Student lecture: Neutrino mass & leptogenesis

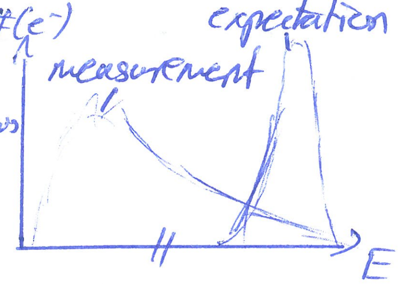
Content: 1 Neutrino mixing and implications of neutrino mass

2 Neutrino mass generation

3 Leptogenesis

1) Neutrino mixing and implications of neutrino mass

→ Historical overview: "ν ↔ anomaly"



- 1930: Pauli's new particle — neutral, bound in nucleus
  β-decay: $n \to p + e^- + \bar{\nu}_e$ → 3-body decay
  ↳ to safe E-conversion (← '29 Bohr)
- 1956: Reines & Cowen – detection of $\bar{\nu}_e$
  via IBD: $\bar{\nu}_e + p \to n + e^+$, $e^+e^- \to 2\gamma$
  $n + {}^{108}Cd \to {}^{109}Cd + \gamma$

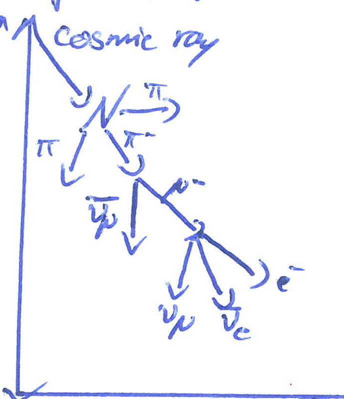
  @ nuclear power plant: $\phi \sim 10^{13} \frac{\bar{\nu}_e}{cm^2 s}$
- 1957: Wu – $\not{P}$ in β-decays
  measure angular distr. of e^- → no preferred direction
  ↳ result: e^- have prefered direction → maximal $\not{P}$!
- 1958: Goldhaber – ν helicity

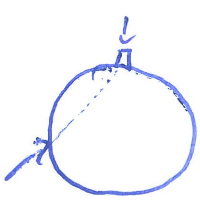
ν_e	LH
$\bar{\nu}_e$	RH

  ↳ (V-A)-theory: Sudarshan, Marschak, Feynman Gell-Mann
  $H = g\, \bar{p}\gamma^\mu(1-\gamma^5)n\, \bar{e}\gamma_\mu(1-\gamma_5)\nu_e$
- 1968: Davis – solar ν-anomaly
  solar ν-flux just 1/3 of expectation

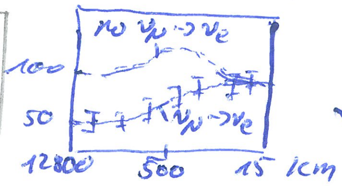
  $\nu_e \to \nu_\mu$
  $\nu_e \to \nu_\tau$

  ↳ Maki, Nakagawa, Sakata: ν's change flavor during propagation
- 1985, 1998 (Super-K): Atmospheric neutrino anomaly
  cosmic rays produce twice as many ν_μ as ν_e, but equal numbers were observed
  ↳ oscillations from ν_μ to ν_e? (L_{osc} > 20km)



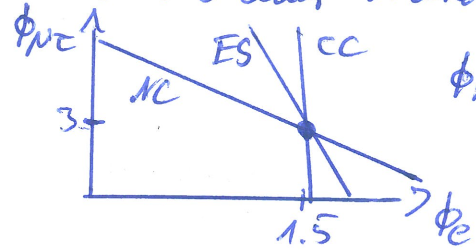


Large enough baseline: upward travelling ν's oscillate



• 2001 (SNO): measurement of solar ν oscillations

↳ solves solar ν-anomaly



ϕ_x: solar ν-flux component (flavor)

