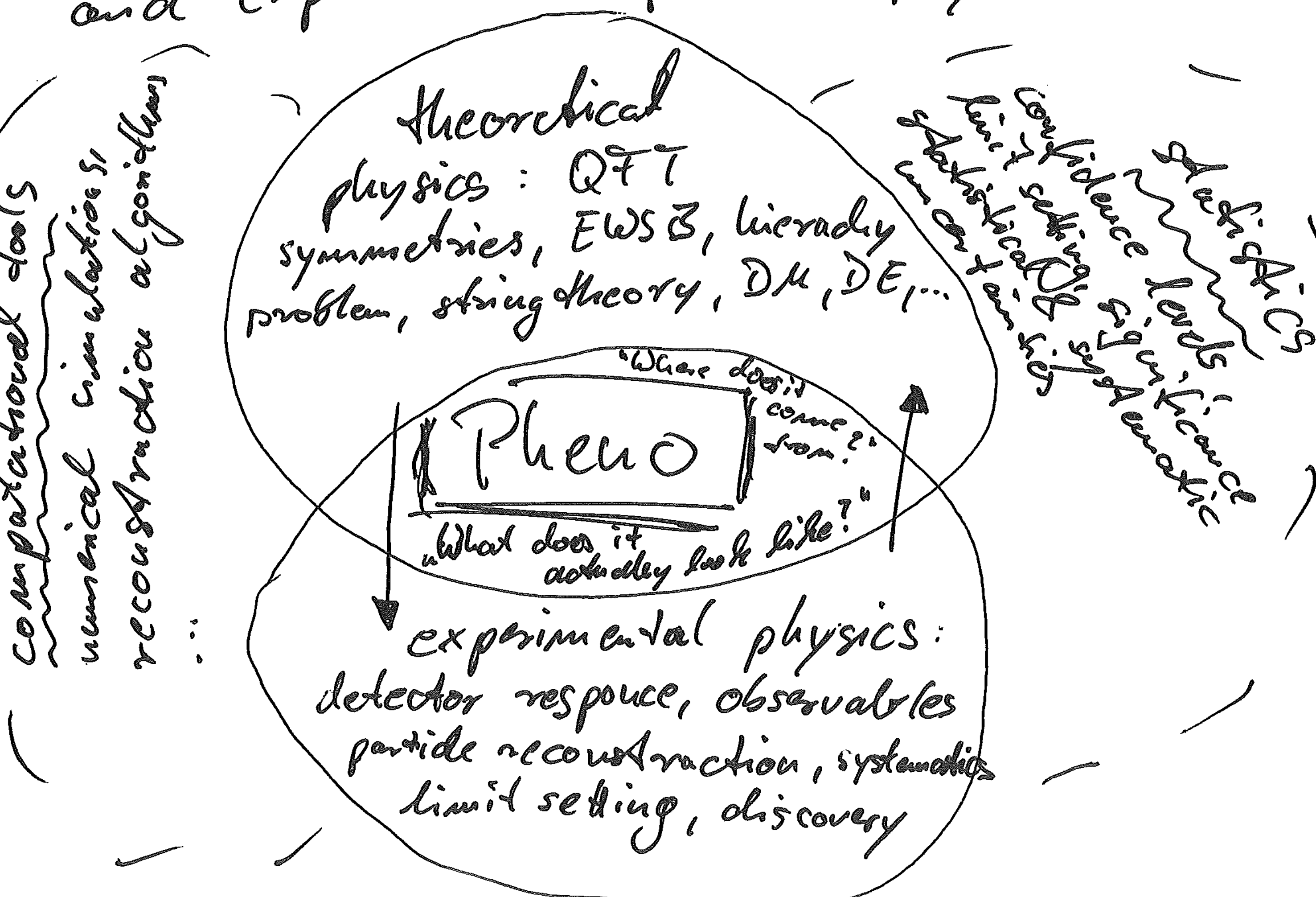
Particle Physics Phenomenology

wikipedia:

„Particle physics phenomenology is the part of theoretical particle physics that deals with the application of theoretical physics to high-energy particle experiments. [...] Phenomenology forms a bridge between the mathematical models of theoretical physics (such as quantum field theory) and experimental particle physics."



personal disclaimer:

These lectures will give a personal and limited view on the field we just like to call: pheno. There is much more to it then what will be presented here. Mostly we are limited by a combination of lack of time/knowledge/interest. I'll try to give hints and some literature for those who'd like to learn and explore more :)

what I would like to bring down to you:

- basic physics guiding hadron colliders („Master equation")
- what is a jet?
- why we need to study the former and how we can learn about (new) physics by doing so

Given this location motivates the following outline of the lecture:

I Know your hood (short)

A Experimental physics (LHC)

LHC, ATLAS & CMS (sorry Stephie)
luminosity, coordinate system
observable & reconstructable objects (trigger?)
first encounter of jets (visual)
[Thompson: Modern Particle Physics, Grupen: Particle Detectors, Evans: The large hadron collider]

B QFT & the standard model (sketchy)

Reminder: Lagrangians & symmetry
Feynman rules: perturbative physics (LO, NLO)
[Srednicki, Peskin & Schröder]
[partial aspects: TASI lecture notes]

Matrix elements & cross sections,
Observables, particle content
Example: $e^+e^- \to$ hadrons
second encounter of jets (origin)

C Tools & statistics

just a short motivation with handwavy examples, more later if we have time

- integration of complicated matrix elements
- signal & background: what's that? how to differentiate, a simple counting example

[MadEvent paper: hep-ph/0208156]

[Barlow: Statistics, Cowan: Stat. Data Analysis, Lyons: Stat. for nuc. & part. physicists]

II QCD at hadron colliders

parton branching, DGLAP evolution eq., pdf's, master equation

parton shower: third encounter of jets (connection A)

jet algorithms: fourth encounter of jets (connection B)

[the bigger picture: a "full" event @ the LHC example: Drell-Yan]

uncertainties!
simulation tools (Sherpa, MadGraph, ...)
[Peskin & Schröder, Ellis: QCD and collider physics, Dissertori: Quantum Chromodynamics, Tilman Plehn: LHC lecture notes, TASI]

III Generating functional formalism & QCD Bremsstrahlung

analytic PS, short QED example
staircase scaling in Drell-Yan at the LHC, pdf effects (maybe),
consequences for uncertainties and other important observables
[Ellis: QCD and collider physics]
[PS: Scaling patterns for QCD jets]
[PS: Jets plus missing energy with an auto-focus]

IV Jets in action

more about our work: - Auto-focus
- IWM
- Scaling in γ+jets

[PS: Jets with missing energy plus an Autofocus]
[PS: Improving Higgs plus Jets]

V Roundup

what else does our group do?

what do other people do?

Toptagger, MadMax, Checkmate

example: Higgs width measurement

[Plehn: Toptagger, PS: MadMax, Tattersall: Checkmate]

I Know your hood

A Experimental physics

We take a short detour into the basic benchmarks of the LHC collider and the two multi-purpose detectors ATLAS & CMS. Although there are two more very interresting experiments, namely LHCb & ALICE, we will not study any of their special physics program.

Some facts about the LHC:

27 km long ring

colliding protons at $\boxed{13\ \mathrm{TeV}}$ CME

high magnetic gradiand

$\rightarrow$ needs superconducting magnets

luminosity: $\mathcal{L} = \boxed{10^{34}\ 1/\mathrm{cm}^2\mathrm{s}}$

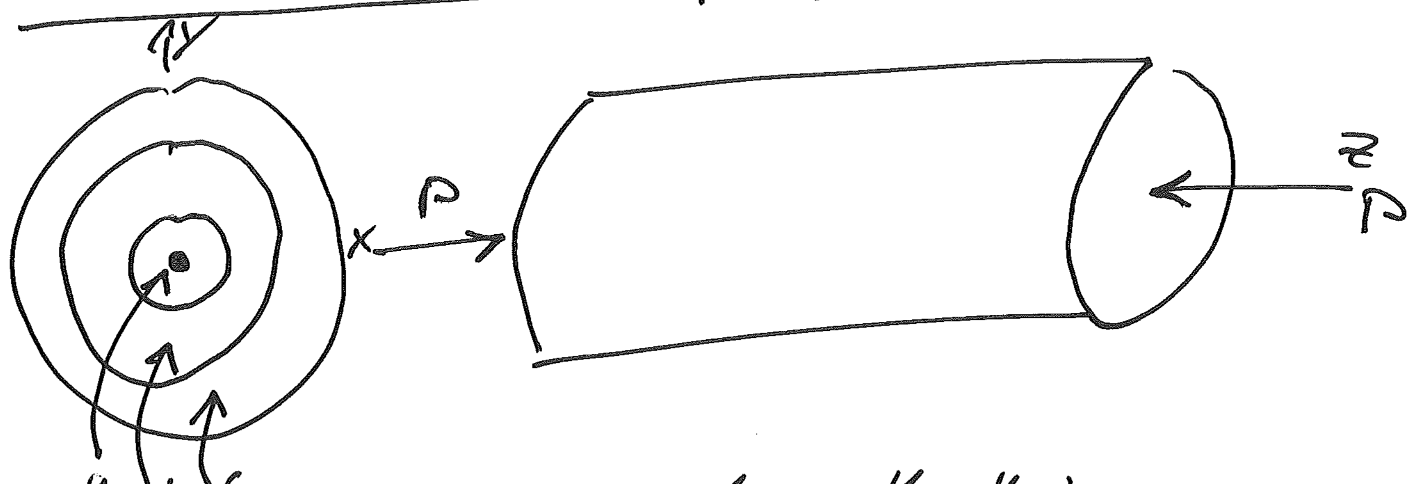
- bunches of ~~20 to 40 protons~~ 10^{11} protons

collision rate ca. 40 MHz

ATLAS & CMS : Full 3D 40MHz

Digicam

schematical set-up (show some pictures)



different detector parts with their own tasks

a) inner detector, measure momentum & tracks

b) calorimeter : two parts
electromagnetic and
hadronic calorimeter

measures energy (energy threshold)

c) muons are special -> extra muon spectroscope

-> four-momentum + interaction pattern allows to conclude on the particle

show $\boxed{\text{CMS - slice}}$

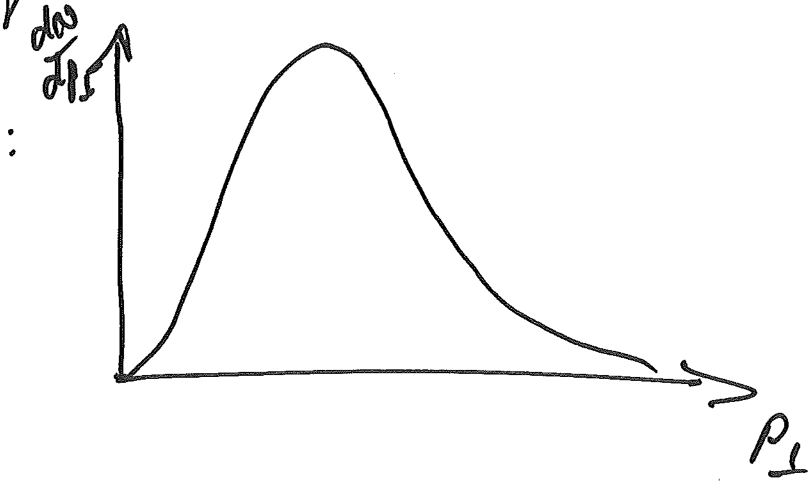
observables

we observe (integrated) rates: how many particles did we get?

