Physics of Extra Dimensions (in Field and String Theory)

Plan:
- Kaluza-Klein theory — lecture 1
- Randall-Sundrum model — lecture 2
- axions (from p-forms)
- 10d compactifications (on CYs) — lecture 3

Motivation:
- top-down: String Theory (a consistent 10d quantum gravity)
- bottom-up: Phenomenology (hierarchies & axions)

We take the latter approach and study phenomenology first and put everything together in the end in string-theory motivated example.

Note: no string theory needed, will phrase everything in field theory language.

1) Kaluza-Klein Compactifications

Will look at 5d example $M_4 \times S^1$ (M_4: ordinary 4d spacetime, S^1: circle):

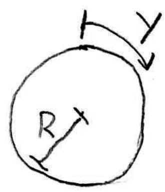
1.1) Scalar Fields

Look at real, massless, free scalar field in 5d

$$S_{5d}[\Phi] = \int_{M_4 \times S^1} d^5x \left(-\tfrac{1}{2} \partial_m \Phi(x^m) \partial^m \Phi(x^m) \right), \qquad m = 0, \dots, 4.$$

Split the coordinates into x^μ on M_4, $\mu = 0, \dots, 3$, & $y = x^4$ on S^1

S^1 is defined by a relation $y \simeq y + 2\pi R$.



For Φ to be well-defined, we need

$$\Phi(x^\mu, y) \overset{!}{=} \Phi(x^\mu, y + 2\pi R)$$

$\Rightarrow$ Expansion into $(2\pi R)$-periodic functions $e^{i\frac{ny}{R}}$, $n \in \mathbb{Z}$.

$$\phi(x^\mu, y) = \sum_{n\in\mathbb{Z}} \phi_n(x^\mu)\, e^{i\frac{ny}{R}} .$$

Reality requires $\phi^* = \phi \Rightarrow \phi_{-n} = \phi_n^*$

$$\Rightarrow \quad \phi(x^\mu, y) = \phi_0(x^\mu) + \sum_{n=1}^{\infty} \left(\phi_n(x^\mu) e^{i\frac{ny}{R}} + \phi_n^*(x^\mu) e^{-i\frac{ny}{R}}\right)$$

With this, we now compuctify explicitely. That is, we integrate out the S^1-dimension to find a 4d spectrum.

Some preliminary calculations:

$$\partial_y \phi(x^\mu, y) = 0 + \sum_{n=1}^{\infty} \left(i\frac{n}{R}\right)\left(\phi_n(x^\mu) e^{i\frac{ny}{R}} - \phi_n^*(x^\mu) e^{-i\frac{ny}{R}}\right)$$

$$(\partial_y \phi)^2 = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \left(-\frac{nm}{R^2}\right)\left(\phi_n e^{i\frac{ny}{R}} - \phi_n^* e^{-i\frac{ny}{R}}\right)\left(\phi_m e^{i\frac{my}{R}} - \phi_m^* e^{-i\frac{my}{R}}\right)$$

$$= \sum_{n=1}^{\infty} 2\frac{n^2}{R^2} \phi_n \phi_n^* + \left(\text{terms w/ } e^{i\frac{ky}{R}},\ k \neq 0\right).$$

$$\int_0^{2\pi R} dy\, (\ldots)\, e^{i\frac{ky}{R}} = 0 \quad \text{for any } k \neq 0$$ (integral of periodic fct. over entire period)

We put the general form of a 5d solution into the action:

$$S_{5d}[\phi] = \int_{M_4\times S^1} d^5x \left[-\frac{1}{2}\partial_m \phi(x^\mu, y)\, \partial^m \phi(x^\mu, y)\right]$$

$$\overset{\text{expand}}{\underset{\text{sum}}{=}} \int_{M_4} d^4x \int_{S^1} dy \left[-\frac{1}{2}\partial_y \phi\, \partial^y \phi - \frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi\right]$$

$$\overset{\text{insert}}{\underset{\text{above result}}{=}} \int_{M_4} d^4x \int_{S^1} dy \Big[-\sum_{n=1}^{\infty} \frac{n^2}{R^2} \phi_n(x^\mu)\phi_n^*(x^\mu) - \sum_{n=1}^{\infty} \partial_\mu \phi_n(x^\mu)\, \partial^\mu \phi_n^*(x^\mu)$$
$$- \frac{1}{2}\partial_\mu \phi_0(x^\mu)\, \partial^\mu \phi_0(x^\mu)\Big]$$