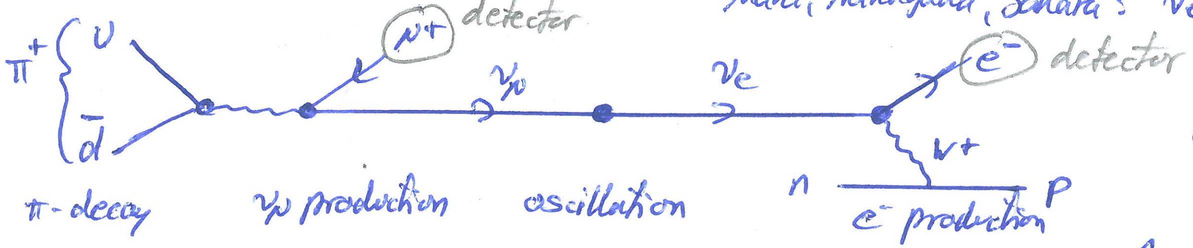
CC: $\nu_e + d \to p + p + e^-$

NC: $\nu_x + d \to \nu_x + p + n$

ES: $\nu_x + e^- \to \nu_x + e^-$

→ **ν flavor oscillations**: Pontecorvo ('57): $\nu_e \to \bar{\nu}_e$

Maki, Nakagawa, Sakata: $\nu_e \leftrightarrow \nu_\mu$



Lepton flavor violation ↯

↳ forbidden in SM

→ BSM physics

Assume: flavor eigenstate $\neq$ mass eigenstate
detection propagation

$$\text{weak interaction}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \underbrace{U}_{\text{unitary trafo}} \cdot \begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}\text{Hamiltonian}$$

⇒ flavor $\overset{(-)}{\nu}_l = \sum_{i=1}^{3} U_{li}^{(*)} \overset{(-)}{\nu}_i$
mass $\overset{(-)}{\nu}_i = \sum_{l=e,\mu,\tau} U_{il}^{(*)} \overset{(-)}{\nu}_l$
} superposition

– $m_i \hat{=}$ mass eigenvalue corresp. to ν_i, $i = 1,2,3$

↳ effective mass: $m_l = \sum_{i=1}^{3} |U_{li}|^2 m_i$ → probability of ν_l having mass m_i

↳ analogue: QM 2-state-system

$$\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

– dynamical states = eigenstates of H = mass eigenstates

naive plane wave solution of $H\nu_i = E\nu_i$

$$\nu_i(t) = \exp(-iE_i t)\,\nu_i(0) \equiv \exp(-iE_i t)\,\nu_i$$

wrong derivation, but correct result

BUT: we can only detect flavor states!

$$\nu_l(t) = \sum_{i=1}^{3} U_{li}^{*} \exp(-iE_i t)\,\nu_i = \sum_{i=1}^{3}\sum_{l'=e,\mu,\tau} U_{li}^{*} \exp(-iE_i t)\, U_{l'i}\, \nu_{l'}$$

$$\equiv \sum_{l'=e,\mu,\tau} A(\nu_l \to \nu_{l'})\,\nu_{l'}$$

→ transition amplitude for flavor change (flavor l @ $t=0$ → flavor l' @ $t>0$)

⟹ probability for flavor transition:

$$P_{ee'} \equiv P(\nu_e \to \nu_{e'}) = |A_{ee'}|^2 = |U_{ei}^* U_{e'i}\, e^{-iE_i t}|^2$$

$$= \sum_{i,j=1}^{3} U_{ei}^* U_{e'i} U_{ej} U_{e'j}^* \exp(-i(E_i - E_j)t)$$

↪ probabilities fulfill unitarity condition $\sum_e P_{ee'} = \sum_{e'} P_{ee'} = 1$

↪ unitarity violation: smoking gun for new physics (ν vertex correction or sterile ν's)

↪ ν's are ultrarelativistic → modifications

① $E_i \simeq p_i$ $\qquad p_i = p_j = p = E$

$$E_i - E_j = \sqrt{p_i^2 + m_i^2} - \sqrt{p_j^2 + m_j^2} \simeq p\left(1 + \frac{m_i^2}{2p^2} - 1 - \frac{m_j^2}{2p^2}\right)$$

$$\overset{!}{=} \frac{m_i^2 - m_j^2}{2E} \equiv \frac{\Delta m_{ij}^2}{2E}$$ → can be negative!?

short baseline ≲ 10 km
intermediate regime 10–100 km
long baseline ≳ 100 km

② $t = L$ (baseline)

define $\Delta_{ij} \equiv \frac{L \Delta m_{ij}^2}{4E}$

$$\Longrightarrow \boxed{P_{ee'} = U_{ei}^* U_{e'i} U_{ej} U_{e'j}^* \exp(-2i\Delta_{ij})}$$

→ oscillation → $\Delta m^2 \neq 0$

→ no sensitivity on absolute mass!

