Soft Diffraction

Student Lecture GRK 1940

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Elastic and inelastic hadronic scattering cross-sections are dominated by soft interactions. For these there is no large momentum transfer or mass present that would give rise to a hard scale at which $\alpha_s \ll 1$. Consequently, these processes cannot be calculated in perturbative QCD and alternative models are needed to calculate them. In Table 1 the total pp cross-section at the $\sqrt{s} = 13$ TeV is compared to some example hard processes; even the dominant inclusive jet production cross-section lies several orders below the total cross-section.

σ_{pp}(total)	~ 110 mb
σ_{pp}(elastic)	~ 30 mb
σ_{pp}(incl. jet $R = 0.4, \lvert y\rvert < 3$)	~ 1.9 μb
$\sigma_{pp}(W)$	~ 190 nb
$\sigma_{pp}(t\bar{t})$	~ 800 pb

Table 1: Cross-sections for selected processes at $\sqrt{s} = 13$ TeV. The large gap between typical perturbative processes and the total cross-section is filled by soft interactions, a significant contribution to which comes from elastic scattering.

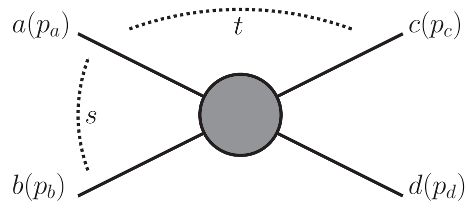
In the lecture we'll first have a look at some qualitative properties of soft diffractive interactions. Then we'll try to investigate the energy dependence of hadronic cross-sections in the framework of Regge Theory. Finally, we'll have a look at my own research and the measurement of diffractive vector-meson photoproduction at HERA.

1 Phenomenology of Soft Diffraction

Before we have a look at some key features of diffractive scattering events, we'll quickly need to define some kinematic variables.

1.1 Kinematics of $2 \to 2$ Scattering

Let's consider $2 \to 2$ scattering $a + b \to c + d$.



From the four-momenta of the particles three Lorentz-invariant variables, the Mandelstam variables can be formed:

$$s = (p_a + p_b)^2$$
$$t = (p_a - p_c)^2$$
$$u = (p_a - p_d)^2$$

Only two of them are independent because

$$\sum_{i=a,b,c,d} m_i^2 = s + t + u.$$

Here, we will use the center-of-mass energy s and the momentum transfer t. In particular, t is related to the scattering angle between a and c in the center-of-mass frame. If we assume all participating masses are equal to m, the relation is simply

$$\cos\theta = \frac{\vec{p}_a \cdot \vec{p}_c}{|\vec{p}_a|\ |\vec{p}_c|} = 1 + \frac{2t}{s - 4m^2}.$$

Under the equal mass assumption, the phyical values for the Mandelstam variables are $s > 4m^2$, $t < 0$, and $u < 0$; which is still mostly true in the general mass case.

1.2 Differential Diffractive Cross-Sections

The total and elastic pp scattering cross-section are shown in Figure 1 as a function of the center-of-mass energy s. Both are driven by soft diffractive interactions and exhibit the same qualitative behaviour to be expected for all diffractive processes. At first, the cross-sections fall off with some power of s at low energies before they start to slowly rise with s at higher energies. Noticeably, the elastic cross-section changes more steeply with s.

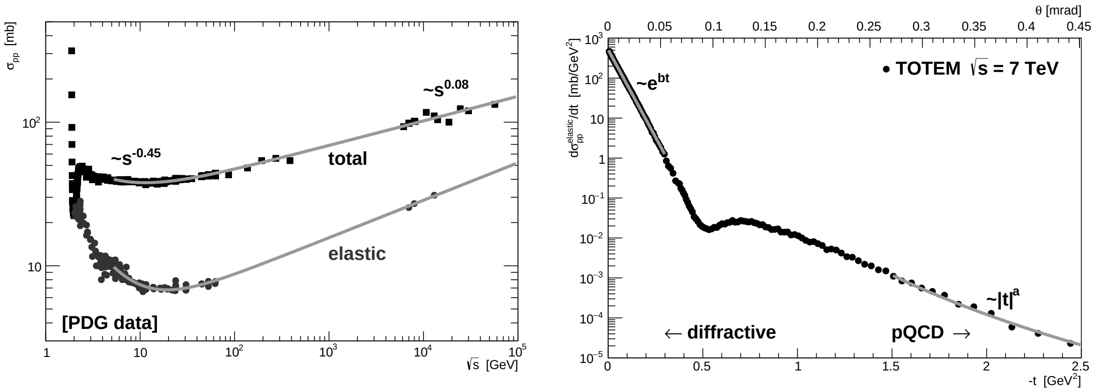


Figure 1: Total and elastic pp cross-section as a function of the center-of-mass energy $\sqrt{s}$ (left) and differential elastic cross-section $d\sigma_{pp}/dt$ as a function of momentum transfer t at $\sqrt{s} = 7$ TeV (right).

