Quantum gravity constraints on inflation

by Philipp Henkenjohann, December 2018

1 Introduction to cosmology and inflation

1.1 Cosmology

Visible universe is spatially homogeneous and isotropic on scales $\gg 100\,Mpc$.

$\Rightarrow$ FRW metric: $ds^2 = dt^2 - \underset{\nearrow}{a^2(t)}\,\underbrace{(dx^2+dy^2+dz^2)}_{\text{could allow curvature here}}$

scale factor

$a(t)$ describes expansion/contraction of universe.
2 observers at fixed coordinates $\vec{x}_1$ and $\vec{x}_2$ have physical distance $d = a(t)\,|\vec{x}_1 - \vec{x}_2|$.
$\rightarrow$ like two points on stretched rubber band:

$a(t)$ is determined by matter content of universe via Einstein equation: $\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{depends on } a} = \frac{1}{M_P^2} T^{\mu\nu}$.

energy-momentum tensor

At large scales we can describe matter as perfect fluid

$\Rightarrow T^{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$

energy density pressure

$$\Rightarrow \begin{cases} \text{Friedmann eq.: } \left(\frac{\dot a}{a}\right)^2 = \frac{\rho}{3M_P^2} \\ \quad + \\ \text{continuity eq.: } \dot\rho = -3\frac{\dot a}{a}(\rho + p) \end{cases}$$

$H := \frac{\dot a}{a}$ Hubble parameter, measures speed of expansion /contraction

Specifying eq. of state $p(\rho)$ allows us to solve this for $\rho(a)$:

radiation (relativistic particles) : $\rho \propto \frac{1}{a^4}$

cold matter (non-relativistic ") : $\rho \propto \frac{1}{a^3}$

cosmological constant : $\rho = \text{const.}$

Universe is filled with all types of matter:

$$\rho_{tot} = \rho_{rad} + \rho_{mat} + \rho_\Lambda \sim \frac{1}{a^4} + \frac{1}{a^3} + \text{const.}$$

$\Rightarrow$ history of universe:

big bang $(a=0)$

radiation domination — photons interact with charged particles $(e^-, p^+, \dots)$

recombination (neutral atoms)

