi)_S$
  $D \to \rho(770)\,(\pi\pi)_S$
- non-resonant
  $D \to (\pi\pi)_S\,(\pi\pi)_S$
  $D[P] \to (\pi\pi)_V\,(\pi\pi)_S$

$\approx$ **100 in total !**

Amplitude model selection

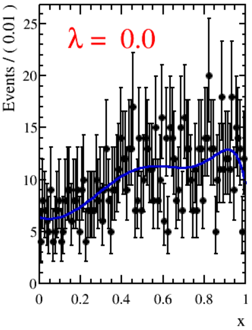
- Overwhelmingly high number of possible amplitudes
- Adding more fit parameters will describe **this** data better
  $\Rightarrow$ **Overfitting**

- **LASSO:** Data-driven method for model selection
  [M. Williams, arXiv:1505.05133]
- Include "all" amplitudes, but penalize complexity in the likelihood:

  $-2\cdot\log(L) \to -2\cdot\log(L) + \lambda \cdot \sum_i |r_i|$
- Larger λ value produces simpler model

LASSO: Toy experiment

- Generated: pdf $= 1 + x$
- Fitted pdf $= 1 + \sum_{i=1}^{10} c_i\, x^i$
- $-2{\cdot}\log(L) \to -2{\cdot}\log(L) + \lambda \cdot \sum_i |c_i|$



$c_1 = -0.9500 \pm 2.1580$

$c_2 = 10.6545 \pm 7.1052$

$c_3 = -6.9912 \pm 11.8217$

$c_4 = -10.2646 \pm 13.3412$

$c_5 = -3.4468 \pm 14.7392$

$c_6 = 5.2567 \pm 15.4221$

$c_7 = 10.3363 \pm 16.0175$

$c_8 = 9.1735 \pm 16.8335$

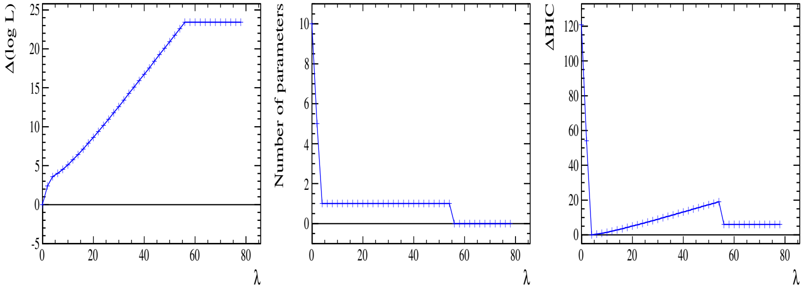
$c_9 = 0.9337 \pm 16.6720$

$c_{10} = -14.1801 \pm 12.4028$

LASSO

How to choose λ ?

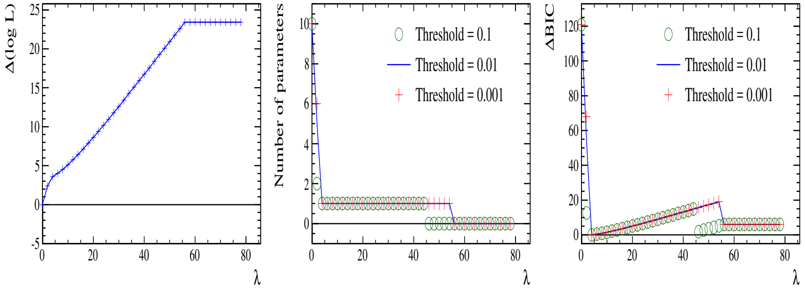
- $\mathrm{BIC}(\lambda) = -2\cdot\log(L) + r\cdot\log(N_{events})$
  r = Number of parameters with: $|c_i| > \text{threshold } (1\%)$
- Balances **gain in fit quality vs. complexity**
- Optimal value $\lambda \approx 4$



LASSO

How to choose λ ?

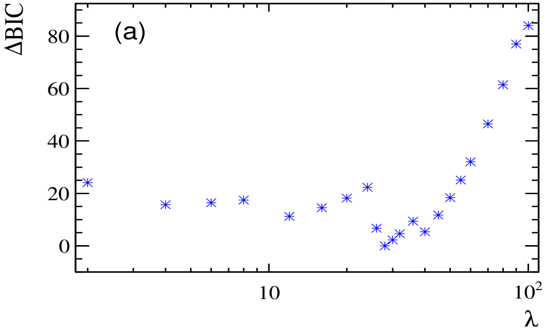
- $\text{BIC}(\lambda) = -2\cdot\log(L) + r\cdot\log(N_{events})$
  r = Number of parameters with: $|c_i| >$ threshold
- Balances **gain in fit quality vs. complexity**
- Optimal value $\lambda \approx 4$



LASSO

How to choose λ ?

