RTG Student Lecture: Measurement of $|V_{ub}|$ at the LHCb experiment, lecture I

Michael Kolpin, Physikalisches Institut Heidelberg

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This series of lectures will treat the measurement of the least well-known CKM matrix element $|V_{ub}|$ at the LHCb experiment. This lecture notes will be a brief summary of the topics taught, taking additional notes is advisable.
In this first lecture I will give a short reminder of CKM mechanism and a general introduction of how its parameters can experimentally be measured.

1 The CKM mechanism

The electroweak interaction is the only possibility in the Standard Model of Particle Physics, in which flavour changing transitions can occur. Due to the weak eigenstates $|q\prime\rangle$ not being identical to the mass eigenstates $|q\rangle$, these transitions can also appear in between different quark families.
The Cabibbo-Kobayashi-Maskawa (CKM) matrix, describes the relation between these sets of eigenstates:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{1}$$

where

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{2}$$

Using this CKM matrix the charged current Lagrangian can then generally be written as

$$\mathcal{L}_{CC} \subset -\frac{g_2}{\sqrt{2}} \left(\overline{u}_L, \overline{s}_L, \overline{t}_L\right) \gamma^\mu W_\mu^+ V_{CKM} \begin{pmatrix} d_L \\ c_L \\ b_L \end{pmatrix} + h.c. \tag{3}$$

Due to unitarity conditions and rephasing of the quark fields, the original 18 parameters of this complex 3×3 matrix can be reduced to 4 free parameters, 3 rotational angles and 1 complex phase. The latter is the only source of CP violation in the Standard Model, which will not be covered by this lecture (but was e.g. discussed last semester in Max' lecture).
One possible parametrisation of the CKM matrix, which shows the hierarchy of the matrix elements, is the Wolfenstein parametrisation:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4), \tag{4}$$

where

$$\lambda = sin\theta_{12}, A\lambda^2 = sin\theta_{23}, A\lambda^3(\rho - i\eta) = sin\theta_{13}e^{-i\delta}.$$

Using the latest world average values, the magnitude of this matrix is

$$\begin{aligned} |V_{CKM}| &= \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \\ &= \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}, \end{aligned} \tag{5}$$

where the colours indicate the high(est) relative uncertainties.

The 6 off-diagonal conditions of unitarity can be used to draw so-called unitarity triangles, where most commonly the relation

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \tag{6}$$

is used (as it results in the most readable visualisation). The construction of the triangle from this relation is shown in figure 1, the current measurement status of it in figure 2. The length of the left side of the triangle is experimentally heavily influenced by the measurement of $|V_{ub}|$, which will be discussed below.
While the measurement of the CP violating phase was discussed in earlier lectures, we will in the following see how the magnitude of the CKM elements can be determined.

2 Measurement of CKM matrix elements

2.1 Measurement principal

As the CKM matrix elements directly show up in the effective Lagrangian, their magnitude can be determined from measuring the respective flavour changing transition. The

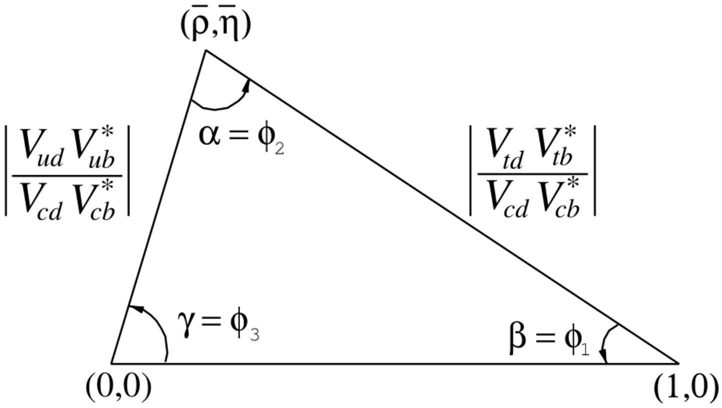

Figure 1: Schematic of the CKM triangle [1]

