...e 2: Type IIB superstring

): Reviews & Adds on last lecture

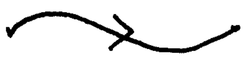
* We've described the propogating of a string in flat space-time $R^{1,D-1}$ w/ Polyakov action

$$S_p = \frac{-T}{2} \int_\Sigma d^2\xi \sqrt{-h}\, h^{ab}\, \partial_a X^\mu\, \partial_b X^\nu\, \eta_{\mu\nu}$$

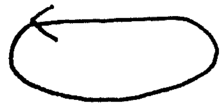
* At the Quantum level:

①: $D = 26$ & ground state is tachyon.

②:



open string



closed string

massless spin 1 : A_μ ; ○ massless spin 2 : $G_{\mu\nu}$.

$\boxed{\text{Yang-Mills}} \xleftrightarrow{\text{AdS/CFT}} \boxed{\text{Gravity}}$

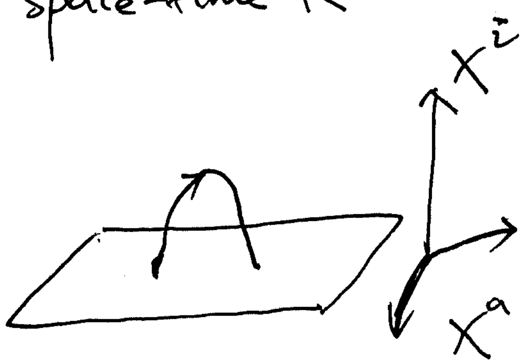
deep reason: D-brane in superstring { Solution of SuperGravity, YM.

③: Dp-brane:

*): As a (p+1) dim hyperplane in space-time $R^{1,D-1}$ where open string can end.

(NN): X^a : $a = 0, 1, \cdots, p$

(DD): X^i : $i = p+1, \cdots, D-1$



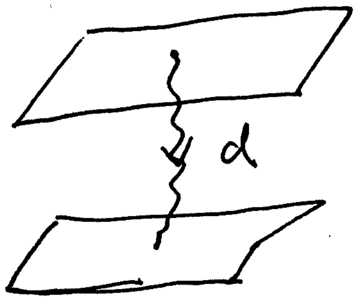
in the perspective of D-brane, First excited states associated with the open string fall into two class

①: $\alpha_{-1}^{a} |0; p\rangle \longrightarrow A_a$: Gauge field living on the wolrd-volume of Dp-brane

②: $\alpha_{-1}^{i} |0; p\rangle \longrightarrow \Phi^i$: Massless scalar.

Goldstone Bosones $\Rightarrow SO(1, D-1) \longrightarrow SO(1, p) \times SO(D-p-1)$

i): 2 Dp-brane



For: A_a. Φ^i

$M_{mass}^2 = T^2 d^2$

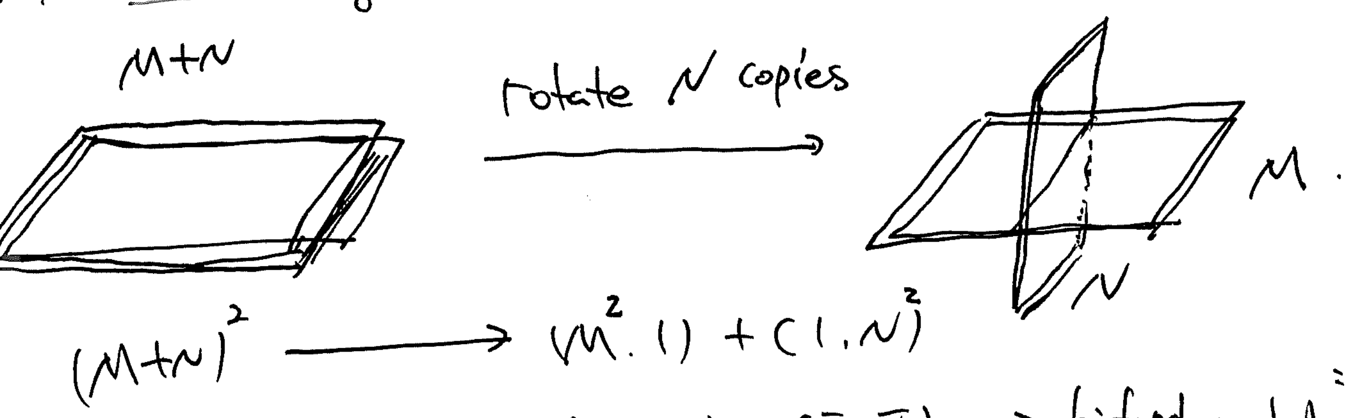
ii): coincident N Dp-brane:

$(A_a)^m_n$. $(\Phi^i)^m_n$. $m, n = 1, \ldots, N$.

"Higgsing"

Carries a "$U(N)$ gauge theory"

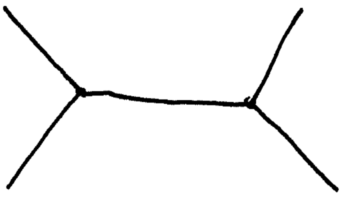
iii): intersecting Dp-brane



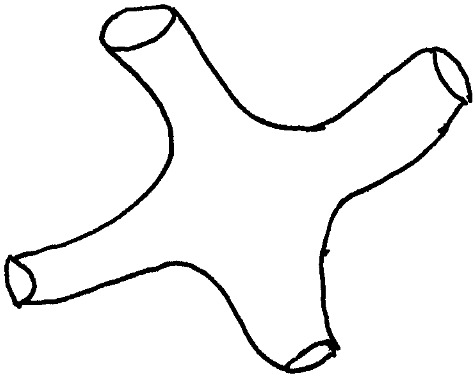
$(M+N)^2 \longrightarrow (M^2, 1) + (1, N^2)$

$(M, N) + (\bar{M}, \bar{N}) \rightarrow$ bifundamental "matter" is: <u>why free string ?</u>

All the information about interacting strings is alredy contained in the free theory (almost true!!!)



particle



string.

For string propagate. the 2d smooth world-sheet locally can be viewed as a free string propagates.

Only ~~got~~ <u>globally</u> do we see the diagram describes interactions

<u>UV finiteness</u>: (we <u>believe so</u>, valid in 5-loops!!)

* Strings on curved backgrounds

1): Curved space-time w/ metric $G_{\mu\nu}$

Action: $$S_\sigma = \frac{1}{4\pi\alpha'}\int_\Sigma d^2\xi \sqrt{-h}\, h^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}.$$

• $G_{\mu\nu}$: understood as a coherent state of gravitons describing the fluctutions of the metric around flat space-time $\eta_{\mu\nu}$!

: better understanding : Anology : Quantum optics

We set out in perturbative QED to Quantize the electromagnetic vacuum and describing its fluctuations by photons. A coherent state of these vacuum fluctuations represents a laser field.

likewise, $G_{\mu\nu}(X)$ can be viewed as a coherent excitations of gravitons.

2): generalise to other fields $B_{\mu\nu}$, Φ

$$S_\sigma = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\xi \sqrt{h} \{ (h^{ab} G_{\mu\nu} + i\epsilon^{ab} B_{\mu\nu}) \partial_a X^\mu \partial_b X^\nu + \alpha' R^{(2)} \Phi \}$$

Consistency condition

Focusing only on $G_{\mu\nu}$

$$S = \frac{1}{4\pi\alpha'} \int d^2\xi \, G_{\mu\nu} \partial_a X^\mu \partial^a X^\nu$$ (weyl invarince at classical level)

Quantum level: Anomaly !!