- Now for amplitude fit for real data
- Optimal value $\lambda \approx 30$



[JHEP 05 (2017) 143]

$D^0 \to \pi^+\pi^-\pi^+\pi^-$ Amplitude fit

16 amplitudes selected

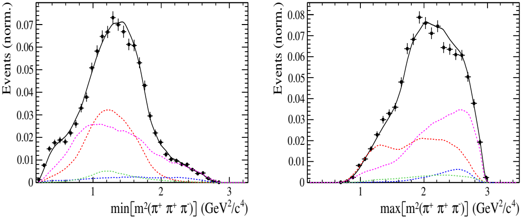
$D \to \pi^- a_1(1260)^+$

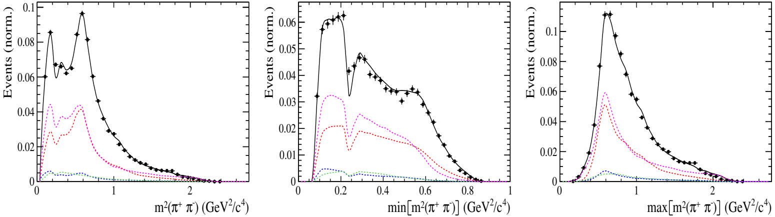
$D \to \pi^- \pi(1300)^+$

$D \to \pi^- a_1(1640)^+$

others

$\chi^2_{5D}/\nu = 1.4$





[JHEP 05 (2017) 143]

Fit Fractions

Fit fraction: $F_i = \frac{\int |r_i A_i|^2 d\Phi}{\int |\sum_j r_j e^{i\delta_j} A_j| d\Phi}$

Decay channel	F_i (%)
$D^0 \to \pi^- \left[a_1(1260)^+ \to \pi^+ \rho(770)^0\right]$	$38.1 \pm 2.3 \pm 3.2 \pm 1.7$
$D^0 \to \pi^- \left[a_1(1260)^+ \to \pi^+ \sigma\right]$	$10.2 \pm 1.4 \pm 2.1 \pm 2.5$
$D^0 \to \pi^+ \left[a_1(1260)^- \to \pi^- \rho(770)^0\right]$	$3.1 \pm 0.6 \pm 0.5 \pm 0.9$
$D^0 \to \pi^+ \left[a_1(1260)^- \to \pi^- \sigma\right]$	$0.8 \pm 0.2 \pm 0.1 \pm 0.4$
$D^0 \to \pi^- \left[\pi(1300)^+ \to \pi^+ \sigma\right]$	$6.8 \pm 0.9 \pm 1.5 \pm 3.1$
$D^0 \to \pi^+ \left[\pi(1300)^- \to \pi^- \sigma\right]$	$3.0 \pm 0.6 \pm 2.0 \pm 2.0$
$D^0 \to \pi^- \left[a_1(1640)^+[D] \to \pi^+ \rho(770)^0\right]$	$4.2 \pm 0.6 \pm 0.9 \pm 1.8$
$D^0 \to \pi^- \left[a_1(1640)^+ \to \pi^+ \sigma\right]$	$2.4 \pm 0.7 \pm 1.1 \pm 1.3$
$D^0 \to \pi^- \left[\pi_2(1670)^+ \to \pi^+ f_2(1270)\right]$	$2.7 \pm 0.6 \pm 0.7 \pm 0.9$
$D^0 \to \pi^- \left[\pi_2(1670)^+ \to \pi^+ \sigma\right]$	$3.5 \pm 0.6 \pm 0.8 \pm 0.9$
$D^0 \to \sigma\, f_0(1370)$	$21.2 \pm 1.8 \pm 4.2 \pm 5.2$
$D^0 \to \sigma\, \rho(770)^0$	$6.6 \pm 1.0 \pm 1.2 \pm 3.0$
$D^0[S] \to \rho(770)^0 \rho(770)^0$	$2.4 \pm 0.7 \pm 1.1 \pm 1.0$
$D^0[P] \to \rho(770)^0 \rho(770)^0$	$7.0 \pm 0.5 \pm 1.6 \pm 0.3$
$D^0[D] \to \rho(770)^0 \rho(770)^0$	$8.2 \pm 1.0 \pm 1.7 \pm 3.5$
$D^0 \to f_2(1270)\, f_2(1270)$	$2.1 \pm 0.5 \pm 0.3 \pm 2.3$
Sum	$122.0 \pm 4.0 \pm 6.4 \pm 7.6$

