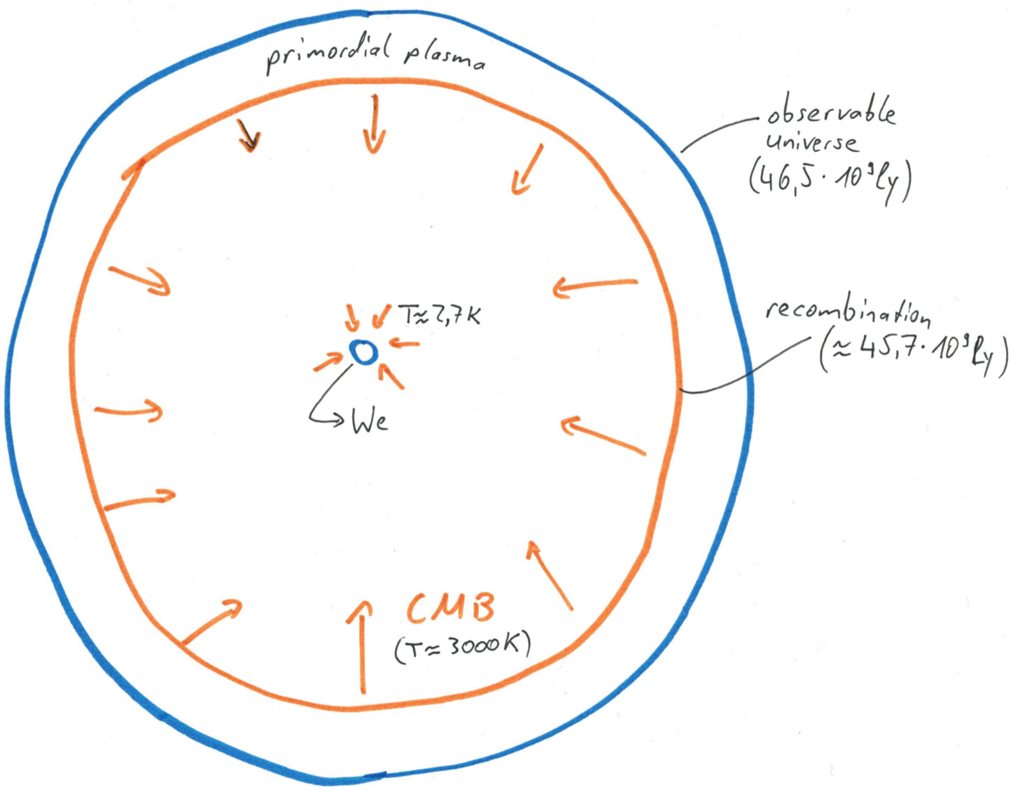
Scalar-tensor theories in cosmology

Student Lecture 2 by Manuel Wittner

- Excursion on Hubble tension: how can Hubble constant be inferred from CMB?



  - CMB almost isotropic but has small anisotropies ($\delta T/T \sim \mathcal{O}(10^{-5})$)
  - Strongest effect are so called baryon acoustic oscillations (BAOs): interplay of gravity and pressure in high density regions leads to oscillations.
  - At recombination, the photons decouple and the anisotropies are "frozen" and define the so called "sound horizon"
  - We can measure the angular distribution of the anisotropies and calculate the angular power spectrum $\rightarrow$ acoustic peak allows us to measure sound horizon during recombination precisely
  - Comparing measurements with theoretical predictions gives the so called angular diameter distance, which depends on the amount of expansion since recombination
    $\Rightarrow$ yields Hubble factor H_0

- There is a large degeneracy of theories with the same background dynamics

  $\Rightarrow$ to distinguish them, we need to look at small perturbations of the otherwise homogeneous quantities:

$$\begin{aligned}\rho &\to \bar{\rho} + \delta\rho \\ p &\to \bar{p} + \delta p \\ \mathrm{d}s^2 &\to -c^2(1+2\Psi)\mathrm{d}t^2 + a(t)^2(1+2\Phi)\mathrm{d}\vec{x}^2,\end{aligned}$$

with $\delta \equiv \delta\rho/\rho, \delta p/p, \Psi, \Phi \ll 1$.