For example: 2-flavor approximation

$$P_{e\mu} = P_{\mu e} = \underset{①}{\sin^2 2\theta}\, \underset{②}{\sin^2}\left(\frac{\Delta m^2}{4E} t\right)$$ ⟶

① amplitude → mixing

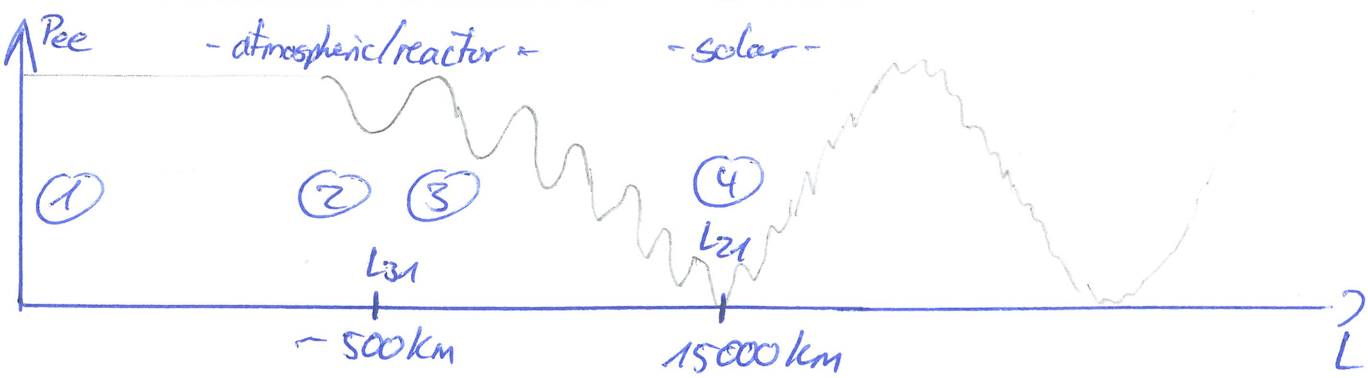
② oscillation frequency → Δm^2

$$P_{ee} = P_{\mu\mu} = 1 - P_{e\mu}$$

$\theta \sim 45°$: maximal mixing

$\theta \sim 0°, 90°$: minimal " → $\nu_e \simeq \nu_i$

→ observations and current status



⟶ we measure two different oscillation pattern → at least two ν's with non-zero mass (→ 2 Δm^2's)

$$U_{PMNS} = \underbrace{\begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Majorana phases}} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric}} \cdot \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{reactor}} \cdot \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar}}$$

Majorana phases: unknown, but irrelevant for oscillations

atmospheric: $\theta_{23} \sim (49 \pm 3)°$ (Super-K '99)

reactor: $\theta_{13} \sim (8.5 \pm 0.2)°$ (Daya Bay, RENO, Double Chooz)

solar: $\theta_{12} \sim (33.6 \pm 0.8)°$ (SNO '02)

– two mass-squared differences:

SNO: $\Delta m_{21}^2 = (7.4 \pm 0.2)\,10^{-5}\,eV^2$ → solar oscillations

Super-K: $|\Delta m_{3\ell}^2| = (2.5x \pm 0.03)\,10^{-3}\,eV^2$ → atmospheric oscillations

↳ sign of $|\Delta m_{3\ell}^2|$ unknown: two different mass orderings possible

NO – $\Delta m_{31}^2 > 0$ 10 – $\Delta m_{32}^2 < 0$

→ octant problem: normal ordering $\theta_{23} < \frac{\pi}{4}$ } shrink experimental
inverted ordering $\theta_{23} > \frac{\pi}{4}$ } uncertainties

question of mass hierarchy should be settled in 2020s!