[JHEP 05 (2017) 143]

Resonance parameters

Resonance		Our Result (MeV)	PDG (MeV)
$a_1(1260)$	m_0	$1225 \pm 9 \pm 16 \pm 10$	1230 ± 40
	Γ_0	$430 \pm 24 \pm 13 \pm 18$	$250 - 600$
$\pi(1300)$	m_0	$1128 \pm 16 \pm 56 \pm 37$	1300 ± 100
	Γ_0	$314 \pm 39 \pm 58 \pm 26$	$200 - 600$
$a_1(1640)$	m_0	$1691 \pm 18 \pm 16 \pm 25$	1647 ± 22
	Γ_0	$171 \pm 33 \pm 20 \pm 35$	254 ± 7

[JHEP 05 (2017) 143]

Resonance parameters

$a_1(1260)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
250 to 600 OUR ESTIMATE				
$367 \pm 9^{+28}_{-25}$	420k	ALEKSEEV	10	COMP	$190\ \pi^- Pb \to \pi^-\pi^-\pi^+ Pb'$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$410 \pm 31 \pm 30$		[18] AUBERT	07AU	BABR	$10.6\ e^+e^- \to \rho^0\rho^\pm\pi^\mp\gamma$
520–680	6360	[19] LINK	07A	FOCS	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
480 ± 20		[20] GOMEZ-DUM..	04	RVUE	$\tau^+ \to \pi^+\pi^+\pi^-\nu_\tau$
580 ± 41	90k	SALVINI	04	OBLX	$\bar{p}p \to 2\pi^+2\pi^-$
460 ± 85	205	[21] DRUTSKOY	02	BELL	$B \to D^{(*)}K^-K^{*0}$
$814 \pm 36 \pm 13$	37k	[22] ASNER	00	CLE2	$10.6\ e^+e^- \to \tau^+\tau^-$, $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$
450 ± 50	22k	[23] AKHMETSHIN	99E	CMD2	1.05–$1.38\ e^+e^- \to \pi^+\pi^-\pi^0\pi^0$
570 ± 10		[24] BONDAR	99	RVUE	$e^+e^- \to 4\pi$, $\tau \to 3\pi\nu_\tau$
$587 \pm 27 \pm 21$	5904	[25] ABREU	98G	DLPH	e^+e^-
$478 \pm 3 \pm 15$	5904	[26] ABREU	98G	DLPH	e^+e^-
$425 \pm 14 \pm 8$	5904	[27,28] ABREU	98G	DLPH	e^+e^-
400 ± 35		BARBERIS	98B		$450\ pp \to p_f\pi^+\pi^-\pi^0 p_s$
$621 \pm 32 \pm 58$		[25,29] ACKERSTAFF	97R	OPAL	$E^{ee}_{cm} = 88$–94, $\tau \to 3\pi\nu$
$457 \pm 15 \pm 17$		[26,29] ACKERSTAFF	97R	OPAL	$E^{ee}_{cm} = 88$–94, $\tau \to 3\pi\nu$

