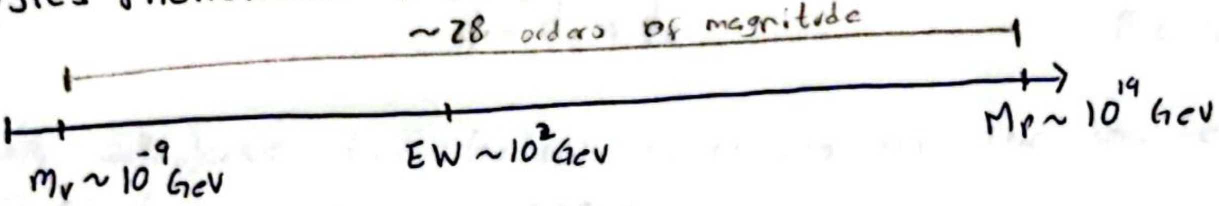
Effective Field Theories

Lecture 1) Introduction to EFTs
Lecture 2) extended DMEFT
Lecture 3) Matching

Motivation

Physics phenomena manifest at multiple energy scales



This makes it impossible for a theory to explain in a single yet consistent and practical framework all physics involved in such a large energy range.

$\Downarrow$

EFTs:
- Systematically separate the effects related to different energy scales keeping only the relevant physics at a given scale.
- Provide a modern approach to renormalization (lecture 3)
- Model independent

Top-down

- Start with the high-energy theory spanned in several scales
- Choose a cutoff Λ such that $\omega_H > \Lambda$, $\omega_L < \Lambda$
- Integrate out the heavy fields
- Theory is then parametrized only by ω_L and Λ

Bottom-up

- Start with the low-energy theory and aim to find the UV theory
- Systematically expand in D
- Make use of experimental results to compute the coefficients
- Specially useful when the UV theory is unknown

Top - down

Consider a QFT theory with a large energy scale $M \gg E$, where E is the energy accessible to experiments. Then, the EFT will only be valid at low energies.

- To construct the EFT we need to follow 3 steps.

1) Fix a cutoff $\Lambda < M$ and divide the fields in heavy and light

$$\phi_H \to \omega_H > \Lambda \qquad \phi_L \to \omega_L < \Lambda$$

$$\phi = \phi_L + \phi_H \qquad\qquad E \ll \Lambda < M$$

We can calculate the Green's functions of the low-energy modes of ϕ in the full theory:

$$\langle 0 | T\{\phi_L(x_1) \cdots \phi_L(x_n)\} | 0 \rangle = \int D\phi_L \underbrace{\int D\phi_H \, e^{iS(\phi_L + \phi_H)}}_{e^{iS_\Lambda(\phi_L)}} \phi_L(x_1) \cdots \phi_L(x_n)$$

2) solving the path integral over ϕ_H

$$\longrightarrow = \int D\phi_L \, e^{iS_\Lambda(\phi_L)} \, \phi_L(x_1) \cdots \phi_L(x_n)$$

3) OPE

where $S_\Lambda(\phi_L) = \int d^4x \, \mathcal{L}^{eff}_\Lambda(x)$

and $\mathcal{L}^{eff}_\Lambda(x) = \sum_i g_i \, Q_i(\phi_L(x))$

expand over the light fields

g_i = Wilson Coefficients

Q_i = All possible operators allowed by symmetries

! In principle $\mathcal{L}_\Lambda^{eff}$ can be composed of an unlimited number of operators. This severely affects the predictivity making predictions impossible.

☺ However, high-dimension operators have rather small contributions to the low observables, in fact only a few operators are relevant. Thus we can truncate the sum by performing "dimensional analysis" and fully recover the predictivity!

Dimensional Analysis

We can write the Wilson coefficients g_i in terms of the full scale M.

$$g_i = \underbrace{C_i}_{O(1)} M^{-(D-d)}$$

D = high-dimension of operators
d = dimension of the theory

Now

$$S_\Lambda(\phi_L) = \int d^dx\, \mathcal{L}_\Lambda^{eff}(x) = \int d^dx \left(\sum_i g_i Q_i\right)$$

$$[\mathcal{L}_\Lambda] = x^{-d} \sim E^d \qquad [g_i] = M^{-(D-d)}$$

$$\Rightarrow [g_i Q_i] \sim E^d$$

$$[M^{-(D-d)} Q_i] \sim E^d$$

$$[Q_i] \sim E^{d+(D-d)} = E^d E^{D-d}$$

The coefficient of the operator with dimension E^d is $C_i \left(\frac{E}{M}\right)^{D-d}$ where $\frac{E}{M} \ll 1$

The contribution of $C_i \left(\frac{E}{M}\right)^{D-d}$ can be derived into three different scenarios

$D = d$	$D < d$	$D > d$
$c_i \left(\frac{E}{M}\right)^{D-d} \sim \mathcal{O}(1)$	For $E \to 0$ $\;c_i \left(\frac{E}{M}\right)^{D-d} \gg 1$	For $E \to 0$ $\;c_i \left(\frac{E}{M}\right)^{D-d} \ll 1$
The operators are equally important at all energy Scales	• The operators grow in the limit $E \to 0$ • Are usually forbidden by Symmetries	• Typically vanish at low energies Numerically suppressed BUT not forbidden
"marginal" (renormalizable)	"relevant" (super-renormalizable)	"irrelevant" (non-renormalizable)

Irrelevant operators are the most important ones
- Can tell something at $\Lambda \sim M$
- Precision measurements
- describe NP

Bottom-Up

Here we start in the low-energy theory (what we know) and construct all possible non-redundant terms to describe our desire theory. For this, we need 3 ingredients:

1) Particle content: All possible fields that can take part in Feynman diagrams as either internal or external legs and $m_p < \Lambda$

2) Symmetries: This reduces the complexity and amount of operators
(gauge, space-time, global etc...)

Escaneado con CamScanner

3) Counting scheme: It is important to decide which operators are relevant in the valid range ($E \ll \Lambda < M$)

(This is necessary because in principle we could also write an infinite sum of operators!)

Generic example

A generic EFT Lagrangian for a SM extension can be written as:

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_i \frac{c_i^{(5)}}{\Lambda} Q_i^{(5)} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} Q_i^{(6)} + \dots$$

- We could remove all D=5 operators as they violate Lepton number
- Additionally D=7 operators violate B-L number
- Furthermore, operators with $D \geq 8$ are strongly suppressed by $\geq \frac{1}{\Lambda^4}$

Therefore, only operators of D = 6 remain, and the Lagrangian reads

$$\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} Q_i^{(6)}$$

The list of operators that are kept must be complete and non-redundant, i.e they must form a basis. The 3 most popular basis are Warsaw, Higgs and SILH.

". The total amount of operators after imposing Lepton and Baryon number conservation is 2499. However, by imposing further symmetries like flavour universality we can reduce the number of operators to only 76 ...

→ What about the Wilson coefficients?

- They must be measured in the experiment!

if $C_i\left(\frac{E}{\Lambda}\right)^{D-d}$

- $\ll 1 \Rightarrow$ Λ is large enough and we can safely neglect higher-order operators
- $\gg 1 \Rightarrow$ Λ is not large enough and the validity gets questionable.

Take away

→ EFTs are awesome for multi scale calculations

→ Model independent

→ "Irrelevant" operators are the most important ones!

→ Valid only at energies below the cutoff Λ

→ 2 differents approaches to build an EFT $\downarrow$ $\uparrow$

#EFT Rules!