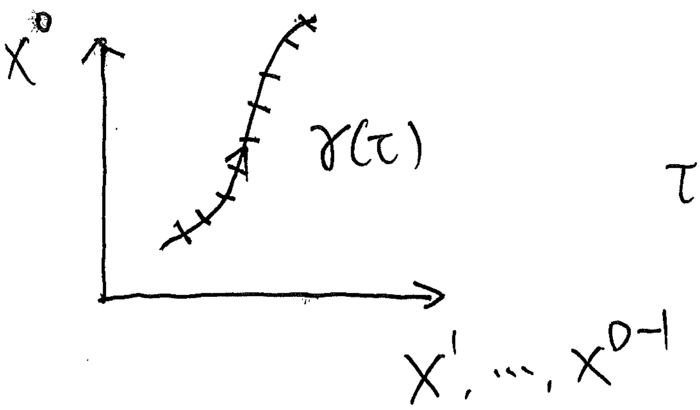
Lecture 1 one hour's tour in (Bosonic) String theory

I. Bosonic Strings

1. classical action

i) recall the case with a relativistic particle.

A propagating of a point-like free particles traces out a world-line γ in flat space-time $R^{1,D-1}$



$$\gamma : \tau \longrightarrow X^{\mu}(\tau) \in R^{1,D-1}.$$

τ: A priori arbitrary parameter labelling the position along the world-line

* Action: $$S_{NG} = -m\int ds = -m\int\sqrt{-\eta_{\mu\nu}dX^{\mu}(\tau)dX^{\nu}(\tau)}$$
$$= -m\int d\tau\sqrt{-\eta_{\mu\nu}\dot{X}^{\mu}\dot{X}^{\nu}}$$

* E.O.M: $\partial_{\tau}^2 X^{\mu}(\tau) = 0$

* Canonical Momentum: $$p^{\mu} = \frac{\delta \mathcal{L}}{\delta(\partial_{\tau}X^{\mu})} = m\frac{\dot{X}^{\mu}}{\sqrt{-\dot{X}^2}}$$

* NOTE: $p^{\mu}p_{\mu} = -m^2$ (Constraint)

The Constraint tells us that $\{X^{\mu}, p^{\mu}\}$ are redundant for describing the real physical phase spaces.

lesson: This is Gauge symmetry!!!

* under world-line γ reparametrization

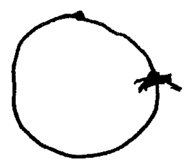
$$\tau \longrightarrow \tau'(\tau)$$

The action S_{NG} is <u>invariant</u>!!!

1.2. Generalitation to classical reletivistic string.

* two topologies associated with 1 dim string,

$\boxed{\text{focusing on oriented string}}$



closed string

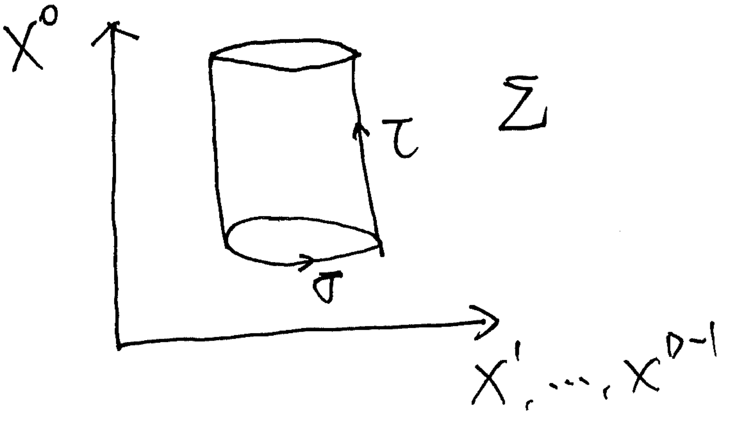
open string

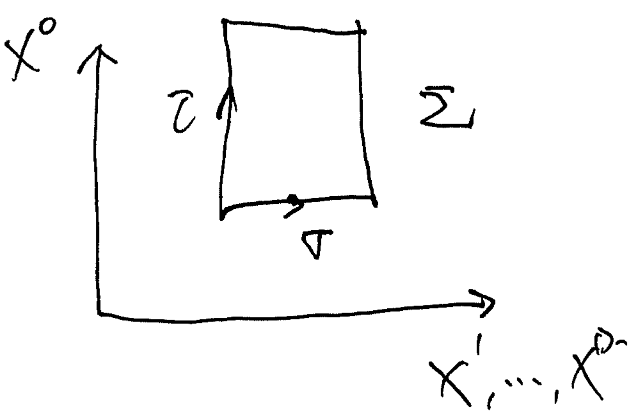
spoiler alert:

$\boxed{\text{associated with Gravity}}$

$\boxed{\text{associated w/ Yang-mills}}$

* world-sheet Σ: $(\sigma, \tau) \longrightarrow X^\mu(\tau, \sigma) \in R^{1,d-1}$





* classical action

③

By anology to the point-like particle, taking action proportional to world-sheet area.

$$S_{NG} = -T\int_\Sigma dA \qquad dA = \sqrt{-\det\left(\frac{\partial X^\mu}{\partial \tau}\frac{\partial X^\nu}{\partial \sigma}\eta_{\mu\nu}\right)}d\tau d\sigma$$

hard to proceding e.g. Quantization

Can also be expressed by

focusing on this one

$$S_p = -\frac{T_p}{2}\int d\tau d\sigma \sqrt{-\det h}\, h^{ab}\,\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}.$$

- h^{ab}: dynamic metric on the 2d world-sheet Σ.

from 2d perspective, this is a 2d Gravity coupled to Scalars $\underline{X}^\mu$!!!, although X^μ is a vector in space-time.

- Symmetries

i): space-time symmetry: poincare symmetry

$$X^\mu(\tau,\sigma) \longmapsto \Lambda^\mu{}_\nu X^\nu(\tau,\sigma) + V^\mu, \quad \Lambda \in SO(1,d-1)$$

From 2d world-sheet perspective, this is global internal symmetry

ii): WS symmetries: denoting $(\tau,\sigma) \to \xi^a$. $a=0,1$

① under $\xi^a \to \tilde{\xi}^a(\xi)$: Reparametrization

$$X^\mu(\xi) \to \tilde{X}^\mu(\tilde{\xi}) = X^\mu(\xi): \quad \text{Scalar}$$

$$h_{ab}(\xi) \to \tilde{h}_{ab}(\tilde{\xi}) = \frac{\partial \xi^c}{\partial \tilde{\xi}^a}\frac{\partial \xi^d}{\partial \tilde{\xi}^b} h_{cd}(\xi): \quad \text{2d metri}$$

②: Weyl invariance

$$h_{ab}(\xi) \to \tilde{h}_{ab}(\xi) = \Omega^2(\xi)\, h_{ab}(\xi)$$

$$X^\mu(\xi) \to X^\mu(\xi)$$

NOTE: both WS symmetries are Gauge Symmetry.

* T_p: String Tension: $\frac{\text{mass}}{\text{length}}$ $\quad T_p := \frac{1}{2\pi\alpha'}$

$[T] = [\text{Mass}]^2$

string length: $l_s = 2\pi\sqrt{\alpha'}$

string mass scale: $M_s = \frac{1}{\alpha'}$

α': the only single parameter in string theory !!!

2. Solving E.O.M

* introducing Momentum-Energy Tensor: T_{ab}

$$T_{ab} := \frac{4\pi}{\sqrt{-h}} \frac{\delta S_p}{\delta h^{ab}} = -\frac{1}{\alpha'}\left(G_{ab} - \frac{h_{ab}}{2} h^{cd} G_{cd}\right)$$

w/ $G_{cd} = \partial_c X^\mu \cdot \partial_d X^\nu \eta_{\mu\nu}$

E.O.M for $h^{ab} \Rightarrow T^{ab} = 0 \Rightarrow G_{ab} = \frac{1}{2}(h\cdot G)\, h_{ab}$

Verifying $\quad S_p[X, h_{ab}]\Big|_{T^{ab}=0} \equiv S_{NG}$.

* E.O.M for $X^\mu(\xi^a)$

5

- by using WS symmetries (Gauge symmetries)

* $\boxed{h^{ab} = \eta^{ab}}$ Gauge fixing.

- $$\delta S_p = -T \int d^2\xi \, [\partial_\tau^2 - \partial_\sigma^2] X \cdot \delta X$$
$$\underbrace{- T \int d\tau \, \partial_\sigma X \cdot \delta X \Big|_{\sigma=0}^{\sigma=l}}_{\text{Boundary term}}$$

- Boundary Conditions

i). for closed string (oriented)

$$X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + l)$$

ii). For open string. For each endpoints $\sigma=0$, $\sigma=l$.

a). Neumann Boundary: $\partial_\sigma X^\mu \big|_{\sigma=0 \text{ and/or } \sigma=l} = 0$

b). Dirichlet Boundary: $\delta X^\mu \big|_{\sigma=0 \text{ and/or } \sigma=l} = 0$

NOTE: Dirichlet Boundary for X^μ Correspond to the string endpoint being fixed in the μ-direction. This breaks poincare-symmetry in space-time!

