# Lecture III: A nutshell on model building in F-theory

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# 1 Model building in Type IIB superstring theory

## 1.1 First step: Compactification

There are many approaches to do model building in string theory/M-theory depending on which theory one works in, but the first step always lies at the compactification. And each approaches has its own virtues and drawbacks. however, it's fair to say F-theory provides a more economic way to do that in terms of well-controlled, calculations, etc.

If we restrict ourselves to the class of theories- Standard model, which are already known to exist, we can make contact with observations by requiring that the extra 6 spatial dimensions $X_{(3)}$ be compact and small - so small that they have not been discovered in any experiment so far. This leads to the notion of compactification

$$M^{(10)} = \mathbb{R}^{1,3} \times X_{(3)} \tag{1.1}$$

And in order to preserving some supersymmetries in 4d effective theories, one usually choose the internal space $X_{(3)}$ as Calabi-Yau three-space, which dubbed as Calabi-Yau compactification.

The type of 4d effective theory (massless spectrum, coupling, etc ) will depends on the geometric properties of Calabi-Yau space such as size, shapes, etc, the detail discussion would be laborious and time-consuming, we will not cover that in this lecture. And every Calabi-Yau manifolds corresponds one vacuum in string theory $\rightarrow$

String Landscape. However, in order to appreciating Calabi-Yau compactification in string theory we start with simple one–Kaluza-Klein compactification in field theory.

### 1.2 Kaluza-Klein compactification

Consider therefore a field theory in 5d spacetime dimensions $M^{(5)}$ with metric $G_{\mu\nu}, \mu, \nu = 0, 1, ..., 4$. Let dimension $x^4$ be rolled up on a circle, i.e. identify

$$x^4 = x^4 + 2\pi R \tag{1.2}$$

This corresponds to the compactification ansatz

$$M^{(5)} \to \mathbb{R}^{1,3} \times S^1 \tag{1.3}$$

The compactified space - here $S^1$ with radius $R$ - is in general called the internal space or the compactification manifold.

Then decomposition (only focus on massless fields here) of various fields after compactification such as metric $G_{\mu\nu}$ will gives rise to

i: A Scalar $\sigma = G_{44}$ [1],

ii: Vector bosons $A_i = G_{i4}, i = 0, 1, ..., 3$,

iii: 4 dim Metric $\tilde{G}_{ij}, i, j = 0, 1, ..., 3$.

The 5d Einstein- Hilbert action decomposes as [2]

$$\begin{aligned} S =& \frac{1}{2k_5^2} \int d^5x \sqrt{-G} R^{(5)} \\ =& \frac{2\pi R}{2k_5^2} \int d^4x \sqrt{-\tilde{G}} e^{\sigma} (R^{(4)} - \frac{1}{4} e^{2\sigma} F_{ij} F^{ij} + \partial_i \sigma \partial^i \sigma) \end{aligned} \tag{1.4}$$

where $F_{ij} = \partial_{[i} A_{j]}$ is the 4d gauge field strength. As one can see that the 4d physics such as coupling, fields are determined by radius of the inner space $S^1$. Note that

$$Vol(S^1) = \int_0^{2\pi} dx^4 \sqrt{G_{44}} = 2\pi\sqrt{\sigma} \tag{1.5}$$

Namely the VEV of the scalar field $\sigma = \sqrt{G_{44}}$, hence known as radion, determines a geometric property of the internal space $S^1$, the volume of $S^1$. And there is no potential in 4d to constraints it hence it is massless. Such flat scalar fields whose VEV determine geometric properties of the compactification space are called moduli fields. One need to stabilize them via other available approaches. In Type IIB/F-theory, There are several trusted scenarios (KKLT/ Large Volume scenario)can service that, which gives much credits for model-building in F-theory.

• Lesson: The geometry/topology of internal space determines the low dimensional physics such as coupling, spectrum, etc. Consider more non-trivial space, like Calabi-Yau, there are much more moduli fields arisen from the geometry of space.

[1] Of course in this case, $\sigma \sim R^2$. But let's stick to this notation since the point we will get is in general.

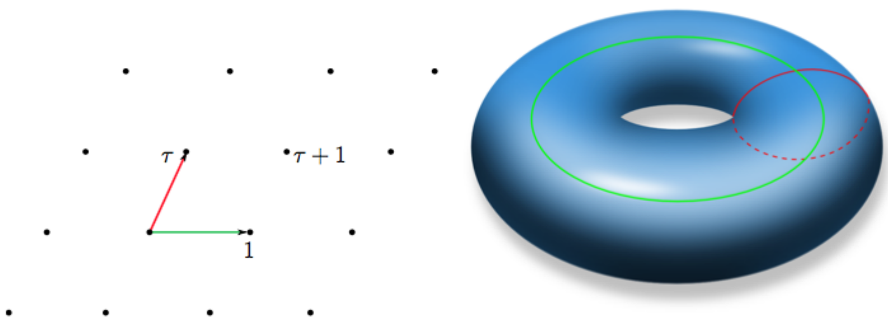
[2] the following action isnt quite of the familiar Einstein-Hilbert form because of that strange factor of $e^{\sigma}$ thats sitting out front. Actually it's known as string frame, one can redefine the fields like $G$ which now depends on $\sigma$ to absorb this term so that it can put as the familiar Einstein-Hilbert form.

