Type IIB Compactifications

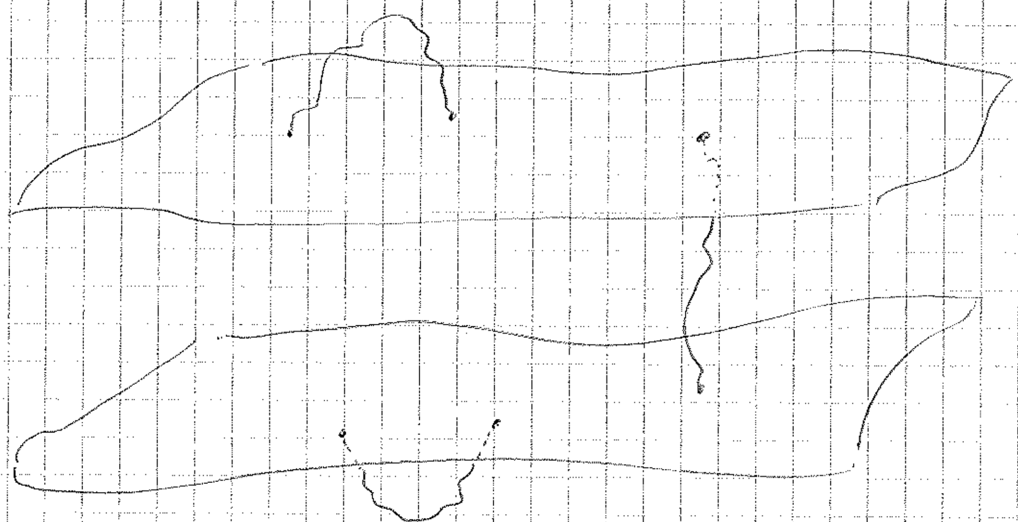
Recap: String theory / M-theory are theories in 10/11 D. To make contact to "real world" physics, need to compactify 6/7 dimensions on a compact space X. Physics in 4D depends on geometry of X.

Importantly: Gauge symmetries are realised on branes.

Gauge symmetries on branes

Last time: branes are scources of p-form potentials, and they are gravitational solutions

A third way: Ending loci of open strings:



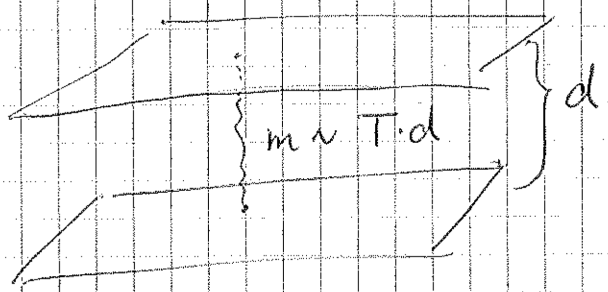
Can explicitly compute: Strings with both ends on one brane give rise to a $U(1)$ gauge field on the brane !!

Heuristically: Strings push & pull at brane

→ small perturbations, corresponding to field excitations

Strings with one end on a brane corresponds to 'charged states' of $U(1)$ gauge field

Strings have tension $T \longleftrightarrow$ mass of charged states corresponds to displacement of branes:



What happens if N branes lie on top of each other ??