N

we can ask for certain regions of phase-space:

$\frac{dN}{dp_\perp}$:



=> distributions

how to interprete?

$$\boxed{N = L \cdot \sigma}$$

nature

integrated luminosity
"experiment"

theory, probability

the coordinate system

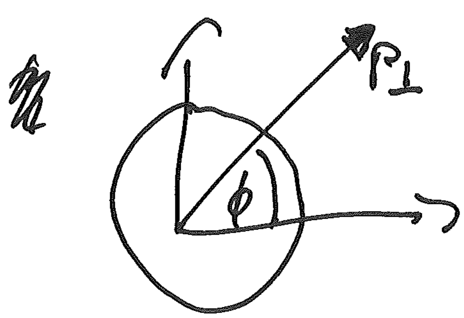
Due to the fact that we have cylindrical detectors and no initial momentum perpendicular to the beam-axis, we use the following coordinate system:

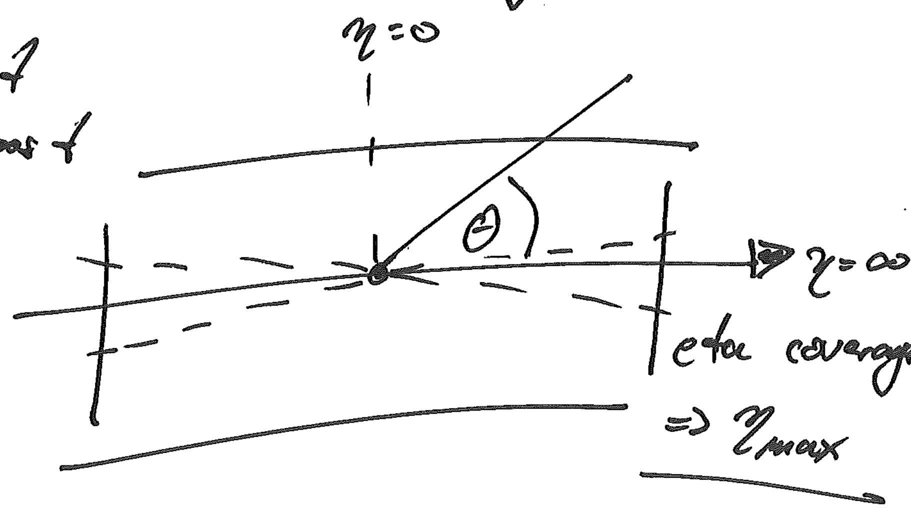
$$E, \; P_\perp, \; \eta, \; \phi$$

calo-info

$\Delta\eta$ is invariant under boost

symmetric only $\Delta\phi$ interesting





$$\eta = -\ln \tan \frac{\theta}{2}$$ "pseudo-rapidity"

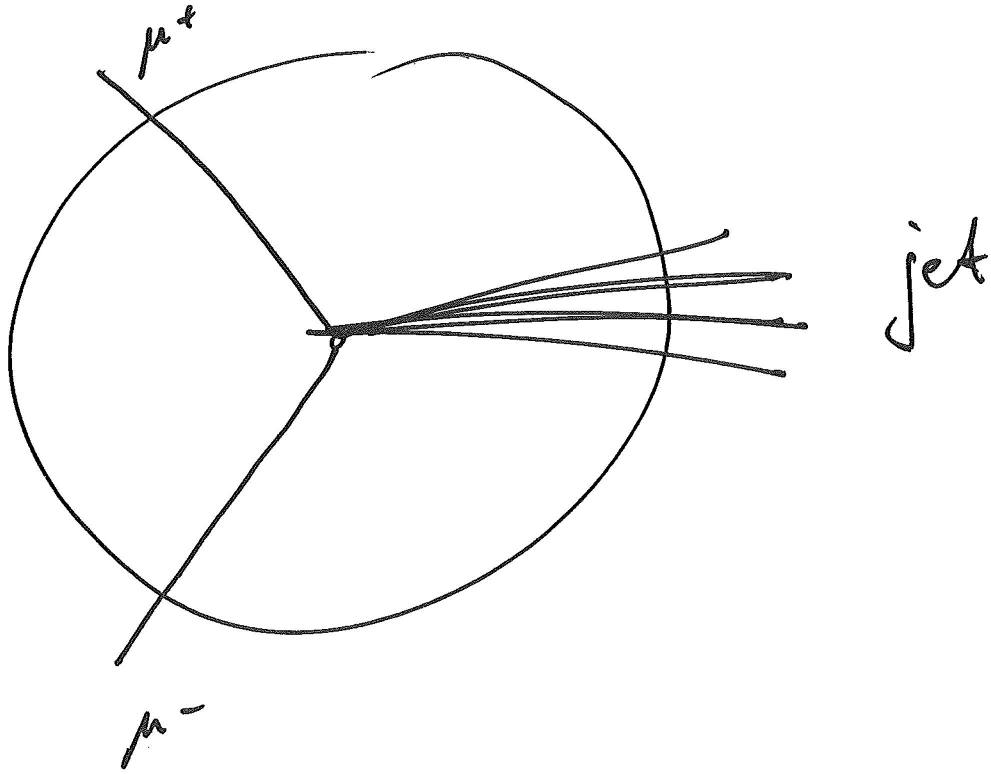
$$P_\perp = (p_x, p_y)$$ "transverse momentum"

$$\sum_n P_{\perp,n} = 0 \; (?)$$ "MET"

an event

A single collission is called an event. Let's take a closer look. (show $pp \to \mu^+\mu^- +$ jets)

$\boxed{\text{jets: collimated spray of hadrons}}$



„Drell - Yan"

3 QFT & the SM

We believe that the fundamental interactions and constituents of matter are guided and described by very simple yet fundamental principles of nature: QM $\oplus$ SR

$\Rightarrow$ QFT

QFT are described by Lagrange-densities obeying certain symmetry conditions: Lorentz & gauge invariance

It will not be our goal to understand or study the beautiful principles and techniques leading to QFT's or how they describe particles and interactions. Rather we will take one of the most useful results of QFT as granted and use it to do some simple computations:

$$\boxed{\text{Feynman rules \& diagrams}}$$

Nevertheless let us very heuristically motivate their origin:
In QM we compute probability amplitudes „$\langle \text{final} | e^{-iHt} | \text{initial} \rangle$"

$$\uparrow_{H \leftrightarrow L}$$

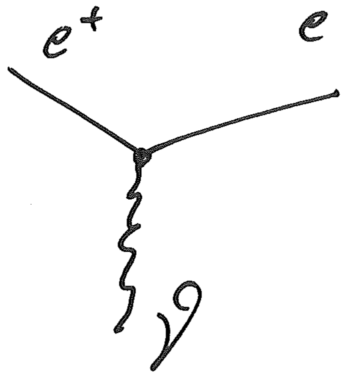
If our Lagrangian contains a small coupling we can expand in this coupling. The single terms in this expansion can be represented by simple lines & vertices => Feynman diagrams. We use them to visualize perturbation theory. The ~~[illegible]~~ connection to the mathematical expression of the probability amplitude, called the matrix element, is done via the so called Feynman rules example QED

$$\mathcal{L} = \bar{\Psi}\left(i\gamma^\mu\left(\partial_\mu + i\overset{\text{coupling}}{e} A_\mu\right) - m\right)\Psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

fields ≙ particles ≙ lines

interaction ≙ vertex

$\Rightarrow$



$= +ie\gamma^\mu$

$\Rightarrow$ external / internal particles

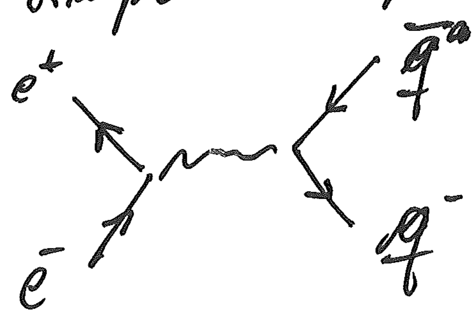
	external	internal
straight line	particle / anti: $u(p)$ / $v(p)$ „spinor"	$\dfrac{i}{\not{p} - m}$
wavy line	$\epsilon^\mu(p)$ „polarization"	$\dfrac{-ig^{\mu\nu}}{p^2}$

This is all very sketchy and by no means formal. You can find the proper way to do it in the literature.

Feynman diagrams are the correct diagramatical expression to compute and put together the matrix element from these Feynman rules:

- select your ~~exitial~~ initial and final state represented by external lines
- draw all possible connections between these using internal lines and vertices at fixed order in the coupling
- sum up all diagrams

simple example: $e^+e^- \to q\bar{q}$ ~~$\mu^+\mu^-$~~ in QED



$$= \bar{v}(e^+)\, ie\gamma^\mu u(e^-)\, \frac{-i}{(e^+ + e^-)^2} \otimes \bar{u}(q)\, ie\gamma_\mu v(\bar{q})\, Q_q$$

e^+ = four-momentum of the positron

e^- = — " — electron

q = — " — ~~muon~~ quark

$\bar{q}$ = — " — anti-~~muon~~ quark

Furthermore, we assume that the QFT describing nature is given by the SM (for the moment). In the end it is our goal to try to falsify this theory at the LHC and learn more about nature itself.

