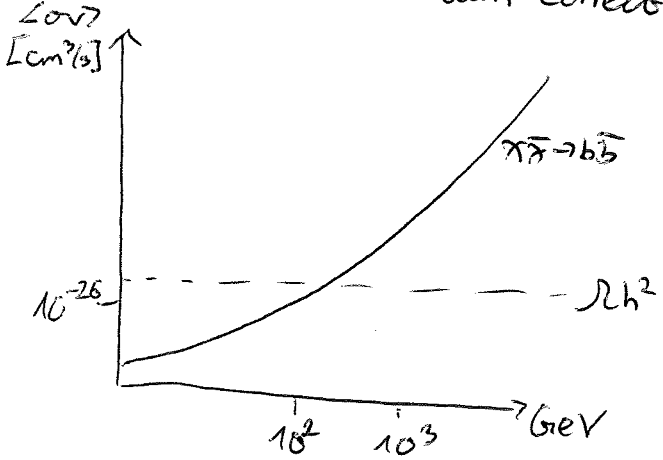
24.07.20 ①

Last Lecture:

Finished with comparison of annihilation cross-section and cross-section to obtain correct relic abundance for thermal WIMPs:



for antiproton detection with AMS

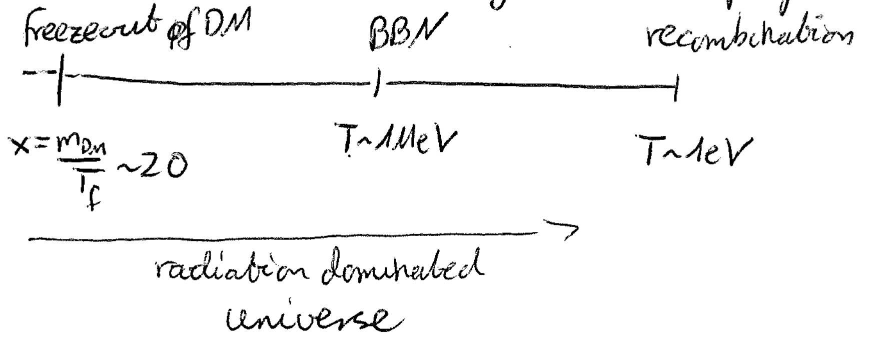
$\Rightarrow$ strong limits on GeV DM!

In more detail: $\frac{1}{A}\frac{dN}{dE\,dt} = \underbrace{\frac{1}{4\pi}\frac{dN}{dE}\begin{cases}\frac{\langle\sigma v\rangle}{2m_{DM}^2} \\ \frac{1}{m_{DM}\tau_{DM}}\end{cases}}_{\text{particle physics}} \underbrace{\begin{matrix}\int dr\, d\Omega\, \rho(\vec{r})^2 \text{ annihilation} \\ \int dr\, d\Omega\, \rho(\vec{r}) \text{ decay}\end{matrix}}_{\text{N-body simulation}}$

N-body simulation
measured by gravitational probes
$\Rightarrow$ much denser regions chosen than example of local halo

More effects of annihilation of DM?

$\Rightarrow$ could influence Early Universe physics



Freezeout shortly: For particles in kinetic equilibrium

$\hookrightarrow$ number density $n = \frac{g}{(2\pi)^3}\int f(\vec{p})\, d^3p$ with $f(\vec{p}) = \frac{1}{e^{(E-\mu)/T} \pm 1}$

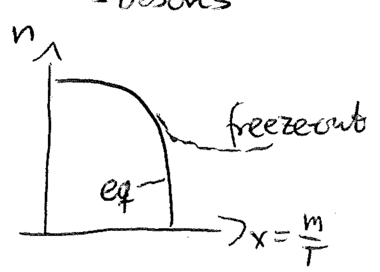
+ fermions
\- bosons

For non-relativistic particles (like DM): $T \ll m, m-\mu$

$E \approx m + \frac{p^2}{2m}$

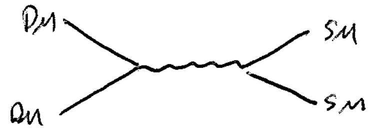
$\Rightarrow e^{(E-\mu)/T} \pm 1 \approx e^{(E-\mu)/T}$ $\Rightarrow$ Maxwell-Boltzmann statistics

$\hookrightarrow n_{eq} = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T}$ $\Rightarrow$ Boltzmann-suppression



number density scales like $\sim a^{-3}$ (scale factor for expansion of universe)
↳ i.e. $\frac{d}{dt}(na^3) = \frac{dn}{dt} + 3Hn = 0$ if no process changes number density

Include $\Gamma_{ann} \sim \langle\sigma v\rangle n_{DM}^2$



$\Rightarrow \quad \frac{dn_{DM}}{dt} + 3H n_{DM} = -\langle\sigma v\rangle (n_{DM}^2 - n_{eq}^2)$ $\quad$ H Hubble parameter

Change of variables $Y = \frac{n}{s}$ with entropy $s \sim a^{-3}$ and $x = \frac{m_{DM}}{T}$

