F-theory : Geometrising the back-reaction of 7-branes

Last time : type IIB compactifications with 7-branes can realise gauge theories in 4D. However, the branes in this setup are treated as probes. Branes are sources for gauge potentials ⟹ their presence have effect ~~of~~ on the compactification configuration!

Due to dimensionality, 7-branes have non-negligible back-reaction! (see hand-out from last time)
How to take it into account?

Observation: type IIB has an $SL(2,\mathbb{Z})$ symmetry:

combine ϕ (dilaton) and C_0 into a complex scalar $\tau := C_0 + ie^{-\phi}$, then type IIB is invariant under

$$\tau \longmapsto \frac{a\tau+b}{c\tau+d}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

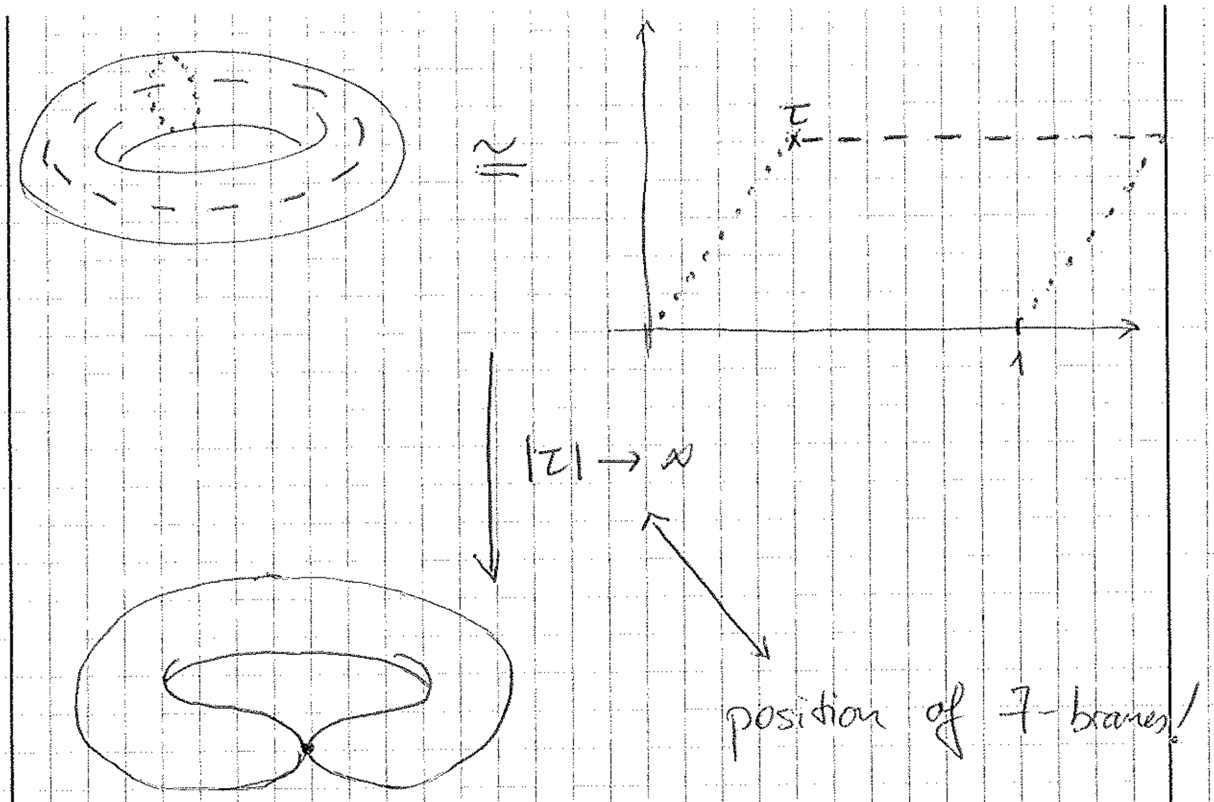
7-branes are sources for τ:
Let z ~~...~~ $= x+iy$ denote the two spatial coordinates orthogonal to the 7-brane world-volume in X_6, then:

$$\tau(z) = \frac{1}{2\pi i} \log \frac{z-z_0}{\lambda} + \dots \quad , \; z_0 \text{ - location of 7-brane}$$