– leptonic CP phase: still unknown, but measurable with next generation of oscillation experiments, e.g. DUNE, Hyper-K

already indications from global fits: $\delta_{NO} = (215^{+40}_{-29})^\circ$

$\delta_{IO} = (284^{+27}_{-29})^\circ$

→ oscillation length $L_{ij} \equiv \frac{4\pi E}{\Delta m_{ij}^2}$ $\left[\exp(-2i\Delta_{ij}) \to \exp(-2\pi i \frac{L}{L_{osc}})\right]$

e.g. $P_{e\mu} = \sin^2 2\theta \sin^2(\pi \frac{L}{L_{osc}})$

↳ estimate at which baseline one is sensitive to Δm_{ij}^2

($\Delta m_{21}^2 \ll \Delta m_{31}^2$ and $\Delta m_{31}^2 \approx \Delta m_{32}^2$: two lengths relevant L_{21} and L_{31} ($\approx L_{32} \ll L_{21}$))

① $L \ll L_{31}$: no oscillations all $m_{ij} = 0$, $P_{ee'} = \delta_{ee'}$

↳ sensitive to active-sterile mixing with $\Delta m^2 \sim 1\,eV$ → flux anomaly!

② $L \approx L_{31}$: atmospheric oscillations $\Delta m_{21}^2 \approx 0$, $s_{12}^2 \approx 0$

$P_{e\mu} = P_{\mu e} = 0$, $P_{\mu\mu} = 1 - \sin^2 2\theta_{23} \sin^2 \Delta_{32}$

③ $L \approx L_{32}$: reactor anti-$\bar\nu$ $\Delta m_{21}^2 \approx 0$

$P_{\bar e \bar e} = 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{32}$, θ_{13} small!

④ $L \approx L_{21}$: solar oscillations

matter effects (add. potential for $\bar\nu_e$) become large

→ vacuum approximation not valid any more

→ leptonic CP violation: ⟶ related to phases of mixing matrices U_{PMNS}

- recap: CP-trafo   CP: $\nu_L \xrightarrow{CP} \bar{\nu}_R$

$\boxed{\begin{array}{l} \nu_L \xrightarrow{C} \bar{\nu}_L \\ \nu_L \xrightarrow{P} \nu_R \end{array}}$

↳ CP invariance directly affects transition probabilities

$$P(\nu_\alpha \to \nu_\beta) \overset{!}{=} P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

→ test experimentally: $\Delta P_{\alpha\beta} = P_{\alpha\beta} - P_{\bar{\alpha}\bar{\beta}} \overset{!}{=} 0$

- degrees of freedom within U:
  - general complex matrix : $2n^2$
  - unitarity condition : $-n^2$

    $\overline{\qquad n^2 \qquad}$ → re-express:

    mixing angles : $\frac{1}{2}n(n-1)$ $\boxed{\text{rotation via reprs of } SO(n)}$

    phases : $\frac{1}{2}n(n+1)$
  - ν-field redefinition : $-(2n-1)$ → n neutrinos, n anti-neutrinos, leaving one overall phase

⟹ mixing angles : $\frac{1}{2}n(n-1)$ ⟶ three flavor case: 3

phases : $\frac{1}{2}(n-1)(n-2)$ Dirac   1 (2D: 0 → no CP)

$\frac{1}{2}n(n-1)$ Majorana   3

- CP-trafo of $\mathcal{L}$: implications on mixing → $U \xrightarrow{C} U^*$

  ↳ CP conservation only if ① U is real or ② further re-phasing of ν's possible

- Low-energy CP : we see ~~CP~~ in nature, hence expect ~~CP~~ also in lepton sector → U_{PMNS} complex and containing at least Dirac phase δ

  ↳ quantify ~~CP~~ : Jarlskog invariant

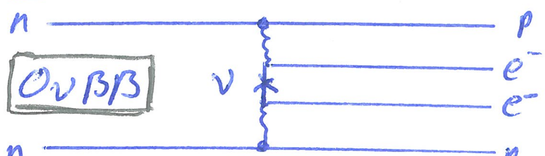
  $\Delta P_{\alpha\beta} \propto J \propto \mathrm{Im}(UUUU)$ → CP cons. if $\theta_i = 0, \frac{\pi}{2}$ or δ, π

  ↳ optimal set-up: $(\nu_e \to \nu_\mu)$-oscillation

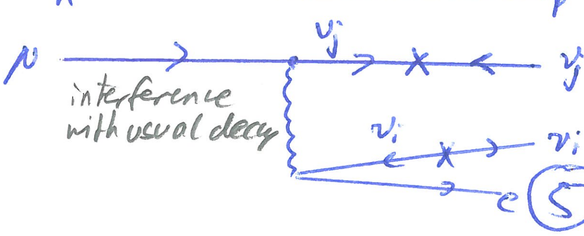
  BUT matter effects mimic low-E ~~CP~~ → since prop. to E_ν, constant for fixed L/E