Anomaly cancellation condition

$$\beta_{\mu\nu}(G) :\doteq \mu \frac{\partial G_{\mu\nu}}{\partial \mu} = 0$$

at one-loop $\quad \beta_{\mu\nu}(G) = \alpha' R_{\mu\nu}$ (5)

$$\Rightarrow \begin{cases} R_{\mu\nu} = 0 \,! \quad \text{Ricci } \underset{\Downarrow}{\underline{\text{flat in}}} \text{ space-time} \\ D = 26 . \qquad\qquad\quad \text{Calabi-Yau} \end{cases}$$

~~In other words~~

In other words, the background space-time where String propagate must obey the vacuum Einstein Equation.

* Remarks: 1): NOTE that here is at one-loop; the perturbative expansion is ~~are~~ in term of $\sqrt{\alpha'}$.

So if the length of space-time is as small as $\sqrt{\alpha'}$, then the above result spoil down.

2): including other fields. $B_{\mu\nu}$. Φ

$\beta_{\mu\nu}(G) = \beta_{\mu\nu}(B) = \beta(\Phi) = 0 \Rightarrow$ low-energy effective action of String

$$S = \frac{1}{2k_0^2} \int d^D x \sqrt{-G}\, e^{-2\Phi} \left(R - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + 4 \partial_\mu \Phi \partial^\mu \Phi\right)$$

perString (Type IIB)

Two Major shortcomings in Bosonic string

1). tachyonic ground state $\longrightarrow$ Vacuum instability + D-brane

2). only bosonic excitation. No Fermions

Superstring

Adding 2d Fermions to the world-sheet, + Supersymmetry

$\Rightarrow$ 5 different ways

We focus on Type II superstring

$$S_{NSR} = -\frac{1}{4\pi}\left(\int_\Sigma d^2\xi \, \frac{1}{\alpha'} \partial_a X^\mu \partial^a X^\nu \eta_{\mu\nu} + 2i \psi_A^\mu \gamma^a_{AB} \partial_a \psi_{\mu B}\right)$$

by anologous procedures, we have ~~(For all superstrings)~~

1). $D=10$ } $\Rightarrow$ For all superstring theories.

2). No tachyon.

3). space-time supersymmetry (differs in different superstring)

$$\text{e.g.} \begin{cases} N=(2,0) & \text{For Type IIB} \\ N=(1,1) & \text{For Type IIA} \end{cases}$$

Massless spectrums in Type IIB

(NS-NS): $G_{\mu\nu}$. $B_{\mu\nu}$. Φ (Same with Bosonic string)

(R-R): $C^{(0)}$. $C^{(2)}_{\mu\nu}$. $C^{(4)}_{\mu\nu\rho\sigma}$ $\in$ p-form ($C^{(p)}$ fields)

plus their superpartner: (Fermions).

* what's the p-form fields C^p?

Mathematical tool: Differential forms

A p-form C^p is a totoally antisymmetric tensor of rank p

$$C^p = \frac{1}{p!} C_{\mu_1 \cdots \mu_p} dx^{\mu_1} \wedge dx^{\mu_2} \cdots \wedge dx^{\mu_p}$$

i). generalization of the electromagnetic 1-form gauge poential

$$A = A_\mu dx^\mu, \quad A_\mu \xrightarrow[\text{transformati}]{\text{Gauge}} A_\mu + \underbrace{\partial_\mu f}_{\equiv\, df}$$

$$F = dA = (\partial_\mu A_\nu - \partial_\nu A_\mu) dx^\mu \wedge dx^\nu \text{ : invariant } \Leftarrow d^2 = 0$$

For $p=2$

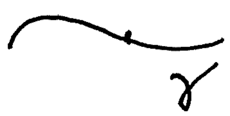
$$C^{(2)} = C^{(2)}_{\mu\nu} dx^\mu \wedge dx^\nu \qquad C^{(2)}_{\mu\nu} \to C^{(2)}_{\mu\nu} + \underbrace{(\partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu)}_{d\Lambda_1}$$

$$H^{(3)} = dC^{(2)} \text{ : invariant}$$

ii). Dp-brane is ~~coupled~~ charged under $C^{(p+1)}$ fields

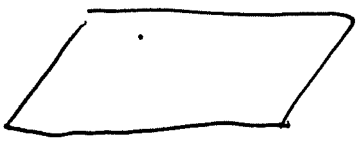
recall electromagnetism: A_μ coupling of a charged particle

$$S \supset q \int dx^\mu A_\mu :$$



$$\Rightarrow \int_\gamma A$$

Similarly: for (p+1) dim object:



I_{p+1}

$$\Longrightarrow \quad S_{Dp} \supset \mu_{p+1} \int_{I_{p+1}} C^{(p+1)}$$

$$F^{(p+2)} = dC^{(p+1)}$$

again in 3+1 dim electromagnetism

* electric charge $\quad q = \int_{S^2} \vec{E} \cdot d\vec{S} = \int_{S^2} {*F_2}$

if there are monopole

* magnetic charge $\quad g = \int_{S^2} \vec{B} \cdot d\vec{S} = \int_{S^2} F_2$

hence

$$\mu_{p+1} = \int_{S^{D-p-2}} {*F^{(p+2)}}$$

Now. a bit fussy.

$$*F^{(p+2)} = \tilde{F}_{D-p-2} = d\tilde{C}_{D-p-3}$$

* magnetically charged under $\tilde{C}_{D-p-3} \Leftarrow D_{D-p-4}$-brane

$$\tilde{\mu}_{D-p-4} \int_{I_{D-p-4}} \tilde{C}_{D-p-3}$$

in Summary (for Type IIB)

$C^{(p)}$-fields

electric coupling $D(p-1)$-brane

magnetic coupling $D(7-p)$-brane

i.e.

	electric	magnetic
$C^{(0)}$:	$D_{(-1)}$-brane	D7-brane
$C^{(2)}$:	D1-brane	D5-brane
$C^{(4)}$:	D3-brane	D3-brane

Remarks: Dp-branes in Type IIB (Superstring) are stable, due to they carry "charge". unlike the ones in Bosonic String (tachyons)

** D-branes are by themselves dynamic objects

i). charged under RR $C^{(p)}$-form fields

ii). gravitate by coupling to NS-NS Sector: $G_{\mu\nu}, B_{\mu\nu}, \Phi$

For i): the action

$$S_{cs} = -\mu_p \int_{D_p} \oplus_{p+1} C_{(p+1)} \wedge ch(F) \wedge \sqrt{\frac{\hat{A}(R_T)}{\hat{A}(R_N)}}$$

lowest level: $S_{cs} \supset -\mu_p \int_{D_p} C_{(p+1)}$

For ii): $S_{DBI} = -T_p \int_{D_p} d^{p+1}\xi \sqrt{-\det(\gamma_{ab} + 2\pi\alpha' F_{ab} + B_{ab})}\, e^{\Phi}$

lowest level: $S_{DBI} \supset -(2\pi\alpha')^2 T_p \int d^{p+1}\xi \left(\cancel{1} + \frac{1}{4} F_{ab}F^{ab} + \frac{1}{2}\partial_a \Phi^2 \partial^a \Phi^2\right)$

* Remarks: The above Dp-branes in Type IIB ($p = 2z+1$) have a property: $T_p = \mu_p$. Gravity = electric force

$\Downarrow$

Coincident Dp-brane

They are <u>BPS state</u>, stable.

$T_p \propto \frac{1}{g_s}$: non-perturbative solitoic solution of SUGRA

III. Model building in Type IIB theory

* First step: Compactification

$M^{10} = M_4 \times M_6$

$M_4 \swarrow$ 4d Minkowski

$M_6 \searrow$ 6d compact. <u>Small</u> space

side note: again, the self-consistency condition of superstring is that the total 2d CFT has $c=15$. in the case that we have 4d Minkowski space whose 2d CFT has $c=6$. So in principle we only need the remaining CFT give rise $c=9$, it doesn't need the 6d 'Geometric' space. Such as Gepnel CFT model, etc.

here for pheno favors, we consider M_6 be a geometric space and we focus on Calabi-Yau space. again the Length of CY should not be as small as stringy length $\sqrt{\alpha'}$.

