# 2) Neutrino mass generation

→ various observables on the market:

- β-decay : $m_{\nu_e} = \sqrt{|U_{ei}|^2 m_i^2} < 2\,\text{eV}$ (KATRIN: $m_{\nu_e} \leq 0.2\,\text{eV}$)
- cosmology : $\sum_i m_i \leq 0.23\,\text{eV}$ (Planck)
- $0\nu\beta\beta$ : $m_{ee} = |U_{ei}^2 m_i| \leq 0.3\,\text{eV}$ (Legend

↳ oscillations indicate $\sum_i m_i \gtrsim 0.05\,\text{eV}$

→ neutrino mass models : options
→ extra fields: $\nu_R$
→ gauge group extentions : LR symmetry
→ new concepts : SUSY, xDim, strings

① Dirac mass term : "business as usual"

↳ add RH neutrino $\nu_R \sim (1,0)$ : $\nu = \nu_L + \nu_R$ (Dirac fermion)

(1F) $$-\mathcal{L}_Y \supset Y^e \bar{L} H e_R + Y^\nu \bar{L}_L \tilde{H} \nu_R + h.c. \qquad \tilde{H} = i\sigma_2 H^*$$

$$\xrightarrow{SSB} \frac{v Y^\nu}{\sqrt{2}} \bar{\nu}_L \nu_R + h.c \qquad = \ldots m_\nu \bar{\nu} \nu$$

↳ works well, but light neutrino mass remains unexplained: $Y^\nu$ a factor $\sim 10^{12}$ smaller than usual Yukawas

(3F) $$-\mathcal{L}_Y \supset Y^e_{\alpha\beta} (\bar{L})_\alpha H (e_R)_\beta + Y^\nu_{\alpha\beta} (\bar{L})_\alpha \tilde{H} (\nu_R)_\beta + h.c$$

$$\xrightarrow{SSB} \frac{v}{\sqrt{2}} Y^e_{\alpha\beta} (\bar{e}_L)_\alpha (e_R)_\beta + \frac{v}{\sqrt{2}} Y^\nu_{\alpha\beta} (\bar{\nu}_L)_\alpha (\nu_R)_\beta + h.c$$

↳ bi-unitary trafo : $l'_{L/R} = V^{l\dagger}_{L/R} l_{L/R}$ , $\nu'_{L/R} = V^{\nu\dagger}_{L/R} \nu_{L/R}$

$$\rightarrow \frac{v}{\sqrt{2}} \Big[ \underbrace{(V_L^{e\dagger} Y^e V_R)_{\alpha\beta}}_{\equiv \tilde{Y}^l_{\alpha\beta}} (\bar{e}'_L)_\alpha (e'_R)_\beta + \underbrace{(V_L^{\nu\dagger} Y^\nu V_R^\nu)_{\alpha\beta}}_{\equiv \tilde{Y}^\nu_{\alpha\beta}} (\bar{\nu}'_L)_\alpha (\nu'_R)_\beta$$

↳ $\nu$-mixing is related to $V_Y^\wedge$ matrices

charged current: $$\mathcal{L}_{CC} = -\frac{g_2}{\sqrt{2}} j^\mu W_\mu + h.c$$

with $$j^\mu = \bar{\nu}_\alpha \gamma^\mu (1-\gamma_5) l_\alpha = 2 \bar{\nu}_L \gamma^\mu l_L$$

$$\rightarrow 2 \bar{\nu}'_L V_L^{\nu\dagger} V_L^e \gamma^\mu l_L \equiv 2 \bar{\nu}'_L U^\dagger \gamma^\mu l_L$$

with PMNS matrix $U^\dagger = V_L^{\nu\dagger} V_L^e$ | for charged leptons : flavor $\hat{=}$ mass (basis)