A further characteristic of hadronic scattering is the generally small momentum transfer t, i.e. particles are predominantly scattered under small angles. As an example, the differential elastic pp cross-section $d\sigma/dt$ measured at $\sqrt{s} = 7$ TeV is also shown in Figure 1 as a function of t and the scattering angle θ between in- and out-going proton. The cross-section falls off exponentially at small $-t$ and then transitions into a softer power law dependence at larger $-t$, where perturbative QCD again becomes applicable. Similarities in the structure of differential hadron-hadron cross-sections to the intensity pattern that can be observed in Fraunhofer diffraction when a light wave is diffracted off an obstacle historically gave rise to the term "diffraction" to describe these kinds of processes. The structure is related to shape and absorptive properties of the proton, but we can't discuss this here.

1.3 Large Rapidity Gaps

A key feature of diffractive scattering is that no strong color flows between the scattering objects and the exchange of other quantum numbers (charge, strong isospin) is suppressed. This is naturally true for elastic scattering but also holds for diffractive dissociation where one of the final state particles disintegrates. As a consequence, a key signature of diffractive scattering events in a particle detector are large rapidity[1] gaps in the measured activity. In a deep inelastic/hard interaction color can flow between the scattering centers. Radiation and hadronization occurs along the way and fills the region between the centers with particles.

Depending on the event toplogy different types of diffractive scattering events can be defined. The most basic are illustrated in Figure 2.

The idea of large rapidity gaps to identify diffractive events has its limitations though. There is a significant chance that the gaps filled with secondary particles arising from additional soft interactions. One thus has to take into account a rapidity gap survival probability.

[1] Pseudorapidity $\eta = -\ln\left[\tan(\theta/2)\right] \in (-\infty, \infty)$

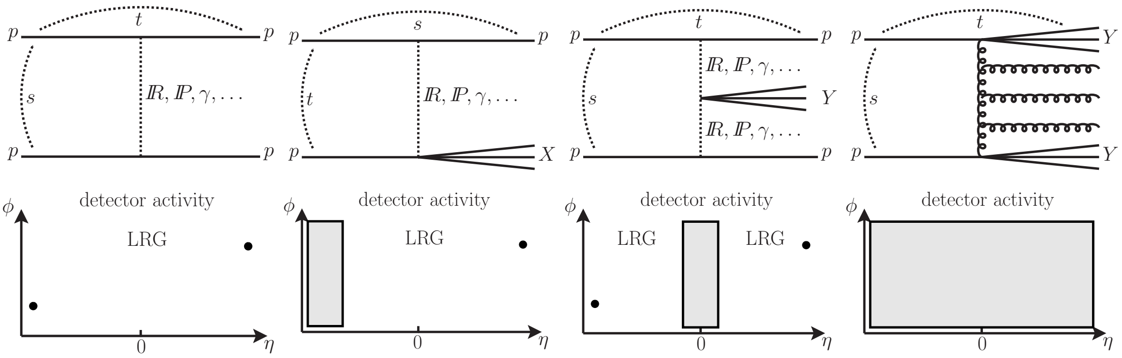


Figure 2: Basic types of proton-proton scattering with the expected activity to be observed in a particle detector: elastic (left), single diffractive dissociation (center left), central diffraction (center right), and non-diffractive (right). The diffractive types can be arbitrarily combined to form more complex topologies.

2 Regge Theory

Regge Theory (after Tullio Regge) is the attempt to describe hadronic cross-sections from basic principles of scattering theory. First developed in the 1950s and '60s it predates QCD. It builds on the idea that hadron-exchange carries the strong force over larger distances but instead of individual hadrons, the correlated exchange of whole hadron families (orbital excitations) is considered. The exchanges are characterized by Regge Trajectories, interpolations of angular momenta from discrete to continuous complex values. We will study some of the key ideas on the example of elastic $2 \to 2$ scattering in order to gain insight on the energy dependence of hadronic cross-sections.