$\Rightarrow \quad \frac{dY}{dx} = -\frac{s x \langle\sigma v\rangle}{H x^2}(Y^2 - Y_{eq}^2) \underset{\text{later times}}{\approx} -\frac{\lambda\langle\sigma v\rangle}{x^2} Y^2$ with constant λ

assume $\langle\sigma v\rangle$ independent of x $\Rightarrow$ $\frac{1}{Y_\infty} = \frac{1}{Y_f} + \frac{\lambda}{x_f}\langle\sigma v\rangle$

↳ $\cancel{Y_\infty} \gg Y_f$ $\quad \boxed{Y_\infty \approx \frac{x_f}{\lambda\langle\sigma v\rangle}}$

x_f determined by $\langle\sigma v\rangle n_{DM}^2 \sim Hn$
(Hubble expansion rate comparable to time needed for DM particles to annihilate)
$x_f \sim 20$

From then on: # of annihilations in comoving volume V_c in one Hubble time

$\frac{dN}{dt} = \cancel{\frac{A}{4\pi r^2}} \frac{\langle\sigma v\rangle}{2} n_{DM}^2 \, dV \rightarrow N_{ann} \approx \frac{n^2\langle\sigma v\rangle}{2} V_c H^{-1}$ $\quad$ $n \sim a^{-3}$, $V \sim a^3$

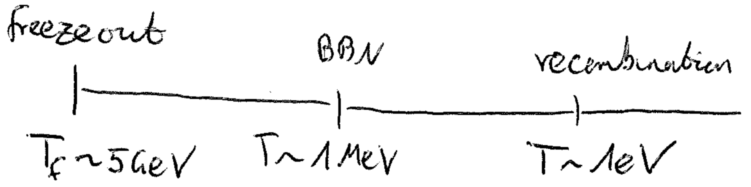
dt ↓ one Hubble time; $\cancel{\frac{A}{4\pi r^2}}$ no detector; dV ↳ V_c

$\propto \frac{\langle\sigma v\rangle}{2} a^{-1}$ $\quad$ rad. dominated $H \sim a^{-2}$

$a \sim T^{-1}$

Per definition: T_f: $\langle\sigma v\rangle n \sim H$ DM particle annihilations on average once per Hubble time at DM freezeout $\Rightarrow N_{anni,f} \sim 1$

Example $m_{DM} = 100\,\text{GeV}$, assume $\langle\sigma v\rangle$ 'constant' ⇝ temperature independent



$\boxed{\text{BBN}}$: production of light nuclei ($\rightsquigarrow$ Helium abundance) ③

- $\frac{N_{ann}(\text{BBN})}{N_{ann}(\text{freeze-out})} \sim \frac{N_{ann}(\text{BBN})}{1} \sim \frac{a_f}{a_{BBN}} \sim \frac{T_{BBN}}{T_f} \sim \frac{1\,\text{MeV}}{5\,\text{GeV}} \sim \frac{1}{5} \cdot 10^{-3}$

energy injection of DM annihilation into BBN

$$N_{ann} \cdot \rho_{DM} \sim \frac{1}{5} \cdot 10^{-3} \cdot 5 \cdot \rho_B \sim 10^{-3} \rho_B \sim 10^{-3} \cdot 1\,\text{GeV} \cdot n_B$$

$\Rightarrow$ 1 MeV per baryon $\Rightarrow$ affects subdominant nuclear abundances

$\boxed{\text{CMB}}$: recombination: electrons and protons form bound state $\rightsquigarrow$ hydrogen atoms
$\hookrightarrow$ universe is neutral $\to$ photons travel freely, i.e. last scattering at time of recombination $\to$ after that cosmic microwave bkg.

$$N_{ann}(\text{CMB}) \sim \frac{1\,\text{eV}}{5\,\text{GeV}} \sim \frac{1}{5} 10^{-9} \Rightarrow N_{ann} \cdot \rho_{DM} \sim 1\,\text{eV} \cdot n_B$$

$\Rightarrow$ 1 eV per baryon, compared to hydrogen ionization ~ 13.6 eV
$\Rightarrow$ DM could significantly ionize universe $\to$ calculated by public codes
$\Rightarrow$ extra electrons broaden surface of last scattering

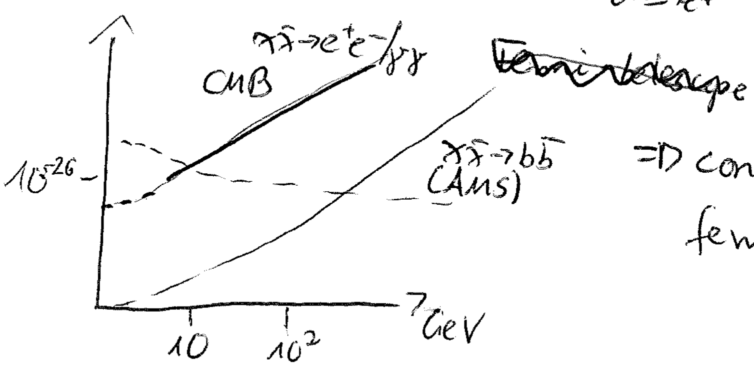
in Detail:

$$\frac{dE}{dt\,dV}(z) \propto (1+z)^6 \underbrace{f(z) \frac{\langle \sigma v \rangle}{m_{DM}}}_{\text{particle physics}} \rho_{DM}^2$$