$\longrightarrow$ N $U(1)$ fields inside the same worldvolume

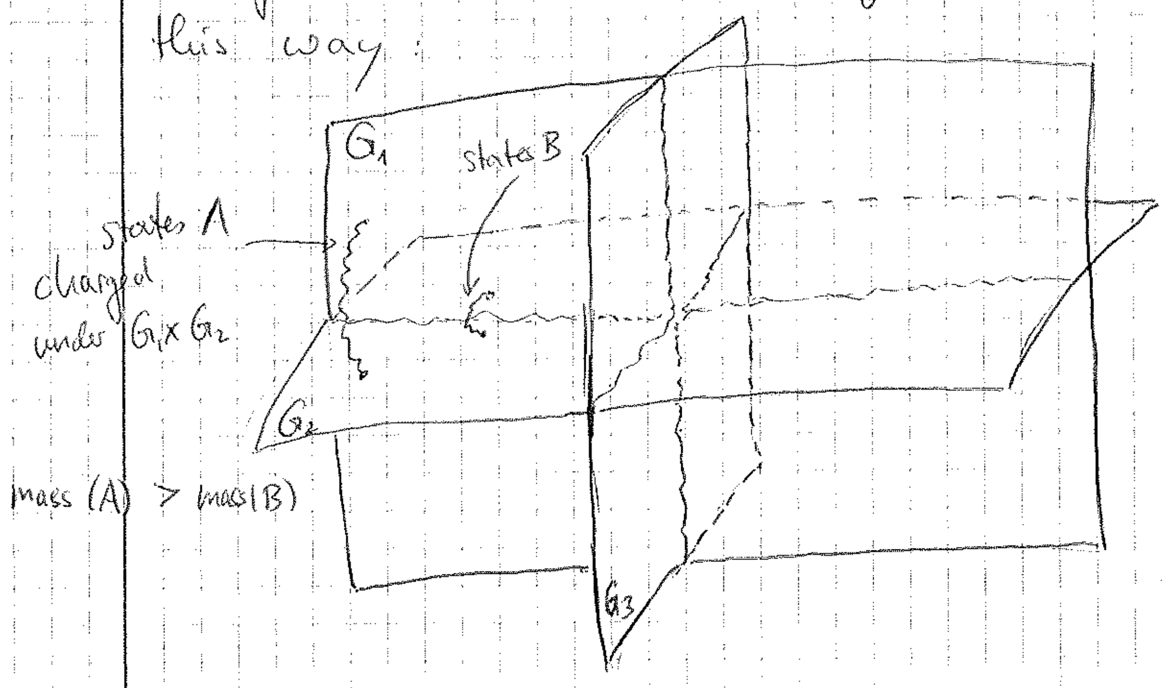
$+2\{(N-1)+(N-2)+\ldots+1\} = N^2 - N$ <u>massless</u> states carrying two $U(1)$-charges

(orientation)

$\longrightarrow$ adjoint representation of $U(N)$

$\Longrightarrow$ open strings on a stack of N branes give rise to a $U(N)$ gauge field!

Also possible to realise charged matter in this way:



→ massless bifundamental matter localises at ~~interaction~~ intersections of branes!

⇒ Standard Model realisable via branes!

However, this naive construction has many phenomenologically unfavourable aspects...

Type IIB string theory

One of the five string theories is called type IIB. Recall from last time that there is a low energy effective field theory (SUGRA); for type IIB, the corresponding SUGRA has:

field	type	electric charges	magnetic charges
ϕ (dilaton)	scalar	–	–
$G_{\mu\nu}$ (metric)	sym. 2-tensor	–	–
B_2	2-form	F1-string	NS5-brane
C_0	0-form	D(-1) instantons	$\boxed{\text{D7-brane}}$
C_2	2-form	D1-string	D5-brane
C_4	4-form	D3-brane	D3-brane

Recap: B_2, C_n have associated gauge symmetry

$$B_2 \to B_2 + d\Lambda_1 \quad , \quad C_n \longrightarrow C_n + d\Lambda_{n-1}$$

fundamental strings in type IIB are charged under B_2, but there are other strings (D1), charged under C_2.

Model building with type IIB

As we will see momentarily, 7-branes have a few particularly nice model building features. In the following, we will focus on these brane types.

1. What types of compactification space?
In principle, any six-dimensional space gives a theory in 4D, but in general, computation to difficult (compare to physical problem of solving any 4D field theory).
→ Need additional symmetries.
One type of Symmetry that has (or had) phenomenological relevance: Supersymmetry (SUSY)

To obtain 4D SUSY theory, compact space X_6 must be a <u>complex</u> manifold, satisfying the so-called Calabi-Yau condition. Sparing technical details: CY-manifolds are Ricci-flat!!
Complex geometry can be described algebraically, i.e. via polynomials → "simple"!

2. Complex submanifolds of X_6 are even (real) dimensional manifolds. Now we want our physical world to be inside branes with gauge symmetries → four of the $(n+1)$ dimensions of a n-brane lie in the non-compact spacetime

$\Longrightarrow$ $(n+1)-4$ are on X_6!

