

Part I: dark matter mystery

① Why do we believe it's a particle?

- Gravitational effects on galaxy scales (rotation curves) and on scales of galaxy clusters (lensing)
- Large Scale Structure of the Universe
- CMB
- Bullet Cluster
- Neutron stars merge (rules out viable modified gravity theories due to strong agreement with GR)

② Dark matter: what kind of beast is it?

We know:

- It constitutes $\sim 85\%$ of matter in the Universe
- Interacts through gravity
- If DM particles ever were relativistic - they should have slowed down early in the history of the Universe
- Electrically neutral (or has tiny charges)
- Stable on cosmological scales

We have no clue:

- Other interactions?
- Mass, spin?
- Several species?

DISCLAIMER: ① suggests that DM is a macroscopic on cosmological scales "particle". It can actually be huge and even baryonic!*

Examples: Massive compact halo objects (MACHOs),
Clouds of neutral gas, black holes...

However, there is not enough such objects (primordial black holes are still under consideration in a small area of parameter space)

All these funny names:

- Cold: Particles were created/decoupled non-relativistic (CDM)
- Warm: Particles were created/decoupled relativistic but became non-relativistic during radiation-dominated epoch. (WDM)
- Hot: Particles were created/decoupled relativistic and became non-relativistic in or around the matter-dominated epoch. (HDM)

Why to care? The "cusp-core/missing satellites/too big to fail" problems

* In cosmology "baryonic" means everything that interacts strongly or electromagnetically, and is not photons.

All these problems have similar roots: it seems that there is fewer small-scale structures than predicted by CDM models.

Why making DM warm can help?

- CDM is pressureless \Rightarrow in the Early Universe structures of all scales can be formed, and they grow from small ones to the larger ones.
- WDM has nonzero pressure \Rightarrow Very small structures cannot be formed, minimal scales formed are set by the DM free-streaming length at that time. Smaller structures are to be created later on.

DISCLAIMER (again): the "problems" mentioned might originate from our incorrect models of baryonic matter in the corresponding objects, and no warm dark matter would be needed.

③ We need some assumptions:

- A1: DM interacts with the SM particles not only gravitationally.
- A2: Let's focus on CDM
- A3: DM used to be in equilibrium with the SM plasma, but decoupled later on (freeze-out). This process sets the DM relic abundance.

Part II: DM relic density. Boltzmann treatment.

① Intuitive picture

Expanding Universe: $ds^2 = -dt^2 + a(t)^2 d\Sigma^2$
 3-dim metric

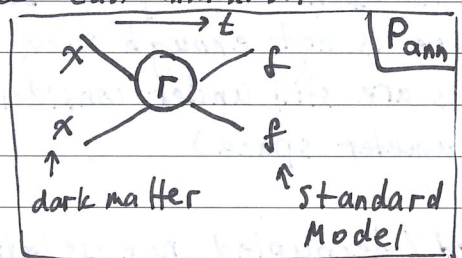
Assume that DM particles can annihilate

with the rate $\Gamma(t)^{comoving}$:

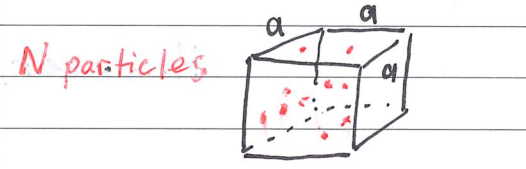
$$\frac{dN}{dt} = -\Gamma(t)^{comoving} N$$

$$\Gamma(t)^{comoving} = \Gamma(t) a^3(t)$$

Hubble parameter $H = \frac{\dot{a}}{a}$



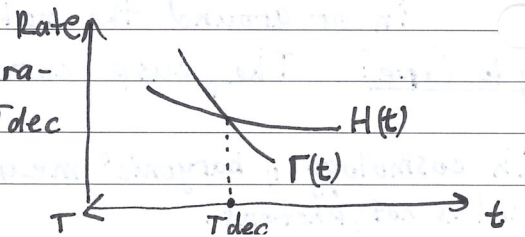
a volume with particles



No interaction: $N = n a^3 = const$

- If $\Gamma \gg H$ particles interact actively \Rightarrow they are in equilibrium
- If $\Gamma \ll H$ particles don't have enough time to find one another as the Universe expands too fast \Rightarrow they are out of equilibrium

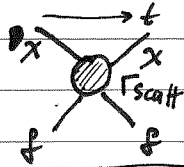
If $\Gamma(t)$ decreases with time (increases with temperature T) faster than $H(T)$: $\exists T_{dec}$ for $T < T_{dec}$ DM particles are out of equilibrium.



Remark : 2 types of equilibrium

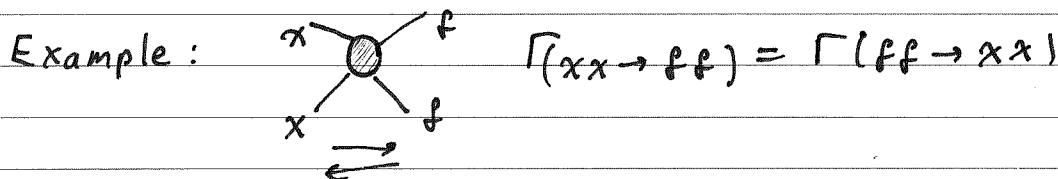
1) Kinetic (thermal) : A particle is in kinetic (thermal) equilibrium if it's distribution function is of an equilibrium shape (Bose-Einstein/Fermi-Dirac).

Remains as long as scattering processes are active ($\Gamma_{scatt} \gg H$)



\Rightarrow particles have the same temperature T

2) Chemical : A process is in chemical equilibrium when it's rate is equal to the reverse one



A particle is out of chemical equilibrium when there is no process in which this particle takes part and that is in chemical equilibrium

• A4 : At the moment of decoupling Γ_{ann} is the only process that keeps DM in chemical equilibrium with plasma (so decoupling means losing this equilibrium). Moreover, ~~throughout the following~~ DM is in thermal equilibrium with plasma during decoupling ($\Gamma_{scatt}(T_{dec}) \gg \Gamma_{ann}(T_{dec})$)

Distribution function after kinetic decoupling :

In the approximation of instant decoupling and in case if there is no DM further interactions popping up, DM distribution function would keep it's shape in comoving coordinates. The only ~~change~~ change would be redshifting of its momentum : $f(|\vec{p}|, t) = f_{decoupling} \left(\frac{a(t)}{a_{decoupling}} |\vec{p}| \right)$