⇒ Leads to perturbed Einstein equations

$$\delta G^\mu_\nu = 8\pi G \delta T^\mu_\nu$$

⇒ In Fourier space, these are given by

$$\begin{aligned}3\mathcal{H}(\Phi' - \mathcal{H}\Psi) + k^2\Phi &= 4\pi G a^2 \delta\rho && \to \quad \text{“Poisson equation”} \\ \Psi &= -\Phi && \to \quad \text{“no anisotropic stress”} \\ k^2(\Phi' - \mathcal{H}\Psi) &= -4\pi G a^2(1+w)\rho\theta && \to \quad \text{“velocity equation”} \\ \Phi'' + 2\mathcal{H}\Phi' - \mathcal{H}\Psi' - (\mathcal{H}^2 + 2\mathcal{H}')\Psi &= -4\pi G a^2 \delta p && \to \quad \text{“pressure equation”}\end{aligned}$$

– Furthermore, conservation equation of EM tensor $\nabla_\mu T^{\mu\nu} = 0$ yields

$$\begin{aligned}\delta' + 3\mathcal{H}(c_s^2 - w)\delta &= -(1+w)(\theta + 3\Phi') && \to \quad \text{“continuity equation”} \\ \theta' + \left[\mathcal{H}(1-3w) + \frac{w'}{1+w}\right]\theta &= k^2\left(\frac{c_s^2}{1+w}\delta + \Psi\right) && \to \quad \text{“Euler equation”}\end{aligned}$$

where $\vec{k}$ is Fourier mode, $\theta \equiv i\vec{k}\cdot\vec{v}$ is velocity divergence, $\delta \equiv \delta\rho/\rho$ is density contrast, $c_s^2 \equiv \delta p/\delta\rho$ is sound speed, $\mathcal{H} \equiv aH$ is conformal Hubble function and $' \equiv \mathrm{d}/\mathrm{d}\tau = a\,\mathrm{d}/\mathrm{d}t$ is the derivative w.r.t. conformal time.

– We could now for example consider small scales $k \gg \mathcal{H}$ and combine these equations to obtain the evolution equation for the density contrast of a pressureless fluid during matter domination:

$$\delta'' + \mathcal{H}\delta' - \frac{4\pi G_N \rho}{a^2}\delta = \delta'' + \mathcal{H}\delta' - \frac{3}{2}\mathcal{H}^2\delta = 0,$$

which is solved by $\delta_1 \sim a$ and $\delta_2 \sim a^{-3/2}$.

Scalar-tensor theories as a modification of GR

- We already mentioned several flaws of standard cosmology and the need to modify the ΛCDM model

  ⇒ One possibility is to modify gravity but that is not so easy: **Lovelock's theorem**: “Einstein tensor + cosmological constant are the only possible field equation of metric tensor and its second order derivatives in 4D.”

- ⇒ To modify general relativity, we must either 1. change number of dimensions or 2. allow higher than second-order derivatives or 3. add degrees of freedom.
  - Scalar-tensor theories do the latter by adding a scalar dof to the gravity sector.

- Most famous example: Quintessence:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{M_P^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \mathcal{L}_{\text{matter}}\right)$$

  - If potential V is flat enough, this resembles a cosmological constant:

$$w_\phi = \frac{p}{\rho} = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V} \to -1 \text{ for } \dot{\phi}^2 \ll V$$

  - However, evolution equation of density contrast becomes

$$\delta'' + \frac{1}{2}(1 - 3w_\phi\Omega_\phi)\delta' - \frac{3}{2}\Omega_m\delta_m = 0,$$

    where now $' \equiv \mathrm{d}/\mathrm{d}N$ is derivative w.r.t. number of e-folds $N \equiv \ln a$.

- Important question: which scalar-tensor theories are we allowed to consider?
  - Ostrogradsky's theorem: "Any system that is described by nondegenerate Lagrangian and has higher than first-order time derivatives has ghost-like instability". (non-degenerate ⇔ $\det(\partial^2 L/\partial\ddot{q}_i\partial\ddot{q}_j) \neq 0$)
  - Simple example:

$$\begin{aligned} L &= \frac{1}{2}\ddot{\phi}^2 - V(\phi) && \Big|\ \psi \equiv \ddot{\phi} \\ &= \psi\ddot{\phi} - \frac{1}{2}\psi^2 - V(\phi) \\ &= -\dot{\psi}\dot{\phi} - \frac{1}{2}\psi^2 - V(\phi) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\psi\dot{\phi}\right) && \Big|\ q \equiv (\phi+\psi)/\sqrt{2},\ Q \equiv (\phi-\psi)/\sqrt{2} \\ &= -\frac{1}{2}\dot{q}^2 + \frac{1}{2}\dot{Q}^2 - U(q,Q) \end{aligned}$$

    ⇒ One of the kinetic terms has wrong sign → gains energy by "climbing up potential"
    ⇒ We must consider stability when we construct a theory!