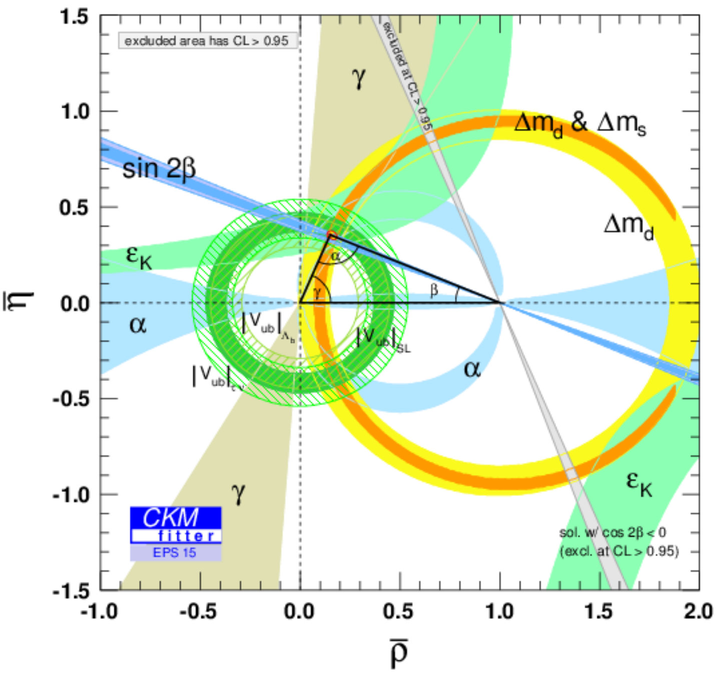

Figure 2: Status of the CKM triangle, as reported by the CKM fitter collaboration [2]

measured cross section (or, in analogy, the branching fraction) is directly proportional to the magnitude squared of the respective matrix element.
The most basic (and also most precise) measurement of any CKM matrix element is obtained by measuring the decay of a neutron into a proton and an electron-neutrino pair (nuclear β decay), where $\sigma \propto |V_{ud}|^2$.

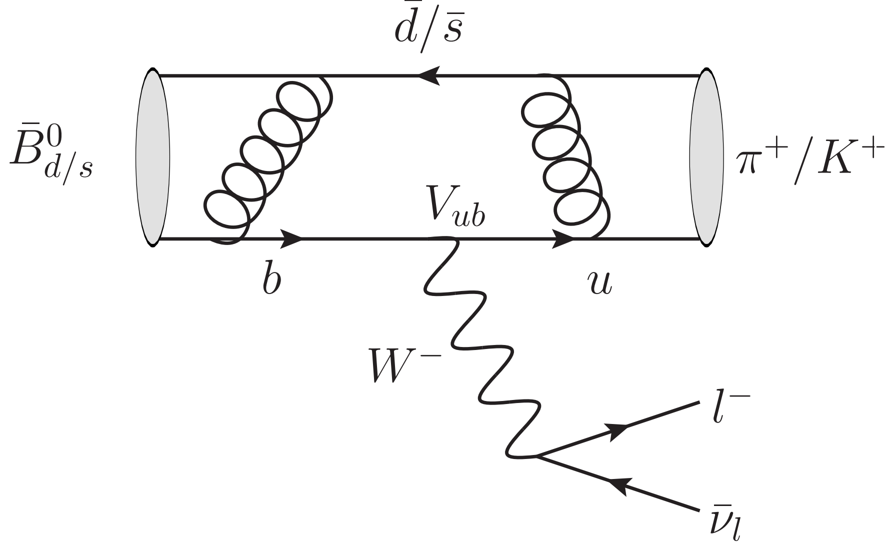

Figure 3: Example of semileptonic decays of B_s^0 mesons

2.2 Off-diagonal elements

As a B-Physicist, I will further focus on the determination of the matrix elements V_{cb} and V_{ub}. To derive the respective magnitude, generally semileptonic decays are used. Fig. 3 shows the diagrams for semileptonic B_s^0 meson decays. These type of decays have 2 large advantages: they have in general very high event yields (about 10% of B meson decays happen that way), and QCD complications can be contained in the form factors of the hadronic part, making theory calculations more reliable.
On the other hand, due to the non-reconstruction of the neutrino, these decays are experimentally very challenging. In the rest of today's lecture I will focus on how these measurements are done at the so-called b factories BELLE and BaBar, while the third lecture will cover the measurement at LHCb.

