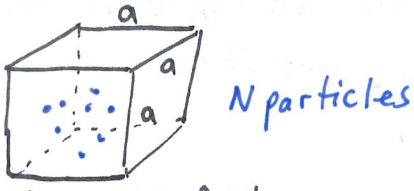
A dark path to long-lived particles

Part II: relic density and miraculous problems with WIMPS

Recap:



$a(t)$ - scale factor

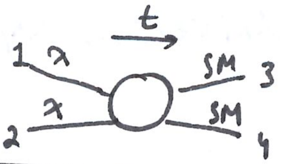
$$\frac{dN(t)}{dt} = -\Gamma^{comoving}_{(t)}$$

$N(t) = n(t)\, a^3(t)$ ($n(t)$: particle density)

$$\boxed{\frac{1}{a^3}\frac{d(na^3)}{dt} = -\Gamma(t)} \quad (*)$$

$\Gamma^{comoving}_{(t)} = \Gamma(t)\, a^3(t)$ ($\Gamma(t)$: interaction rate)

Consider annihilations:



Some more notations:

$M_{ij\to kl}$ - matrix element of process $ij \to kl$

$d\Pi_i \equiv \frac{d^3 p_i}{(2\pi)^3 2E_i}$ - Lorentz invariant phase space

Every particle i is characterized by its distribution function f_i:

$$n_i(t) = \frac{g_i}{(2\pi)^3}\int d^3p_i\, f_i(p,t)$$

g_i - number of internal d.o.f.

$p = |\bar{p}|$ (assume homogenic & isotropic universe)

By the definition of M:

$$\Gamma(t) = \sum_{spins}\int d\Pi_1 \ldots d\Pi_4 (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)\left[f_1 f_2 \underbrace{\cancel{(1\pm f_3)(1\pm f_4)}}_{A2} |M_{12\to 34}|^2 - f_3 f_4 \underbrace{\cancel{(1\pm f_1)(1\pm f_2)}}_{A2} |M_{34\to 12}|^2\right]$$

Let's simplify it!

- A5: CP(T) invariance: $M_{12\to 34} = M_{34\to 12} = M$
- A6: Thermal equilibrium of the SM particles: $f_3 f_4 = f_3^{eq} f_4^{eq}$
- A7: Detailed balance: in a full equilibrium $\Gamma^{(T)}_{12\to 34} = \Gamma^{(T)}_{34\to 12}$
  $\Rightarrow f_1^{eq} f_2^{eq} = f_3^{eq} f_4^{eq}$ (all with the same temperature T)

$$\boxed{\Gamma(t) = \sum_{spins}\int d\Pi_1 \ldots d\Pi_4 (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4) |M|^2 [f_1 f_2 - f_1^{eq} f_2^{eq}]}$$

Note: in principle, $\Gamma(t)$ should include all processes in which χ takes part and which change the number of χ particles (e.g. decays of/to χ)

Define cross-section σ:

$$\sum_{spins}\int \underbrace{d\Pi_3\, d\Pi_4}_{\text{final states}} (2\pi)^4\delta^4(p_1+p_2-p_3-p_4)|M|^2 = 4 g_1 g_2 \sqrt{(p_1 p_2)^2 - m_1^2 m_2^2}\,\sigma$$

$$\Gamma(t) = \underbrace{g_1\int\frac{d^3p_1}{(2\pi)^3} f_1}_{dn_1}\ \underbrace{g_2\int\frac{d^3p_2}{(2\pi)^3} f_2}_{n_2}\ \cancel{4}\sigma \underbrace{\frac{\sqrt{(p_1 p_2)^2 - m_1^2 m_2^2}}{\cancel{4} E_1 E_2}}_{v_{møl}} - (1,2 \to 1^{eq}, 2^{eq})$$

$$\boxed{\Gamma(t) = \int \sigma v_{møl}\,(dn_1\, dn_2 - dn_1^{eq}\, dn_2^{eq})}$$

Note: $v_{møl}$ is a dimension-less quantity but it is <u>not</u> the relative velocity between 1 and 2. However, it is defined such that $v_{møl}\, n_1 n_2$ is Lorentz invariant. In the rest frame of n_2 (n_1) $v_{møl}\, n_1 n_2 = v^{rel}_{lab}\, n_1 n_2$ and v^{rel}_{lab} is indeed the relative velocity between 1 and 2.