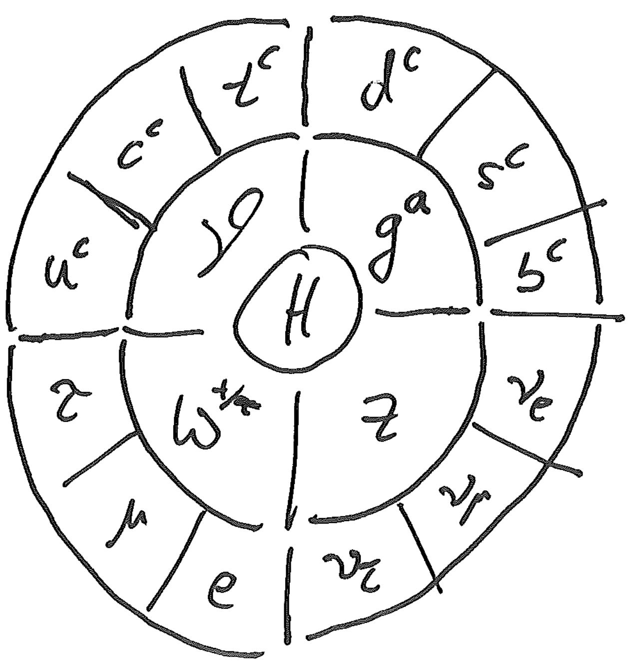
the SM

gauge symmetry: $SU(3)_C \otimes \underbrace{SU(2)_L \otimes U(1)_Y}$

broken via the $\boxed{\text{Higgs}}$

$\Rightarrow$ massive gauge bosons

particle content:



$c = 3$ colors
$a = 8$ color-combinations

$\oplus$ „anti-particles"

cross - sections

matrix element $\hat{=}$ probability „$\mathcal{M}(p)$"
amplitude $\in \mathbb{C}$

cross - section $\hat{=}$ probability

To get the probability of a given process we need to add up all kinematically allowed configurations weighted by their probability density $\mathcal{M}^2$

$$\boxed{\sigma = \int dLips\, |\mathcal{M}|^2}$$

Lorentz invariant phase space measure

In full detail this relation is given by ($A+B \to f$)

$$\sigma = \frac{1}{2E_A 2E_B |v_A - v_B|} \int \prod_f \frac{d^3 p_f}{(2\pi)^3} \frac{1}{2E_f} \cdot |\overline{\mathcal{M}}|^2 \cdot (2\pi)^4 \delta^{(4)}\Big(p_A + p_B - \sum_f p_f\Big)$$

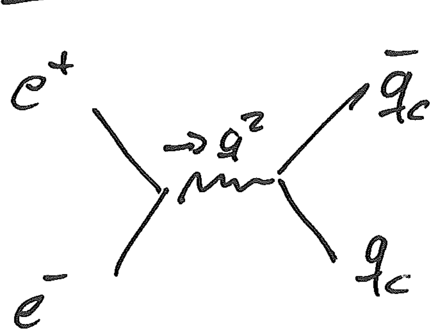
flux factor

spin averaged

energy-momentum conservation

Let us now use all this to compute the process $\boxed{e^+e^- \to \text{hadrons}}$ (no intermediate Z for simplicity)

<u>matrix element</u>: [Peskin & Schroeder for details, Ch 5]



$$= \bar{v}(e^+)\, ie\gamma^\mu\, u(e^-) \otimes \frac{-i}{q^2} \otimes \bar{u}(q)\, ieQ_q \gamma_\mu\, v(\bar{q}) = i\mathcal{M}$$

spin averaged <u>squared matrix element</u>:

$$\frac{1}{4} \sum_{\substack{\text{spins}\\ \text{colors}}} |\mathcal{M}|^2 = 3Q_q^2 \frac{e^4}{q^4} \operatorname{tr}(\not{e}^+ \gamma^\mu \not{e}^- \gamma^\nu) \operatorname{tr}(\not{\bar{q}} \gamma_\mu \not{q} \gamma_\nu)$$

$$= 3Q_q^2 \frac{e^2}{q^4} \left[(e^- q)(e^+ \bar{q}) + (e^- \bar{q})(e^+ q) \right]$$

<u>phase space & energy momentum conservation</u>

$$q = (E, E\sin\theta, 0, E\cos\theta)$$

$$e^+ = (E,0,0,E) \longrightarrow \ast \longleftarrow (E,0,0,-E)\, e^- $$

$$\bar{q} = (E, -E\sin\theta, 0, -E\cos\theta)$$

$$\Rightarrow E_A = E_B = E \quad \& \quad |v_A - v_B| = 2$$

$$E_f = E$$

$$\frac{d^3q \, d^3\bar{q}}{(2\pi)^3(2\pi)^3} \longrightarrow \frac{2\pi \cancel{4\pi} E^2 \, d\cos\theta}{(2\pi)^3 (2\pi)^3}$$

$$|\bar{\mathcal{M}}|^2 = 3 Q_q^2 e^4 \left[1 + \cos^2\theta\right]$$

$$\sigma = \frac{1}{8E^2} \int_{-1}^{+1} \frac{2\pi E^2 \, d\cos\theta}{(2\pi)^3(2\pi)^3} \frac{1}{4E^2} (2\pi)^4 \, 3 Q_q^2 e^4 \left[1 + \cos^2\theta\right]$$

$$= 3 Q_q^2 \frac{4\pi\alpha^2}{3E^2}, \quad \text{with } \alpha = \frac{e^2}{4\pi}$$

Note: $\sigma_{\mu^+\mu^-} = \frac{4\pi\alpha^2}{3E^2}$

The ratio $R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = "3 \sum_i Q_i^2" = N_c \frac{11}{9}$

is very well measured.

Number of colors ☺

Fact of nature:

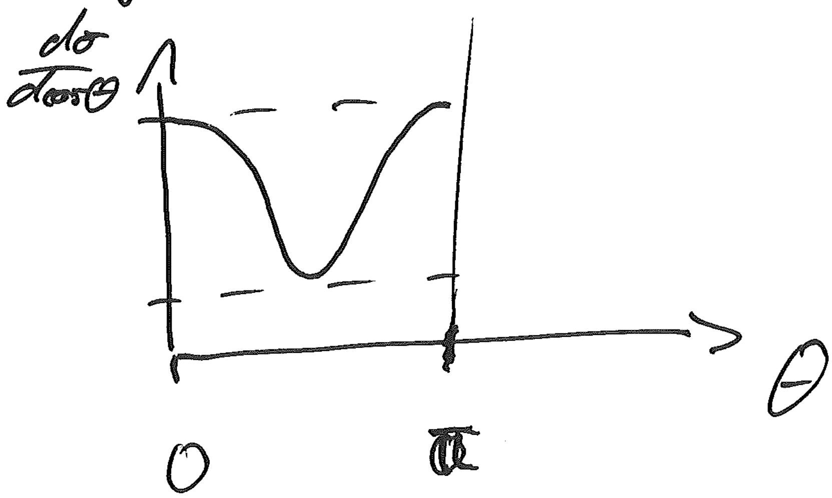
We only observe color neutral objects. The quarks we computed must transform into hadrons. There is not such thing as a $q \to p$ connection. Therefor, we expect a single quark do transform into several hadrons. However, due to energy momentum conservation these align all in on direction. We observe a collimated spray of hadrons. A so called $\boxed{\text{jet.}}$

observables & differential cross-sections

Of course we ~~do~~ not need to perform the integration step above. We also could stop at

$$\frac{d\sigma}{d\cos\theta} = 3 Q_q^2 \frac{4\pi\alpha^2}{8E}(1+\cos^2\theta)$$

This is the production rate as a function of the polar angle.



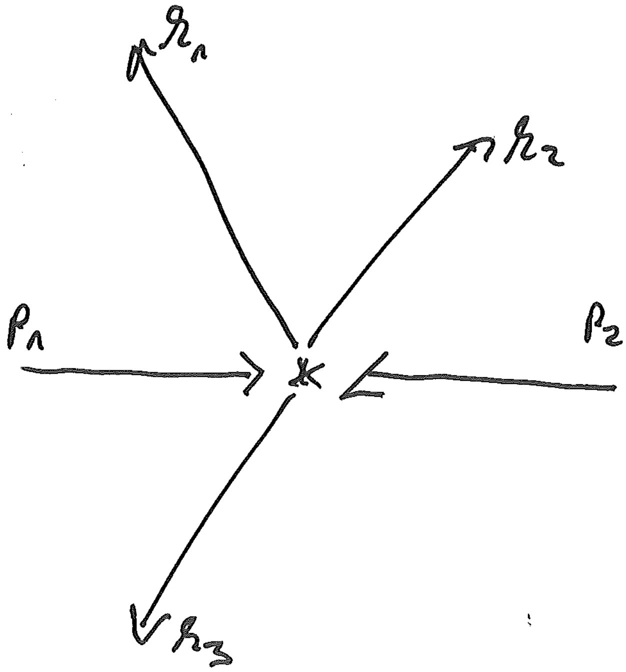
$\longrightarrow$ strongly peaked in the forward direction

C Tools & stastistic & uncertainties

Imagine we would not compute a simple 2→2 process as we did above, but an 2→n, where n is big, process. It is rather obvious that this is impossible. In that case we rely on numerical simulation tools, so called Monte Carlo tools. The offer the additional advantage that we can extract events:

$$\sigma = \int d\sigma(p) \longrightarrow \sigma = \sum_n \Delta\sigma(p)$$

finite set of phase-space configurations just as in a real experiment

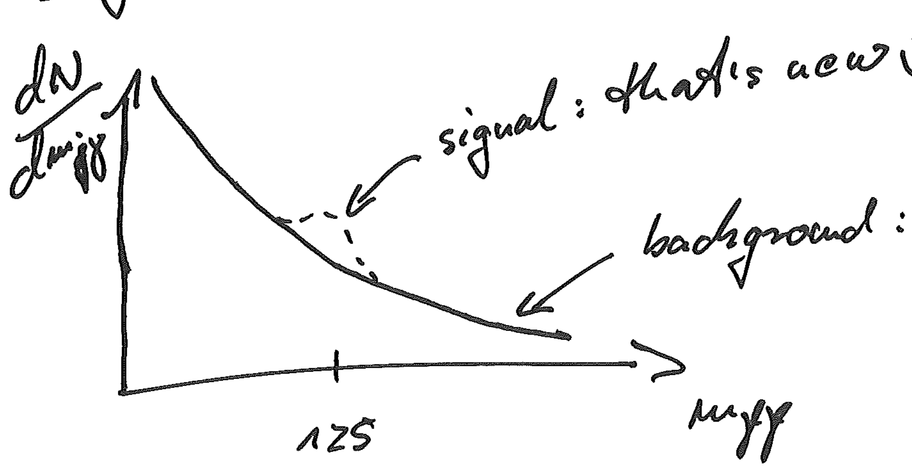


$$p = \{p_1, p_2; k_1, k_2, k_3\}$$

Why statistics?

We already have a very nice and accurate description of nature given by the SM. As scientists it is our goal to bring this model to its limits and to falsify it. We ~~are~~ know that the SM does a good job. It is deviations we look for. And we do this by the means of searching for a so called signal.

signal & background



sth. we already know, but contributes to the observable we are interested in.

e.g.: the Higgs discovery

statistical fluctuations

We only have a finite set of events to map our observable $\Rightarrow \sqrt{N}$ error

systematic uncertainties

The experiment and the detector bring additional uncertainties, which have to be considered.

theoretical uncertainties

Theoretical calculations have a lot of different sources for uncertainties. One example is that we rely on a perturbative expansion. NLO $\sim \mathcal{O}(1.5)$
There are more examples later.

$$\boxed{N = L \cdot \sigma}$$

We need to understand and controll all these uncertainties if we want to claim discovery.