## 1.3 Torus

More non-trivial geometry involves 2d Torus $T^2$.

Recall that a (two-dimensional) Torus $T^2$ can be defined as the quotient $\mathbb{C}/\Lambda$, where $\Lambda$ is a lattice in $\mathbb{C}$. The lattice $\Lambda$ can be defined by two vectors $\vec{a}, \vec{b} \in \mathbb{C}$, such that the torus is given by identifying $\vec{x} \cong \vec{x} + \vec{a} \cong \vec{x} + \vec{b}$. The area of the torus is given by the area of the parallelogram spanned by $\vec{a}$ and $\vec{b}$, while the shape is determined by the angle and the relative length between the two vectors. In complex geometry, the area is known as the moduli of Kahler parameter of the torus, and the shape is the moduli of complex structure. Hence one can expect that kahler moduli and complex moduli will appear as scalar fields in low-dimensional compactified fields.

In the figure (1), we set one of the vectors defining $\Lambda$ to be $1 \in \mathbb{C}$. Then, the complex structure of the torus is completely determined by the other vector, which we call $\tau$. In this form, it can be easily shown that the transformations $\tau \to \tau + 1$ and $\tau \to \frac{\tau}{\tau+1}$ leave the size of lattice $\Lambda$ invariant(A Kahler deformation leaves the shape fixed, but changes the volume. A complex structure deformation leaves the volume fixed, but changes the shape) . In fact, these transformations generate the full group of $SL(2,\mathbb{Z})$, which is thus the symmetry group of the torus.




(a) Lattice defining the torus. (b) Torus embedded in $\mathbb{R}^3$.

**Figure 1:** *Torus, Adapted from [5]*

## 1.4 Local model with D7-branes

In order to get 4d $N = 1$ effective theories, one favorable way in Type IIB that is consider Calabi-Yau compactification with space-filling D7-brane, namely the D7-branes spans 4d Minkowski space $\mathbb{R}^{1,3}$ and a complex 2 surface inside Calabi-Yau. As we discussed in the previous two lecture, the matter and couplings of standard model can arise from intersecting brane model as the below picture shows:

This is commonly way to do model building as a bottom-up approach, in which we first construct a local configuration realizing a particular gauge sector (Standard model) of physical interest. We then embed this local sector in a consistent global compactification (incorporate Gravity), which defines a complete string vacuum. However there are several problems with this local model:

• Like the disadvantage of usual bottom-up approach, it does not keep track of all the global consistency conditions required by the full string theory. For example, the decoupling from gravity usually gives a constraint on the possible spaces where the local configuration is located. Thus, local models narrow down the possible vacua in the string landscape and can give possible requirements of the global geometry. one also should notice when the 6d internal space is compact, the D7-brane needs O7-plane to cancel its RR charge and tension since the compact properties of $X_3$ makes the flux run nowhere, which makes $X_3$ losing the property of Calabi-Yau. Instead, it would be kahler manifold $B_3$.

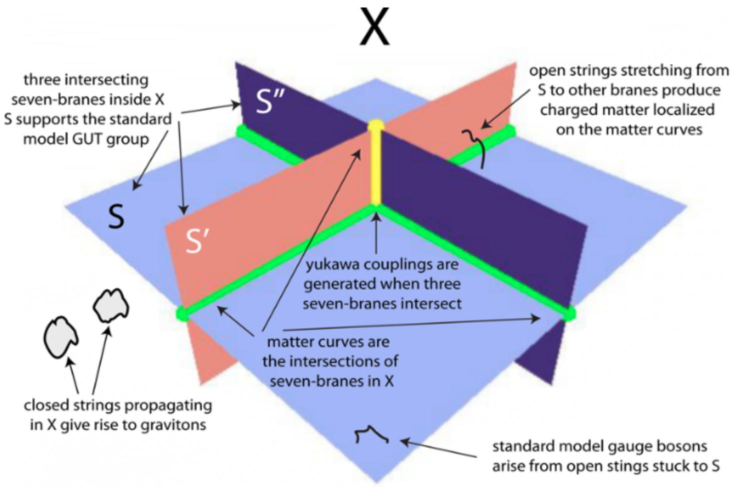



**Figure 2:** *Intersecting D-brane model, Adapted from [3]*

Except the above common disadvantage of bottom-up approach, there are two main issues associated with type IIB model building needing to be addressed :

• D-branes are often treated in the probe approximation for perturbative descriptions of Type IIB, where the backreaction on the geometry and form-potentials due to non-zero tension and RR-charge are neglected far from the brane. For sources (branes) of high codimension this is often a valid approximation, but in the case of 7-branes of co-dimension 2 that is no longer true due to the speciality of Possion eqaution in 2d. Namely, $\Delta\Phi(r) = \delta(r) \to \Phi(r) = \frac{1}{r^{n-2}}$ for $n > 2$.

• Though this is technique issue, the exceptional gauge groups $E_n$ (which is pistol in Grand Unified theories, such as $E_6$ in SU(5) GUT model) are hard to generated in the intersecting brane model in perturbative Type IIB string, one needs go to non-perturbative limit to generate the exceptional gauge group.