$$\left(+\text{ terms w/ } e^{i\frac{ky}{R}}\right)$$

$$\overset{\int dy}{=} \int_{M_4} d^4x \; (2\pi R) \left[-\frac{1}{2} \partial_\mu \Phi_0 \partial^\mu \Phi_0 - \sum_{n=1}^{\infty} \left(\partial_\mu \Phi_n \partial^\mu \Phi_n^* + \frac{n^2}{R^2} \Phi_n \Phi_n^* \right) \right]$$

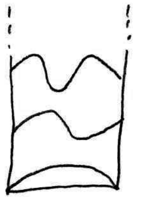
$$\underset{\text{fields}}{\overset{\text{rescale}}{=}} \int_{M_4} d^4x \left[-\frac{1}{2} \left(\partial_\mu \tilde{\Phi}_0 (x^\mu) \right)^2 - \frac{1}{2} \sum_{n=1}^{\infty} \left(|\partial_\mu \tilde{\Phi}_n (x^\mu)|^2 + m_n^2 \, |\tilde{\Phi}_n (x^\mu)|^2 \right) \right]$$

$$= S_{4d} [\tilde{\Phi}_n]$$

$\Rightarrow$ tower of 4d fields $\tilde{\Phi}_n$ w/ masses $m_n^2 = \frac{n^2}{R^2}$, $n \in \mathbb{N}_0$.

Compare: energy levels of periodic solutions in infinite potential well:

$$E_n^2 \sim \frac{n^2}{R^2}$$



n = 2
n = 1
n = 0

At energies $E \ll \frac{1}{R}$, all massive fields $\tilde{\Phi}_1, \tilde{\Phi}_2, \ldots$ are not observable. We only probe $\tilde{\Phi}_0$, the theory seems like an ordinary 4d theory.

If $\Phi(x^\mu, y)$ was massive w/ mass M, the spectrum would be

$$m_n^2 = M^2 + \frac{n^2}{R^2}, \quad n \in \mathbb{N}_0 .$$

What do we make of this? Take $\Phi_0(x^\mu)$ as, e.g., the Higgs. We have not observed heavier scalar fields at $E \sim 10$ TeV.

$$\Rightarrow \quad m_1, m_2, \ldots > 10 \text{ TeV}$$

$$\Leftrightarrow \quad R < \frac{1}{10 \text{TeV}} \sim 10^{-20} \text{ m}$$ in this simple model.

1.2) Gravity

For contractions in both 4d & 5d we implicitely used metrics. Let's consider them in more detail:

- In a curved space, the metric can be in general different at every spacetime point: $g_{\mu\nu} = g_{\mu\nu}(x^\mu)$. $\Rightarrow$ It is a field!
- GR tells us that its dynamics is governed by a very specific Lagrangian: $S_{EH} = \frac{M_P^2}{2}\int d^4x \sqrt{-g}\, R[g]$, $R[g] = g^{\mu\nu}\partial_\rho(g^{\rho\sigma}\partial_\nu g_{\sigma\mu}) + \dots$

It will suffice to consider schematically

$$S[g_{\mu\nu}] = \frac{M_P^2}{2}\int d^4x\, (\partial g)^2 .$$

Here, $M_P^2 = \frac{1}{8\pi G}$ is the Planck mass measuring the strength of gravity.

The same is true for the metric G_{mn} of 5d-spacetime:

$$S[G_{mn}] = \frac{M_5^3}{2}\int d^5x\, (\partial G)^2 , \quad M_5 : \text{5d Planck mass.}$$

For $M_4 \times S^1$ we have again:

$$G_{mn}(x^\mu, y) = \sum_{k\in\mathbb{Z}} G^k_{mn}(x^\mu)\, e^{i\frac{ky}{R}} .$$

$\Rightarrow$ We have a 4d massless field $G^0_{mn}(x^\mu)$.