matter domination — free-streaming photons $\uparrow$ CMB

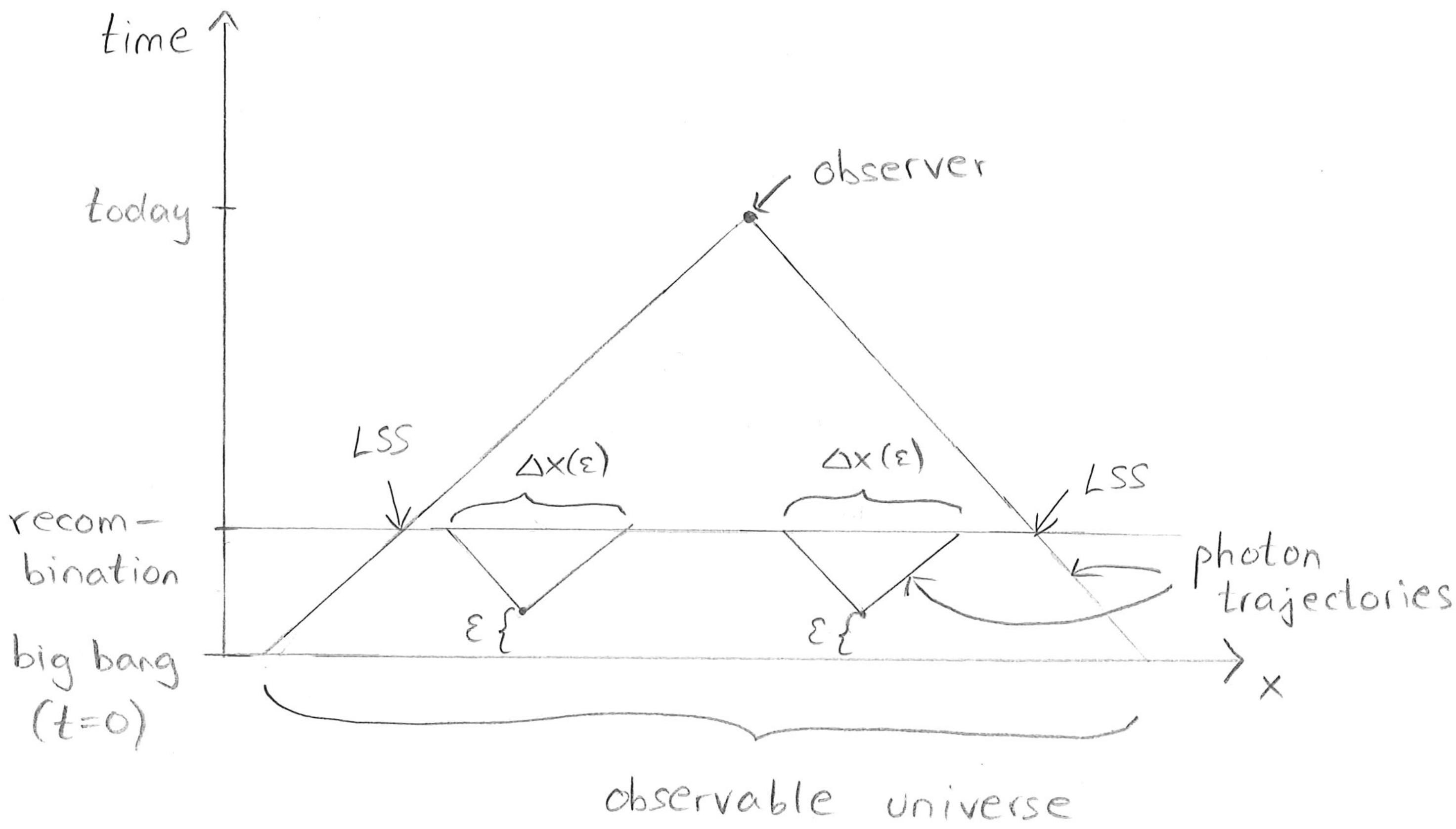
cosmological constant domination

today

time

CMB spectrum is thermal with $\langle T \rangle \approx 2.7\,K$ and $\frac{T - \langle T \rangle}{\langle T \rangle} \sim 10^{-5}$ $\Rightarrow$ almost perfectly isotropic!

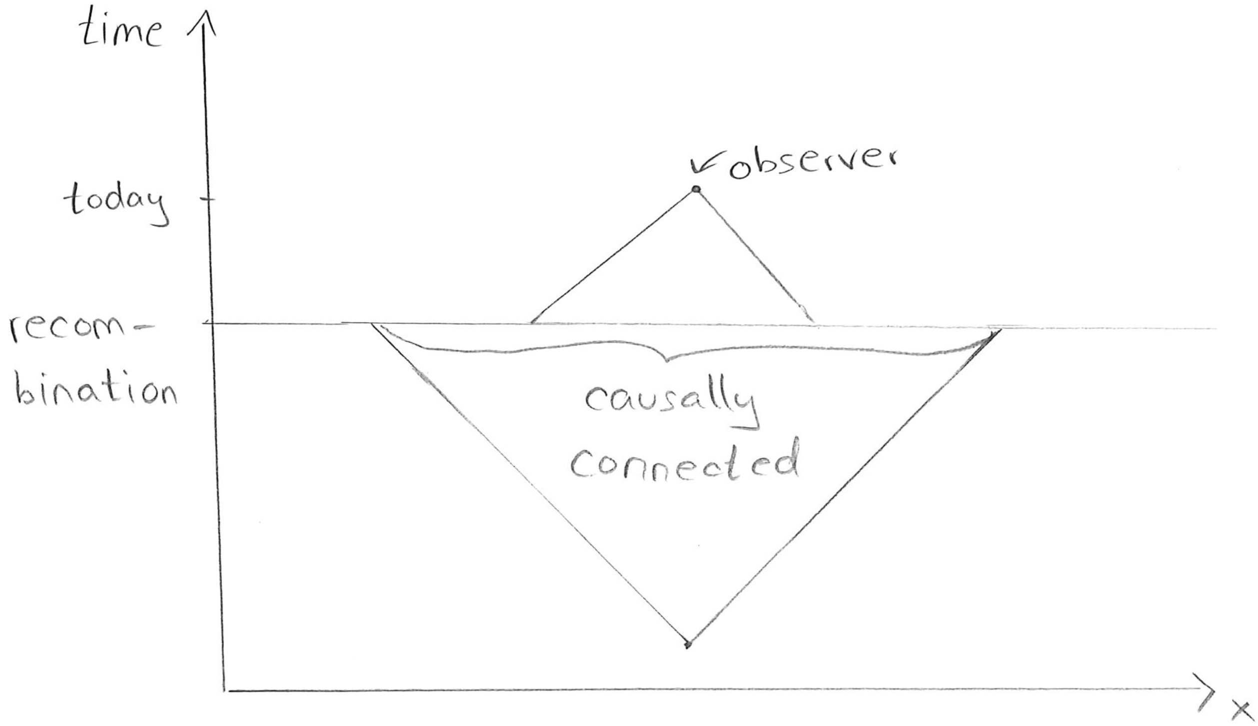
This is quiet surprising because photons seperated by $\gtrsim 1°$ have never been in causal contact in the past. Generically, we would therefore expect the CMB temperature to fluctuate considerably on this scale.