## 2 Back-reaction of D7-brane in Type IIB

Warning: We will only talk about the part from charge back-reaction. For the mass backreaction on background geometry, this is believed that the 10d background geometry is warped with factors instead being a direct product.

We have seen that 7-branes carry magnetic charges under $C_0$ (electric charges under $C_8 \leftarrow S_{CS} = \mu \int C_8$)inside [3] $\tau := C_0 + ie^{-\phi}$, Now we want to see what's the back-reaction of D7-brane. Define the complex coordinate $z \in \mathbb{C}$ parametrises the $z = x^8 + ix^9$ plane that is orthogonal to the D7-brane where D7-brane is pointlike source , the equations of motion (2-dim Possion eqaution) for $C_8$ in presence of a 7-brane at $z = z_0$ takes the form

$$d * F_9 = \delta^2(z - z_0) \tag{2.1}$$

Gauss Law tells us the integrated form should be ( in the normalized unit )

$$1 = \int_C d * F_9 = \oint_{S^1} F_1 = \oint_{S^1} dC_0 \tag{2.2}$$

Also with constraints from supersymmetry, which turns out that the axio-dilaton $\tau$ must be a holo-morphic function in $z$, makes the simple solution:

$$\tau(z) = \tau_0 + \frac{1}{2\pi i}\log(z - z_0) + \text{regular at } z_0 \tag{2.3}$$

[3]the complexities of $\tau$ is required by supersymmetry, where the image part is string coupling $g_s := e^{-\phi}$

in the vicinity of a D7-brane. Note that at $z = z_0$, where D7-branes locate, the value of $\tau$ diverges. Hence we can view the degenerations of $\tau$ as a "detection" to signal the D7-branes.

The apparent issue arising from the monodromy of the logarithm, i.e. $\tau \to \tau + 1$ when we move around $z_0$ in a circle. At first sight this might come as a shock as it seems to make a consistent interpretation of the background solution impossible.

The deus ex machina approaching to our rescue is the fact that Type IIB is invariant under $SL(2,\mathbb{Z})$. The Monodromy above can be given by the matrix $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ i.e. a symmetry of the theory. This suggests that one can identify D7-branes by their monodromy effect on the axio-dilaton profile $\tau$.

### 2.1 $SL(2,\mathbb{Z})$ invariance of Type IIB

We have discussed the the low-energy limit of Type IIB in the last lecture, here just review the basics. Its bosonic field content is summarised in the following table (3).

| field | (name) | type | electric BPS state | magnetic BPS state |
|---|---|---|---|---|
| $\phi$ | (dilaton) | scalar | – | – |
| $G_{\mu\nu}$ | (metric) | symmetric 2-tensor | – | – |
| $B_2$ | ($B$-field) | 2-form | F1-string | NS5-brane |
| $C_0$ | (RR 0-form) | 0-form | D(−1) instanton | D7-brane |
| $C_2$ | (RR 2-form) | 2-form | D1-string | D5-brane |
| $C_4$ | (RR 4-form) | 4-form | D3-brane | D3-brane |

**Figure 3:** *Bosonic field content of 10D type IIB SUGRA*

Its action takes the form

$$S_{SUGRA} = \frac{2\pi}{l_s^8} \int d^{10}x (\sqrt{-G}R - \frac{1}{2(Im\tau)^2} d\tau \wedge *d\bar{\tau} + \frac{1}{Im\tau} dG_3 \wedge *d\bar{G}_3 + \frac{1}{2}\tilde{F}_5 \wedge \tilde{F}_5 + C_4 \wedge H_3 \wedge F_3) \qquad (2.4)$$

with $G_3 = F_3 - \tau H_3, \tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$ and $\tau = C_0 + ie^{-\phi}$. Here the field strength tensors we have introduced in last lecture as $F_{n+1} = dC_n, H_3 = dB_2$.

One crucial property of this action is an $SL(2,\mathbb{R})$ symmetry, which acting the fields as

$$\begin{pmatrix} C_4 \\ G \end{pmatrix} \to \begin{pmatrix} C_4 \\ G \end{pmatrix}, \quad \tau \to \frac{a\tau + b}{c\tau + d}, \quad \begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} C_2 \\ B_2 \end{pmatrix}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{R}) \qquad (2.5)$$

When going to quantum level, due to the D(-1)-instanton, only $SL(2,\mathbb{Z})$ survives. Note that the S-duality $\tau \to \frac{-1}{\tau}$ is included in this symmetry.

## 3 Introduction to F-theory

Hence how to describe the back-reaction of D7-brane on the geometry? Given the fact we discussed above, the crucial observation one can make is that by identifying the axio-dilaton $\tau$ with the modular parameter of an auxiliary torus, which we briefly introduced in section 1, the $SL(2,\mathbb{Z})$ invariance of Type IIB can be seen as simply the symmetry transformation of that torus, which is fibered over the 10-dimensional Type IIB geometry $\mathbb{R}^{1,3} \times B_3$ so that the Torus fibered over $B_3$ gives rise to a Calabi-Yau four-fold [4].