2.1 Scattering Theory

The transition of the asymptotic initial state $|i\rangle = |a, b\rangle$ to the asymptotic final state $|f\rangle = |c, d\rangle$ is described by a scattering matrix $\boldsymbol{S}$ that relates to the transition probability for i going to f:

$$P_{i\to f} = |\langle f|\hat{S}|i\rangle|^2$$

Typically, $\boldsymbol{S}$ is written in terms of a scattering amplitude $\boldsymbol{A}$ by factoring out the case where no interaction takes place:

$$S_{if} = \delta_{if} + i(2\pi)^4\delta^4(p_i - p_f)A_{if},$$

which is related to the scattering cross-section

$$d\sigma(i \to f) = \frac{1}{\Phi}|A_{if}|^2 d\Pi$$

Here, Φ is a flux-factor and $d\Pi$ the Lorentz-invariant phasespace available to the final state. For $2 \to 2$ scattering one finds in the high-energy limit:

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2}|A_{if}(s,t)|^2$$

Irrespective of the underlying interaction, $\boldsymbol{S}$ is expected to have several fundamental properties:

1. **Lorentz invariance:**
   $\boldsymbol{S}$ should be Lorentz invariant and thus be a function of only Lorentz invariant variables, i.e., $\boldsymbol{S} \equiv \boldsymbol{S}(s,t)$.

2. **unitarity and the optical theorem:**
   To conserve total probability, $\boldsymbol{S}$ should be unitary, i.e.:

   $$1 = \boldsymbol{S} \cdot \boldsymbol{S}^\dagger = \boldsymbol{S}^\dagger \cdot \boldsymbol{S}$$

   To ensure the unitarity condition the scattering matrix $\boldsymbol{A}$ must satisfy:

   $$2Im\,[A_{if}] = (2\pi)^4\delta^4(P_i - P_f)\sum_k A_{ik}A^\dagger_{kf}$$

An interesting consequence arises when we consider the case of elastic forward scattering ($f = i$, $t = 0$) with vanishing momentum transfer. Then

$$2Im\left[(A_{ii}(s, t = 0)\right] = (2\pi)^4 \sum_k \delta^4(P_i - P_k)|A_{i\to k}|^2 \propto \sigma_{tot},$$

i.e. the total cross-section is given by the imaginary part of the forward elastic amplitude. This relation is called the optical theorem and central to the study of hadronic cross-sections. In the high energy limit the proportionality constant is a flux factor $\Phi \simeq 2s$ and we get:

$$\sigma_{tot}(s) = \frac{1}{s} Im\left[A_{elas}(s, t = 0)\right].$$

3. **analysticity and crossing-symmetry:**
The assumption that $\boldsymbol{S}(\boldsymbol{A})$ is a (complex) analytic function is a bit more involved. It is connected to causality and has has multiple consequences such as on the singularity structure of $\boldsymbol{S}$. For this lecture it is only relevant in relation to crossing-symmetry. First, we assume that $\boldsymbol{A}(s,t)$ can be continued beyond the physical region of $ab \to cd$, which is given by $s > 4m^2$, $t < 0$, and $u < 0$ to all values of s, t, and u. Then we require the crossed process $a(p_a)\bar{c}(-p_c) \to \bar{b}(-p_b)d(p_d)$ with:

$$s_t = (p_a + (-p_c))^2 = t > 4m^2$$
$$t_t = (p_a - (-p_b))^2 = s < 0$$

to be described by the same amplitude once s and t are interchanged:

$$A_{ab\to cd}(s,t) = A_{a\bar{c}\to\bar{b}d}(t,s).$$

Crossing symmetry holds order by order in perturbative quantum field theory; take for example $e^-e^- \to e^-e^-$ Coulomb-scattering in the t-channel vs $e^-e^+ \to e^+e^-$ Bhabha-scattering s-channel.

One consequence of crossing-symmetry is the idea that an s-channel resonance in $a\bar{c} \to \bar{b}d$ can be exchanged in the t-channel in the scattering $ab \to cd$ as is depicted in Figure 3.

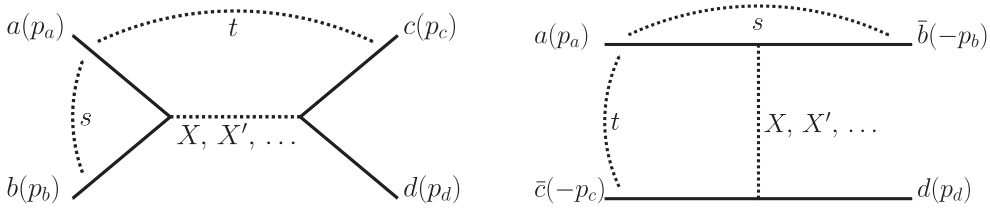


Figure 3: Diagrams for an s-cannel resonance in $ab \to cd$ (left) that is exchanged in the t-channel in the crossed process $a\bar{c} \to \bar{b}d$ (right).