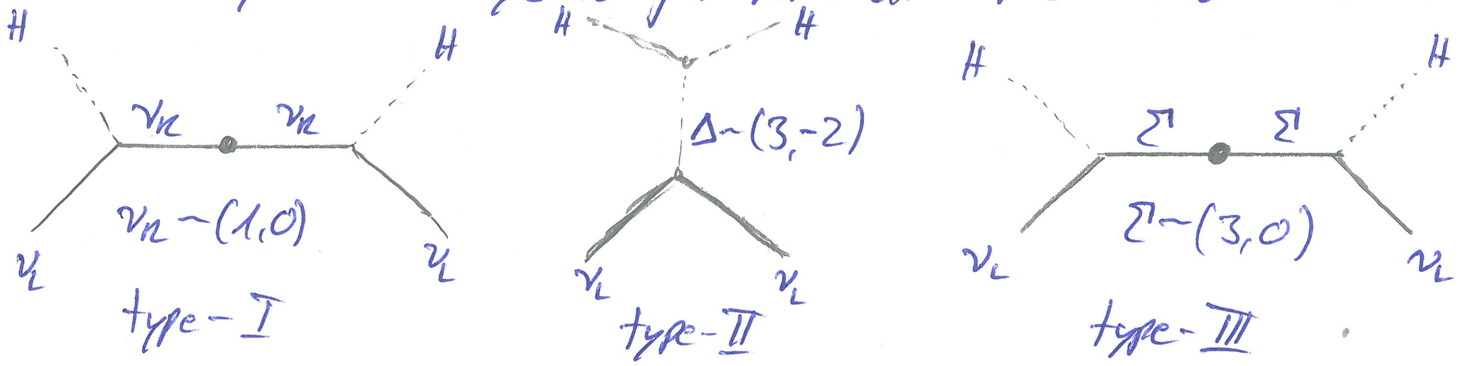
$U^\dagger = V_L^{\nu\dagger}$ | $V^l_{L/R} = \mathbb{1}$

② Majorana mass term : Majorana fermion $\nu = \nu^c$
(only one chiral component $\nu = \nu_L + \nu_L^c$)
↳ no new field, but $\nu$ as Majorana particle :
interesting pheno : $\nu = \bar{\nu}$, L violation

Majorana mass term : $-\frac{1}{2} m \, \overline{\nu_L} \nu_L^c + h.c$
↳ direct inclusion forbidden by $SU(2)_L$ gauge invariance
↳ different origin : generated by integrating out higher scales → EFT

→ Weinberg operator : $O_5^W = \frac{f}{\Lambda} (L^T (i\sigma_2) H) C^+ (H^T (i\sigma_2) L)$
↳ NP scale ($\Lambda$)

→ only three ways to generate at tree-level : seesaws




→ type-I seesaw mechanism :
↳ include RH component $\nu_R$ and assume Majorana neutrinos :
- RH Majorana mass term does not break $SU(2)_L$ : $-\frac{1}{2} \overline{\nu_R^c} M_R \nu_R + h.c$
- Dirac mass contribution from Higgs : $\bar{\nu}_L m_D \nu_R + h.c$, $m_D = \frac{Y^\nu}{\sqrt{2}} v$

→ interplay between two scales : $\Lambda_{EW} \sim v \sim O(100)$ GeV
$\Lambda_{\nu_R} \sim M_R \sim$ much higher

- Dirac mass term in detail :

$$\bar{\nu}_L m_D \nu_R = \frac{1}{2} ( \bar{\nu}_L m_D \nu_R + (\bar{\nu}_L m_D \nu_R)^T )$$
$$= \frac{1}{2} \bar{\nu}_L m_D \nu_R + \frac{1}{2} \overline{\nu_R^c} m_D^T \nu_L^c$$

with $\overline{\psi^c} = \psi^T C$, $\psi^c = \gamma_0 C \psi^*$

$C^+ = C^{-1} = C^T = -C$

- Combine all mass terms in one matrix : $s$ - #($\nu_R$'s)
$m_D$ : $(3 \times s)$, $M_R$ : $(s \times s)$, $0$ : $(3 \times 3)$

$$-\mathcal{L}_I = \frac{1}{2}\,\overline{\nu_L}\, m_D\, \nu_R + \frac{1}{2}\,\overline{\nu_R^c}\, m_D^T\, \nu_L^c + \frac{1}{2}\,\overline{\nu_R^c}\, M\, \nu_R + h.c.$$

$$= \frac{1}{2}\,\underbrace{(\overline{\nu_L}, \overline{\nu_R^c})}_{\overline{n_R^c}} \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + h.c.$$

$$= \frac{1}{2}\,\overbrace{n_R^T\, C^{-1}}\, \underset{(3+s)\times(3+s)}{\mathcal{M}}\, n_R + h.c. \qquad \text{with } n_R = \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} \begin{matrix} \}3 \\ \}s \end{matrix}$$