From a 4d perspective the indices $\{m,n\}$ should be decomposed into $\{\mu,\nu\}$, $\{\mu,4\}$, $\{4,\nu\}$, $\{4,4\}$, i.e.

$$G^0_{mn}(x^\mu) = \left(\begin{array}{c|c} G^0_{\mu\nu} & G_{4\nu} \\ \hline G_{\mu 4} & G_{44} \end{array}\right) =: \left(\begin{array}{c|c} g_{\mu\nu}(x^\mu) & A_\nu(x^\mu) \\ \hline A_\mu(x^\mu) & e^{2\sigma(x^\mu)} \end{array}\right)$$

$\Rightarrow$ 4d massless metric $g_{\mu\nu}(x^\mu)$, vector $A_\mu(x^\mu)$, scalar $\sigma(x^\mu)$.

Interpretation of σ:

(Infinitesimal) lengths are measured in 5d by the (square root of the) line element:

$$ds^2_{5d} = G_{mn}\, dx^m dx^n = g_{\mu\nu}\, dx^\mu dx^\nu + e^{2\sigma} dy^2 + (\text{non-diag.})$$
$$\sim ds^2_{4d} + ds^2_{S^1}$$

$\rightarrow$ ds_{S^1} is the proper measure of lengths along the S^1-direction.

For example, the S^1-circumference is given by

$$\mathrm{Vol}(S^1) = \int_{S^1} ds_{S^1} = \int_{S^1} \sqrt{G_{44}}\, dy = \int_0^{2\pi R} e^\sigma dy = 2\pi R\, e^\sigma$$

$\Rightarrow$ σ rescales volume of S^1.

With some calculation, we arrive at

$$S_{5d}[G] = \frac{M_5^3}{2} \int d^5x\, (\partial G)^2 = \ldots \text{ as before, more involved now} \ldots$$
$$= M_5^3\, \pi R \int d^4x \left[(\partial g)^2 - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

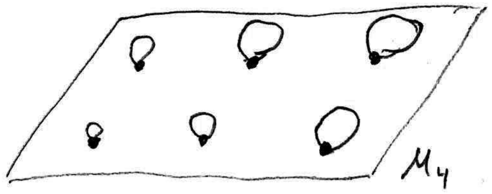
$\Rightarrow$ the 4d massless scalar field $\sigma(x^\mu)$ is dynamical

('modulus' or 'radion' in this special case)

$\rightsquigarrow$ internal spacetime (S^1) is dynamical due to gravity!

By comparison w/ 4d action for $g_{\mu\nu}$, one finds:

$$M_p^2 = 2\pi R\, M_5^3 .$$



$\mathrm{Vol}(S^1)$ depends on x^μ!

The strength of 4d gravity decreases w/ growing volume of extra dimensions

Can have e.g. $M_5 \sim m_{Higgs} \sim 100\ GeV$.

$$M_p^2 \sim R\, M_5^3 \quad \text{requires} \quad R \sim \frac{M_p^2}{M_5^3} \sim \frac{(10^{18}\ GeV)^2}{(100\ GeV)^3} \sim \frac{10^{30}}{GeV} \sim 10^{14}\, m$$

Do same thing in 6 extra dimensions:

$$R \sim \left(\frac{M_p^2}{M_{10}^8}\right)^{1/6} \sim \left(\frac{(10^{18}\ GeV)^2}{(100\ GeV)^8}\right)^{1/6} \sim \frac{10^3}{GeV} \sim 10^{-13}\ m.$$

Summary

Extra dimensions lead to

- tower of states $m_n^2 \sim \frac{n^2}{R^2}$
- moduli $\sigma(x^\mu)$ determining shape and size of extra dimensions
- dilution of gravity: $M_p^2 \sim Vol_d(\text{extra dimensions})\ M_{4+d}^{2+d}$

2) Warped Compactifications

Motivation: Matter backreacts on full geometry.

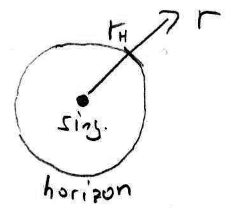
The general ansatz capturing this are warped products.

We look at this in the following example:

2.1) Black Branes

We look at 'Black Hole' solution of Einstein's equations: vacuum with a singularity at $r=0$.