2.2 t-Channel Exchange Amplitude

We are interested in the high energy behavior of the amplitude $A_{ab\to cd}(s,t)$ for t-channel exchange. Using crossing symmetry we can express $A_{ab\to cd}(s,t)$ in terms of $A_{a\bar{c}\to\bar{b}d}(t,s)$ continued to the regime $s > 4m^2$, $t < 0$. In general, we can expand $A_{a\bar{c}\to\bar{b}d}(s,t)$ as a series of Legendre polynomials in a so-called partial wave expansion:

$$A_{a\bar{c}\to\bar{b}d}(s,t) = \sum_{l=0}^{\infty}(2l+1)a_l(s)P_l(\cos\theta(s,t)),$$

where the sum is over the contributing angular momenta l, $a_l(s)$ is the so-called partial wave amplitude and $P_l(x)$ a Legendre polynomial of order l. If there is resonance of spin J and mass M_J it gives rise to a pole in s around which it becomes the dominant contribution[2]:

$$A_{a\bar{c}\to\bar{b}d}(s,t) \sim A_{res}(s,t) \sim \frac{P_J(\cos\theta(s,t))}{s - M_J^2}$$

[2]The argument that follows is very handwaving. One can do this more thoroughly but that is beyond this lecture. The formalism to do so is to make the angular momentum complex and, using Cauchy's integral formula, write the partial wave equation as a contour integral in the complex l-plane. By choosing appropriate boundary conditions $a_l(t)$ can be uniquely continued to $a(l,t)$ with $a(l,t) = a_l(t)$ for integer l such that the integral/series converges. Also, do resonances have a width?

Using crossing symmetry and replacing $\cos\theta(s,t) = 1 + \frac{2t}{s-4m^2}$ we can use this to obtain the amplitude for t-channel exchange of the resonance in $ab \to cd$:

$$A_{ab\to cd}(s,t) = A_{a\bar{c}\to\bar{b}d}(t,s) \sim \frac{P_J(1+\frac{2s}{t-4m^2})}{t-M_J^2}.$$

As $|t| \ll s$ (for diffractive events) for large enough $s \to \infty$ we can express P_J by the leading exponent, $P_J(1+\frac{2s}{t-4m^2}) \sim s^J$. Using the optical theorem we then find for the energy dependence of single particle exchange:

$$\sigma_{tot} \overset{s\to\infty}{\propto} \frac{s^J}{s} = s^{J-1} \begin{cases} s^{-1}, & \text{for J=0} \\ s^0, & \text{for J=1} \\ s^1, & \text{for J=2} \end{cases}$$

Which does not match neither the low nor high energy dependence that we observed for example in the pp scattering cross-section in Figure 1. Even worse, for higher spin resonances even violates unitarity.

2.3 Regge Trajectories and Resonance Families

The way out of this problem lies in realizing that the resonances/hadrons that are exchanged are not elementary particles but composite objects. As such they can have orbital excitations which can also be exchanged. For example for the spin-1 $\rho(770)$ meson in the ground state, one can also observe a spin-3 $\rho(1690)$ and spin-5 $\rho(2350)$ excited state. In order to get the right amplitude, we have to sum them all up. At first glance it's not clear how one would correctly do that, however it turns out that the orbital excitations are strongly correlated.