There are 2 conceptually different ways to measure semileptonic B decays: exclusive and inclusive measurements, which will be elaborated upon further below.

2.3 Exclusive measurements

Exclusive measurements aim to fully reconstruct a decay chain, such as e.g. $B^0 \to \pi^- e^+ \nu$, $B^0 \to D^- \mu^+ \nu$, $B_s^0 \to K^- \mu^+ \nu$, etc.
The magnitude of the matrix element involved can be extracted from the measured branching ratio, taking into account form factors, which can be calculated from theory (lattice QCD, light cone sum rules). This branching ratio can be calculated via

$$\mathcal{B}(B \to \text{sig}) = \frac{\mathrm{N_{B\to sig}}}{\mathrm{N_{B,produced}}} = \frac{\mathrm{N_{B\to sig,measured}}}{\mathrm{N_{B,produced}}} \cdot \frac{1}{\epsilon_{\text{sig,meas.}}}, \tag{7}$$

where $N_{B\to\text{sig}/B\to\text{sig,measured}/\text{B,produced}}$ are the number (or yield) of B mesons decaying into the signal, decaying into the signal and being measured, and of B mesons produced. $\epsilon_{\text{sig,meas.}}$ is the efficiency in measuring the signal decay in case it ocurrs.
Ways to derive the latter will be discussed in the second lecture, as it similar through all branching ratio measurements. However, determining the yield in semileptonic decays is especially challenging, due to the non-reconstruction of the neutrino. Therefore, the original b hadron is only partially reconstructed, and the hadron mass cannot be fitted in order to extract the signal yield.
In the following, I will describe the method to account for this as used by the so-called "B-factories" BELLE and BaBar.
These experiments, located at the KEK and SLAC colliders, respectively, use e^+e^- collisions, operating at the $\Upsilon(5S)$ resonance, which decays mostly into pairs of $B\overline{B}$ mesons. Due to quantum entanglement, the states of each of these B mesons are directly connected. Additionally, the initial state (and all it's properties) of two leptons is completely known. The detectors covers (almost) the full 4π spatial angle, which allows the reconstruction of particles independently of their direction.
Figure 4 shows the example of what a signal candidate event looks like in the BELLE detector.

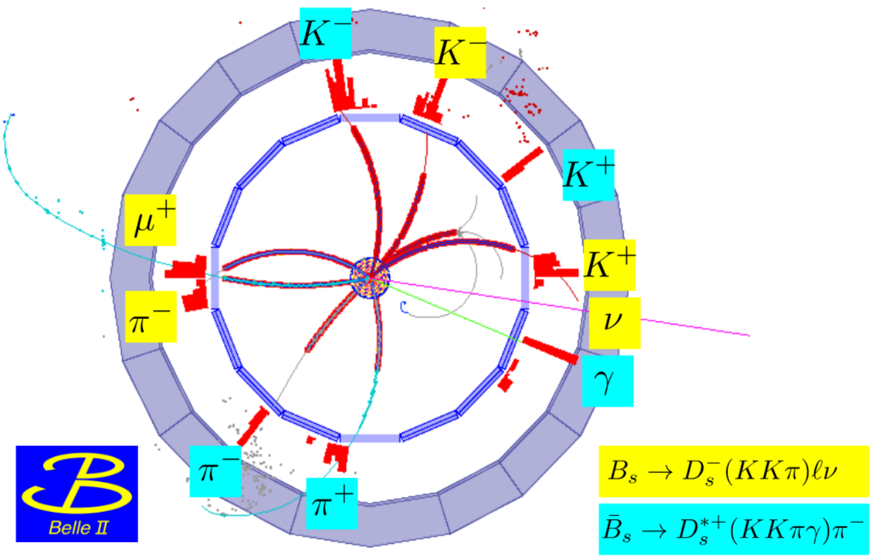

Figure 4: BELLE event display of the decay $B_s^0 \to D_s^- \ell\nu$

For signal candidates it is required that the non-signal B meson, the so-called "tag B " ($\overline{B}_s^0$ in this case), is fully reconstructed. That way, the only non-reconstructed particle in the event is the neutrino. Using the full kinematic information of the remaining particles, combined with the precise knowledge of the initial state, the 4-momentum of the neutrino can be reconstructed. The variable used to destinguish signal from background is the so-called missing mass M_X, defined via