From NOW on. We mainly focus on open string.

* general solutions for open-strings w/ ~~Neumann~~ various B.C.

(NN) : $X^\mu(\tau,\sigma) = x^\mu + \frac{2\pi\alpha'}{\ell} p^\mu \tau + i\sqrt{2\alpha'} \sum_{n\neq 0 \in \mathbb{Z}} \frac{1}{n}\alpha_n^\mu e^{-\frac{\pi}{\ell} n\tau} \cos\left(\frac{n\pi\sigma}{\ell}\right)$

- x^μ : position of Center-of-mass of the String
- p^μ : Momentum ....
- α_m^μ : the oscillator modes of the Strings

(DD) : $X^\mu(\sigma=0) = x_0^\mu$. $X^\mu(\sigma=\ell) = X_1^\mu$

$$X^\mu(\tau,\sigma) = x_0^\mu + \frac{1}{\ell}(X_1^\mu - X_0^\mu)\sigma + \sqrt{2\alpha'} \sum_{n\neq 0 \in \mathbb{Z}} \frac{1}{n}\alpha_n^\mu e^{-\frac{i\pi n\tau}{\ell}} \sin\left(\frac{n\pi\sigma}{\ell}\right)$$

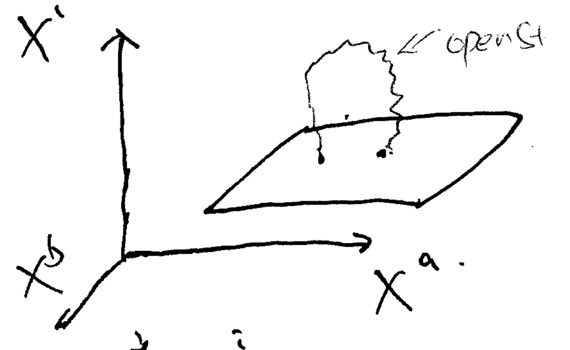
(ND) ....

upshot : The E.O.M for classical string action hence => S

$$\begin{cases} (\partial_\tau^2 - \partial_\sigma^2) X^\mu(\xi^a) = 0 \quad \text{plus Boundary condition} \\ T_{ab} = 0 \quad (\text{Constraint}) \end{cases}$$

3 : first introducing D-brane

Considering the following Configuration.



i) (NN) for X^a, $a = 0, \dots, p$

ii) (DD) for X^i. $i = p+1, \dots, D-1$, & $X_0^i = X_1^i$

A Dp-brane is a (p+1) dim hyperplane of space-time on which open strings can end. 7

4. Constraint $T_{ab} = 0$

* For open string, non-diagonal parts vanish automatically

$$T_{\pm\pm} = 4\alpha' \sum_{m=-\infty}^{+\infty} L_m e^{-2im(\tau \pm \sigma)}$$

$\alpha_0^\mu = \sqrt{\frac{\alpha'}{2}}\, p^\mu$

w/ $L_m = \frac{1}{2}\sum_n \alpha^\mu_{m-n}\alpha^\nu_n \eta_{\mu\nu}$, classical level.

$$T_{\pm\pm} = 0 \implies \boxed{L_m = 0 \quad \forall m}$$

for $m=0$. $\begin{cases} L_0 = 0 \\ M^2 = -p^2 \end{cases} \implies \boxed{M^2 = \frac{1}{\alpha'}\sum_{n=1}^{\infty}\alpha_{-n}\alpha_n}$

5: Quantization.

* Canonical quantization.

promoting the classical fields X^μ & its Canonical Momentum Π^μ to operators and replace

$$\{\,,\,\}_{PB} \longrightarrow \frac{1}{i}[\,,\,]$$

Canonical equal time commutation relations

$[\hat{x}^\mu, \hat{p}^\nu] = i\eta^{\mu\nu}$, $[\hat{\alpha}^\mu_m, \hat{\alpha}^\nu_n] = m\,\delta_{m+n,0}\,\eta^{\mu\nu}$

Hermicity $X^\mu \implies \hat{X}^\mu = (\hat{X}^\mu)^\dagger$. $\hat{\Pi}^\mu = (\hat{\Pi}^\mu)^\dagger \implies \alpha^\mu_m = (\alpha^\mu_{-m})^\dagger$

Applying similar story from harmonic oscillator in QM

* $\alpha^{\mu}_{-|m|}$: creation operators
  $\alpha^{\mu}_{|m|}$: annihilate operators
  $\Bigg\} +$ normal ordering $:\alpha^{\mu}_{m}\alpha^{\nu}_{n}: = \begin{cases} \alpha^{\mu}_{m}\alpha^{\nu}_{n} & \text{if } m\leq \\ \alpha^{\nu}_{n}\alpha^{\mu}_{m} & \text{if } n< \end{cases}$

Number operator $N := \sum_{m} \alpha^{\mu}_{-m}\alpha^{\nu}_{m}\eta_{\mu\nu}$

whose eigenvalues count the number of excited modes.