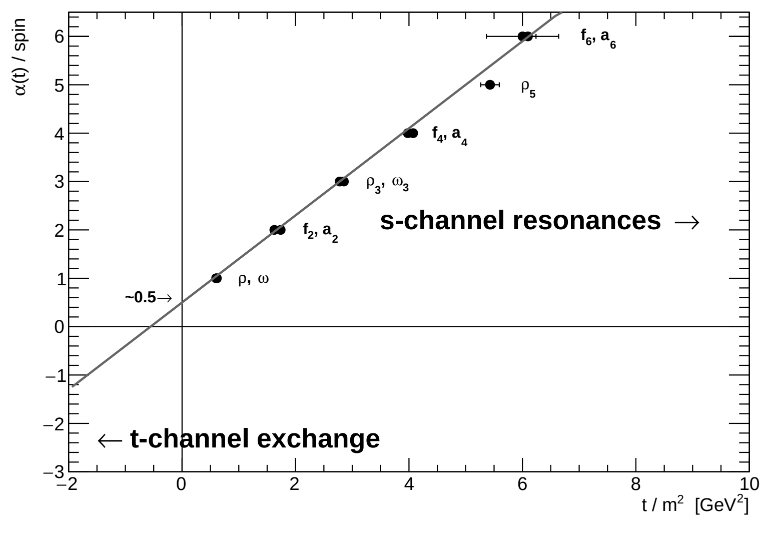


Figure 4: Chew-Frautchi plot for the four dominant and degenerate Regge trajectories from the ρ, ω, f_2 and a meson families

In Figure 4 the spin of mesons from the ρ, ω, f_2 and a families is plotted against their mass squared. And as you can see there's a clear linear relation[3]. The line in the spin-mass plane the mesons from one family lie on is called a Regge-Trajectory:

$$\alpha_{I\!R} = \alpha_0 + \alpha'\, t,$$

where for a resonance $\alpha_{I\!R}(t = M_J^2) = J$. The most important Regge trajectories indeed come from ρ, ω, f_2 and a as are shown. They are degenerate with $\alpha_0 \sim 0.5$ and $\alpha' \sim 0.9$ GeV^{-2}.

[3] You can take this as an experimental observation. However, it also follows from simple hadron models where they are assumed to be rotating systems of quarks connected by an open color string. If the force between any two points of the string is constant, then the contribution from the string to J is proportional to its contribution to the mass squared: $J_{string} \propto M^2_{string}$. As a consequence the Regge trajectories are linear with a universal slope.

If we insert the Regge trajectorie in the sum over all resonance contributions to the amplitude, i.e. we replace

$$t - M_J = \frac{\alpha_{I\!R}(t) - J}{\alpha'}$$

we can calculate it:

$$A_{ab\to cd}(s,t) \overset{s\to\infty}{\to} \sum_{l=0}^{\infty}(2l+1)\frac{\beta_l(t)}{t - M_l^2}s^l = \sum_{l=0}^{\infty}(2l+1)\frac{\beta_l(t)\alpha'}{l - \alpha_{I\!R}(t)}s^l.$$

The dominant contribution comes from the pole at $l = \alpha_{I\!R}(t)$ and we get:

$$A_{ab\to cd}(s,t) \overset{s\to\infty}{\sim} s^{\alpha_{I\!R}(t)}.$$

This looks very similar to the exchange of a single resonance with t-dependent spin $\alpha_{I\!R}(t)$ which is why one sometimes speaks of Reggeon exchange. However, Reggeons are not real particles but the combined effect of multiple resonance exchange. With the Reggeon intercept α_0, Reggeon exchange nicely describes hadronic cross-sections at low energies $s \sim 10$ GeV.

2.4 The Pomeron

There is no clear evidence for a hadronic Regge trajectory with an intercept close to 1 that could describe the observed high energy rise of hadronic cross-sections. That is why an ad-hoc trajectory, called Pomeron ($I\!P$, after Isaak Pomerantschuk), is introduced to fill that role. It's parameters are roughly given by:

$$\alpha_{I\!P} \simeq 1.0808 + 0.25\ \mathrm{GeV}^{-2}t.$$

While it is difficult to say what kind of physical object the (soft) Pomeron exactly is, it's quantum numbers are known: It has no electric charge, even spin and is even under charge conjugation. Intuitively, one could think of it as a colorless object of two bound gluons. This idea is supported by a perturbatively calculable hard Pomeron whose structure can be studied in hard diffractive scattering such as diffractive jet or heavy vector meson production.

2.5 Some Remarks

Intuitively, a rising cross-section seems to violate unitarity. In fact, the total cross-section is only constraint by unitarity by the Froissart bound:

$$\sigma_{tot} \leq \frac{\pi}{m_\pi^2}\log^2(s/s_0)$$

While s_0 is not known, assuming $s_0 = 1$ GeV gives $\sigma_{tot}(13\ \mathrm{TeV}) \lesssim 22$ barns so in practice one need not worry about the rising cross-section anytime soon. Nonetheless, the Pomeron picture can not be complete or completely correct and at the very least needs additions or corrections at higher energies. Another problem with Regge theory is that Regge trajectories can not be continued to arbitrarily large masses because eventually the heavy hadrons will fragment into multiple lighter ones. However, because it describes a wide range of data it is still useful.