$$M_X^2 = (p_{e^+} + p_{e^-} - p_{B_{tag}} - p_D - p_\ell)^2, \tag{8}$$

where the p_i are the respective 4-momenta. Per construction, signal events (where only the neutrino is missing) have M_X^2 values around 0 (smeared by resolution effects), while background events typically show much higher values. Figure 5 shows the M_X^2 distributions of this BELLE measurement for different B decay modes. The separation of signal from background is cleary visible.

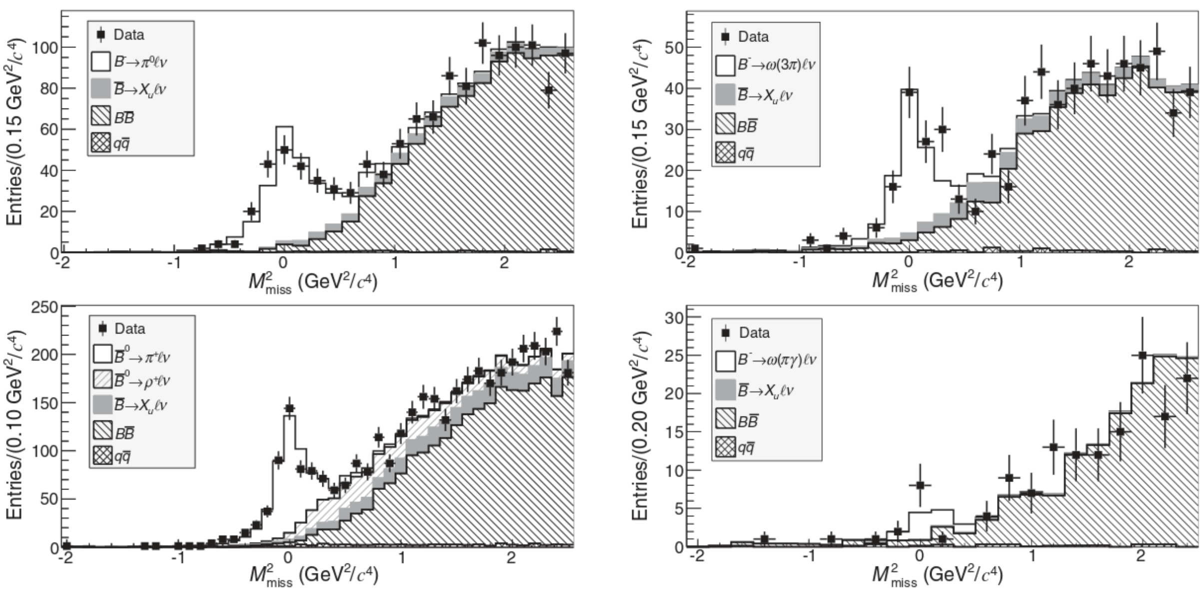

Figure 5: BELLE missing mass squared distributions for different B decays [3]

2.4 Inclusive measurements

In order to make an inclusive measurement of $|V_{u/cb}|$, all transitions of the type $b \to uc\ell\nu$ need to be measured simultaneously. The way this is obtained at the BELLE experiment is briefly laid out in the following.
Again, the entanglement of the $B\overline{B}$ pair originating from an Υ decay is exploited. The tag B is required to decay via $\overline{B}_s^0 \to D_s^+ X^-$, while at the same time looking for signal decays of the type $B_s^0 \to \ell^+ X-$. The desired branching ratio can then be extracted by comparing all events with a D_s^+ originating from a $\overline{B}_s^0$ decay with the ones, where in addition a lepton with the same charge is reconstructed:

$$\frac{N(D_s^+\ell^+)}{N(D_s^+)} \propto \frac{N(B_s^0 \to X\ell\nu)}{N(B_s^0)} = \mathcal{B}(B_s^0 \to X\ell\nu) \tag{9}$$

Figure 6 shows the way the required yields are extracted: Fits to the invariant mass of $D_s^+ \to K^+ K^- \pi^+$ are used to determine the yield of D_s^+, while a simultaneous fit to the kinematic variable $x(D_s^+)$ is used to assure these D_s^+ actually originate from a $\overline{B}_s^0$ decay. The distribution of the lepton momentum is used to validate the lepton originates from a B_s^0 decay.