II QCD at hadron colliders

In the last section we succeeded with our first meaningful collider computation. We took a short-cut neglecting the Z boson but as long as $\sqrt{s} \ll m_Z$ at our electron-positron collider, that's not a problem. So why the hell a whole section about hadron colliders? Let's answer this question by proceeding straight forward, but naive. Let's turn the computation $e^+e^- \to q\bar{q}$ around. The only thing we have to do is to average over colors, too. $\sigma = \frac{1}{3} Q_q^2 \frac{4\pi\alpha^2}{3E^2}$
Nice! That was easy. However, we cannot prepare quarks as initial states in nature! There is color confinement. We only observe color neutral states. How about $p\bar{p}$ as initial state?

$$p = (u, u, d)$$

$$\bar{p} = (\bar{u}, \bar{u}, \bar{d})$$

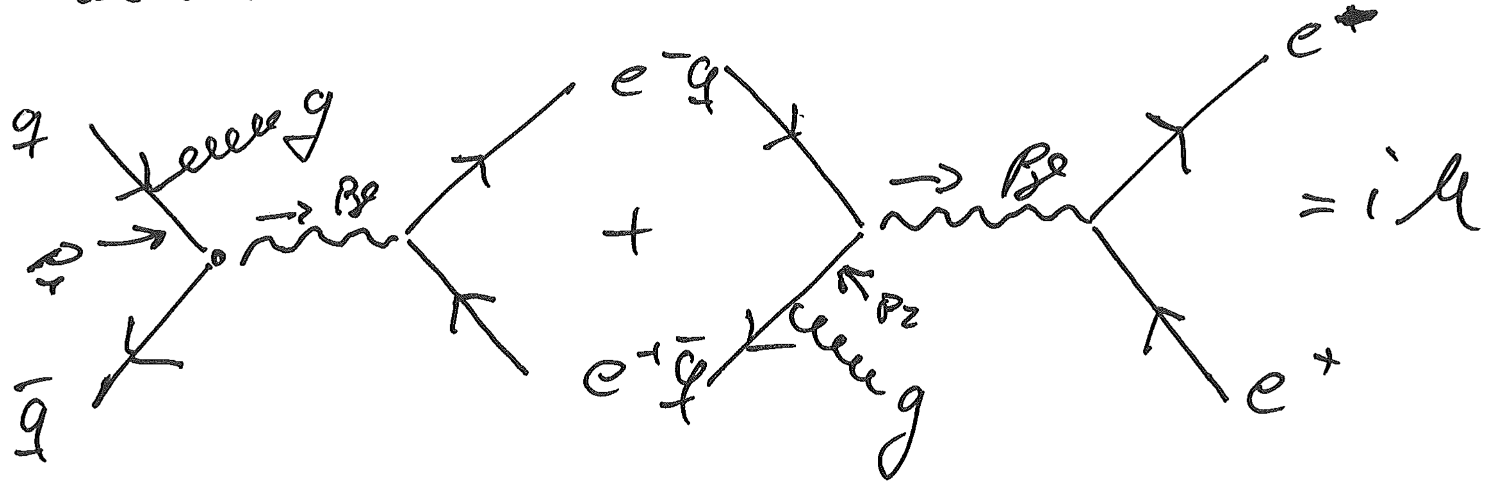
That helps we find the quarks we need, for example $q\bar{q} = d\bar{d}$, within the proton. There is a small caveat about E^2. Before, this was the energy of the incoming electron beam. However, it is clear that a single quark cannot hold the total energy of the proton, because they have to fly together. It has to be that $E_q = x E_p$. Looking at the composition of the proton let's just guess that $x = 1/3$ for d-quark and $x = 2/3$ for u-quarks.

$$\Rightarrow \sigma_{q\bar{q}\to e^+e^-} = \frac{1}{3} Q_q^2 \frac{4\pi\alpha^2}{3 x_q x_{\bar{q}} E^2}$$

So far so good?!

Let us now observe what happens if we radiate an addition gluon from the quarks.

In the language of Feynman diagrams we have:



$$= \bar{v}(\bar{q})\, ie Q_q \gamma^\mu \frac{i\not{p}_1}{p_1^2} \gamma^\nu\, ig\, u(q)\, \epsilon^*_\nu(g)\, \frac{-i}{p_\gamma^2}$$

$$\otimes\, \bar{u}(e^-)\, ie\gamma_\mu\, v(e^+) + \bar{v}(\bar{q})\, ie Q_q \gamma^\nu\, ig$$

$$\otimes\, \frac{i\not{p}_2}{p_2^2}\, ie\gamma^\mu u(q)\, \epsilon^*_\nu(g)\, \frac{-i}{p_\gamma^2}\, \bar{u}(e^-)\, ie\gamma_\mu v(e^+)$$

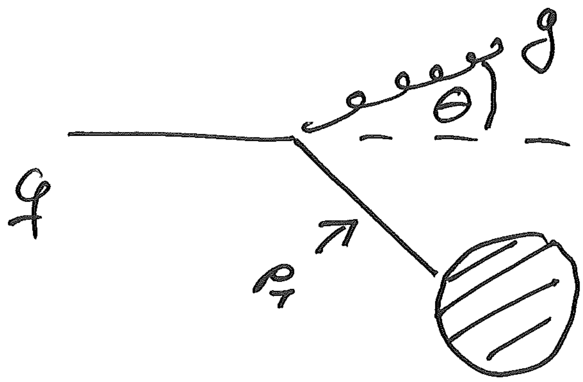
strip constants

$$\overset{\Downarrow}{\simeq} \bar{v}(\bar{q})\left(\gamma^\mu \frac{\not{p}_1}{p_1^2}\gamma^\nu + \gamma^\nu \frac{\not{p}_2}{p_2^2}\gamma^\mu\right) u(q)\, \epsilon^*_\nu(g)\, \frac{1}{p_\gamma^2}\, \bar{u}(e^-)\gamma_\mu v(e^+)$$

It is not impossible to compute $\mathcal{M}^2$ and do the phase space integration by hand. However, it is lengthy and error-prone. We will therefor take a short cut.

soft collinear limit & factorization theorem

To proceed further let us now focus on the new propagator structure $1/p^2$



$\sim \frac{1}{p_1^2}$

$$q = (E_q, 0, 0, E_q)$$

$$g = (E_g, 0, E_g \sin\theta, E_g \cos\theta)$$

$$p_1 = q - g \quad \Rightarrow \quad p_1^2 = -2qg$$

$$\boxed{\Rightarrow \quad \frac{1}{p_1^2} = \frac{1}{-2 E_q E_g (1 - \cos\theta)}}$$

We make the following observations:

- divergent for $E_g \to 0$ (<u>soft limit</u>)
- divergent for $\theta \to 0$ (<u>collinear limit</u>)

Let's try to translate this to hadron collider coordinates

$$p_\perp = E_g \sin\theta$$

$$\eta = -\log\tan\frac{\theta}{2}$$

$$\boxed{\text{soft collinear limit: } p_\perp \to 0,\ \eta \to \pm\infty}$$

However, our detector has an energy threshold as well as an eta-coverage. To further study what the actual consequences are we rephrase our observation in a more formal way.

Intuitively: We expect a contribution (divergent) to the cross section. We need to check how that spoils our previous computation.

Factorization theorem

If any final state particle becomes collinear to any of the other external particles the cross section factorizes.[1]

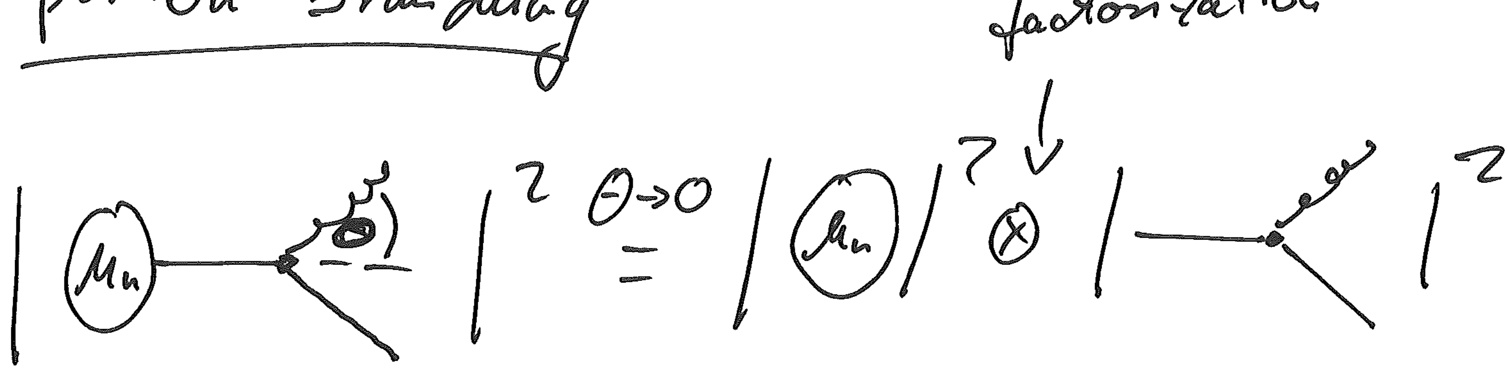
$$\boxed{d\sigma_{n+1} \simeq d\sigma_n \frac{dp_\perp^2}{p_\perp^2} \, dz \, \frac{\alpha_s}{2\pi} P(z)}$$

The quantities in this formula need clarification. From the previous discussion it should be clear why we expect a factor $\sim 1/p_\perp^2$

[Altarelli: Asymptotic freedom in parton language]

1) This is a non-trivial statement. It means that there is no interference. In addition it implies that also the phase space factorizes.

parton branching

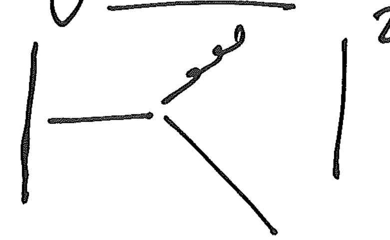


1.) What is z?

z is the energy fraction of the gluon: $\underline{E_g = z E_q}$

2.) What is $P(z)$?

$P(z)$ is the so called splitting kernel.