↳ to get mass matrices of LH and RH neutrinos, use unitary trafo for block-diagonalization: $U^\dagger U = U U^\dagger = \mathbb{1}$

assume: $U = \begin{pmatrix} A & D^\dagger \\ -C & B^\dagger \end{pmatrix} \longrightarrow$ ① $A^\dagger A + CC^\dagger = DD^\dagger + BB^\dagger = AA^\dagger + DD^\dagger = CC^\dagger + B^\dagger B = \mathbb{1}$
② $DA - BC = -CA^\dagger + B^\dagger D = 0$

→ off-diagonal ⟷ active-sterile mixing → small
↳ neglect higher order terms

$$U^T \mathcal{M}\, U \overset{!}{=} \begin{pmatrix} m_\nu & 0 \\ 0 & M_N \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} -A^T m_D C - C^T m_D^T A + \cancel{C^T M_R C} & A^T m_D B^\dagger - \cancel{C^T m_D^T D^\dagger} - C^T M_R B^\dagger \\ \cancel{-D^* m_D C} + B^* m_D^T A - B^* M_R C & D^* m_D B^\dagger + B^* m_D^T D^\dagger + B^* M_R B^\dagger \end{pmatrix}$$

off-diagonal: $C = M_R^{-1} m_D^T A = (M_R^{-1})^T m_D^T A$ (since Majorana mass matrix is symmetric: $m_M = m_M^T$)

with ② we get $D = B M_R^{-1} m_D^T$

$$\Rightarrow \begin{pmatrix} -A^T m_D M_R^{-1} m_D^T A & 0 \\ 0 & B^*(M_R + M_R^{-1} m_D^\dagger m_D + m_D^T m_D^*(M_R^{-1})^\dagger) B^\dagger \end{pmatrix} \overset{!}{=} \begin{pmatrix} m_\nu & 0 \\ 0 & m_N \end{pmatrix}$$

↳ $\Lambda_{EW} \ll \Lambda_R$, hence $M_R \gg m_D^2 M_R^{-1}$:

$$\Rightarrow \begin{pmatrix} -A^T m_D M_R^{-1} m_D^T A & 0 \\ 0 & B^* M_R B^\dagger \end{pmatrix}$$

procedure works for arbitrary matrices $A, B \to A, B \,\hat{=}\, \mathbb{1}$

$$\hat{=} \begin{pmatrix} -m_D M_R^{-1} m_D^T & 0 \\ 0 & M_R \end{pmatrix}$$ with eigenstates $\nu_{light} \approx \nu_L$, $\nu_{heavy} \approx \nu_R$

$m_N \approx M_R$

⇒ type-I seesaw formula: $\boxed{m_\nu = -m_D M_R^{-1} m_D^T}$


example: 1 generation

$$m_\nu \approx \frac{m_D^2}{M_R} \quad \text{with} \quad m_D \sim 100\,\text{GeV},\ M_R \sim 10^{16}\,\text{GeV} \qquad \underline{m_\nu \sim \mathcal{O}(10^{-3})\,\text{eV}}$$

↳ active-sterile mixing: $U = \begin{pmatrix} 1 & m_D^* M_R^{-1*} \\ -M_R^{-1} m_D^T & 1 \end{pmatrix}$

$$P(\nu_A \to \nu_S) \sim |U_{AS}|^2 \sim \frac{m_D^2}{M_R^2} \sim \frac{m_\nu}{M_N} \sim \frac{m_\nu}{0.1\,\text{eV}} \cdot \frac{M_N}{1\,\text{eV}} \cdot 10^{-13}$$

→ suppressed by very heavy $M_R$!