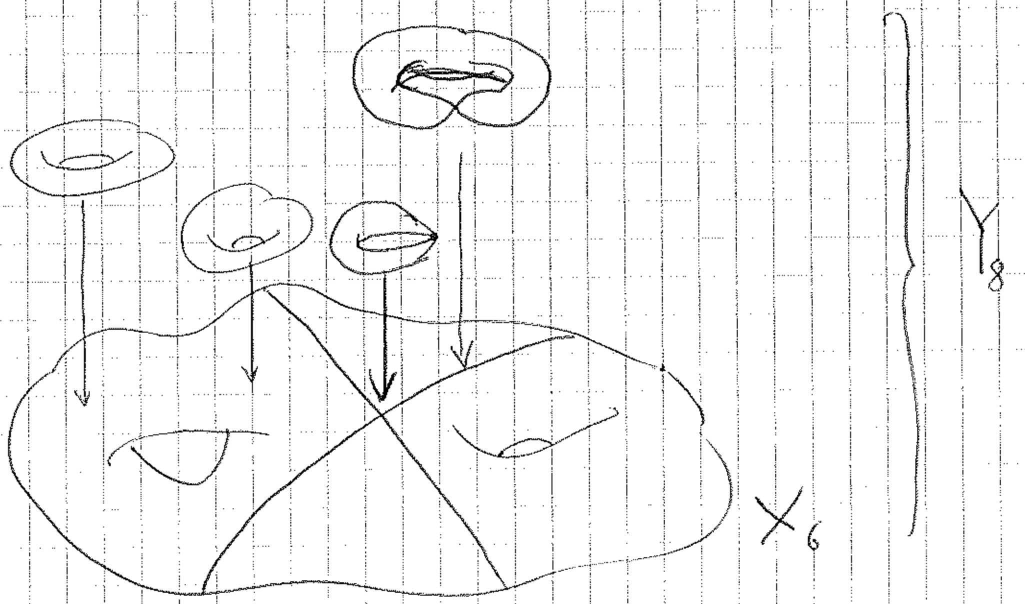
λ some constant.

$\Big[SL(2,\mathbb{Z}) \longleftrightarrow$ monodromy of τ around $z_0 \Big]$

idea: identify τ with "complex structure" of a torus



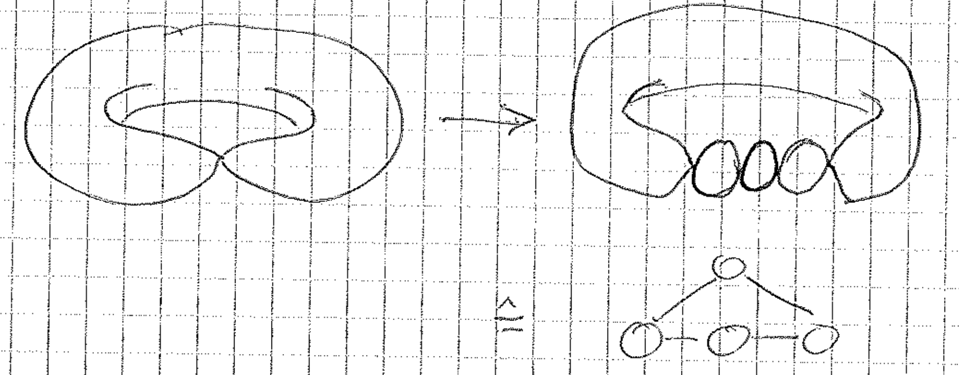
mathematically: <u>torus - fibration</u> Y_8