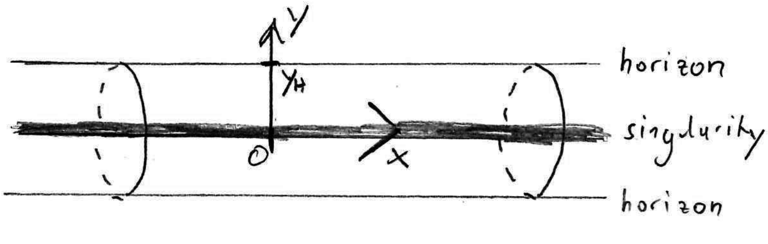
$$ds^2 = -\underbrace{\left(1-\frac{r_H}{r}\right)}_{\substack{\text{time dilatation/} \\ \text{redshift}}} dt^2 + \left(1-\frac{r_H}{r}\right)^{-1} \underbrace{g_{ij}(\vec{x})}_{\substack{\text{metric on} \\ \text{spatial slices of} \\ \text{constant time}}} dx^i dx^j$$



$$= e^{-A(r)} \left(-dt^2\right) + e^{A(r)} \left(g_{ij}(\vec{x})\, dx^i dx^j \right)$$

The 2^{nd} line is just a rewriting with $e^{-A(r)} := 1 - \frac{r_H}{r}$.

... but why only a singular point and not a singular surface?



$p=1$ - dim. sing. surface

'Black String'

Solution:

$$ds^2 = e^{-A(y)} \underbrace{\left(-dt^2 + dx^2\right)}_{\substack{\text{metric along} \\ \text{black string}}} + e^{A(y)} \underbrace{g_{ij}(\vec{y})\, dy^i dy^j}_{\substack{\text{metric orthogonal} \\ \text{to black string}}}$$

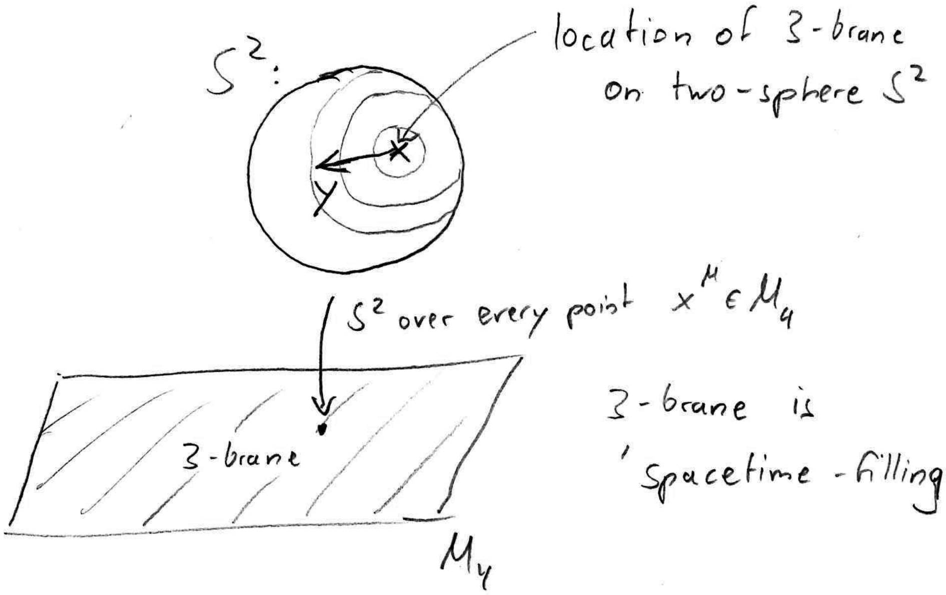
$$y := \sqrt{\Sigma\, y_i^2}, \qquad e^{-A(y)} = \left(1 - \frac{y_H^2}{y^2}\right)$$

Finally: 'Black p-Brane'

$$ds^2 = e^{-A(y)} \underbrace{\left(-dt^2 + d\vec{x}^{\,2}\right)}_{(p+1)\text{-dim. metric}} + e^{A(y)} \underbrace{g_{ij}(\vec{y})\, dy^i dy^j}_{\substack{\text{metric orth. to} \\ \text{black p-brane}}}$$

$$e^{-A(y)} = \left(1 - \left(\frac{y_H}{y}\right)^{p+1}\right).$$

E.g. $p = 3$ on M_4 of $M_4 \times S^2$:



Note the appearence of a y-dep. fct. $e^{-A(y)}$ in front of y-indep. metric of space parallel to p-brane.

$\Rightarrow$ more than a simple product.