* Fock space : spanned by the set of states

$$\{ \prod_{m} (\alpha^{\mu}_{-m})^{n_{m,\mu}} |0;p\rangle \}$$

where $|0;p\rangle$ is ground state satisfying

a) $\hat{p}^{\mu}|0;p\rangle = p^{\mu}|0;p\rangle$

b) : $\alpha^{\mu}_{m}|0;p\rangle = 0 \qquad \forall m>0$

$\Rightarrow$ oscillators of the string appear as particle in space-time

* However, similar to the QED covariant quantization.

We have negative norm states

$$\langle p';0|\alpha^{0}_{1}\alpha^{0}_{-1}|0;p\rangle \sim -\delta^{D}(p-p').$$

reason, We haven't imposed Constraints $T_{ab}=0$.

We've seen in classical level $\cdot$ $T_{ab}=0 \Rightarrow L_m = 0 \ \forall m$.

However, in Quantum level, We should impose

$$\langle phys | L_m | phys \rangle = 0 \qquad \forall m$$

Since $L_m^\dagger = L_{-m}$

$$\Rightarrow \quad L_m | phys \rangle = 0 \qquad \forall m \neq 0$$

~~While~~ For $m=0$. L_0 Case, there is ordering ambiguity.

For $m \neq 0$. Since $[\alpha_{m-n}, \alpha_n] = 0$, $L_m = \frac{1}{2} \sum_n \alpha^\mu_{m-n} \alpha^\nu_n \eta_{\mu\nu}$

is well-defined. but for $m=0$, this is not the case!!

This seems not to much surprise. In QFT, This is anologous

to the Zero-energy

$$H = \int \frac{d^3p}{(2\pi)^3} E_p a_p^\dagger a_p + \Delta \delta(0)$$

from $[a_p, a_p^\dagger]$

hence for $m=0$ We instead impose

$$(L_0 - a) | phys \rangle = 0 \quad \text{for Some Constant } a.$$

Spoiler alert: in Bosonic String. The absence of these negative norm States require $\begin{cases} a = 1 \\ D = 26 \end{cases}$

remarks: The fixed dimension of string theory can be deduced from CFT perspective.

Actually, the world-sheet 2d theory with flat space-time is a 2d CFT, in Quantum level, There is poential Conformal symmetry Anomaly: $D=26$ ($D=10$ for superstring) is the condition for the Anomaly cancellation !!!.

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* Mass Formula,

For open string:

$$M^2 = \frac{1}{\alpha'}(N-1)$$

side NOTE:

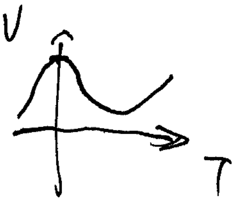
$$\boxed{\begin{array}{l} J_{max} = N = \alpha' M^2 + 1 \\ \text{: Regge trajectors.} \\ \text{why } \alpha' \text{ called "Regge Slope"} \end{array}}$$

* spectrum:

$N=0 \Rightarrow$ ground state $|0;p\rangle$. $M^2 = -\frac{1}{\alpha'} < 0$

tachyon $\Rightarrow$ the vacuum is not stable

$$M^2 = \frac{\partial^2 V(T)}{\partial T^2}\Big|_{T=0}$$



$N=1 \Rightarrow (\alpha_{-1}^{\mu})|0;p\rangle$. space-time Vector: massless vector

$M^2 = 0$

$N=2$: $\left\{ \begin{array}{l} (\alpha_{-2})^{\mu}|0;p\rangle \\ \alpha_{-1}^{\mu}\alpha_{-1}^{\nu}|0;p\rangle \end{array} \right.$ $M^2 = \frac{1}{\alpha'} \sim (M_{string})^2$

Very heavy!!!

* A remark for closed string spectrum ($D=26$).