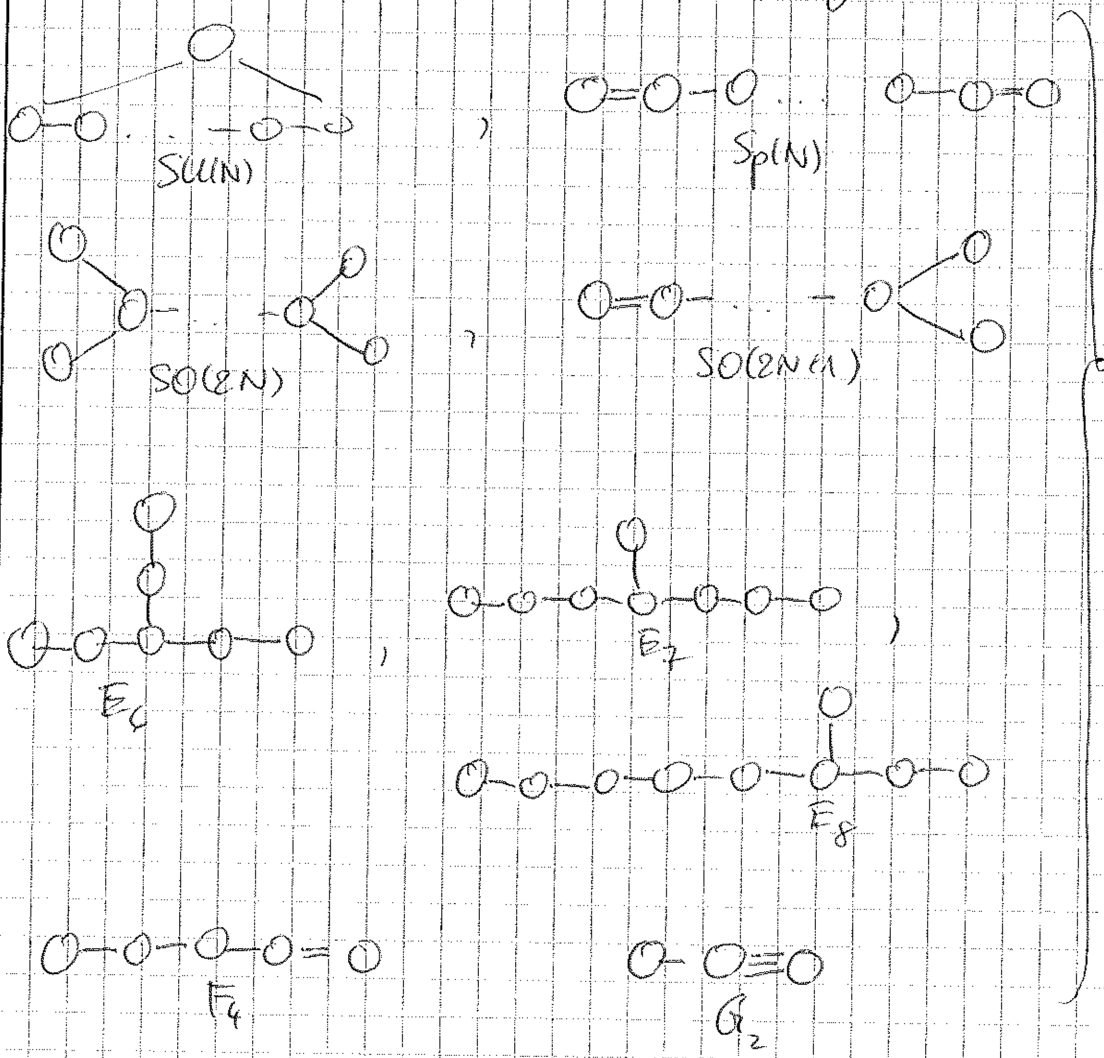
A mathematically consistent Y_8 <u>automatically</u> takes care of all back-reaction effects of 7-branes!

Type of singular fiber determines the gauge group on branes!