Note that since $\tau$ various on the $B_3$ and $\tau := C_0 + ie^{-\phi}$, hence the string coupling $g_s := e^{-\phi}$ can be infinities over some place on the $B_3$. This leads to

• Definition: F-theory is a non-perturbative formulation of Type IIB compactification with 7-branes back-reaction on the geometry [5].

We should stress that the $B_3$ is the physical space-time where the Torus $T^2$ is only a auxiliary space which encoded the physics of the 7-branes. Hence the in F-theory the space-times is 12, the real physical space-time is still 10.

## 3.1 Elliptic fibration of CY

Algebraic speaking, a elliptic curve (Torus with a marked point) can be defined as the loci $\{[x:y:z]\}$ inside the weighted projective space $\mathbb{P}_{231}$ [6] satisfying the so-called Weierstrass form [7]:

$$y^2 = x^3 + fxz^4 + gz^6 \tag{3.1}$$

and the point where $\tau$ diverges is described by zeros of $\Delta := 4f^3 + 27g^2$.

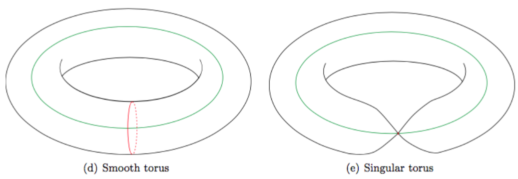



**Figure 4:** *Torus degeration, Adapted from [5]*

Armed with this representation of a single elliptic curve, we can proceed to elliptic fibrations [8]. Suppose we have some n-complex dimensional manifold $B_n$, covered by local coordinates $u_i$. Then a fibration of the Weierstrass curve over $B_n$ is obtained by promoting the constants $f$ and $g$ to suitable polynomials in the coordiates $u_i$ of $B_n$, $f = f(u_i), g = g(u_i)$ and further satisfying certain condition in order to be a Calabi-Yau space [9]. Hence the complex structure $\tau$ is now dependent on the base coordinates $u_i$. In particular, the elliptic fiber degenerates on a codimension-one sublocus on $B_n$ determined by the vanishing of the likewise $u_i$-dependent discriminant $\Delta = 27g^2 + 4f^3$. In view of what we said before, this vanishing locus must be interpreted as a divisor (codimension 1 surface inside $B_n$) wrapped by a stack of 7-branes.

From a physics perspective the most essential data of an elliptic fibration are the locus and the type of fiber degenerations because these allow us to deduce the nature of the 7-branes wrapping the corresponding

[4] Note that Minkowski space does not being 4 dimension. In principle F-theory can defined on $\mathbb{R}^{1,11-2d} \times X_d$ with $d = 1,2,3,4,5$, here in order to get a phenomenological viable model we need to consider compactifications of F-theory down to 4 real dimensions.

[5] There are various ways to define f-theory depending on which angle one look at it. For example, one can also view it as a limit of vanishing size of elliptic in elliptic fibered Calabi-Yau manifold compactification of M-theory.

[6] Namely, $(x,y,z) \cong (\lambda^2 x, \lambda^3 y, \lambda z), \lambda \in \mathbb{C} - 0$

[7] There are different ways to describe an elliptic curve, here we use the simplest one, namely being as a hypersurface or, more generally, as a complete intersection of some weighted projective space. Every elliptic fibration with a section can be represented by a Weierstrass model defined in terms of f and g.

[8] A fibration is a generalization of the notion of a fiber bundle, except that the fibers need not be the same space, nor even homeomorphic; rather, they are just homotopy equivalent. One without those backgrounds can roughly view this as relation between hair and head

[9] The condition is $f \in \Gamma(B_n, L), g \in \Gamma(B_n, L), c_1(L) = c_1(B_n)$

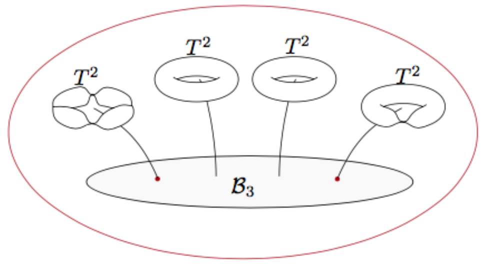



**Figure 5:** *A Calabi-Yau four-fold as elliptic fibration over $B_3$*

divisor. The different ways how the complex structure of the elliptic fiber can degenerate have been classified by Kodaira for the case of a Weierstrass model.