A8: $\sigma v_{møl}$ does not depend strongly on n_1 and $n_2 \Rightarrow$ we can replace it with the average value $\langle \sigma v_{møl} \rangle \equiv \dfrac{\int \sigma v\, dn_1^{eq}\, dn_2^{eq}}{\int dn_1^{eq}\, dn_2^{eq}}$ to factor it out

$$(*) \Rightarrow \boxed{\frac{1}{a^3}\frac{d(na^3)}{dt} = -\langle \sigma v_{møl} \rangle (n^2 - n_{eq}^2)}$$

where we used $n_1^{(eq)} = n_2^{(eq)} = n^{(eq)}$ for the two ~~sa~~ identical DM particles

Let's get ~~my~~ rid of the dependence on the Universe's expansion $\Rightarrow$ change to the comoving coordinates.

Entropy of the (closed) system: $S = \text{const}$ (no heat flow from/to the system)

Entropy density $s = S/a^3$ behaves as $n = N/a^3$

$\Rightarrow$ Introduce $Y = \frac{n}{s}$ – does not depend on $a(t)$.

A2 assumes also that DM was produced well before structures were formed $\Rightarrow$ during the radiation-dominated epoch.

<u>Rad. dom</u>: Hubble parameter $H(t) = \frac{1}{2t} = \frac{T^2}{M_{Pl}^*}$ $\left(M_{Pl}^* = \frac{M_{Pl}}{1.66\sqrt{g_*}}\right.$ – effective Planck Mass
(from Friedman's equations)

g_* – number of relativistic degrees of freedom $\left.\right)$

$$\Rightarrow \begin{cases} \dfrac{dx}{dt} = \dfrac{H(T=m)}{x} \\ \dfrac{d(na^3)}{dt} = \dfrac{d}{dt}\left(\dfrac{na^3 \overbrace{s}^{S=\text{const}}}{s}\right) = s a^3 \dfrac{dY}{dt} \end{cases} \rightarrow \boxed{\frac{dY}{dx} = -\frac{Y_{eq}}{x}\frac{\Gamma_{eq}}{H}\left(\frac{Y^2}{Y_{eq}^2} - 1\right)} \quad (**)$$

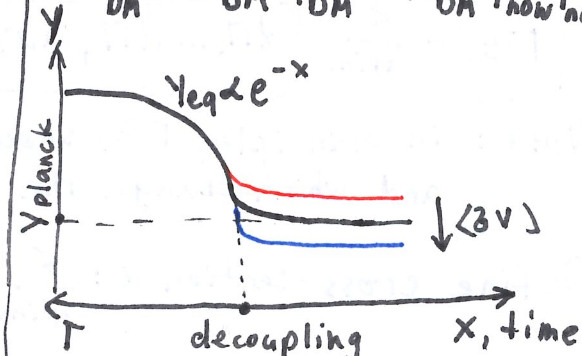
where $\Gamma_{eq} = n_{eq} \langle \sigma v \rangle$

- $\frac{\Gamma_{eq}}{H} \gg 1 \Rightarrow Y = Y_{eq}$ (Maxwell-Boltzmann)
- $\frac{\Gamma_{eq}}{H} \ll 1 \Rightarrow Y = $ const – decoupled DM

Decoupling happens at $\Gamma_{eq}(t) \sim H(t)$

(more precisely at $\frac{\Gamma_{eq}}{H} \sim \frac{x}{Y_{eq}} \sim x \frac{n_{eq}}{s_{eq}} \sim x\, O(1) \frac{T^3}{T^3} \sim 20$)

$\Omega_{DM} = m_{DM} n_{DM} \propto m_{DM} Y_{now} T_{now}$



decoupling
$n_{eq}\langle \sigma v \rangle \sim (20 \div 30) H$

<u>Why electroweak scale?</u>

It appears that from (**):

$$\Omega_{DM} h^2 \sim 0{,}12 \; \frac{10^{-26}\, cm^3 s^{-1} \;(\text{or } 10^{-9}\, GeV^{-2})}{\langle \sigma v \rangle}$$