$f(z)$* characterizes fraction of energy released into gas (almost no redshift dependence)

$\rightsquigarrow$ effectively

$$f_{eff} \sim \frac{1}{2 m_{DM}} \int_0^{m_{DM}} E\,dE \left[2 f_{eff}^{e^+e^-}(E) \left(\frac{dN}{dE}\right)_{e^+} + f_{eff}^{\gamma}(E) \left(\frac{dN}{dE}\right)_{\gamma} \right]$$



$\Rightarrow$ constraints on few GeV DM

* Calculation cooling of electromagnetically interacting particles, and fraction of energy converted to hydrogen ionization and how long cooling process takes

Questions: (1) What assumptions have been made?

> velocity-averaged cross-section is dominated or fully determined by its velocity-independent term → no temperature dependence

> thermal relic abundance is obtained by one single annihilation process

> the total relic abundance is determined by thermal DM annihilation process(es)

⇒ (2) So what do the exclusion plots tell us?

→ That the relic abundance is not determined by one single velocity-independent annihilation process!

⇒ Do we have more than one annihilation process?

> Is one of them velocity-suppressed?

> Is one of them invisible to ID in the final states?

⇒ How much can a vel.-independent annihilation contribute to the total relic abundance? Is WIMP just a subcomponent of DM?

(⇒ is DM not completely symmetric between particles/anti-particles?)

⇒ for everything that weakens ID constraints especially the GeV/sub-GeV region is interesting.

Velocity dependence of cross-section

"Task": expand $\langle \sigma v \rangle$ in $x = \frac{m_{DM}}{T}$

We have: $\sigma v_{rel} = \frac{1}{4E_1E_2} \underbrace{\int dLIPS\, |M|^2}_{\text{Lorentz-invariant}}$, $dLIPS = (2\pi)^4 \delta^{(4)}(p_1+p_2 - \sum_i p_i) \times \prod_i \frac{d^3 p_i}{(2\pi)^3 2p_i^0}$

$\hookrightarrow w(\hat{s})$ and $\hat{s} = (p_1+p_2)^2$
$= 2(m_{DM}^2 + E_1E_2 - p_1p_2\cos\theta)$

thermal average:

$$\langle \sigma v_{rel} \rangle = \frac{1}{n_0^2} \int d^3p_1\, d^3p_2\, f(E_1) f(E_2) \frac{1}{E_1E_2} w(\hat{s}) \quad , \; n_0 = \int d^3p\, f(E)$$

$$= \frac{g^2}{8\pi^4 n_0^2} \int_{m_{DM}}^{\infty} dE_1\, dE_2\, p_1 p_2\, e^{-E_1/T} e^{-E_2/T} \int d\cos\, w(\hat{s})$$

Change of variables: $E_i = m_{DM}(1 + x^{-1} y_i) \quad y_i \in [0,\infty]$

$$p_i = m_{DM}(2x^{-1})^{1/2} \left(y_i + \tfrac{1}{2}x^{-1} y_i^2\right)^{1/2}$$

$$\hookrightarrow \frac{\hat{s}}{4m_{DM}^2} = 1 + \tfrac{1}{2}(y_1+y_2)x^{-1} + \tfrac{1}{2} y_1 y_2 x^{-2} - x\left(y_1 + \tfrac{1}{2}x^{-1}y_1^2\right)^{1/2}\left(y_2 + \tfrac{1}{2}x^{-1}y_2^2\right)^{1/2} \cos\theta$$

expand $w(\hat{s})$ in x^{-1} by Taylor exp. of $w(\hat{s})$ around $\frac{\hat{s}}{4m_{DM}^2} = 1$
+ n_0 expressed in terms of x

$$\Rightarrow \langle \sigma v_{rel} \rangle = \frac{1}{m_{DM}^2}\left[w - \tfrac{3}{2}(2w - w')x^{-1} + \tfrac{3}{8}(16w - 8w' + 5w'')x^{-2} + \mathcal{O}(x^{-3}) \right.$$

$$=: a + bx^{-1} + cx^{-2} + \mathcal{O}(x^{-3})$$ and with $v_{rel}^2 \sim \frac{T}{m_{DM}}$

$\Rightarrow$ partial wave expansion $\langle \sigma v_{rel} \rangle = \sum_L a^{(L)} v^{2L}$ with orbital angular momentum L of initial state

$$= \underset{\text{s-wave}}{a^0} + \underset{\text{p-wave}}{a^1 v^2} + \ldots$$

$\Rightarrow$ partial wave expansion helps to make statement about coefficients of expansion. If process has

$L=0$ component $\Rightarrow$ velocity-independent (example Dirac $\chi\bar{\chi} \to Z' \to f\bar{f}$)
$L=0$ absent $\Rightarrow$ velocity-suppressed (complex scalar $\chi\chi^+ \to Z' \to f\bar{f}$)