| sing. type | discr. deg(Δ) | gauge enhancement type | group | coefficient vanishing degrees $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_6$ |
|---|---|---|---|---|---|---|---|---|
| $\mathrm{I}_0$ | 0 | | — | 0 | 0 | 0 | 0 | 0 |
| $\mathrm{I}_1$ | 1 | | — | 0 | 0 | 1 | 1 | 1 |
| $\mathrm{I}_2$ | 2 | $A_1$ | $SU(2)$ | 0 | 0 | 1 | 1 | 2 |
| $\mathrm{I}_{2k}^{\mathrm{ns}}$ | $2k$ | $C_{2k}$ | $SP(2k)$ | 0 | 0 | $k$ | $k$ | $2k$ |
| $\mathrm{I}_{2k}^{\mathrm{s}}$ | $2k$ | $A_{2k-1}$ | $SU(2k)$ | 0 | 1 | $k$ | $k$ | $2k$ |
| $\mathrm{I}_{2k+1}^{\mathrm{ns}}$ | $2k+1$ | | [unconv.] | 0 | 0 | $k+1$ | $k+1$ | $2k+1$ |
| $\mathrm{I}_{2k+1}^{\mathrm{s}}$ | $2k+1$ | $A_{2k}$ | $SU(2k+1)$ | 0 | 1 | $k$ | $k+1$ | $2k+1$ |
| II | 2 | | — | 1 | 1 | 1 | 1 | 1 |
| III | 3 | $A_1$ | $SU(2)$ | 1 | 1 | 1 | 1 | 2 |
| $\mathrm{IV}^{\mathrm{ns}}$ | 4 | | [unconv.] | 1 | 1 | 1 | 2 | 2 |
| $\mathrm{IV}^{\mathrm{s}}$ | 4 | $A_2$ | $SU(3)$ | 1 | 1 | 1 | 2 | 3 |
| $\mathrm{I}_0^{*\mathrm{ns}}$ | 6 | $G_2$ | $G_2$ | 1 | 1 | 2 | 2 | 3 |
| $\mathrm{I}_0^{*\mathrm{ss}}$ | 6 | $B_3$ | $SO(7)$ | 1 | 1 | 2 | 2 | 4 |
| $\mathrm{I}_0^{*\mathrm{s}}$ | 6 | $D_4$ | $SO(8)$ | 1 | 1 | 2 | 2 | 4 |
| $\mathrm{I}_1^{*\mathrm{ns}}$ | 7 | $B_4$ | $SO(9)$ | 1 | 1 | 2 | 3 | 4 |
| $\mathrm{I}_1^{*\mathrm{s}}$ | 7 | $D_5$ | $SO(10)$ | 1 | 1 | 2 | 3 | 5 |
| $\mathrm{I}_2^{*\mathrm{ns}}$ | 8 | $B_5$ | $SO(11)$ | 1 | 1 | 3 | 3 | 5 |
| $\mathrm{I}_2^{*\mathrm{s}}$ | 8 | $D_6$ | $SO(12)$ | 1 | 1 | 3 | 3 | 5 |
| $\mathrm{I}_{2k-3}^{*\mathrm{ns}}$ | $2k+3$ | $B_{2k}$ | $SO(4k+1)$ | 1 | 1 | $k$ | $k+1$ | $2k$ |
| $\mathrm{I}_{2k-3}^{*\mathrm{s}}$ | $2k+3$ | $D_{2k+1}$ | $SO(4k+2)$ | 1 | 1 | $k$ | $k+1$ | $2k+1$ |
| $\mathrm{I}_{2k-2}^{*\mathrm{ns}}$ | $2k+4$ | $B_{2k+1}$ | $SO(4k+3)$ | 1 | 1 | $k+1$ | $k+1$ | $2k+1$ |
| $\mathrm{I}_{2k-2}^{*\mathrm{s}}$ | $2k+4$ | $D_{2k+2}$ | $SO(4k+4)$ | 1 | 1 | $k+1$ | $k+1$ | $2k+1$ |
| $\mathrm{IV}^{*\mathrm{ns}}$ | 8 | $F_4$ | $F_4$ | 1 | 2 | 2 | 3 | 4 |
| $\mathrm{IV}^{*\mathrm{s}}$ | 8 | $E_6$ | $E_6$ | 1 | 2 | 2 | 3 | 5 |
| $\mathrm{III}^*$ | 9 | $E_7$ | $E_7$ | 1 | 2 | 3 | 3 | 5 |
| $\mathrm{II}^*$ | 10 | $E_8$ | $E_8$ | 1 | 2 | 3 | 4 | 5 |
| non-min | 12 | | — | 1 | 2 | 3 | 4 | 6 |

**Table 3.1:** *Refined Kodaira classification resulting from Tate's algorithm, Adapted from [4].*

For our purpose, we introduce the so-called Tate form ( A general Weierstrass form can be locally viewed as a Tate form )

$$y^2 = x^3 + xyza_1 + x_2z^2a_2 + yz^3a_3 + xz^4a_4 + z^6a_6 \tag{3.2}$$

where the coefficients $a_i$ is the function in the base and $f, g$ in Weierstrass model can be expressed with $a_i$. Hence the Kadaira classification of the type of singularities can be labeled by the $a_i$, which listed in the (3.1).

• Important Fact: The types of singularities dictates the types of gauge group living on the D7-branes, which can be seen from (3.1).

The above fact, dubbed as ADE classification, can be understood from dual M-theory perspective, which beyond the scope of our lecture, hence we skip the discussion and refer to references like [1]. However, one should bear in mind that the physical space $B_3$ is always smooth in F-theory compactification, the singularities we discussed above only refers to the extra 2d elliptic curve.

As we have stressed above, the location of 7-branes are determined by where the discriminant vanishes

$$\Delta = \prod_i p_i(u_i) = 0 \tag{3.3}$$

where we have factorized into several polynomials $p_i$ and each polynomial $p_i$ define a 4-cycle (Divisor $S_i := p_i$) inside the base space $B_3$ wrapped by a stack of 7-branes.