LSS = surface of last scattering

Even for $\varepsilon \to 0$, $\Delta x(\varepsilon) \ll$ observable universe at recombination. $\Rightarrow$ Horizon problem

Solution: Extend the time period before recombination:



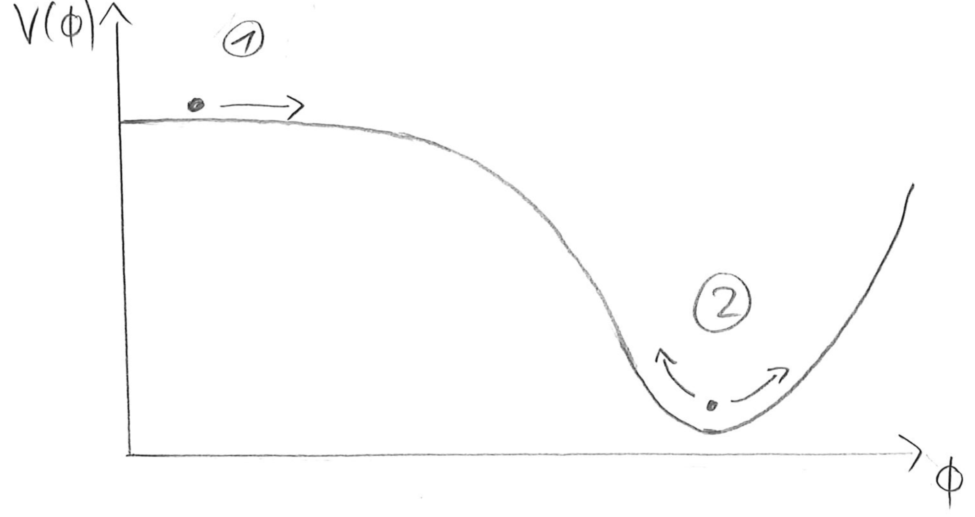
This can be realized by having an early phase of accelerated expansion, e.g. $H = \frac{\dot{a}}{a} = \text{const.}$
$\Rightarrow a \propto e^{Ht} \Rightarrow$ range of t is extended from $(0, \infty)$ to $(-\infty, \infty)$ (heuristic argument)

Such a phase is called inflation.

1.2 Inflation

Friedmann eq.: $H^2 = \text{const.} = \frac{\rho}{3M_P^2} \Rightarrow \rho = \text{const.}$

Realize this with a spatially homogeneous real scalar field ϕ with potential $V(\phi)$:



① slow rolling in flat region $\Rightarrow \rho \approx \text{const.}$

② damped oscillations and decay into SM particles energy density: $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$

Inflation requires:

1) $\rho \approx$ const. $\Rightarrow \quad \dot{\phi}^2 < V(\phi)$

2) $\rho \approx$ const holds long enough, i.e $\dot{\phi}$ increases very little in typical time $\frac{1}{H}$: $\Delta\dot{\phi} = |\ddot{\phi}|\frac{1}{H} < |\dot{\phi}|$