* The ground state is still tachyon $M^2 < 0$

* $N=1$. $M^2 = 0$. $\xi_{ij}\, \tilde{\alpha}^i_{-1} \alpha^j_{-1} |0;p\rangle$ $i,j = 1,\cdots,24$

The other two D.O.F is elliminated by Gauge fixing condit.

decomposing ξ_{ij} as

$$\xi_{ij} = \xi_{(ij)} + \xi_{[ij]} + \xi^{(0)}$$

$\xi_{(ij)}$: symmetric traceless; $\xi_{[ij]}$: antisymmetric; $\xi^{(0)}$: Scalar (trace-part)

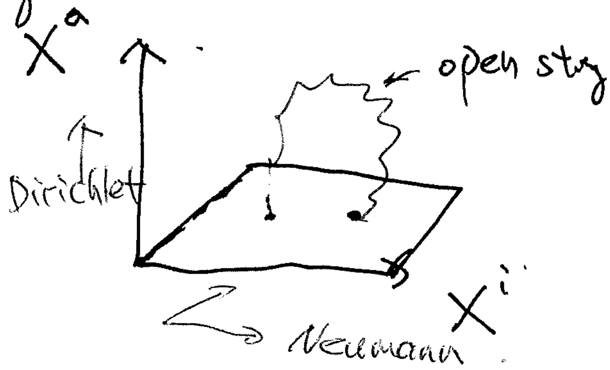
$\xi_{(ij)}$: $G_{\mu\nu}$: $\boxed{\text{Graviton}}$!!!

$\xi_{[ij]}$: $B_{\mu\nu}$: 2-form field

$\xi^{(0)}$: Φ : dilaton : $g_s \sim e^{\langle \Phi \rangle}$

Spectrum along D-brane

(i): single D-brane (A world of light)



(NN): $i = 0, \cdots, p$

(DD) : $a = p+1, \cdots 25$

We distinguish.

(1): excitations along the brane in directn X^i. $i = 0, \cdots, p$

(2). excitations orthogonal . . . . X^a. $a = p+1, \cdots, 25$

First Lorentz group $SO(1,25) \longrightarrow SO(1,p) \times SO(25-p)$.

Spectrum

* $N=0$. still tachyon :

* $N=1$.

1): $\alpha_{-1}^{i'}$ $\quad i' = 0, \cdots, p-1$. $m^2 = 0$

The space-time indices lie within the brane. and transform as fundamental representation $\square$ of $SO(p-1)$.

This must be gauge poential by general arguments of QFT! We introduce a gauge field A_i with $i=0, \cdots, p$ lying on the brane whose quata are identifield w/ photo.

2). $\alpha_{-1}^{a}|0;p\rangle$ $\quad a = p+1, \ldots, D-1$, $M^2=0$

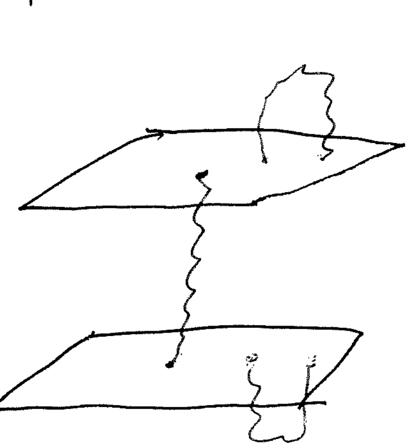
Since they don't carry an index "i" along the D-brane.

Those form a collection of 24-p massless scalar fields from the perspective of the Dp-brane.

remark: They're Goldstone Bosons associated to spontaneous Breaking of 26 dim poincare-symmetry.

** parallel Dp-branes (A world of glue)

↑ Dirichlet



(NN). X^i $\quad i=0,\ldots p$

(DD) X^a: $a=p+1,\ldots,25$ $\quad \rightleftarrows$ Neumann.

$X_1^a \neq X_2^a$

* We now find 2 sets of massless states.

$$
\left.\begin{array}{ll}
\text{i). } \alpha_{-1}^{i}|0_{11},p\rangle & M_{11}^2=0 \\
\alpha_{-1}^{i}|0_{22},p\rangle & M_{22}^2=0
\end{array}\right\} \text{vector} \;\&\; \text{Scalar} \left\{\begin{array}{l}
\alpha_{-1}^{a}|0_{11},p\rangle \\
\alpha_{-1}^{a}|0_{22};p\rangle
\end{array}\right.
$$

in addition

* $\alpha_{-1}^{i}|0_{12};p\rangle$ $\quad \alpha_{-1}^{i}|0_{21};p\rangle$ Massive Vectors

$$M_{12}^2 = M_{21}^2 = \frac{1}{2\pi\alpha'}\frac{(X_2^a - X_1^a)^2}{2\pi\alpha'}$$

as well as Massive Scalars

* Coincident Dp-brane $(X_2^a - X_1^a)$

$\Rightarrow$ 4 massless vector $\quad \alpha_{-1}^{i'} |0_{ij}; p\rangle \quad i, j = 1, 2.$

4 massless Scalar $\quad \alpha_{-1}^{a} |0_{ij}; p\rangle$

in general, Consider N ~~para~~ coincident Dp-branes.