$a_1(1260)$ WIDTH

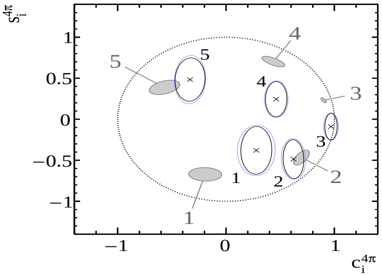
VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
250 to 600 OUR ESTIMATE				
389 ± 29 OUR AVERAGE Error includes scale factor of 1.3.				
$430 \pm 24 \pm 31$		DARGENT	17	RVUE	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
$367 \pm 9^{+28}_{-25}$	420k	ALEKSEEV	10	COMP	$190\ \pi^- Pb \to \pi^-\pi^-\pi^+ Pb'$
• • • We do not use the following data for averages, fits, limits, etc. • • •				
$410 \pm 31 \pm 30$		[18] AUBERT	07AU	BABR	$10.6\ e^+e^- \to \rho^0\rho^\pm\pi^\mp\gamma$
520–680	6360	[19] LINK	07A	FOCS	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
480 ± 20		[20] GOMEZ-DUM..	04	RVUE	$\tau^+ \to \pi^+\pi^+\pi^-\nu_\tau$
580 ± 41	90k	SALVINI	04	OBLX	$\bar{p}p \to 2\pi^+2\pi^-$
460 ± 85	205	[21] DRUTSKOY	02	BELL	$B \to D^{(*)}K^-K^{*0}$
$814 \pm 36 \pm 13$	37k	[22] ASNER	00	CLE2	$10.6\ e^+e^- \to \tau^+\tau^-$, $\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$
450 ± 50	22k	[23] AKHMETSHIN	99E	CMD2	1.05–$1.38\ e^+e^- \to \pi^+\pi^-\pi^0\pi^0$
570 ± 10		[24] BONDAR	99	RVUE	$e^+e^- \to 4\pi$, $\tau \to 3\pi\nu_\tau$
$587 \pm 27 \pm 21$	5904	[25] ABREU	98G	DLPH	e^+e^-
$478 \pm 3 \pm 15$	5904	[26] ABREU	98G	DLPH	e^+e^-
$425 \pm 14 \pm 8$	5904	[27,28] ABREU	98G	DLPH	e^+e^-
400 ± 35		BARBERIS	98B		$450\ pp \to p_f\pi^+\pi^-\pi^0 p_s$
$621 \pm 32 \pm 58$		[25,29] ACKERSTAFF	97R	OPAL	$E^{ee}_{cm} = 88$–94, $\tau \to 3\pi\nu$
$457 \pm 15 \pm 17$		[26,29] ACKERSTAFF	97R	OPAL	$E^{ee}_{cm} = 88$–94, $\tau \to 3\pi\nu$

Decay Rate Asymmetries for $D \to \pi^+\pi^-\pi^+\pi^-$

$$\mathcal{A}^i_{CP} = \frac{F_i - \bar{F}_i}{F_i + \bar{F}_i}$$

Decay channel	$\mathcal{A}^i_{CP}$ (%)	Significance (σ)
$D^0 \to \pi^- \, a_1(1260)^+$	$+4.7 \pm 2.6 \pm 4.3 \pm 2.4$	0.9
$D^0 \to \pi^+ \, a_1(1260)^-$	$+13.7 \pm 13.8 \pm 9.8 \pm 5.8$	0.8
$D^0 \to \pi^- \, \pi(1300)^+$	$-1.6 \pm 12.9 \pm 5.0 \pm 4.4$	0.1
$D^0 \to \pi^+ \, \pi(1300)^-$	$-5.6 \pm 11.9 \pm 25.6 \pm 10.3$	0.2
$D^0 \to \pi^- \, a_1(1640)^+$	$+8.6 \pm 17.8 \pm 16.0 \pm 10.8$	0.3
$D^0 \to \pi^- \, \pi_2(1670)^+$	$+7.3 \pm 15.1 \pm 8.0 \pm 6.6$	0.4
$D^0 \to \sigma \, f_0(1370)$	$-14.6 \pm 16.5 \pm 9.3 \pm 1.3$	0.8
$D^0 \to \sigma \, \rho(770)^0$	$+2.5 \pm 16.8 \pm 13.8 \pm 14.6$	0.1
$D^0 \to \rho(770)^0 \, \rho(770)^0$	$-5.6 \pm 5.0 \pm 2.2 \pm 1.9$	1.0
$D^0 \to f_2(1270) \, f_2(1270)$	$-28.3 \pm 12.3 \pm 18.5 \pm 9.7$	1.2

$\Rightarrow$ **No** evidence for CP violation
[JHEP 05 (2017) 143]



[JHEP 1801 (2018) 144]

- Strong phase measured in 5 phase-space bins
  $c_i^{4\pi} = \int_{\text{bin i}} |A_D||A_{\bar{D}}| cos(\delta_D) \propto \frac{N_{CP+}}{N_{CP+}+N_{CP-}}$
  $s_i^{4\pi} = \int_{\text{bin i}} |A_D||A_{\bar{D}}| sin(\delta_D)$
- Binning optimized for sensitivity to γ (gain of factor 2.2)
- Good consistency with model prediction:
  $\chi^2/ndof = 13.7/10\,(p = 0.19)$
- Estimated sensitivity $\sigma(\gamma) \approx 12^\circ(5^\circ)$ for LHCb-Run-II(III)

Summary

Amplitude analysis of $D \to \pi^+\pi^-\pi^+\pi^-$

- Performed data-driven model selection
- Determined mass and width of $a_1(1260)$ meson
- **No** evidence for *CP* violation
- Crucial input for future measurements of CKM angle γ and charm mixing at LHCb