* intuitive example

Kaluza-Klein compactification.

* 5d Gravity on $R^{(1,3)} \times S^1$ w/ metric $G_{\mu\nu}$, $\mu,\nu = 0,1,\cdots 4$

$$ds^2 = \tilde{G}_{ab}\, dx^a dx^b + e^{2\sigma}(dx^4 + A_a dx^a)^2 \quad . \ a,b = 0,1\cdots 3$$

$$G_{\mu\nu} \longrightarrow (\ \underset{\substack{\downarrow \\ \text{4d metric}}}{\tilde{G}_{ab}}\ ,\ \underset{\substack{\downarrow \\ \text{4d Gauge field}}}{A_\mu = G_{\mu 4}}\ ,\ \underset{\substack{\searrow \\ \text{4d Scalar}}}{\sigma = G_{44}}\ )$$

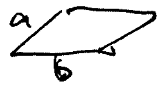
$$S = \frac{1}{2k_5^2}\int d^5x \sqrt{-G}\, R^{(5)}$$

$$= \frac{2\pi R}{2k_5^2}\int d^4x \sqrt{-\tilde{G}}\, e^{\sigma}\left(R^{(4)} - \frac{1}{4}e^{2\sigma}F_{\mu\nu}F^{\mu\nu} + \partial_\mu\sigma\partial^\mu\sigma\right)$$

lesson:

We can see the $\underline{\text{Geometry of } S^1 \text{ (Radius } R\text{)}}$ determines

4d theory couplings.



More non-trivial. Torus

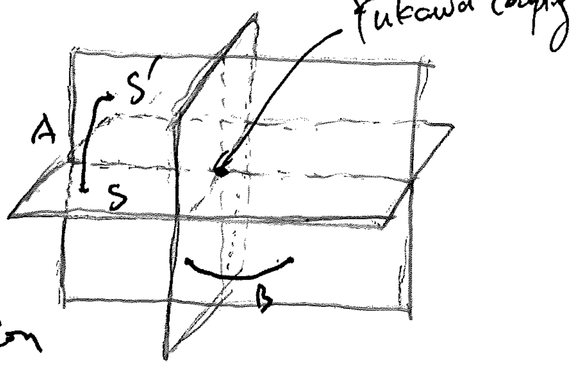
$\frac{a}{b}$. a, b

We call massless field σ as moduli in compactification

* In Calabi-Yau compactification, there are plenty of moduli depending on which CY one choose. due to they're massless. They poentially cause some problems such as fifth force. overclosing of the universe. e.t.c. One need to find a way to stabilize it

moduli stabilization: In type IIB. Fluxes

* Matters living on the D7-brane (space-filling)



We've discussed in Bosonic String.

~~bifundamental fields lives on the~~

bifundamental fields ~~comes~~ from exicitation from string connecting different D-brane. The Mass is determined by the distance of two Dp-brane

~~[illegible]~~ Same stories apply in superstring.

* ~~[illegible]~~. We have D7-brane fills 4d Minkowski space and 4d surface in 6d Calabi-Yau. $\Leftarrow$ Supersymmetry $=1$.

holomorphic surfaces

* massless charged matter lives on the loci of intersecting D7-brane. Complex 1 dim curves in Calabi-Yau

* coupling between states (Yukawa) corresponds to intersecting matter curve : complex 0 dim point in Calabi-Yau.

* Summary: For model building in Type IIB w/ D7-brane

    * Choose a Calabi-Yau X.
    * space-filling D7-branes. $\rightarrow$ orientifold
    * Fluxes $\Rightarrow$ { moduli stabilization, Chirality }

} many other consistent constraints e.g : charge conservation, Anomaly.