Generally, one can show: Any matter that respects 4d Poincaré invariance leads to a solution of Einstein's equations of a 'warped product' form

$$ds^2 = e^{-A(y)} \eta_{\mu\nu}\, dx^\mu dx^\nu + e^{A(y)} g_{ij}\, dy^i dy^j$$

$e^{-A(y)}$: 'warp factor'

2.2) Randall-Sundrum model

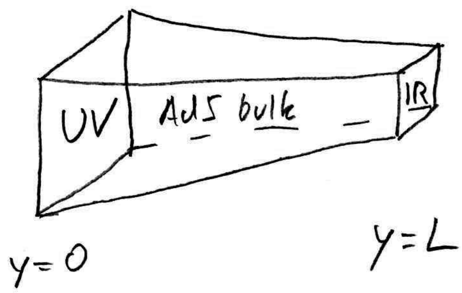
Back to 5d. Now compactify 5th dimension on an intervall. $[0,L]$

General form of metric:

$$ds^2 = e^{-A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + \underbrace{dy^2}_{e^{\tilde{A}(\tilde{y})} d\tilde{y}^2}$$

We add 2 3-branes (of opposite tension) on 4d space at $y=0$ & $y=L$. One finds solution

$$ds^2 = e^{-2y/\ell} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2 \quad , \quad \ell = \frac{M_5^3}{T}$$



5d 'bulk' is AdS w/ $\Lambda = -\frac{T^2}{M_5^3}$

$\pm T$: tension of branes

Can have matter live on the IR brane only:

$$S_{5d} = \underbrace{\frac{M_5^3}{2} \int d^5x \sqrt{-g}\, (\partial g)^2}_{\text{5d gravity}} + \underbrace{\int_{y=L} d^4x \sqrt{-g|_{y=L}}}_{\substack{\text{matter action is}\\ \text{constraint to brane}\\ \text{at } y=L.}} \left(-\frac{1}{2} g^{\mu\nu}|_{y=L} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2\right)$$

insert metric/ metric determinants

$$\overset{\downarrow}{=} \frac{M_5^3}{2} \int d^4x\, dy \sqrt{-\eta}\, e^{-2y/\ell} (\partial \eta)^2$$

$$+ \int_{IR} d^4x \sqrt{-\eta}\, e^{-2L/\ell} \left(-\frac{1}{2} e^{2L/\ell} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2\right)$$

Integrate $\int_0^L dy$; only keep 0-modes of KK-tower.

$$S_{4d,IR} = \frac{M_p^2}{2} \int_{IR} d^4x \sqrt{-\eta}\, (\partial\eta)^2$$

$$+ \int_{IR} d^4x \sqrt{-\eta} \left(-\frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi - \frac{1}{2} m_{IR}^2 \phi^2 \right)$$

We end up w/ 4d theory of gravity and matter with

$$M_p^2 = \int_0^L dy\, e^{-2y/\ell} M_5^3 = \frac{1}{2} M_5^3 \ell \left(1 - e^{-2L/\ell}\right)$$

$$m_{IR}^2 = e^{-2L/\ell} m^2$$

For e.g. $M_5 = M_p = \frac{1}{\ell} = m$, one needs

$L = 37\,\ell$ to achieve

$m_{IR} = m_{Higgs} = 100$ GeV

The UV-brane theory would have

$m_{UV}^2 = m^2$, M_p^2 = as above

$$\Rightarrow \frac{m_{IR}^2}{m_{UV}^2} = e^{-2L/\ell}$$ redshifting of IR quantities

Moving a particle from UV- to IR-brane has same effect as moving a particle towards black hole horizon!

3) Axions

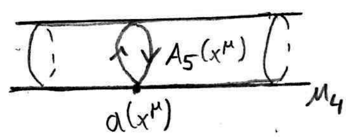
3.1) From Gauge Potentials

Consider a 5d $U(1)$ gauge theory on $M_4 \times S^1$.