1) + 2) + e.o.m. gives slow-roll conditions:

$$\varepsilon := \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 < 1 \quad , \quad \eta := M_P^2 \left|\frac{V''}{V}\right| < 1$$

simple example: $V(\phi) = \frac{1}{2} m^2 \phi^2$

$\Rightarrow \quad \varepsilon = \eta = 2\left(\frac{M_P}{\phi}\right)^2$

$\Rightarrow$ inflation for $\phi > M_P$

This is called large-field inflation and works for models with $V \propto \phi^n$, $n > 1$

remarks:

- ϕ is called inflaton
- quantum fluctuations of inflaton are seeds for structure formation and lead to CMB fluctuations

1.3 Natural inflation

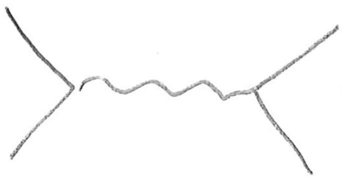
Problem: Unknown physics, e.g. heavy particles with mass M, induce corrections of the form ϕ^n/M^{n-4} to the inflaton potential $V(\phi)$:

$$V(\phi) \to V(\phi) + \sum_n a_n \frac{\phi^n}{\Lambda^{n-4}} + \sum_n b_n \frac{(\partial\phi)^{2n}}{\Lambda^{4n-4}} + \dots$$

Λ = scale of new physics

$a_n, b_n = \mathcal{O}(1)$

This is similar to Fermi's 4-fermion-interaction:

 $\sim \dfrac{g^2}{E_{CM}^2 - m_W^2} \xrightarrow{E_{CM} \ll m_W}$  $\sim \dfrac{g^2}{m_W^2}$

Large field inflation: $\phi > M_p \gtrsim \Lambda \Rightarrow$ lose control!

(Approximate) symmetries forbid (suppress) the dangerous operators $\propto \phi^n$, e.g. shift symmetry:

$\phi \to \phi + c, \quad c \in \mathbb{R}$

concrete realization via massless scalar ϕ (axion) coupled to Yang-Mills theory:

$$\mathcal{L} = \frac{1}{2g^2} \operatorname{tr} F^2 + \frac{1}{2}(\partial\phi)^2 + \frac{1}{16\pi^2} \frac{\phi}{f} \operatorname{tr} F\tilde{F}$$

f = axion decay constant

$$\phi \to \phi + c \Rightarrow \mathcal{L} \to \mathcal{L} + \underbrace{\frac{1}{16\pi^2} \frac{c}{f} \operatorname{tr} F\tilde{F}}_{\text{total derivative}}$$

$\Rightarrow$ operators like ϕ^n not allowed

However, non-perturbative gauge field configurations, called instantons, generate a potential for ϕ:
$$V(\phi) = V_0 e^{-S}\left(1 - \cos\frac{\phi}{f}\right) + \mathcal{O}(e^{-2S})$$

and hence break shift symmetry to discrete symmetry: $\phi \to \phi + 2\pi f \Rightarrow \phi$ is periodic.

$S = \frac{8\pi^2}{g^2}$ is the instanton action and we need $S \gg 1$ to control potential.

slow-roll conditions:

$$\varepsilon = \frac{1}{2}\frac{M_p^2}{f^2}\frac{1+\cos\frac{\phi}{f}}{1-\cos\frac{\phi}{f}} < 1, \quad \eta = \frac{M_p^2}{f^2}\frac{\cos\frac{\phi}{f}}{1-\cos\frac{\phi}{f}} < 1$$

$\Rightarrow f \gtrsim M_p$

Literature:

Kolb & Turner, The Early Universe

Liddle & Lyth, Cosmological inflation and Large-Scale Structure

Baumann & McAllister, Inflation and String Theory

2 Gravity and symmetries

2.1 Global continuous symmetries

QFT with global symmetry:
$\longrightarrow$ conserved charge & charged particles

Throw total charge Q ($Q \in \mathbb{Z}$) into black hole (BH)

BH evaporation: Hawking radiation with $T = \frac{M_p^2}{M}$

(M = BH mass)

thermal radiation : # particles = # anti-particles

$\Rightarrow$ no discharge, BH loses only mass

Let BH evaporate to mass $M_* \leq M$

$\Rightarrow$ BH with mass M_* and charge Q

Repeat this procedure for any charge Q.