It is given by $\frac{1}{2} \frac{z(1-z)}{p_\perp^2}$ 

color-factor

$$\boxed{P(z)_{q \to gq} = C_F \frac{1+(1-z)^2}{z}}$$

soft divergence

other splitting kernels: $g \to q\bar{q}$, $g \to gg$

[For details look into the original Altarelli & Parisi paper or Ellis: QCD]

Let's use this knowledge and go back to our lepton production:

$$\sigma_{l^+l^-} = \frac{1}{3} Q_q^2 \frac{4\pi\alpha^2}{3x_q x_{\bar{q}} E^2}$$

$$\Rightarrow d\sigma_{l^+l^- g} = \frac{1}{3} Q_q^2 \frac{4\pi\alpha^2}{3x_q x_{\bar{q}} E^2} \frac{dp_\perp^2}{p_\perp^2} \frac{\alpha_s}{2\pi} dz\, P(z)$$

$$= \frac{1}{3} Q_q^2 \frac{4\pi\alpha^2}{3x_q x_{\bar{q}} E^2} \frac{\alpha_s}{2\pi} C_F \frac{1+(1-z)^2}{z} dz \frac{dp_\perp^2}{p_\perp^2}$$

Let us integrate this from $p_\perp^{min}$ to $p_\perp^{max}$ and z^{min} to z^{max}. We get for the last part modulo pre-factors

$$\log\left(\frac{p_\perp^{max}}{p_\perp^{min}}\right)\left[\log\left(\frac{z^{max}}{z^{min}}\right) + \mathcal{O}(z^{1})\right]$$

$\Rightarrow$ adding one gluon changes the cross section by a factor

$$\boxed{\alpha_s \log^2(\ldots)} \approx \underline{\mathcal{O}(1)}$$

That is bad news, becaus it spoils the convergence of our perturbative series! Looking at the factorization theorem it is clear what happens if we add a second parton: we'll get another term of the form $\alpha_s \log^2$. This means that an infinite series of jet-radiation contributes to our cross section and we can not just simply use the „inverted" LEP process at a hadron collider. Luckily, there is a way out. The solution is to be found in the proton which we modeled wrong. We know that the quarks in the proton need to interact to form a bound state. Therefor, there must also be gluons in the proton. In addition we know that what we see depends on the resolution we use. $R \sim \frac{1}{\mu_F}$
The resolution is inverse to the momentum transfer of the process we study, e.g. the scale at which it takes place.

DGLAP evolution eq. & pdf's

In our naive example we just put $x_q = 1/3$. However, we saw that there are a lots of gluons around. They are dynamical and their appereance should be dependent on which scale we probe them. This means, however, that x_q cannot be fixed, but must be given by some distribution $f_q(x)$. In addition we expect this also do depend on the scale at which we „probe" the proton

$$\Rightarrow \quad \boxed{f_{q/p}(x, \mu_F) = \text{parton density function pdf}}$$

„What is the probability to find a parton q in a proton p with a given x at an momentum transfer of order μ_F"

This modifies how we compute cross sections:

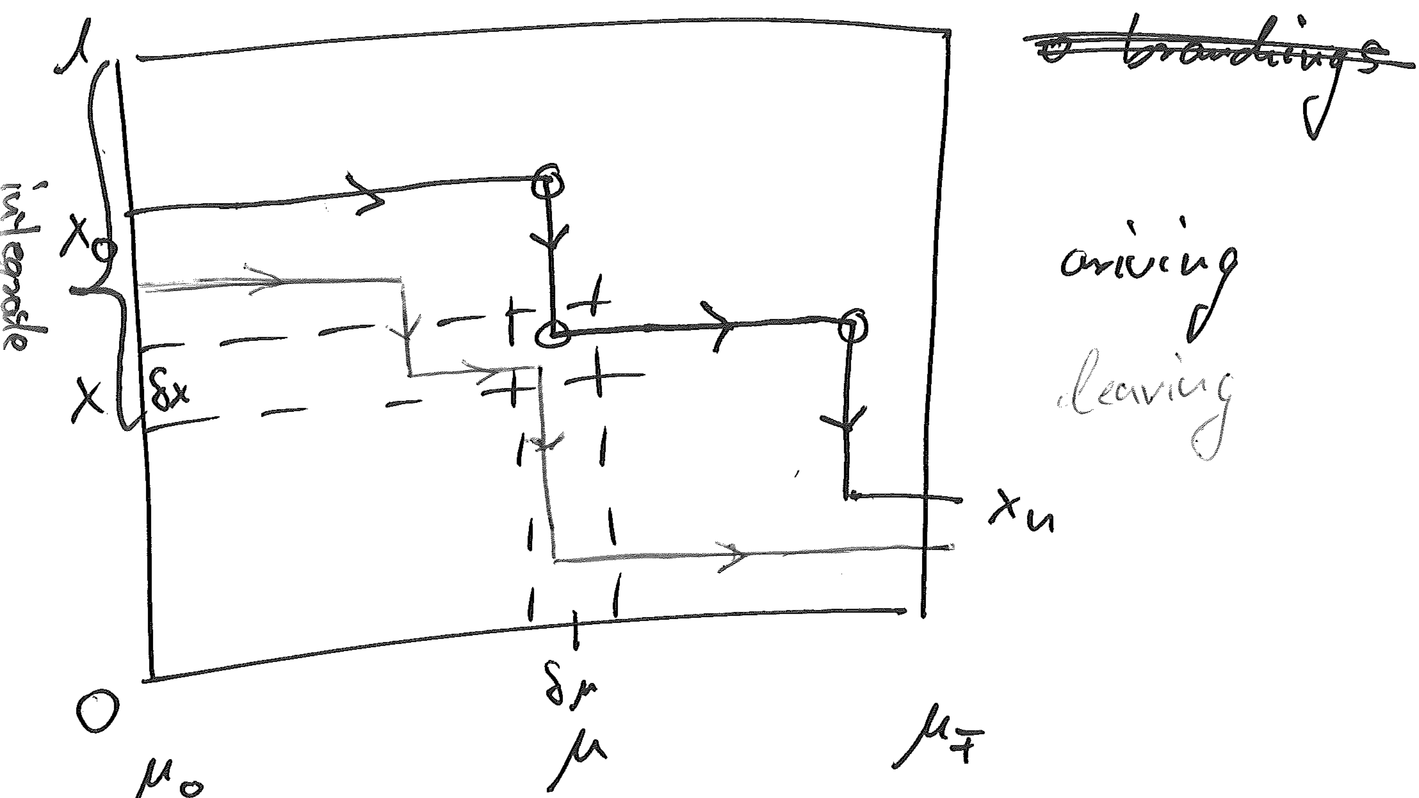
$$\boxed{\sigma(pp \to X, \sqrt{s}) = \int dx_1 dx_2 \sum_{i,j=q,\bar{q},g} f_i(x_1, \mu_F) f_j(x_2, \mu_F) \otimes \hat{\sigma}(ij \to X, x_1 x_2 \sqrt{s}, \mu_F)}$$

Master equation to compute cross sections at the LHC.

How does that solve the problem with the convergence of the perturbative series and where do we get $f(x)$ from?

Let's start with the first part of the question. Imagine we would know the pdf at some (low) scale μ_0. What happens if we radiate an additional (collinear!) gluon?

[Ellis: QCD Ch 5]



At μ_0 the x values have a given distribution $f(x,\mu_0)$. To find $f(x,\mu_F)$ we just need to follow all the pathes. Let's consider a small step $\delta\mu$. δf is just the number of pathes arriving in the element $(\delta x, \delta\mu)$ minus those leaving divided by δx.

The probability of a splitting is given by $P(z)$ and costs an amount of energy z.

$$\Rightarrow \quad \text{in}: \int_x^1 dx' \, \delta(x - zx') \quad \text{out}: \int_0^x dx' \, \delta(x' - zx)$$

We get:

$$\delta f_{in} = \frac{\delta\mu}{\mu} \int_x^1 dx' dz \frac{\alpha_s}{2\pi} P(z) f(x', \mu) \delta(x - zx')$$

$$\delta f_{out} = \frac{\delta\mu}{\mu} f(x, \mu) \int_0^x dx' dz \frac{\alpha_s}{2\pi} P(z) \delta(x' - zx)$$

integrate the $\delta()$:

$$\delta f_{in} = \frac{\delta\mu}{\mu} \int_0^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z}, \mu\right)$$

$$\delta f_{out} = \frac{\delta\mu}{\mu} f(x, \mu) \int_0^1 dz \frac{\alpha_s}{2\pi} P(z)$$

$$\Rightarrow \boxed{\delta f(x, \mu) = \frac{\delta\mu}{\mu} \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) \left[\frac{1}{z} f\left(\frac{x}{z}, \mu\right) - f(x, \mu)\right]}$$

introducing the Sudakov-form factor Δ

$$\Delta(\mu) \equiv \exp\left[-\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \int_0^1 dz \frac{\alpha_s}{2\pi} P(z)\right]$$

with $\mu \frac{\partial}{\partial\mu} \Delta(\mu) = -\int dz \frac{\alpha_s}{2\pi} P(z) \Delta(\mu)$

now: convert $\delta f \to \partial f$ & $\delta\mu \to \partial\mu$

$$\Rightarrow \mu \frac{\partial f(x,\mu)}{\partial \mu} = \int dz \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu\right) + \frac{f(x,\mu)}{\Delta(\mu)} \mu \frac{\partial \Delta(\mu)}{\partial \mu}$$

This we can manipulate to become

$$\mu \frac{\partial}{\partial \mu}\left(\frac{f(x,\mu)}{\Delta(\mu)}\right) = \frac{1}{\Delta(\mu)} \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu\right)$$

which we can integrate to

$$f(x,\mu) = \Delta(\mu) f(x,\mu_0) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \frac{\Delta(\mu)}{\Delta(\mu')} \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu'\right)$$

This is an integral eq. for $f(x,\mu)$ starting from a given $f(x,\mu_0)$. The first term tells us that the Sudakov form-factor describes a <u>non-splitting</u> probability.

There exists also a differential form of this equation. However, we would have needed to regularize the poles in $P(z)$, thus I skipped it. It reads:

$$\boxed{\mu \frac{\partial}{\partial \mu} f_i(x,\mu) = \sum_{i \to jk} \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} \hat{P}_{i \to jk}(z) f_j\left(\frac{x}{z},\mu\right)}$$

The DGLAP evolution equation.

How did this solve our problem? We resummed all the collinear radiation and got an evolution eq. for the proton. The initial value has to be measured. It is an non-perturbative object. However, from there we can use solid perturbative methods to evolve these distributions up to scale where we probe them μ_F.

$$\boxed{\sigma(pp \to X) = \int dx_1 \, dx_2 \, f(x_1,\mu_F) f(x_2,\mu_F) \hat{\sigma}(qq \to X, \mu_F)}$$

Recapitulation

- experimental environment & reconstructable objects, jets: visual
- observable „ $N = L \cdot \sigma$ "
- theoretical basis: QFT & the SM
- Feynman rules & phase space integration allow us to compute (differential) cross sections. $\sigma(e^+e^- \to q\bar{q}) = 3 Q_q^2 \frac{4\pi\alpha^2}{3E^2}$

partons $(q, \bar{q}, g)$: jets: origin

C Tools & uncertainties

1.) complicated processes $(2 \to n)$ cannot be computed by hand: use automated numerical integration tools (MadGraph, Sherpa Alpgen)

2.) goal: identify a $\boxed{\text{new}}$ signal above backgrounds

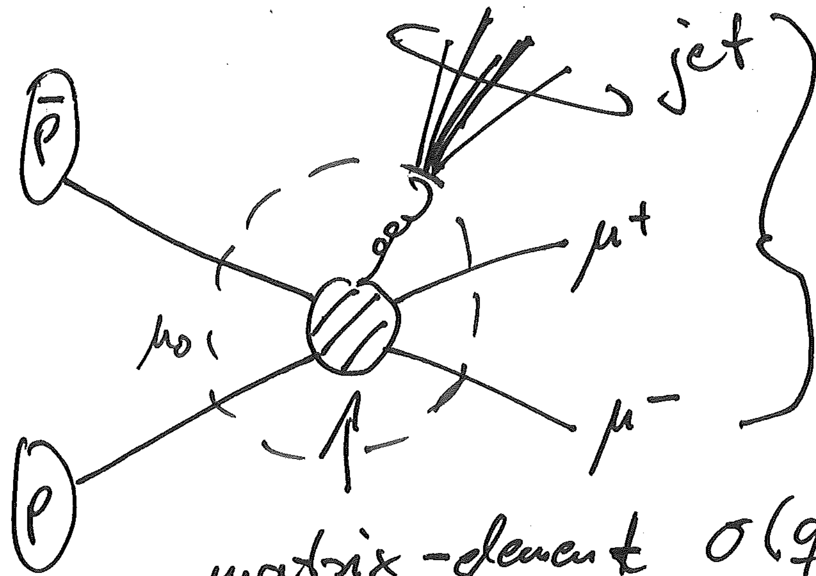
quantitatively: significance, confidence level

„How good can we distinguish the data from a given background hypothesis and the given uncertainties?"