⇒ Only branes with odd n give rise to "complex" submanifolds on X_6.
→ Reason why type IIB is so sucessful! (because can use complex geometry tools)

E.g. D3-branes are of central interest in the AdS/CFT correspondence.

For particle physics model building: 7-branes

- These "wrap" (real) 4-dimensional or (complex) 2-dimensional subspace
Geometry of these spaces determine the gauge symmetry on the branes, e.g. volume of this subspace ⟷ gauge coupling strength

- Massless Matter states localise on intersection of branes, i.e. on (complex) 1-dim., or (complex) curves in X_6. So-called "matter curves"

- Coupling between states (Yukawa type) correspond to intersections of matter curves!

Many physical quantities, e.g. Yukawa matrices, scalar potentials etc. depend on details of geometry.

<u>One aspect</u>: chirality

Recall that Weyl fermions have notion of chirality: eigenfunctions of Dirac operators

$\sigma^\mu \partial_\mu \rightarrow$ right-handed , $\bar\sigma^\mu \partial_\mu \rightarrow$ left-handed

$(\sigma^\mu \partial_\mu \psi_R = 0)$ $(\bar\sigma^\mu \partial_\mu \psi_L = 0)$

chirality of fermions in a representation R:

$\chi(R) := \#(\text{left-handed in } R) - \#(\text{right-handed } R)$

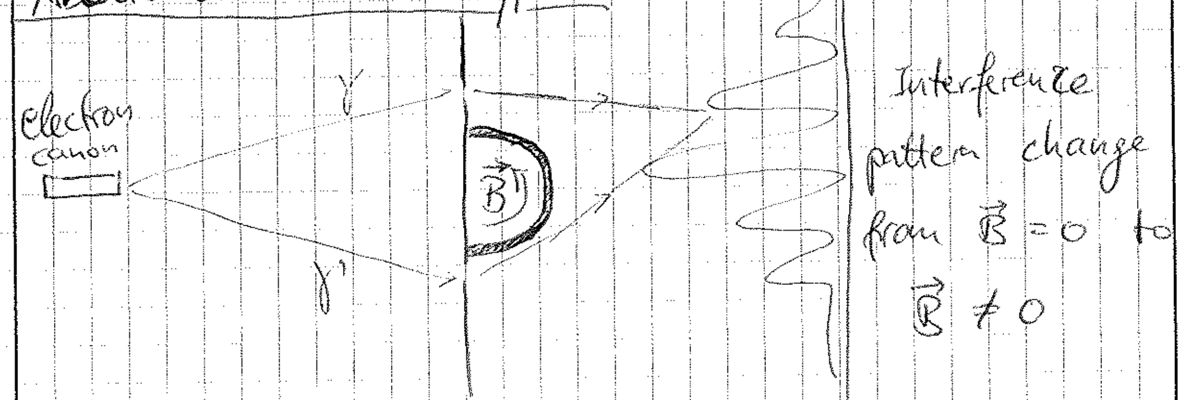
In mathematics, the zero eigenfunctions of the Dirac operator are counted by cohomology groups.

This difference is a priori 0! But is altered by non-zero background value of gauge field strengths $F_{n+1} = dC_n$ (called <u>fluxes</u>)

"Background" $\triangleq$ vev, around which we can excite small perturbations (cf. Higgs field and boson)

Derivation very technical, <u>but</u> compare for analogy with

<u>Aharonov-Bohm effect</u>:



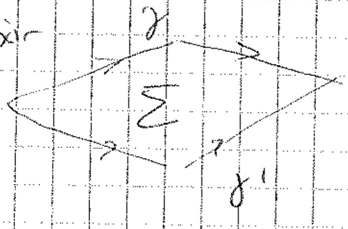
In electromagnetism, $\vec{B} = \vec{\nabla}\times\vec{A}$ is the field strength of a gauge field. A non-zero value of $\vec{B}$ $\hat{=}$ flux.