And two stacks of 7-branes can intersect on 2-cycles $\Sigma_{ij}$:

$$\Sigma_{ij} = S_i \bigcap S_j \tag{3.4}$$

which are dubbed as matter curves inside $B_3$, aka the matters located. We have discussed the reason in the last lecture, Generally we can simply identify the localized matter by considering a general breaking pattern for the intersecting branes

$$\begin{aligned} &G_\Sigma \to G_i \times G_j \\ &G_\Sigma \to (Ad(G_i), 1_j) \oplus (1_i, Ad(G_j)) \oplus_m [(R_i^m, R_j^m) \oplus c.c] \end{aligned} \tag{3.5}$$

However, in order to obtaining the chiral matter, the essential piece should be added is $G_4$ flux (the part descending to gauge fluxes when going back to Type IIB perturbative limit)

$$\xi_R = n_R - n_{R^*} \sim \int_\Sigma G_4 \tag{3.6}$$

where $G_4 = dC_3$ fluxes can be understood as VeV of background field strength $G_4$, which also heavily depends on the Calabi-Yau manifolds which F-theory compactificaties).

Yukawa points hence are given by three matter curves $\Sigma_i$ intersecting at a point

$$p = \Sigma_i \cap \Sigma_j \cap \Sigma_k \tag{3.7}$$

This can be understood that the Yukawa coupling is determined by an overlap integral of the internal wave functions on the 7-brane, which depends on the local properties of the $p$ on the 7-branes. The texture of Yukawa couplings crucially depends on the geometry.

In Summary,

| Dimension | Internal Dimension | Feature |
|---|---|---|
| 10 | $6 = \dim(\mathcal{B}_3)$ | Gravity |
| 8 | $4 = \dim(S)$ | Gauge theory |
| 6 | $2 = \dim(S \cap S')$ | Matter |
| 4 | $0 = \dim(S \cap S' \cap S'')$ | Interactions |

**Figure 6:** *The dimension of geometry corresponds the various physics in F-theory*

# 4 Toy model-SU(5) GUT

To give a basic idea of how it works, we now try to building SU(5) GUT in F-theory. Firstly defining $w$ as one of the coordinates in the Base $B_3$ we can realize the SU(5) gauge group on a divisor $S : w = 0$, which corresponds to a singularity type $I_5$ in the above Table, the following vanishing degrees dictates the configuration:

$$a_1 = \mathbf{b}_5,\ a_2 = \mathbf{b}_4 w,\ a_3 = \mathbf{b}_3 w^2,\ a_4 = \mathbf{b}_2 w^3,\ a_6 = \mathbf{b}_0 w^5 \tag{4.1}$$

This results in a discriminant given by

$$\Delta = \frac{w^5}{16}(\mathbf{b}_5 P + w\mathbf{b}_5^2(8\mathbf{b}_4 P + \mathbf{b}_5 R) + w^2(16\mathbf{b}_3^2\mathbf{b}_4^2 + \mathbf{b}_5 Q) + \mathscr{O}(w^3)) \tag{4.2}$$

where $P = \mathbf{b}_3^2\mathbf{b}_4 - \mathbf{b}_2\mathbf{b}_3\mathbf{b}_5 + \mathbf{b}_0\mathbf{b}_5^2$ and likewise $R$ and $Q$ involve simple terms of $\mathbf{b}_i$.

Here $\Delta$ factorizes such that the first term $\propto w^5$, which is consistent with the location of a divisor $S$ with gauge group $SU(5)$, and the term in brackets does not generically factorize more and therefore corresponds to a simple abelian $I_1$- singularity away from S, denoted as $S'$ with gauge group $U(1)$. Now further enhancements can occur at sub-loci $S \cap S'$, such that higher rank gauge groups localize along matter curves $\Sigma := S \cap S'$ where the enhanced gauge group can be identified by Tate's algorithm.

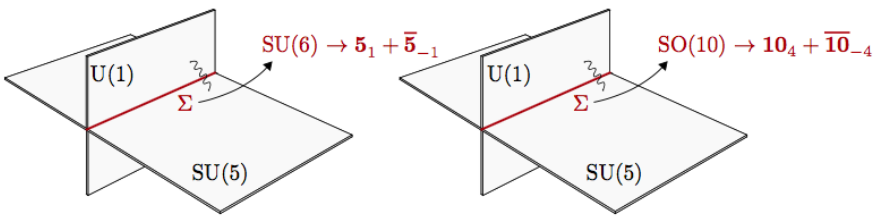



**Figure 7:** *Matter curves, Adapted from [6]*

Since the $S'$ carries no non-abelian gauge group, the singularity gets enhanced by rank one. the matter curves and its associated spectrum are:

• The 5 and its conjugate arises on matter curves $\Sigma_{SU(6)} : \Sigma_{SU(6)} : w = 0 \cap P = 0$ under a gauge enhancement to SU(6), along which the discriminant $\Delta$ scales like $w^6$, as required for a $SU(6)$ singularity:

$$\begin{aligned} SU(6) &\to SU(5) \times U(1) \\ 35 &\to 24_0 \oplus 1_0 \oplus 5_1 \oplus \bar{5}_{-1} \end{aligned} \tag{4.3}$$