$\Rightarrow$ N^2 Massless Vectors $\quad \alpha_{-1}^{i'} |0_{ij}, p\rangle. \quad i, j = 1 \ldots$

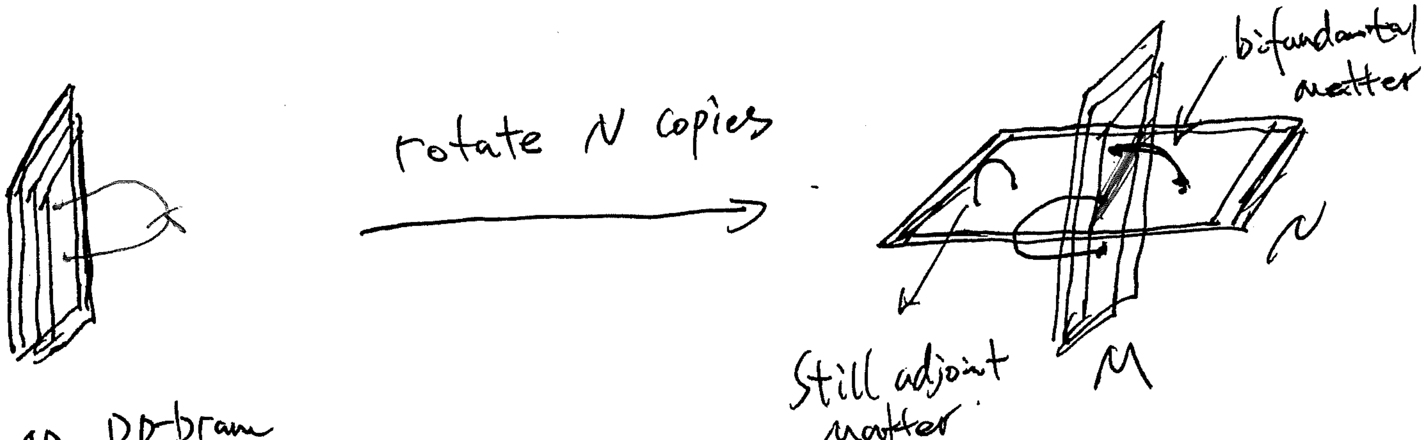
N^2 $\cdots$ Scalars $\quad \alpha_{-1}^{a} |0_{ij}; p\rangle \quad i, j = 1 \cdots N$

Adjoint scalars

$\boxed{\text{A Stack of } N \text{ Coincident Dp-brane Carries a } U(N) \text{ gauge theory}}$

Final remarks: How to get charged matter ???

rotate N copies



$N+M$ Dp-brane

$U(N+M) \longrightarrow U(M) \times U(N)$

Adjoint representation

$(M+N)^2 \longrightarrow M^2 + N^2 + \boxed{(M, \bar{N}) + (\bar{M}, N)}$

bifundamental matters

6: Strings on curved backgrounds

1: Curved target space-time w/ metric $G_{\mu\nu}(X)$.

$$S_\sigma = \frac{1}{4\pi\alpha'}\int_\Sigma d^2\xi \sqrt{h}\, h^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X)$$

- $G_{\mu\nu}$: understood as a coherent state of gravitons describing the fluctuations of the metric around $\eta_{\mu\nu}$

* Anology: Quantum optics

We set out in perturbative QED to Quantize the electromagnetic vacuum and describing its fluctuations by photons. A coherent state of these vacuum fluctuations represents a laser-field. likewise $G_{\mu\nu}(X)$ can be viewed as a coherent excitation of gravitons

: general lesson:
The string propagates in a background described by a coherent state of its own massless fluctuations!

* generalise to other fields $B_{\mu\nu}$, Φ

$$S_\sigma = \frac{1}{4\pi\alpha'}\int_\Sigma d^2\xi \sqrt{h}\,\{ (h^{ab}G_{\mu\nu} + i\epsilon^{ab}B_{\mu\nu})\partial_a X^\mu \partial_b X^\nu + \alpha' R^{(2)}\Phi \}$$

: "non-signal model"

f Coinstency condition.

i). Focusing only $G_{\mu\nu}$:

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma\, G_{\mu\nu}\partial_a X^\mu \partial^a X^\nu$$

(Weyl invariance at classical level)

Quantum level: poential Anomaly!!!