3.) sources of uncertainty:

- statistical (finite counting)
- systematic (detector effects & understanding)
- theoretical (perturbative expansion, ...)

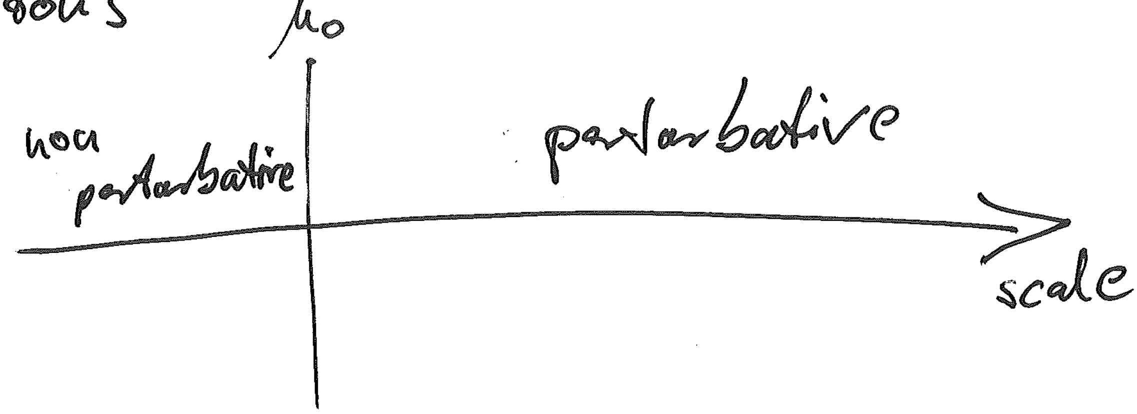
II QCD at hadron colliders



final state
→ detector

matrix-element $\sigma(q\bar{q} \to \mu^+ \mu^- g)$

initial state
→ protons



scale = energy transfer, virtuality/off-shellness, mass-threshold, resolution, proton-mass assumption: $p = (u, u, d)$ & $\bar{p} = (\bar{u}, \bar{u}, \bar{d})$

$$\Rightarrow \boxed{\sigma(\bar{q}q \to \mu^+\mu^-) = \frac{1}{3} Q_q^2 \frac{4\pi\alpha^2}{3 x_q x_{\bar{q}} E^2}}$$

~~$x_q = 1/3$~~ $\quad f(x_q)$ pdf $\quad$ ~~$(= \frac{1}{3}\delta)$~~ ~~$(= \delta(1/3)\,?)$~~

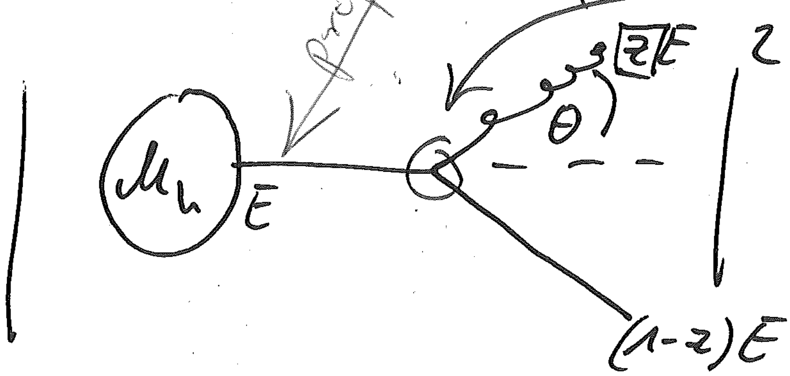
integrate over x_q values, simplest model: $x_q = 1/3$

additional jet radiation: factorization theorem

$$\boxed{d\sigma_{n+1} = d\sigma_n \frac{dp_\perp^2}{p_\perp^2} \, d\boxed{z} \, \frac{\alpha_s}{2\pi} P(\boxed{z})}$$

propagator^2

$|$vertex$|^2$



θ small ($p_\perp \to 0$)

$\to$ collinear limit

feature of massless gauge theories

$$\boxed{P(z) = C_F \frac{1 + (1-z)^2}{z}}$$

- neglects interference
- exact in limits

z small ($E_g \to 0$)

$\to$ soft limit

$$\Rightarrow \sigma(q\bar{q} \to \mu^+\mu^- g) = \sigma(q\bar{q} \to \mu^+\mu^-) \otimes \text{const} \otimes \log\left(\frac{p_\perp^{max}}{p_\perp^{min}}\right)\left[\log\left(\frac{z^{max}}{z^{min}}\right) + \mathcal{O}(z)\right]$$

$$\Rightarrow \boxed{\Delta\sigma \sim \alpha_s \log^2(\ ) \simeq \mathcal{O}(1)}$$

Our perturbative expansion is spoiled!

additional jet radiation is not suppressed

two jets : $\Delta\sigma \sim \alpha_s^2 \log^4(\ ) \simeq \mathcal{O}(1)$

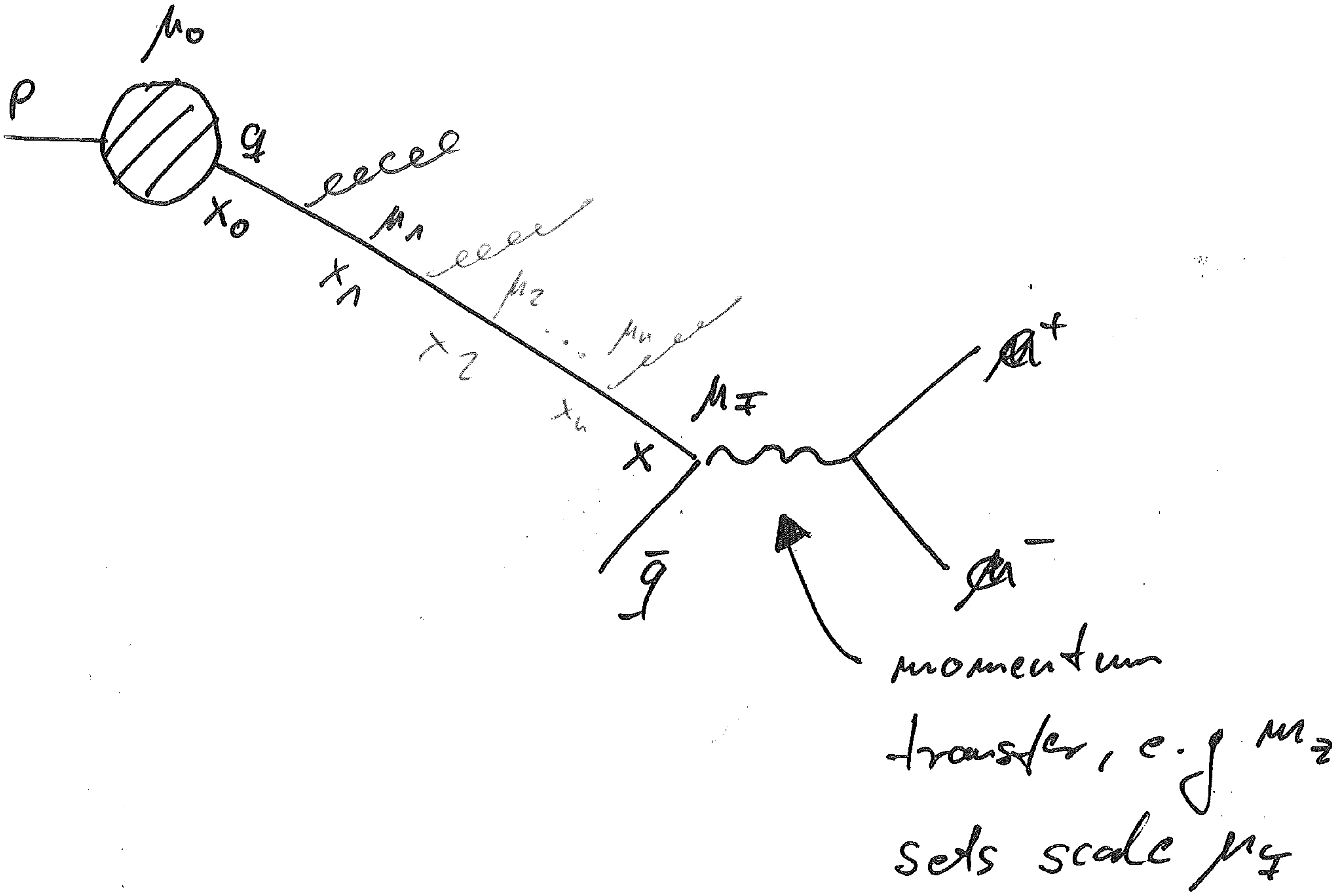
perturbative expansion $\longrightarrow$

large logarithms $\downarrow$

$$\begin{array}{ccc} \alpha_s \log^2 & \alpha_s^2 \log^4 & \alpha_s^3 \log^6 \\ \alpha_s \log & \alpha_s^2 \log^2 & \cdots \\ \vdots & \ddots & \end{array}$$