• The 10 and its conjugate arises on matter curves $\Sigma_{SO(10)} : w = 0 \cap \mathbf{b}_5 = 0$ under a gauge enhancement to $SO(10)$, along which the discriminant $\Delta$ scales like $w^7$, as required for a $SO(10)$ singularity.

$$\begin{aligned} SO(10) &\rightarrow SU(5) \times U(1) \\ 45 &\rightarrow 24_0 \oplus 1_0 \oplus 10_4 \oplus \bar{10}_{-4} \end{aligned} \tag{4.4}$$

and Yukawa coupling point:

• The **10 10** $5_H$ Yukawa (for generating mass of down quarks) is localized at a point $p_{E_6} : w = 0 \cap \mathbf{b}_5 = \mathbf{b}_4 = 0$. Namely:

$$\begin{aligned} &E_6 \rightarrow SU(5) \times U(1) \times U(1) \\ &78 \rightarrow 24_{0,0} \oplus 1_{0,0} \oplus 1_{0,0} \oplus [10_{-1,-3} \oplus 10_{4,0} \oplus 5_{-3,3} \oplus c.c] \oplus 1_{5,3} \oplus 1_{-5,-3} \end{aligned} \tag{4.5}$$

• The **10** $\bar{5}_m\ \bar{5}_H$ Yukawa (for generating mass of up quarks) is localized at a point $p_{SO(12)} : w = 0 \cap \mathbf{b}_5 = \mathbf{b}_3 = 0$ .

$$\begin{aligned} SO(12) &\rightarrow SU(5) \times U(1) \times U(1) \\ 66 &\rightarrow 24_{0,0} \oplus 1_{0,0} \oplus 1_{0,0} \oplus [10_{4,0} \oplus \bar{5}_{-2,2} \oplus \bar{5}_{-2,-2} \oplus c.c] \end{aligned} \tag{4.6}$$

•The coupling $5_H \bar{5}_m 1$ Yukawa is localized at a point $p_{SU(7)} : w = 0 \cap P = R = 0$, while $\mathbf{b}_5, \mathbf{b}_4 \neq 0$, the singularity type enhances to $A_6$. The state 1 represents a (possibly localised) GUT singlet, can be viewed as neutrinos. Hence one can utilize this type of Yukawa coupling to generate masses for Neutrino masses.

$$\begin{aligned} SU(7) &\rightarrow SU(5) \times U(1) \times U(1) \\ 48 &\rightarrow 24_{0,0} \oplus 1_{0,0} \oplus 1_{0,0} \oplus 5_{0,6} \oplus 5_{-7,1} \oplus c.c] \end{aligned} \tag{4.7}$$

The global picture looks like the following: Note here we skip the Base $B_3$ and instead look into the GUT

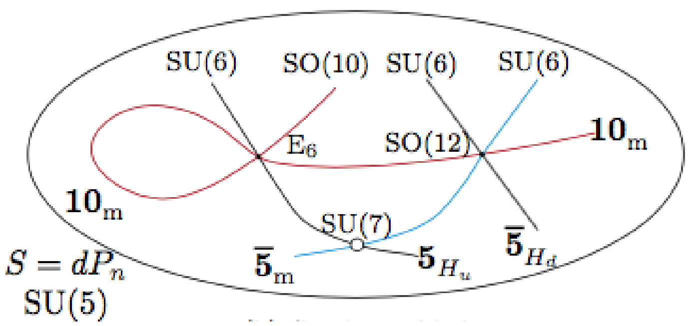



**Figure 8:** *Illustration of SU(5) GUT in F-theory. Adapted from [6]*

divisor $S$ which we take del Pezzo surfaces $dP_n$ for reason of decoupling gravity. The red matter curve denotes $SO(10)$ which can be arisen from $S$ intersecting with $\mathbf{b}_5 = 0$ as above shows. The two black matter curve enhance the gauge group to $SU(6)$, which can also be arisen like above shows. The enhancement of $SU(7)$ can be obtained by tuning on a matter curves perpendicular to GUT divisor $S$.

Note that for generic $SU(5)$ geometries above, the matter curve for the 5 representation is a single connected object; in this situation all three generations of $5_m$ and the vector-like pair $5_H + \bar{5}_H$ are localised on the same curve. this is unacceptable for phenomenological reasons such as generating dangerous proton decay. This can in principle fixed by introducing extra symmetries as a selection rule, such as U(1) symmetries, which is the focus in last past years. The abelian symmetry is F-theory is connected with the global properties of Calabi-Yau manifold, more precisely is global section, and this is beyond the scope of the lecture, one can refer to vast references.

We also have not discussed the chirality of the matter. This is dictated by the choices of the $G_4$ fluxes, as above section mentioned. The $G_4 := dC_3$ fluxes heavily depends on the geometry of Calabi-Yau four-fold. In precise term, $G_4 \in H^{(2,2)}(\hat{X}_4, \frac{1}{2}\mathbb{Z}) \cap H^{(2,2)}(\hat{X}_4)$. In this SU(5) model building, there are also other requirements on gauge fluxes for addressing the issue with the common GUT such as avoiding bulk exotic matter when breaking GUT group $SU(5)$, the gauge coupling unification in MSSM. etc. We refer to [1] for details.