  ⟶ large baseline : $\frac{L}{E} > 1000 \frac{\mathrm{km}}{E}$

- ~~CP~~ from Majorana phases:



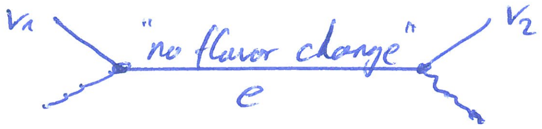
  - generally expected in processes involving Majorana mass contribution but highly suppressed



  - high-energy ~~CP~~ → leptogenesis

→ charged lepton oscillations*:

- flavor of charged lepton defined by its mass → no flavor change
  <u>BUT</u>: oscillations analogous to ν's possible



<u>ToDo</u>: measure m_1 and m_2 very precisely

→ shortest oscillation length: $L_{\mu e} = \frac{4\pi E}{m_\mu^2 - m_e^2} \approx 2\cdot 10^{-11} \frac{E}{\text{GeV}}\,\text{cm}$

for $L_{\mu e} \sim 1\text{m}$: $E \sim 10^{12}\,\text{GeV}$

→ <u>useful approximation for V_{PMNS}*</u>: tri-bimaximal mixing

↳ neglect θ_{13}, since it is small

solar · reactors, nonzero but small

$$V = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

with $\theta_{12} = \frac{\pi}{6}$, $\theta_{23} = \frac{\pi}{4}$, $\theta_{13} \approx 0$

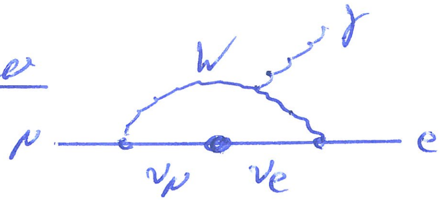
→ atmospheric oscillations

unitarity · unitarity · missing elements from orthogonality of ν-states

↳ <u>important</u>: no $\cancel{CP}$, since δ and $\sin\theta_{13}$ are always grouped together!

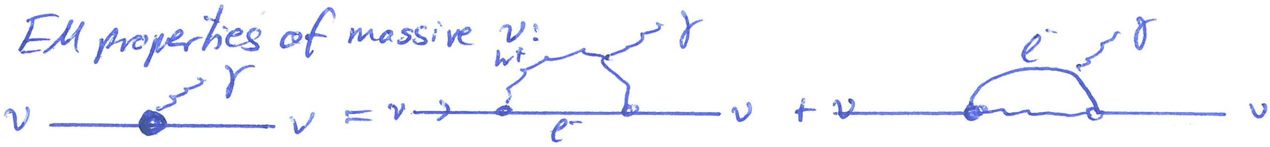
↳ theoretically interesting since structure can be motivated by group theory → discrete groups

→ <u>Appetiser: plethora of ν-mass pheno</u>

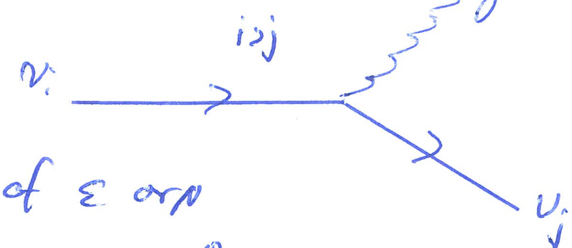


- lepton flavor violation (LFV):
  - new decays possible: $\mu \to e\gamma$, $\mu \to 3e$
  - strict bounds: $BR(\mu \to e\gamma) \propto \left(\frac{m_i}{m_W}\right)^4 \lesssim 10^{-50}$ for $m_i \sim O(1)\,\text{eV}$

- EM properties of massive ν:



  - charge radius, magnetic moment, el. dipole moment, anapole moment → m_i
  - very small, but interesting pheno



- ν-decay:
  - non-degenerate ν-mass + existence of ε or μ
  - lifetime $\tau = \frac{1}{\Gamma} = \frac{10^{43}\,\text{s}}{\left(\frac{m_i}{\text{eV}}\right)^5} \gg 10^{17}\,\text{s} = \tau_{\text{Universe}}$