$\Rightarrow$ infinitely many BHs with mass M_*

no-hair conjecture: cannot measure Q

$\Rightarrow$ macroscopic state of BH only determined by M_*

Calculate BH entropy in two ways:

1) Bekenstein-Hawking : $S = \frac{1}{2} \frac{M^2}{M_P^2}$

2) S(BH with mass M)

$=$ log (# microscopic states with mass M)

$\longrightarrow \infty$

$\Rightarrow$ 2) contradicts 1)

Assuming 1) is correct, have to give up charge conservation.

$\Rightarrow$ Gravity is incompatible with global continuous symmetry.

warning: • don't know mechanism of charge conservation violation

• no-hair conjecture not proven in full generality

Even more speculative:

2.2 The weak gravity conjecture (WGC)

U(1) gauge theory with coupling g, coupled to gravity.

Recover global symmetry for $g \to 0$.
$\Rightarrow$ problem with gravity

WGC avoids this as follows:

Any U(1) gauge theory with coupling constant g and coupled to gravity must contain a charged particle with mass m and charge q such that $m < g q M_p$.

This ensures

gravitational force $\left(\frac{m^2}{M_p^2 r^2} \right) \leq$ electric force $\left(\frac{g^2 q^2}{r^2} \right)$

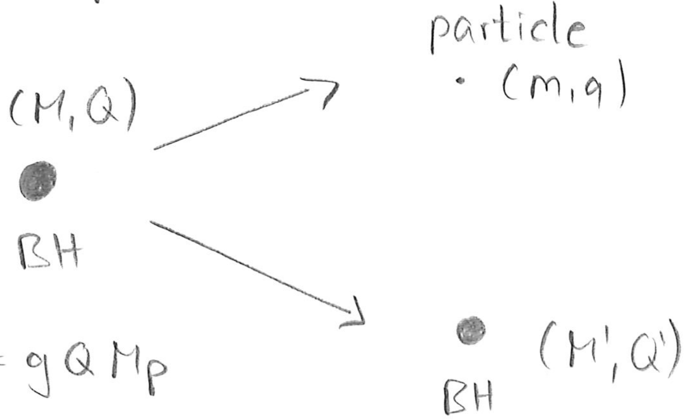
"derivation/motivation":

BHs charged under U(1) have 2 parameters: mass M, charge Q, with $M \geq g Q M_p$

Q measurable via electric field

$\longrightarrow$ cannot repeat global symmetry argument

Instead, people tried to argue that extremal BHs, $M = gQM_p$, should not be stable:



$M = gQM_p$ $\qquad\qquad$ $M' \geq gQ'M_p$

charge & energy conservation:

$$Q = q + Q' \quad , \quad M > m + M'$$

$$\Rightarrow \quad m < M - M' = gQM_p - M'$$

$$= gqM_p + gQ'M_p - M' \leq gqM_p$$

Arguments are not too convincing.
Much motivation comes from string theory where WGC seems too hold.

literature: arXiv: hep-th/0601001

3 WGC and natural inflation

3.1 Natural inflation from 5 dimensions

no charged particles in natural inflation → no WGC
However, natural inflation is 4d EFT of 5d $U(1)$ gauge theory compactified on circle (S^1)

$$S = -\frac{1}{4g_5^2}\int d^5x \, F_{IJ}F^{IJ} + \text{charged particles} + \text{gravity}$$

$$= \int d^5x \, \mathcal{L}_{5d} \qquad (I,J = 0,1,2,3,4)$$

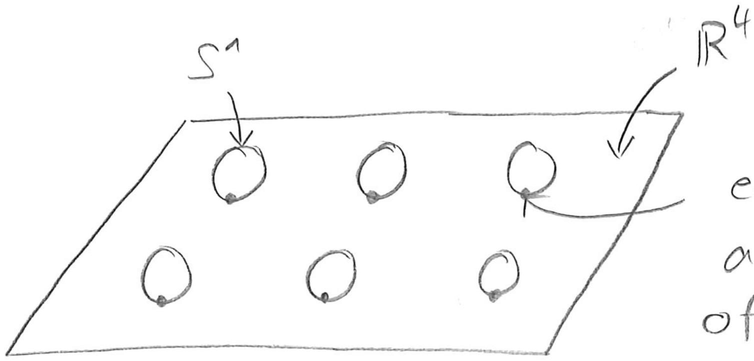
Compactify x^4 on S^1 with radius R.