There are still many GUT problem, we will not discus here due to time reason, such as GUT breaking (via hypercharge flux localized on the GUT brane), Proton decay (for example, avoiding through U(1) symmetries to forbid some dangerous couplings, implies that the 5 representation of $SU(5)$ localized on different curves ) , doublet-triplet splitting (e.g Imposing certain conditions on matter curves gauge fluxes $F$ ), Absence of massless bulk exotics, etc. In principle, F-theory model can provide nice and clear methods (e.g. considering $SU(5) \times U(1)$ theories in 4d) to attack those GUT problems [4], that is one of exciting sides of F-theory.

However, one should notice that there are general issues associated with model building in string theory. The first prominent one is the string landscape. Namely there are too much vacuum (corresponding to the number of Calabi-Yau spaces as well as some background configurations like fluxes, In F-theory it's estimated as around $10^{500}$) that satisfying features of standard model or MSSM but there is no first principle to pick up one, if people do not satisfy the Anthropic principle. The smallness of cosmological constants is also an obstinate one in string theory. As so far, there is no satisfying solution to this problem.

# A Standard model

The Standard model spectrum are listed in the following figure

| Field | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|
| $Q^i = (u^i, d^i)$ | 3 | 2 | +1/6 |
| $U^i = (u^c)^i$ | $\bar{3}$ | 1 | -2/3 |
| $D^i = (d^c)^i$ | $\bar{3}$ | 1 | +1/3 |
| $L^i = (\nu^i, e^i)$ | 1 | 2 | -1/2 |
| $E^i = (e^c)^i$ | 1 | 1 | +1 |
| $H = (H^-, H^0)$ | 1 | 2 | -1/2 |

**Figure 9:** *Standard model*

It produce the masses for quarks $Q^i, U^i, D^i$ and leptons $L^i, E^i$ through the Yukawa coupling

$$L_Y = Y_U^{ij} \bar{Q}^i U^j H^* + Y_D^{ij} \bar{Q}^i D^j H + Y_L^{ij} \bar{L}^i E^j H + h.c. \tag{A.1}$$

where $i, j$ are generation indices and $Y_{U,D,E}$ are coupling constant.

# B SU(5) GUT

SU(5) is the mother of all GUT groups. In Georgi-Glashow SU(5) models, the embedding of the MSSM gauge group $SU(3) \times SU(2) \times U(1)_Y$ rests on the identification of the $U(1)_Y$ generator with the Cartan generator $T = \text{diag}(2, 2, 2, -3, -3)$ within $SU(5)$. The MSSM matter is organised into $SU(5)$ multiplets as

$$10_M \leftrightarrow (Q_L, u_R^c, e_R^c), \;\; \bar{5}_m \leftrightarrow (d_R^c, L) \;\; \bar{5}_H \leftrightarrow (T^d, H_d) \;\; 5_H \leftrightarrow (T^u, H_u), 1 \leftrightarrow \nu_R^c \tag{B.1}$$

where the triplets $T_u, T_d$, which are not present in the MSSM, must receive high-scale masses via doublet-triplet splitting. The interaction terms for up and down type quarks are respectively controlled by

$$QUH_u \leftarrow 10_M \cdot 10_M \cdot 5_H \tag{B.2}$$

$$LEH_d + QDH_D \leftarrow 10_M \cdot \bar{5}_M \cdot \bar{5}_H \tag{B.3}$$

# C $SU(5)$ GUT breaking via Hypercharge fluxes

The philosophy behind this scenario is turning on hypercharge fluxes associated with $U(1)_Y$ so that the commutator is $SU(3) \times SU(2) \times U(1)_Y$.

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# D Soft SUSY breaking

The general philosophy of soft SUSY breaking is that the SUSY is dynamically broken in the hidden sector and communicate with visable sector through Messages fields. In the hidden sector supersymmetry is broken dynamically and can be parametrised by chiral super eld, $X$, that has a non-zero vev

$$X = x + \theta F \tag{D.1}$$

The scale of the supersymmetry breaking in the hidden sector is then set by $\sqrt{F}$.

There are two types of soft SUSY breaking scenario in MSSM. one is gravity mediation breaking and the other is gauge mediation breaking. the form will generate troublesome Flavor changing neutral current (FCNC). The later the FCNC is suppressed but work has to be done with the $\mu$ problem.

In an SU(5) GUT model arising from F-theory compactification, the messengers fields $f, \bar{f}$ are comprised of vector-like pairs $5+\bar{5}$ or $10+\bar{10}$. The $X$ field is a GUT singlet and the correspond matter curve normal to GUT Brane. The the superpotential term

$$W \supset \lambda X f \bar{f} \tag{D.2}$$

will generate a mass for $f, \bar{f}$ through the VEV of $X$. The SUSY breaking is then communicated to the visible sector because the MSSM fields also interact with the messenger fields.

The $\mu$ problem is arisen from the term

$$W \supset \mu H_u H_d \tag{D.3}$$

The natural scale for $\mu$ is the GUT scale but the electroweak symmetry breaking of the Standard Model requires it to lie at the weak scale.

In F-theory one can utilize the fact that $H_u$ and $H_d$ live on different matter curves via PQ symmetry to evading this coupling. And a weak scale term can then be generated by coupling the messenger fields to the Higgs.

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