Planck

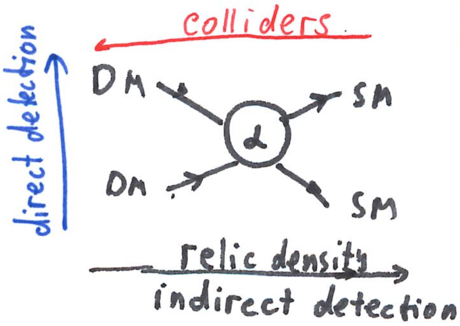
The stronger interactions are, the longer is DM in equilibrium $\Rightarrow$ the smaller is Ω_{DM}

$\Rightarrow \langle \sigma v \rangle \sim$ weak scale gives correct relic abundance (WIMP Miracle)

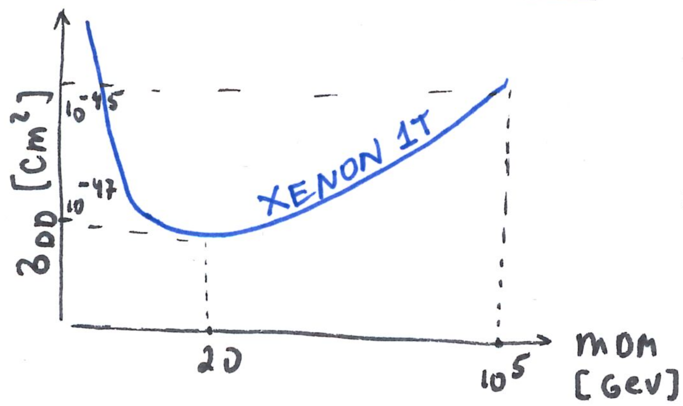
Testability

$\langle \sigma v \rangle_{ann} \propto \alpha^2$

$\langle \sigma v \rangle_{DD} \propto \alpha^2$



Main problem: direct detection



$\Rightarrow$ α is tiny

$\Downarrow$

$\langle \sigma v \rangle_{ann}$ is so small that we have too much DM

overabundance

Ways out?

Of course, there are many. Many of the modified models are still called WIMP or WIMP-like models. However, there are no „classical" WIMPs in there.

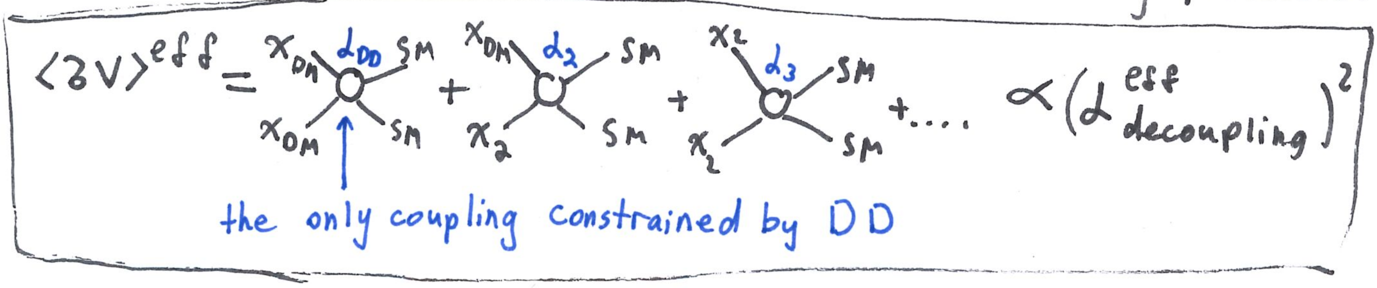
Example: co-annihilation

Note: | DD experiments constrain α_{now}
| Relic density is set by $\alpha_{decoupling}$

Idea: Let's increase $\alpha^{(eff)}_{decoupling}$ without changing α_{now}!

Realization: Add more states (yes, introducing more parameters always helps :))

Relic density is set by ~~[illegible]~~ the sum of the following processes:



Later, all χ_i states decay to DM $\Rightarrow$ today only χ_{DM} is present $\Rightarrow$ direct detection is indeed sensitive to α_{DD} only.

Requirements:

- χ_i are close in mass (for them to be present at $T_{decoupling}$)
- $\alpha_{2,3..}$ are not too small (or χ_i becomes DM itself)