As before: $$A_m(x^\mu, y) = \sum_{k\in\mathbb{Z}} A_m^k(x^\mu)\, e^{i\frac{ky}{R}}$$

Consider 0-mode $A_m^0(x^\mu)$ and define $a(x^\mu) \equiv A_5^0(x^\mu)$.

i.e. $$A_m^0(x^\mu) = (\underbrace{A_\mu(x^\mu)}_{\text{4d gauge field}}, \underbrace{a(x^\mu)}_{\text{4d scalar}})$$



In a theory w/out matter, A_m only appears, due to gauge invariance, in the combination $$F_{mn} \supset \partial_m A_n^0 - \partial_n A_m^0 .$$

We again compactify and track the field $a(x^\mu)$:

$$S_{5d} = \int d^4x \int dy \left(-\tfrac{1}{4} F_{mn} F^{mn}\right) = -\tfrac{1}{4}\int d^4x \int dy \,(\partial_m A_n - \partial_n A_m)(\partial^m A^n - \partial^n A^m)$$

collect terms w/ only massless stuff $$\supset -\tfrac{1}{2}\int d^4x \int dy \left[\partial_m A_n^0\, \partial^m A^{0,n} - \partial_m A_n^0\, \partial^n A^{0,m}\right]$$

everything with A_5^0 $$\supset -\tfrac{1}{2}\int d^4x \int dy \left[\partial_\mu a(x^\mu)\, \partial^\mu a(x^\mu) - \partial_\mu a(x^\mu) \underbrace{\partial^5 A^{0,\mu}(x^\mu)}_{=0}\right]$$

$$= -\tfrac{1}{2}\int d^4x \int dy \,(\partial_\mu a(x^\mu))^2$$

$$= \int d^4x \left[-\tfrac{1}{2} f_a^2 (\partial_\mu a)^2\right] \quad \text{w/} \quad f_a^2 = 2\pi R.$$

Since only F_{mn} appears in original action, $a(x^\mu)$ will only ever appear as $\partial_\mu a(x^\mu)$ in the 4d action $\Rightarrow$ shift symmetry!

a is an axion, $a \to a + \text{const.}$, f_a: axion decay constant.

3.2) From p-Form Gauge Potentials

Instead of a vector gauge potential A_m with field strength F_{mn} one can have '2-form' gauge potential B_{mn} with field strength H_{mno}:

	vector = 1-form	2-form
gauge field	$A_m = A_{[m]}$	$B_{mn} = B_{[mn]}$
field strength	$F_{mn} = \partial_{[m} A_{n]}$	$H_{mno} = \partial_{[m} B_{no]}$
gauge transf.	$A_m + \partial_{[m]} \chi$	$B_{mn} + \partial_{[m} \chi_{n]}$
action	$\int d^dx \; F_{mn} F^{mn}$	$\int d^dx \; H_{mno} H^{mno}$

Here, $[\cdot]$ indicates antisymmetrization:

$$F_{mn} = \partial_{[m} A_{n]} = \frac{1}{2!} \left(\partial_m A_n - \partial_n A_m \right)$$

or
$$H_{mno} = \partial_{[m} B_{no]} = \frac{1}{3!} \big(\partial_m B_{no} + \partial_n B_{om} + \partial_o B_{mn} - \partial_m B_{on} - \partial_n B_{mo} - \partial_o B_{nm} \big)$$

(An object w/ just one index is automatically antisymmetrized:
$A_{[m]} = A_m$.)

More general: p-form gauge potentials $C_{m_1 \dots m_p}$:

Gauge transf. $C_{m_1 \dots m_p} \to C_{m_1 \dots m_p} + \partial_{[m_1} \chi_{m_2 \dots m_p]}$

leaves field strength $F_{m_1 \dots m_p m_{p+1}} = \partial_{[m_1} C_{m_2 \dots m_{p+1}]}$ and

therefore action $\int d^dx \; F_{m_1 \dots m_{p+1}} F^{m_1 \dots m_{p+1}}$ invariant.

Note, that we often look at 2-form gauge theories in 4d:

Just like 1-form gauge theory has a dual description

$$\tilde{F}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \qquad w/ \quad \tilde{F}_{\mu\nu} = \partial_{[\mu} \tilde{A}_{\nu]}$$

in terms of dual gauge vector $\tilde{A}_\nu$, one can dualize

$$\partial_\mu a = \varepsilon_{\mu\nu\rho\sigma} \partial^{[\nu} H^{\rho\sigma]}$$

The gauge symmetry of the gauge field $a(x^\mu)$ is then

$$a(x^\mu) \longrightarrow a(x^\mu) + \text{const.}$$

$\Rightarrow$ The electromagnetic dual to a 2-form gauge theory is an axion theory!