Concretely: resolution of singularity:



In general, one has the following possibilities:



(affine) Dynkin diagrams

$\rightarrow$ 1-to-1 correspondence between singularity types of torus-fibrations and semi-simple Lie-algebras !!

$\hookrightarrow$ $SU(N)$, $Sp(N)$, $SO(2N)$, $SO(2N+1)$, E_6, E_7, E_8, F_4, G_2

Note: In type IIB, the torus is just a book-keeping device, and not part of real space !!

<u>But</u>: Via duality to M-theory, the torus becomes part of the 11D spacetime

$$11D \xrightarrow{Y_8} 3D \xrightarrow[\text{limit}]{\text{special}} 4D \overset{\text{equivalent}}{\longleftrightarrow} \text{(non-perturbative) type IIB}$$

In this picture, the degrees of freedom of gauge fields arise from M2-branes wrapping the $\mathbb{P}^1$'s in the fiber ! $\longrightarrow$ <u>simple roots</u> $\hat{=}$ generators of gauge symmetry

How is this an extension of type IIB ?

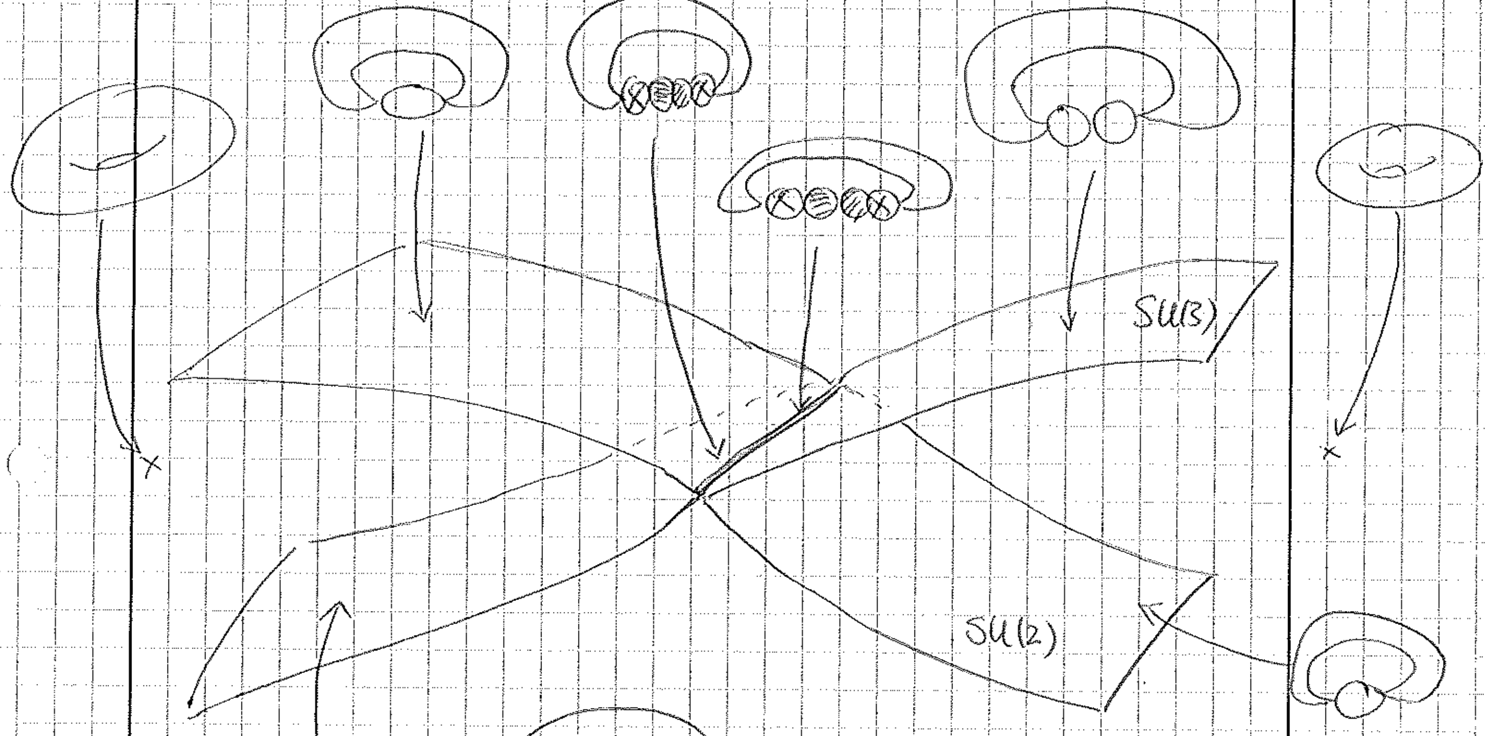
- string coupling $g_s = e^{+\phi} = \mathrm{Im}(\tau)^{-1}$
- in type IIB: $g_s = 0$ ($\longleftrightarrow$ $\mathrm{Im}(\tau) = \infty$)
  "perturbative" limit; only possible gauge groups:
  $SU(N)$, $SO(2N)$, $Sp(N)$ (no E_6 !!)
- in F-theory: g_s finite $\longleftrightarrow$ τ finite,
  "non-perturbative" corrections (g_s-corrections)
  <u>geometrised</u> $\longrightarrow$ can realise other groups