Interpretation (heuristic): Wave-function of electrons (carrying information whether left- or right-handed) is affected by flux of $\vec{B}$

Interference $\rightsquigarrow$ sensitive to relative phase of the two possible paths γ and γ':

$$\Delta\varphi \sim \exp\left(i\int_\gamma \vec{A} - i\int_{\gamma'} \vec{A}\right) = \exp\left(i\oint_{\gamma\gamma'} \vec{A}\right)$$

$$= \exp\left(i\int_\Sigma \vec{B}\right),$$ Σ: interior of paths



In type IIB string theory compactified on X_6, matter in rep R on a matter curve C has chirality

$$\chi = \int_C F$$

where F is a 2-form describing the flux configuration.

Technically, both the A.-B.-effect and chirality in IIB are due to a line bundle with non-trivial first Chern-class $c_1 \equiv F$

For a full type IIB compactification, need:

- compactification space X_6
- submanifolds wrapped by 7-branes
- fluxes on X_6

} subject to many, many consistency conditions

Fluxes have, in addition to giving rise to chirality, many other purposes for phenomenology, e.g.:

- generate non-trivial Yukawa matrices (with hierarchies!)
- stabilise compactification configuration

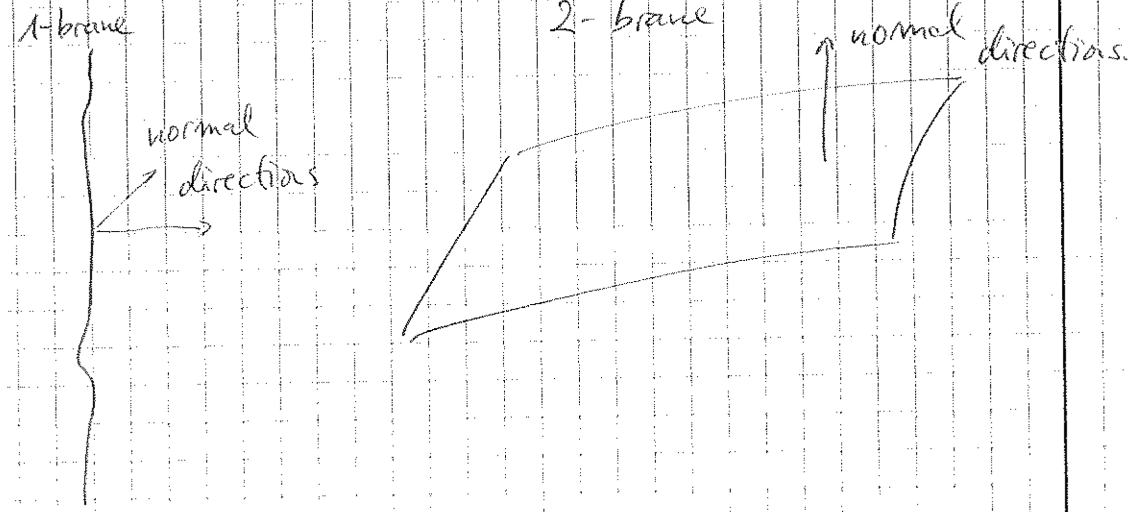
Going beyond probe-brane limit

For many years, model building has advanced along lines above. **But**, there the branes are treated as **probes** (e.g. earth in Sun's orbit). Question: can back-reactions be neglected?

$\rightarrow$ Different branes have different answers!

Similar to classic electrodynamics or Newtonian gravity, branes as scources (charged or massive) have back-reaction governed by ~~Laplace~~ Poisson-like equation: $\Delta \Phi \sim \delta(\vec{r}_0)$

$\delta(\vec{r}_0)$ describes the distribution (charge or mass) of the brane:



If we have $p > 2$ normal directions, then

$$\Phi \sim \frac{1}{|\vec{r}-\vec{r}_0|^{p-2}} \quad \left(\text{familiar: point-like in 3D} \Rightarrow \frac{1}{r} \atop \text{plane in 3D} \Rightarrow r \right)$$

$$p = 2 \longrightarrow \Phi \sim \ln(|\vec{r}-\vec{r}_0|)$$

$\rightarrow$ back-reaction does not fall off asymptotically

$\Longrightarrow$ 7-branes wrapping 4-dimensions on X_6 have non-negligible effect on $\Phi = C_0$!! $\rightsquigarrow$ Need better description