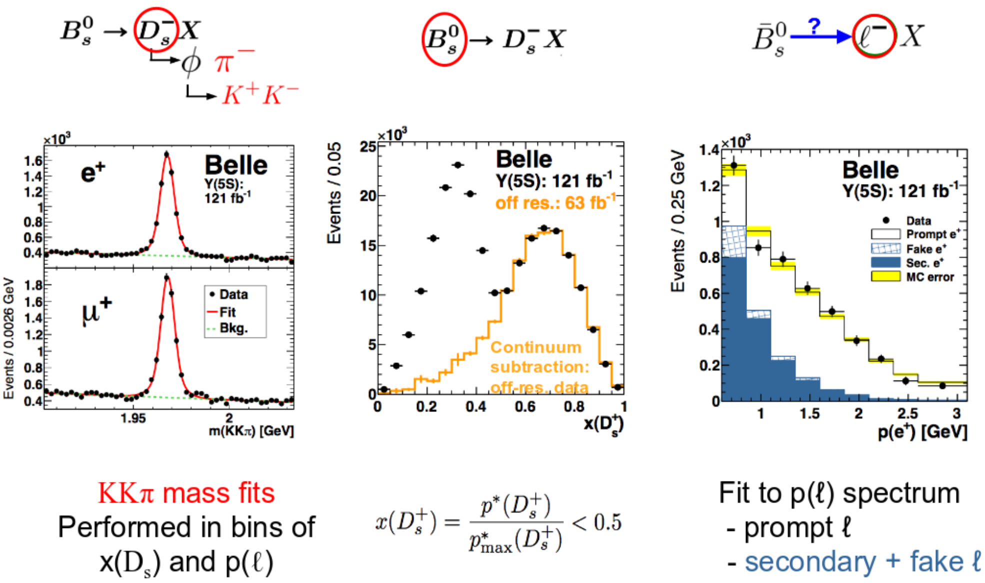

Figure 6: Fitted distributions used at BELLE to measure inclusive $B \to X\ell\nu$decays [4]

Combined with theory input to distinguish $b \to u\ell\nu$ from $b \to c\ell\nu$ decays, the inclusve values for $|V_{u/cb}|$ can be extracted.

2.5 Exclusive vs. inclusive

Figure 7 shows the comparison of exclusive and inclusive measurement results for $|V_{cb}|$. One can see that there is a clear tension between these two approaches, where the inclusive result is clearly higher than the exclusive one. The same is occurring for measurements of $|V_{ub}|$, fig. 7 also shows the development of inclusive and exclusive world averages over the last 6 PDG editions, showing that the discrepancy became more significant with higher precision.

While statistical fluctuations or measurement errors cannot yet be excluded as a reason for these discrepancies, another possible explanation was proposed by Bernlochner et al. [5]. A New Physics model where the effective Lagrangian for a $b \to u\ell\nu$ transitions is expanded by a right-handed charged current:

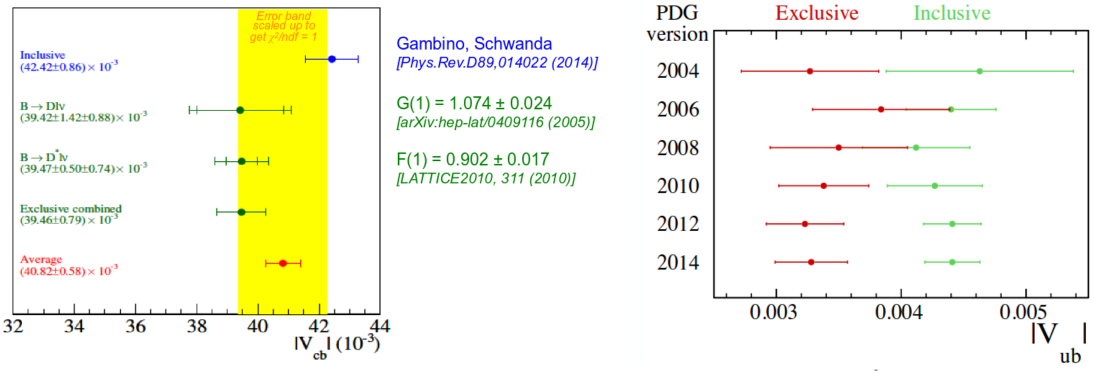

Figure 7: Discrepancies between exclusive and inclusive measurements appear both for $|V_{cb}|$ (left) and $|V_{ub}|$ (right).

$$\mathcal{L}_{\text{eff}} = -\frac{g}{\sqrt{2}} V_{ub} \left(\bar{u}\gamma_\mu P_L b + \epsilon_R \bar{u}\gamma_\mu P_R b \right) \left(\bar{\nu}\gamma^\mu P_L \ell \right) + h.c. \tag{10}$$

Introducing this right-handed charged current with a proportionality constant ϵ_R modifies the vector and axial form factors by

$$V \to (1 + \epsilon_R)V \tag{11}$$

and

$$A_i \to (1 - \epsilon_R)A_i, \tag{12}$$

thus affecting directly the extraction of the matrix element magnitudes from the measured branching fractions. Figure 8 shows the dependency of the extracted values of the size of ϵ_R, as well as fits to extract the most probable value for ϵ_R, assuming that inclusive and exclusive measurement results should be identical.
This yields a results of $\epsilon_R = -0.15 \pm 0.06$. More precise measurements will be needed to be able to support or reject this hypothesis.

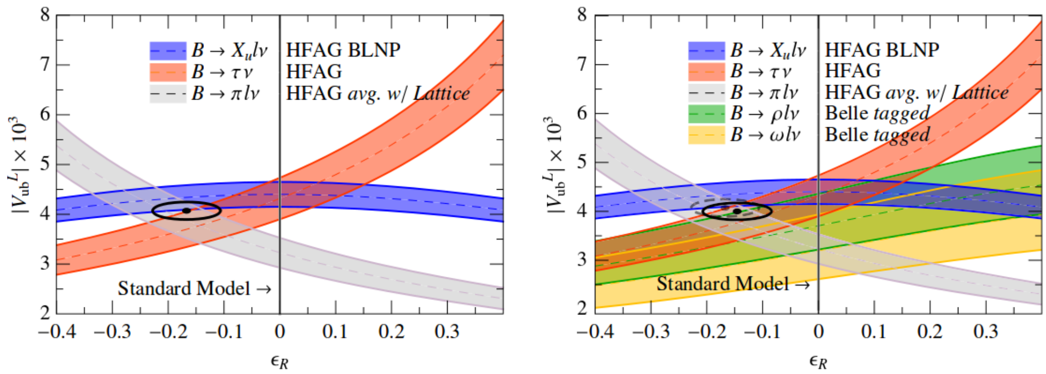

Figure 8: Fit for the most probable value of ϵ_R [5]

References

[1] Particle Data Group, J. Beringer *et al.*, *Review of particle physics*, Phys. Rev. **D86** (2012) 010001, and 2013 partial update for the 2014 edition.

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[3] Belle Collaboration, A. Sibidanov *et al.*, *Study of exclusive $b \to X_u \ell\nu$ decays and extraction of $|V_{ub}|$ using full reconstruction tagging at the belle experiment*, Phys. Rev. D **88** (2013) 032005.

[4] Belle Collaboration, C. Oswald *et al.*, *Measurement of the inclusive semileptonic branching fraction $\mathcal{B}(B_s^0 \to X^- \ell^+ \nu_\ell)$ at belle*, Phys. Rev. D **87** (2013) 072008.

[5] F. U. Bernlochner, Z. Ligeti, and S. Turczyk, *New ways to search for right-handed current in b -¿ rho ell nu decay*, 1408.25161408.2516.