One uses the latter description since it is simpler (just a scalar theory).

Back to axions from extra dimensions:

The axion discussed before comes from the internal component of a 1-form gauge field.

It's shift symmetry comes from the fact that it always appears w/ derivatives, since only the field strength appears in 5d.

In e.g. 6d $\rightarrow$ 4d compactifications, a 2-form gauge field B_{mn} can also have a purely internal component

$$b(x^\mu) := B^0_{56}(x^\mu)$$

For the same reason as above, it also enjoys a shift symmetry.

$\Rightarrow$ p-forms in $d>4$ dimensions + complicated internal geometry = plethora of axions!

3.3) Discrete Shift Symmetries

still shift symm.!

Non-pertubative effects (induced, e.g., by couplings $\overbrace{a F \tilde{F}}$) will break the continuous shift symmetry by inducing a potential

$$V(a) = \Lambda^4 \left(1 - \cos\left(a/f_a\right)\right)$$

$$\Lambda = \mu \, e^{-S_{inst}}$$

S_{inst}: instanton action

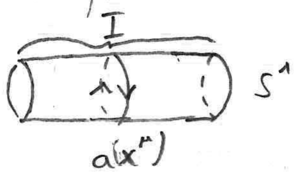
μ: some energy scale

3.4) Monodromies

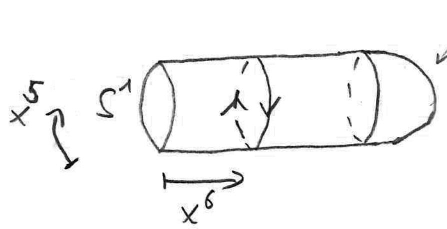
Some effects can induce non-periodic potentials.

Take e.g. $M_4 \times (I \times S^1)$

$\longrightarrow$ axion on S^1.



Now consider 'cigar' instead of $I \times S^1$



S^1 is a boundary of the 'cigar' Σ

We have:

$$a(x^\mu) = \frac{1}{2\pi R} \int_{S^1} dx^5 \, A_5 = \frac{1}{2\pi R} \int_{\partial\Sigma} dx^5 \, A_5 \sim \frac{1}{2\pi R} \int_{\Sigma} dx^5 dx^6 \, \partial_6 A_5$$

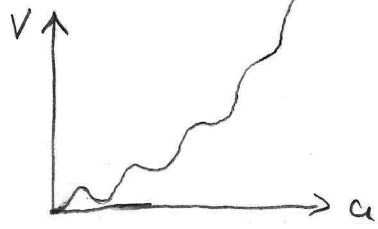
Stokes' theorem

$$\sim \frac{1}{2\pi R} \int_{\Sigma} dx^5 dx^6 \, F_{56}$$

$\Rightarrow$ $a(x^\mu) \neq 0$ induces $F_{56} \neq 0$ on Σ

$$\Rightarrow \quad S_{6d}[A_m] = \int d^6x \left(-\tfrac{1}{4} F_{mn} F^{mn}\right) \supset \int d^4x \; m_a^2 \, a(x^\mu)^2$$

$$\Rightarrow \quad V_{mon.}(a) = \Lambda^4 \left(1 - \cos\left(a/f_a\right)\right) + m_a^2 \, a^2$$



4) Calabi-Yau Compactifications (in type IIB string theory)

String Theory requires 10 dimensions for consistency.

The 10d (vacuum) Einstein equation reads

$$R_{mn}[G_{mn}] = 0. \quad \text{(vanishing 'Ricci-curvature')}$$

$\Rightarrow$ Need six, 'Ricci-flat' extra dimensions to connect string theory with our universe.

These two (+ some other conditions) imply the extra dimensions to be 'Calabi-Yau' 3-manifolds CY_3.