$\Rightarrow$ new expansion parameter

$\Rightarrow$ we need to re-sum large logarithms to all orders

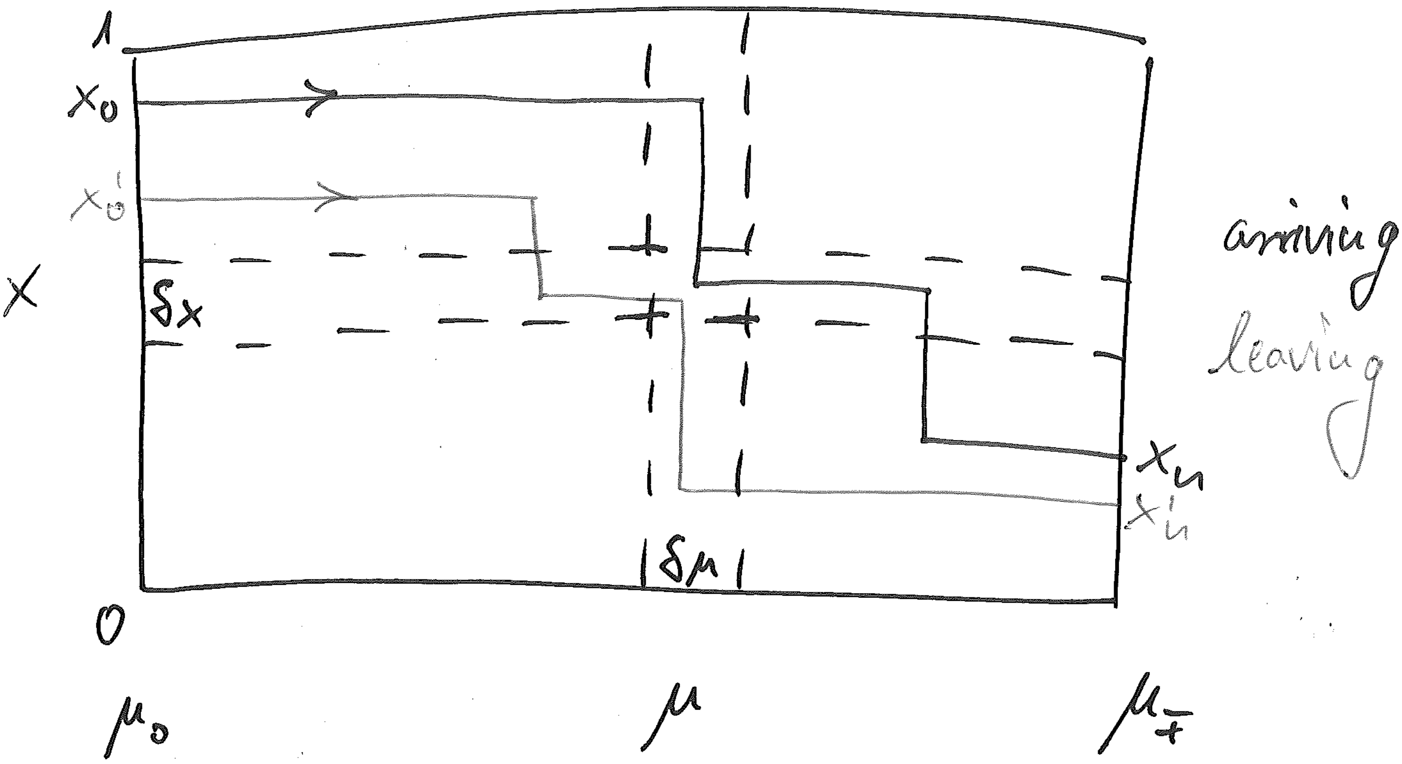


Drell-Yan at the LHC

Add one gluon

Add n gluons

$\Rightarrow$ additional gluon radiation moves the actual quark we use through the $\mu - x$ plane



$$\delta f_{\text{in}} = \frac{\delta\mu}{\mu} \int_x^1 dx'\, dz\, \frac{\alpha_s}{2\pi} P(z)\, \delta(x - z x')\, f(x', \mu)$$

$$\delta f_{\text{out}} = \frac{\delta\mu}{\mu} f(x, \mu) \int_0^x dx'\, dz\, \frac{\alpha_s}{2\pi} P(z)\, \delta(x' - z x)$$

$$\Rightarrow \boxed{\delta f = \frac{\delta\mu}{\mu} \int_0^1 dz\, \frac{\alpha_s}{2\pi} P(z) \left[\frac{1}{z} f\left(\frac{x}{z}, \mu\right) - f(x, \mu) \right]}$$

divergent structure

plus-perscription:

$$\int_0^1 dx \frac{g(x)}{(1-x)_+} = \int_0^1 dx \frac{g(x) - g(1)}{1-x}$$

$$\& \quad \frac{1}{(1-x)_+} = \frac{1}{1-x} \quad \text{if } 0 \leq x < 1$$

regularize divergence

restoring indices $\overset{i,j}{\Longrightarrow}$

$$\boxed{\mu\frac{\partial}{\partial\mu}f_i(x,\mu)=\sum_j\int_x^1\frac{dz}{z}\frac{\alpha_s}{2\pi}P_{ij}^{+}(z)f_j\left(\frac{x}{z},\mu\right)}$$

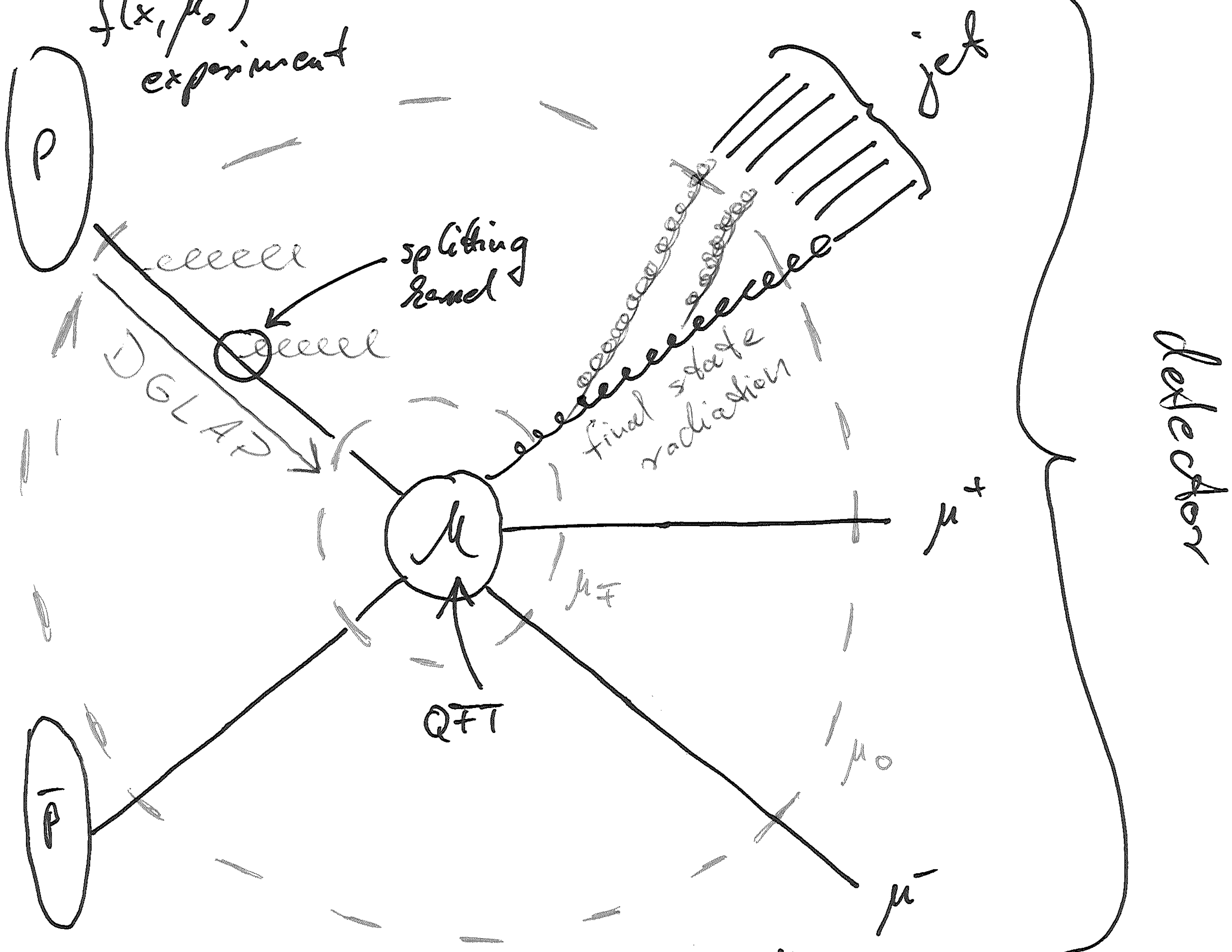
DGLAP evolution equation

pdf's are non-perturbative objects which have to be measured at low μ_0. With perturbative computation we can evolve these to any scale μ_F of the physics we are interested in.

Master equation for cross-sections

$$\boxed{\begin{aligned}\sigma(pp\to X,\sqrt{s})=\int_0^1 dx_1dx_2\sum_{i,j}f_i(x_1,\mu_F)f_j(x_2,\mu_F)\\ \otimes\hat{\sigma}(ij\to X,x_1x_2\sqrt{s},\mu_F)\end{aligned}}$$

Recapitulation:



$$\boxed{\text{scales}: \quad T = R = 1/E = 1/\mu}$$

Z production:

$\mu_0 = 1\,\text{GeV}$, $\mu_F = 90\,\text{GeV}$

$t_0 = 6.6 \cdot 10^{-25}\,s$, $t_F = 7.3 \cdot 10^{-27}\,s$

DGLAP eq.

$$\mu \frac{\partial}{\partial \mu} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} \frac{\alpha_s}{\partial \pi} P_{ij}(z) f_j\left(\frac{x}{z},\mu\right)$$

Master eq.

$$\sigma(pp \to X) = \sum_{ij} \int_0^1 dx_1 dx_2\, f_i(x_1,\mu_F) f_j(x_2,\mu_F) \otimes \hat{\sigma}(ij \to X)$$

inclusive!

Today

- „solving" & interpreting the DGLAP eq.
- final state radiation

If time:

- event simulation: merging, exclusive
- event reconstruction: top tagger (better ask Torben)
- jet scaling patterns

Sudakov form factor

$$\Delta(\mu,\mu_0) = \exp\left(-\int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}\int dz\,\frac{\alpha_s}{2\pi}P(z)\right)$$

re-write and integrate DGLAP:

$$f_i(x,\mu) = \Delta(\mu,\mu_0)\,f_i(x,\mu_0) + \int_{\mu_0}^{\mu}\frac{d\mu'}{\mu'}$$
$$\otimes\,\Delta(\mu,\mu')\int\frac{dz}{z}\,\frac{\alpha_s}{2\pi}\,P(z)\,f\left(\frac{x}{z},\mu'\right)$$

= nothing happened + one-splitting at μ' convoluted with $f(\mu')$

$\Rightarrow$ iterative solution resumming all splittings

$\Rightarrow$ Sudakov form factor is a non-splitting propability

$\to$ Marcov chain computer program!