Weyl (conformal) invariant at Quantum level.

$$\beta_{\mu\nu}(G) := \mu \frac{\partial G_{\mu\nu}}{\partial \mu} = 0$$

at one-loop $\quad \beta_{\mu\nu}(G) = \alpha' R_{\mu\nu}$

$\Rightarrow R_{\mu\nu} = 0$!!! Ricci flat!!!

in other words, the background space-time where string propagate must obey the Vacuum Einstein equation!!!

Application: in ~~str~~ Superstring theory. We consider Calabi-Yau

ii). including other fields $B_{\mu\nu}$. Φ

$$\beta_{\mu\nu}(G) = \alpha' R_{\mu\nu} + 2\alpha'\nabla_\mu\nabla_\nu\Phi - \frac{\alpha'}{4}H_{\mu\lambda k}H_\nu{}^{\lambda k} \quad w/\ H = dB$$

$$\beta_{\mu\nu}(B) = -\frac{\alpha'}{2}\nabla^\lambda H_{\lambda\mu\nu} + \alpha'\nabla^\lambda\Phi H_{\lambda\mu\nu}$$

$$\beta(\Phi) = -\frac{\alpha'}{2}\nabla^2\Phi + \alpha'\nabla_\mu\Phi\nabla^\mu\Phi - \frac{\alpha'}{24}H_{\mu\nu\lambda}H^{\mu\nu\lambda}$$

$B_{\mu\nu}(G) = B_{\mu\nu}(B) = \beta(\Phi) = 0 \Longrightarrow$

$$S = \frac{1}{2k_0^2}\int d^D x\sqrt{-G}\, e^{-2\Phi}\left(R - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} + 4\partial_\mu\Phi\partial^\mu\Phi\right)$$

$\leftarrow$ (low-energy effective action)

A nod to Superstring:

two major shortcomings associating w/ Bosonic String

i): tachyonic ground state $\longrightarrow$ vacuum instability

ii): only bosonic excitations. No Fermions (matter)

Superstring: Adding 2d Fermions to the Sp + WS ~~Supersym~~ Supersymmetry

e.g. For Type II theory

$$S_{NSR} = -\frac{1}{4\pi}\left(\int_\Sigma d^2\xi \, \frac{1}{\alpha'} \partial_a X^\mu \partial^a X^\nu \eta_{\mu\nu} + 2i \bar{\psi}^\mu_A \gamma^a_{AB} \partial_a \psi_{\mu B}\right).$$

$\psi^\mu_A = \begin{pmatrix} \psi^\mu_+ \\ \psi^\mu_- \end{pmatrix}$: 2d Ferminors (Majorana-Weyl spinors)

by anology procedure (though more laborious task)

~~led~~ leading to

(i): $D = 10$

(ii). No tachyon.

$\boxed{\text{Massless states}}$ including $\boxed{G_{\mu\nu}.\ B_{\mu\nu}.\ \Phi}$: Same as BS

Additionally, Some high p-forms fields

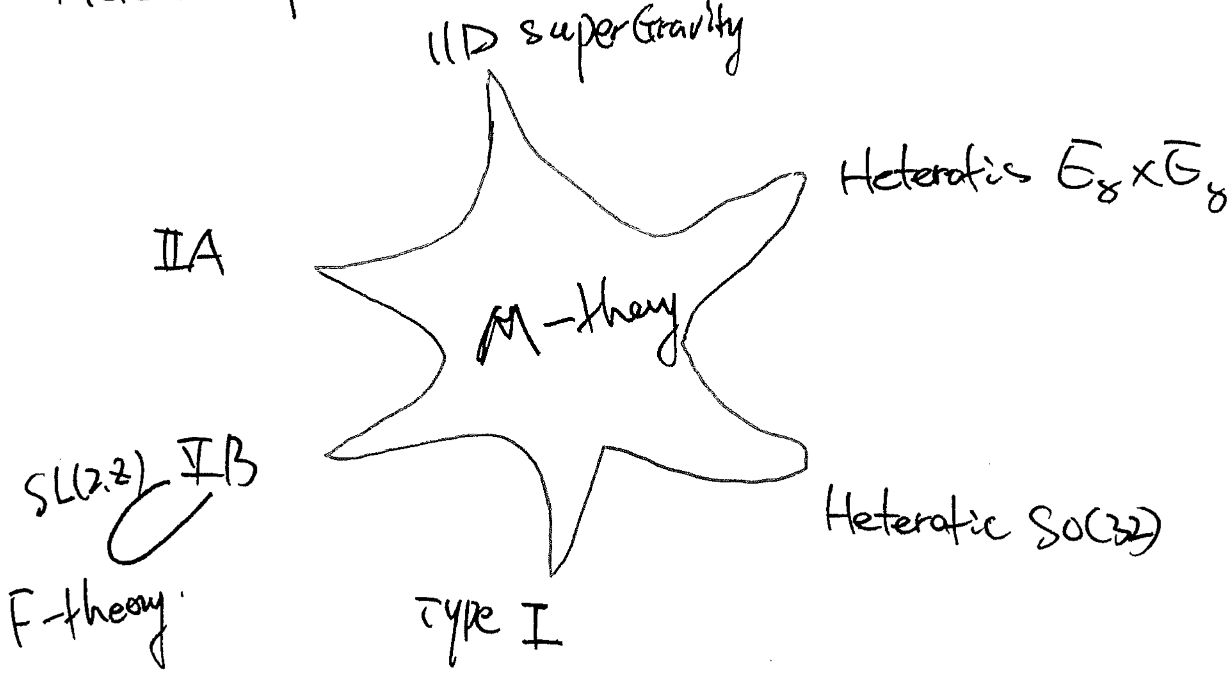
Type IIB : C_0. $C_{\mu\nu}$. $C_{\mu\nu\rho\sigma}$