Some properties giving rise to the phenomena discussed:

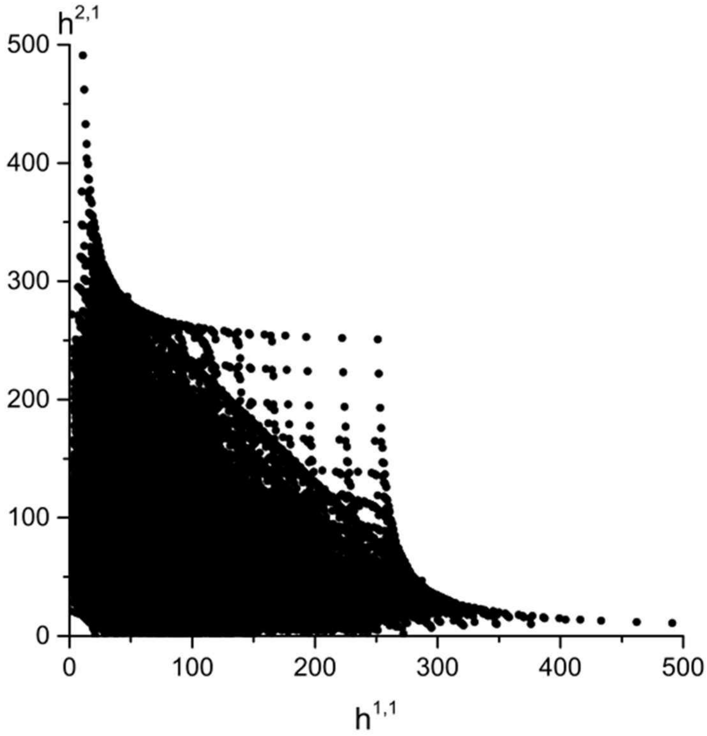
- Characterized by integer numbers $(h_{1,1}, h_{2,1})$, see graph below for a plot of these numbers for a class of CY_3's

  $\longrightarrow$ $\boxed{2h_{1,1} + 1 \text{ axions}}$ arise in a comp. on any of the CY_3's.

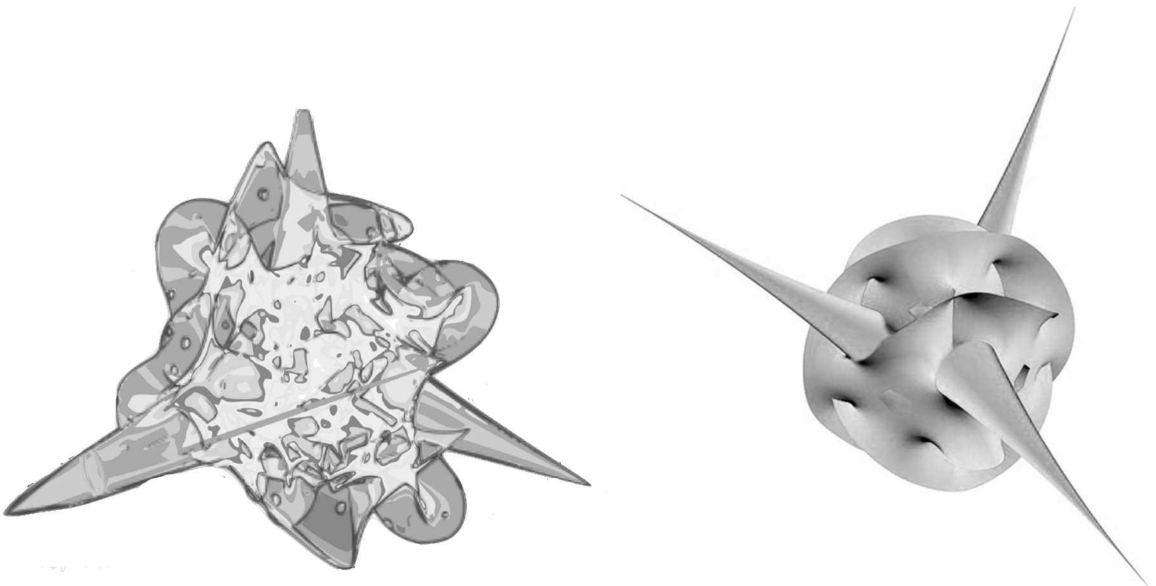
  (This does not include axions whose (discrete) shift symmetry is broken)

- Generically CY_3's have singularities. Once one smoothens these by adding fluxes (branes/matter), this leads to many $\boxed{\text{strongly warped regions,}}$ so-called 'throats', see figure.

  Matter localized in these regions behaves as in the Randall-Sundrum model



The $h_{1,1}$-$h_{2,1}$-plane for CY_3's on the Kreuzer-Skarke list. One finds up to $h_{1,1} \sim 500$.



A sketch of a CY_3 with multiple 'throats', strongly warped regions.