energy $\ll \frac{1}{R}$ ⇒ cannot see S^1

⇒ 4d EFT $\mathcal{L}_{4d} = \int dx^4 \, \mathcal{L}_{5d}$

Wilson line $a = \int_0^{2\pi R} dx^4 A_4$, $\partial_4 a = 0$

⇒ a is 4d field



each point in $\mathbb{R}^4$ is attributed the value of a → 4d field

Use gauge freedom to set $\partial_4 A_4 = 0$

⇒ $a = 2\pi R A_4$

4d kinetic term for a:

$$-\frac{1}{4g_5^2}\int_0^{2\pi R} dx^4\, 2\, F_{\mu 4}F^{\mu 4} \supset \frac{1}{2g_5^2}\int_0^{2\pi R} dx^4\, \partial_\mu A_4 \partial^\mu A_4$$

$$= \frac{1}{2}\,\frac{1}{2\pi R g_5^2}\,\partial_\mu a \partial^\mu a$$

without proof: charged particles going around S^1 induce effective potential for a (like instantons in 4d): $V = V_0 e^{-S}(1-\cos a) + \mathcal{O}(e^{-2S})$

S = action of particle with mass $m = m\int_{WL} ds$
$= 2\pi R m$

$$\Rightarrow \mathcal{L}_{4d} = \frac{1}{2}\,\frac{1}{2\pi R g_5^2}\,\partial_\mu a \partial^\mu a - V_0 e^{-S}(1-\cos a) + \ldots$$

$\rightarrow$ natural inflation with axion decay constant

$$f = \frac{1}{\sqrt{2\pi R}\, g_5}$$

3.2 WGC for axions

WGC in 5d: $m \leq g_5 M_5^{3/2}$

(M_5 = 5d Planck mass , charge $q=1$)

4d Planck mass $M_4^2 = 2\pi R M_5^3$

$$\Rightarrow f \cdot S = \frac{2\pi R m}{\sqrt{2\pi R}\, g_5} \leq \sqrt{2\pi R}\, M_5^{3/2} = M_4$$

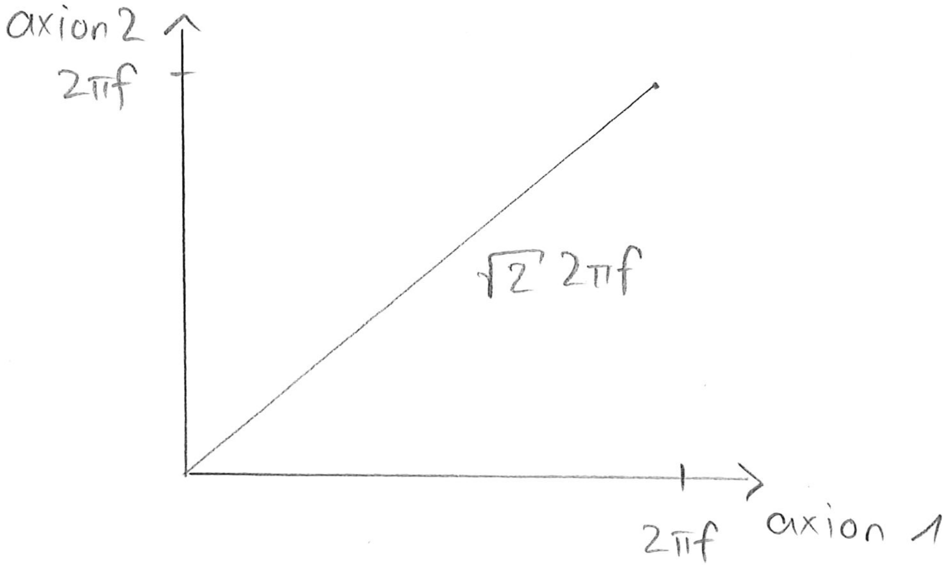
$S > 1$ (control over V) $\Rightarrow f < M_4$