$\rightarrow$ importantly, E_6 is possible !
(responsible for the up-type Higgs Yukawa coupling in SU(5)-GUTs)

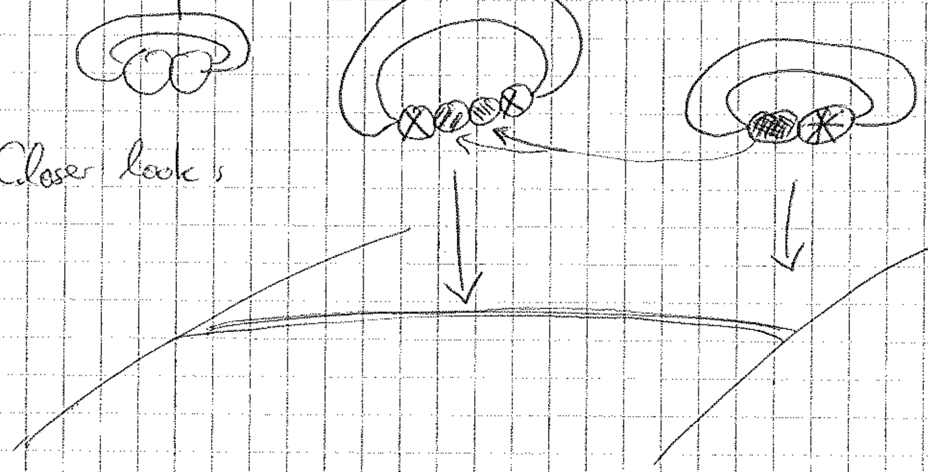
There are explicit and rigorous analyses that precisely derive, how gauge theories arise in F-theory via duality to M-theory

Matter & Couplings in F-theory

In type IIB, matter states arise at intersections of 7-branes. In F-theory – as extension of IIB:



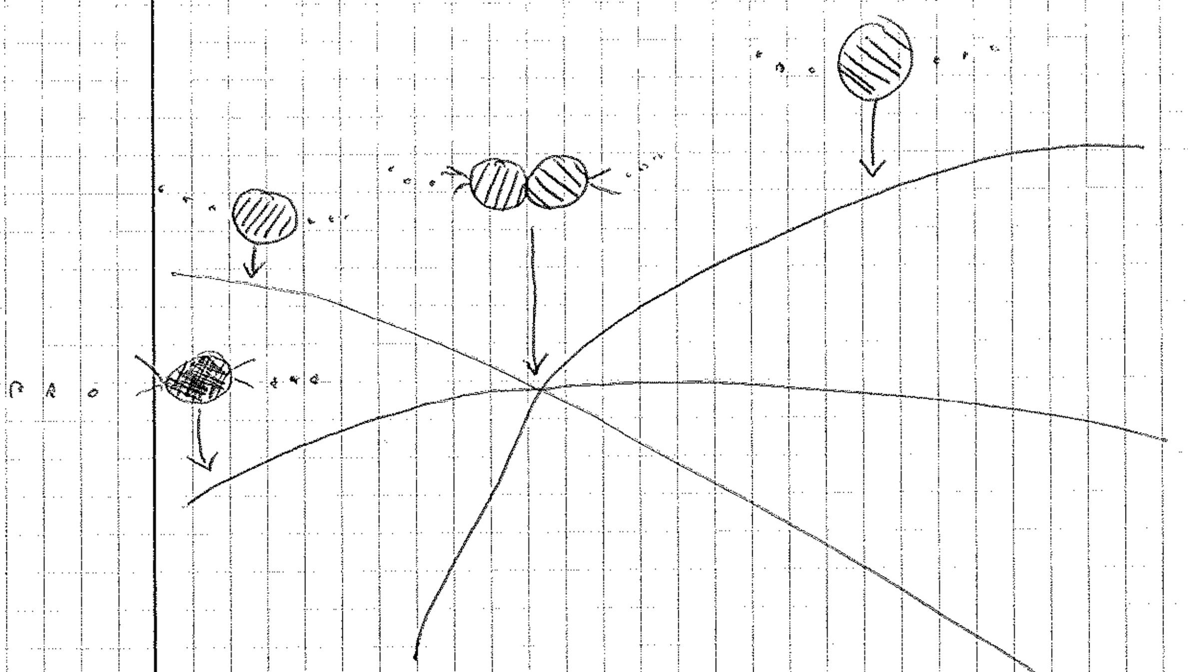
Closer look:

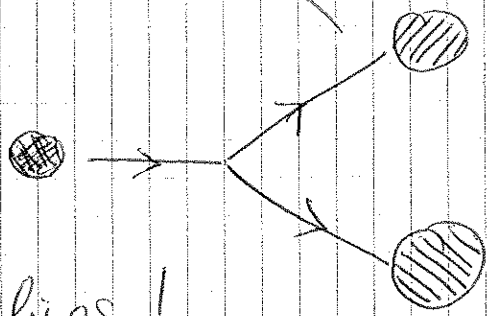


Over matter curves, simple roots split into weights of a representation of the gauge group.

Case at hand: bifundamental states $(\underline{3}, 2)$ of $SU(3) \times SU(2)$

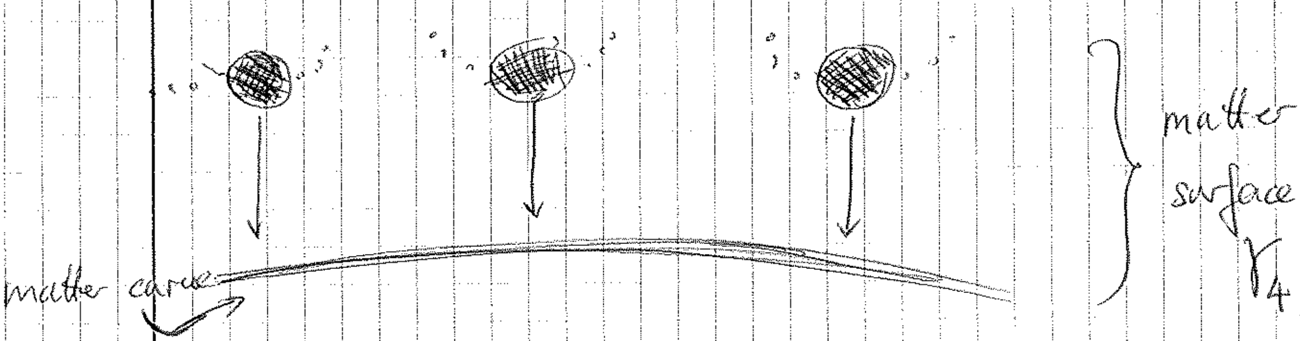
Similar splitting process over intersections of matter curves:



$\cong$ Feynman diagramm : 

$\longrightarrow$ realise Yukawa couplings !