Type IIA . C_μ, $C_{\mu\nu\rho}$

iii): The space-time has <u>supersymmetry</u>.

Means there are some superpartner — <u>Fermions</u>.

There're five version of perturbative superstring.



(non-perturbative strong coupling limit)

§ D-brane in superstring (Type II)

*: not every dimension in space-time has Dp-brane.

P: even in Type IIA
odd in Type IIB

> they're charged under p-form C fields (gauge field ~~(the photo (A) is~~ coupling ~~of photon~~.

recall electromagnetism.

gauge poentital $A_\mu \to A_\mu + \partial_\mu f$

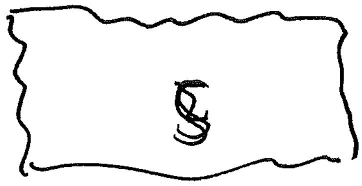
coupling of a charged particle. $S \supset q_e \int dx^\mu A_\mu$ ← charge

$\gamma \longrightarrow \int_\gamma A$ A is a 1-form higher dimensional gauge fields C_p

e.g. $C_{\mu\nu} \xrightarrow[\text{transformati}]{\text{gauge}} C_{\mu\nu} + \underbrace{d\Lambda_1}_{\text{2-form}}$

$$(d\Lambda_1)_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu A_\mu .$$

Can be integrated over a 2-dimensional ~~[illegible]~~ surface S



$\longrightarrow \int_S C_{\mu\nu} dx^\mu dx^\nu$

S is the D1-brane.

IIB: $D(2p+1) \longleftrightarrow C^{(2p+2)}$ $\quad p = -1, 0, \cdots 4$.

IIA: $D(2p) \longleftrightarrow C^{(2p+1)}$ $\quad p = 0, \cdots, 4$

Dp-brane sourced $C^{(p+1)}$-form field !!!

** D-brane are by themselves <u>dynamical objects</u>

(i) are charged under R.R p-form fields $C^{(p)}$.

(ii): gravitate by coupling to closed string in the NS-NS sector

They have <u>mass</u> !

This seems however, not to be much surprise since we've discussed already closed string generates gravity. the gravitational waves passing through the hyper-plane would wrap space-time itself.

So the Dp-brane as a (p+1) dim hyperplane in the space-time could hardly remain rigid!

$$S_{Dp} = - T_p \int d^{p+1}\xi \sqrt{-\det \gamma}$$

$$\gamma_{ab} := \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b} \eta_{\mu\nu}$$

more general, in a background generated by closed string modes

$G_{\mu\nu}$, $B_{\mu\nu}$, Φ

$$\boxed{S_{DBI} = - T_p \int_{Dp} d^{p+1}\xi \sqrt{-\det(\gamma_{ab} + 2\pi\alpha' F_{ab} + B_{ab})}\; e^{\Phi}}$$

This describe D-brane coupled to NS-NS fields

~~For coupling to R-R-fields C^p,~~

$$\boxed{S_{CS} = -\mu_p \int_{Dp} \oplus_{p+1} C_{p+1} \wedge ch(F) \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}}}$$

to lowest order $S_{CS} \supset -\mu_p \int_{Dp} C^{(p+1)}$

$$\boxed{S_{DBI} = -T_p \int d^{p+1}\xi \left(1 + \frac{(2\pi\alpha')^2}{4} F_{ab} F^{ab} + \cdots\right)}$$