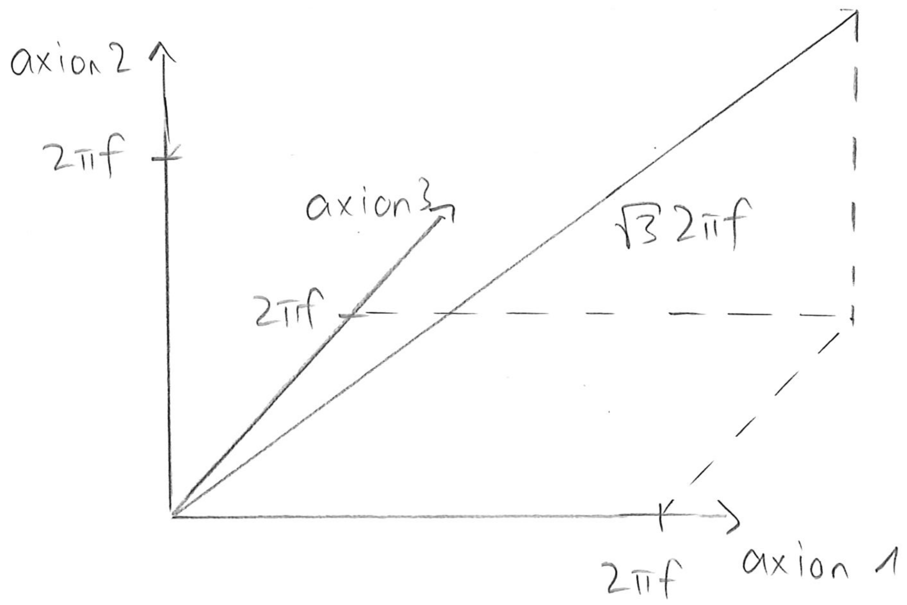
$\Rightarrow$ no natural inflation !?

current research: how to evade this constraint?

2 examples:

N-flation: multiple axions

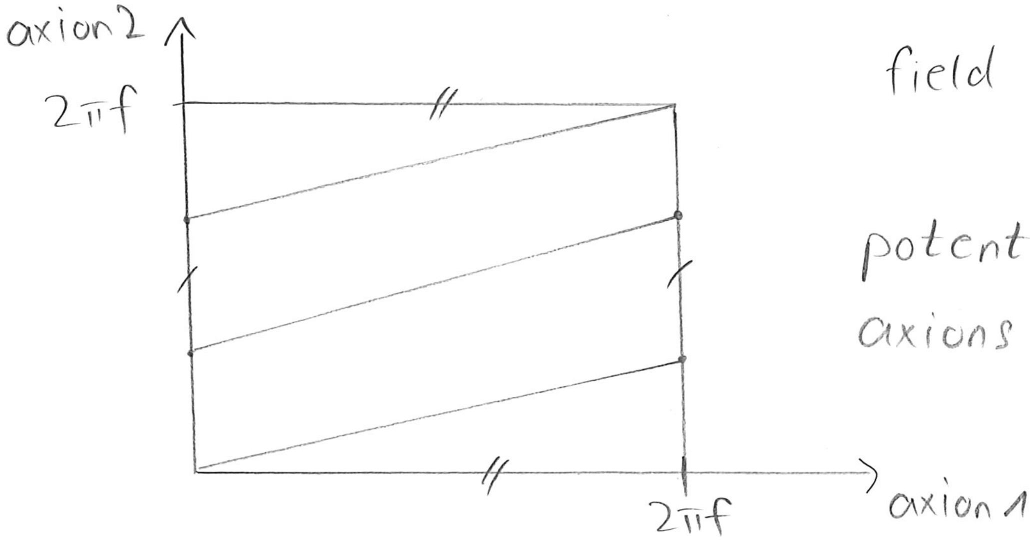




diagonal of N-dimensional cube is $\propto \sqrt{N}$

$\rightarrow$ large field excursion

winding inflation: 2 axions



field space: $S^1 \times S^1$ = torus

potential forces axions on winding trajectory

Literature: hep-th/0507205
hep-ph/0409138
1409.5793