<u>Chiral matter & anomalies in F-theory</u>



The fibration of the matter $\mathbb{P}^1$ over the matter curve gives rise to a (real) four-dimensional subspace γ_4 of Y_8, called <u>matter surface</u>.

Matter on γ_4 are a priori <u>non-chiral</u>, i.e. # left-handed = # right-handed states. Chirality is induced by <u>G_4-fluxes</u>.

G_4-fluxes are further input that specify the ~~the~~ compactification configuration beyond the pure geometrical data of Y_8. (See hand-out of last time for more details.)

In any case: G_4 is a 4-form,

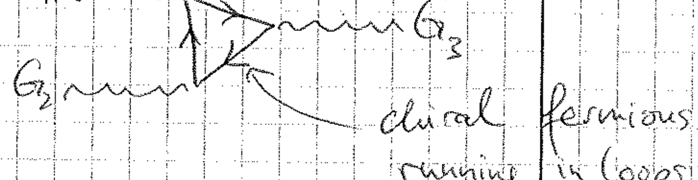
$$\chi(R) = \#\,\text{left-handed} - \#\,\text{right-handed} = \int_{\gamma_4(R)} G_4 ,$$

where $\gamma_4(R)$ is the matter surface associated with representation R.

[Not much is known mathematically about "matter surfaces", i.e. special four-dim. subspaces of a generic Y_8.]

Physical requirement: 4D gauge theories with chiral matter have potential quantum anomalies that need to vanish!

Quantitative description:



$$\Rightarrow \mathcal{A} = \sum_R \lambda(R) \cdot \chi(R) ,$$

where $\lambda(R)$ is a group-theoretic factor depending on the type of anomaly.

Example: Standard model $SU(3) \times SU(2) \times U(1)$

$$SU(3)^3\text{-anomaly:} \quad 2 \cdot \underset{Q_L}{\chi((\underline{3},\underline{2}))} + \sum_i \underset{u_R,\, d_R}{\chi(\underline{3}_i)} \overset{!}{=} 0$$

$$SU(2)^2 - U(1): \quad 3 \cdot \frac{1}{6} \cdot \chi((3,2)) + \left(-\tfrac{1}{2}\right) \cdot \chi(2_L) \overset{!}{=} 0$$

$$SU(3)^2 - U(1): \quad 2 \cdot \frac{1}{6} \cdot \chi((3,2)) + \left(-\tfrac{2}{3}\right) \cdot \chi(\bar{3}_u) + \left(\tfrac{1}{3}\right) \chi(\bar{3}_d) = 0$$

$$U(1)^3 \quad : \quad 6 \cdot \left(\tfrac{1}{6}\right)^3 \chi(3,2) + \sum_R \dim(R)\, q(R)^3\, \chi(R) = 0$$

$$U(1)\text{-grav}: \quad \sum_R \dim(R)\; q(R)\; \chi(R) = 0$$

unique solution: one chiral fermion in each R per <u>family</u>

In F-theory:

$$A = \sum_R \lambda(R)\, \chi(R) = \sum_R \lambda(R) \int_{\gamma_4(R)} G_4$$

$$= \int_{\sum_R \lambda(R) \gamma_4(R)} G_4 \quad \longleftarrow \text{ geometric object!}$$

<u>Physical input</u>: For <u>any</u> consistent G_4, this integral must vanish $\rightsquigarrow$ how is it satisfied?

$\longrightarrow$ We explicitly analysed for many models the subspaces $\sum_R \lambda(R) \gamma_4(R)$ [required some novel techniques]

Result: These subspaces are such that <u>any</u> valid G_4 gives zero anomaly.

$\longrightarrow$ unexpected result from mathematical point of view! Maybe unknown property of Y_8? $\longrightarrow$ requires further analyses...