The parton shower

looking at a single parton in (μ, x) space: $\longrightarrow (\mu_1, x_1)$

how to move to (μ_2, x_2) ?

$$\Delta(\mu_1, \mu_2)^{*} = R \quad \text{(random number)}$$

$$\Rightarrow \mu_2 = \ldots \quad \text{(if outside boundaries} \rightarrow \text{no further branching)}$$

~~* time-like vs. space-like~~

$$\int^{x_2/x_1} dz \, \frac{\alpha_s}{2\pi} P(z) = R' \int dz \, \frac{\alpha_s}{2\pi} P(z)$$

$$\Rightarrow \quad x_2 = \ldots$$

Note: angular distribution $\in [0, 2\pi]$ needs some extra treatment: gluon polarizations

* time-like vs. space-like:
"from or to high μ?"
incoming vs outgoing line

Backward evolution

the line connecting the hard process with the proton is special: pdfs.

$$(\mu_2, x_2) \longrightarrow (\mu_1, x_1)$$

$$\Delta(\mu_2, \mu_1) \frac{f(x_2, \mu_1)}{f(x_2, \mu_2)} = R$$

$\Rightarrow \mu_1$

$$\int^{x_2/x_1} dz \frac{\alpha_s}{2\pi} \frac{P(z)}{z} f\left(\frac{x_2}{z}, \mu_1\right) = R' \int dz \frac{\alpha_s}{2\pi} \frac{P(z)}{z} f\left(\frac{x_2}{z}, \mu_1\right)$$

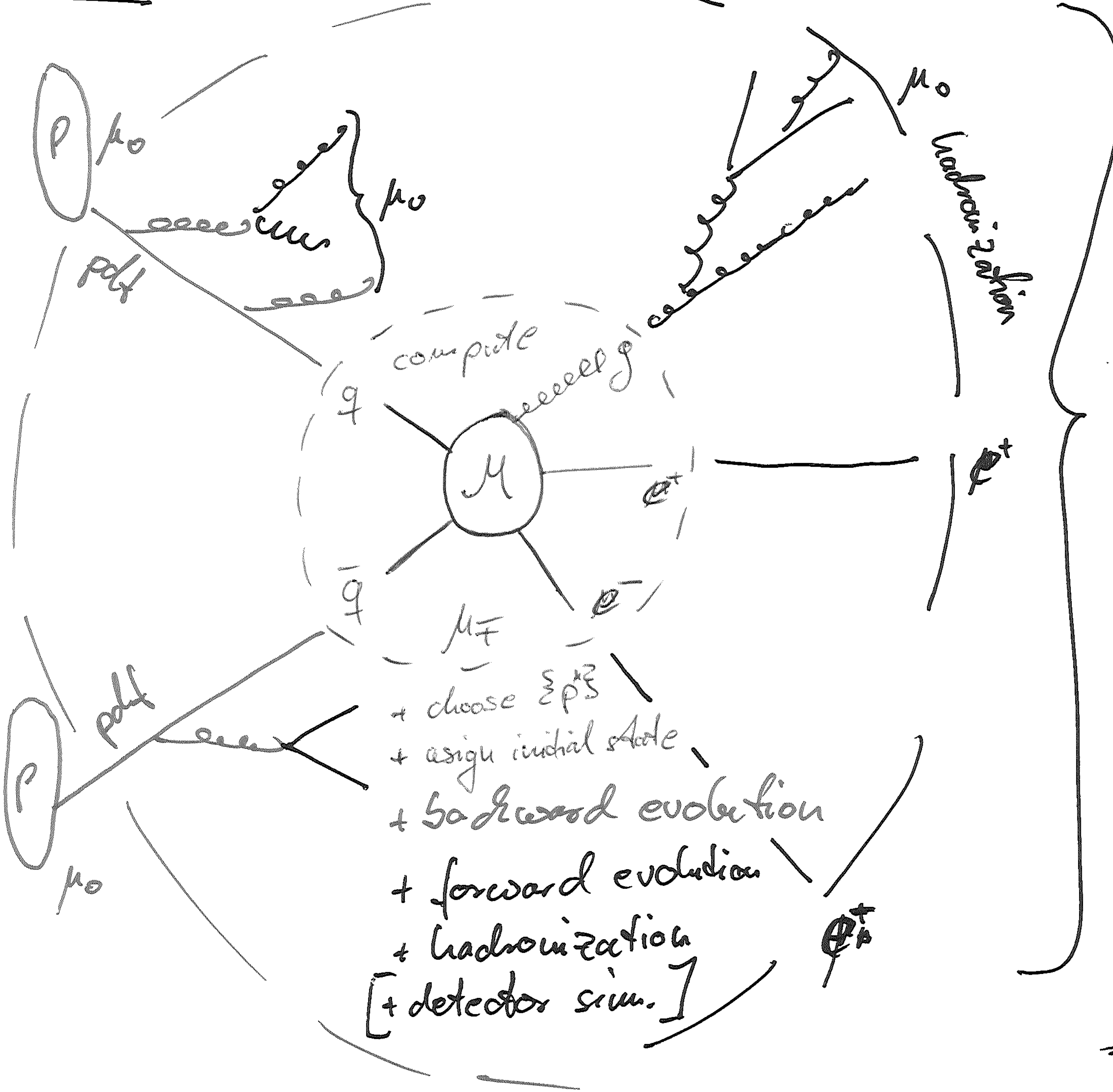
$\Rightarrow x_1$

formal: Ellis QCD chapter 5

What happens at μ_0?

- transition from partons to hadrons
- we need models to do this (data input)

The full picture: „exclusive" event for Drell-Yan plus one jet



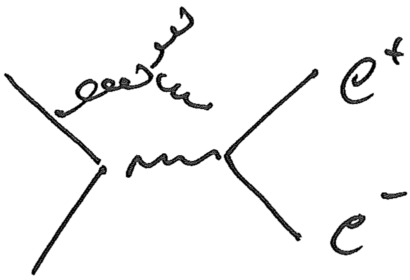
detector: $\boxed{\ell^+\ell^- + \text{jets}}$

$\Rightarrow$ iterate

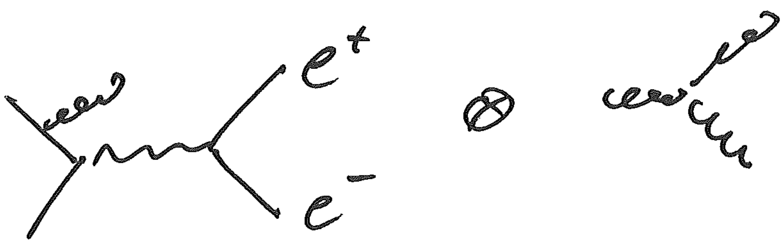
Matching & Merging

How to decide which jets do part into the matrix element and which into the parton shower?

Problem: over-counting



matrix element



parton shower

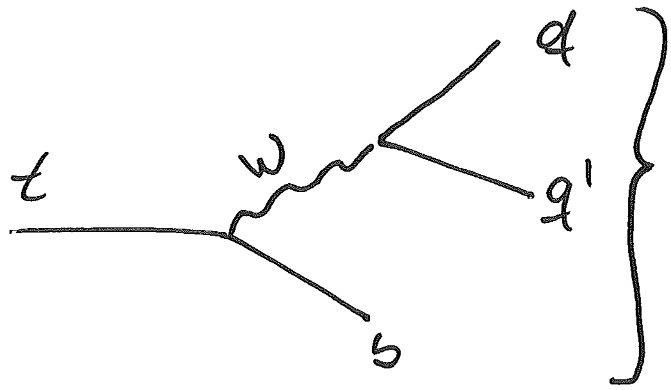
veto strategy	$\neq$ matching scale
veto shower configurations which mimik ME	divide phase space between ME and PS

$\boxed{\text{truely exclusive}}$

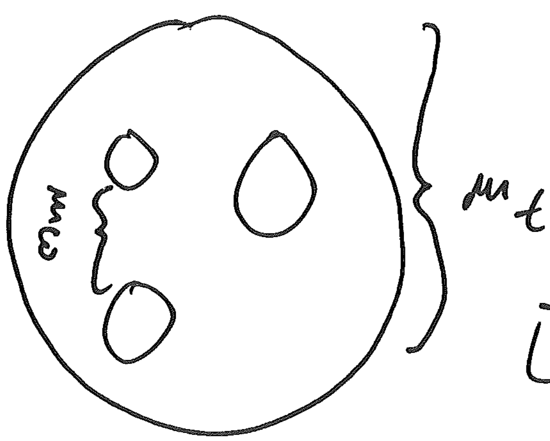
complicated final states : top tagging

expert in the room : Torben

top life time < 10^{-25} s => no hadronic bound states



fat-jet with substructure



$\boxed{\text{signal}}$

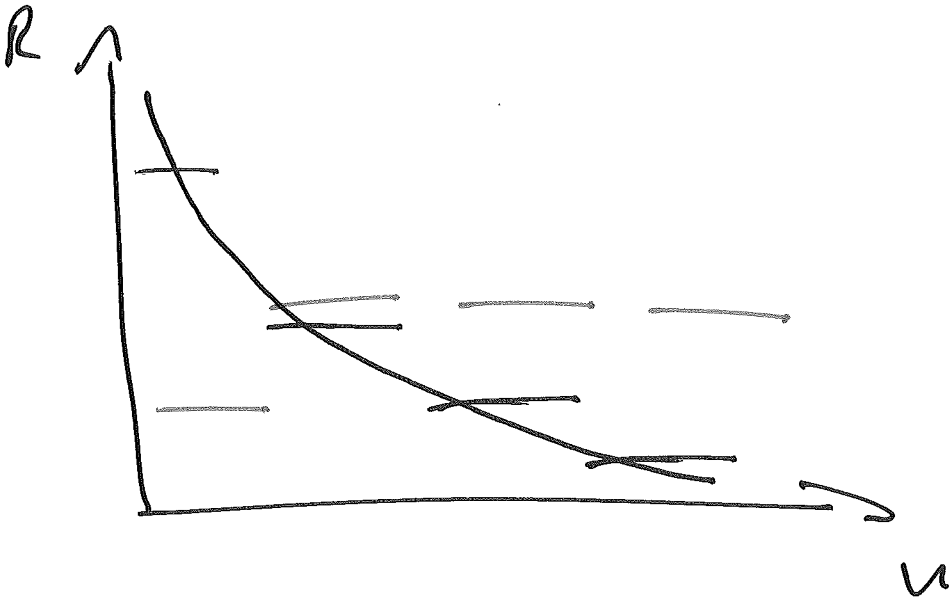
[use detailed decay knowledge for a good detection]

To do this correctly we need to understand how „normal" jets form and behave!

Jet scaling patterns

QED Bremsstrahlung

$$R_{\frac{n+1}{n}} = \frac{\sigma_{n+1}}{\sigma_n} = \frac{c}{n+1}$$



QCD Bremsstrahlung

$$R_{\frac{n+1}{n}} = \frac{\sigma_{n+1}}{\sigma_n} = \text{const}$$

→ understandable in terms of splitting kernels and evolution eq.