## → type-II seesaw mechanism:

↳ assume existence of a scalar $SU(2)_L$ triplet $\Delta = (\Delta_1, \Delta_2, \Delta_3)$

- generate light neutrino mass through symmetry breaking $m_\nu \sim \langle \Delta \rangle$
- $\Lambda_{EW}$ affects neutrino mass indirectly

$$\mathcal{L}_{\mathrm{II}} = \mathrm{tr}\left[(D_\mu \Delta)^\dagger (D^\mu \Delta)\right] - m_\Delta^2 \,\mathrm{tr}\left[\Delta^\dagger \Delta\right] + Y^\nu L_L^T C (i\sigma_2) \Delta L_L$$

$$+ \mu_\Delta H^T (i\sigma_2) \Delta^\dagger H \quad + h.c. + \dots \quad ; \quad [\mu_\Delta] = 1$$

with the scalar triplet in fund. $SU(2)$ repr. $\Delta = \frac{\sigma_i \Delta_i}{\sqrt{2}} = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}$

↳ finding VEV of $\Delta$ / generating neutrino mass

$$V(H,\Delta) = -m_H^2 H^\dagger H + \frac{\lambda_H}{4}(H^\dagger H)^2 + m_\Delta^2\, \mathrm{tr}\left[\Delta^\dagger \Delta\right] - \mu_\Delta H^T (i\sigma_2) \Delta^\dagger H + h.c.$$

$$V\left(\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \Delta\right) = -\frac{1}{2} m_H^2 v^2 + \frac{\lambda_H v^4}{16} + m_\Delta^2\, \mathrm{tr}(\Delta^\dagger \Delta) - \mu_\Delta \left(0, \tfrac{v}{\sqrt{2}}\right) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Delta^\dagger \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} + h.c.$$

$$= m_\Delta^2 \left( |\Delta^{++}|^2 + |\Delta^+|^2 + |\Delta^0|^2 \right) - \frac{1}{2} v^2 \mu_\Delta \Delta^{0*} - \frac{1}{2} v^2 \mu_\Delta \Delta^0 + \text{constants}$$

$$\frac{\partial V}{\partial \Delta^{0*}} \overset{!}{=} 0 \longrightarrow \langle \Delta^0 \rangle \equiv v' = \mu_\Delta \frac{v^2}{2 m_\Delta^2}$$

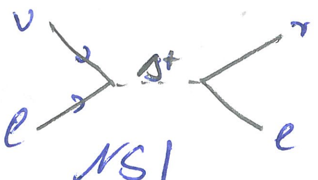
$$\Rightarrow Y^\nu L_L^T C (i\sigma_2) \Delta L_L \xrightarrow{\langle \Delta^0 \rangle} \frac{1}{2} Y^\nu \frac{\mu_\Delta v^2}{m_\Delta^2} \overline{\nu_L^c} \nu_L \equiv \frac{1}{2} m_{\mathrm{II}} \overline{\nu_L^c} \nu_L$$

$\Rightarrow$ type-II seesaw formula: $\boxed{m_{\mathrm{II}} = Y^\nu \frac{\mu_\Delta v^2}{m_\Delta^2}}$

three options for small $m_\nu$:
- small Yukawa → our intention is to avoid this
- small $\mu_\Delta$ → t'Hooft natural ($\mathcal{L} - \mu_\Delta$, $\mu_\Delta \to 0$: more symmetry
- heavy $\Delta$

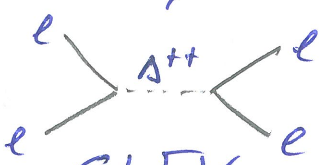
↳ further pheno:

interactions [redacted] induced by $L_L^T C (i\sigma_2) \Delta L_L$



NSI

$M_\Delta \gtrsim 750$ GeV



CLFV

e.g. $\mathcal{L}_{NSI} = \frac{Y Y^\dagger}{M_\Delta^2} (\bar{\nu}_L \gamma^\mu \nu_L)(\bar{e} \gamma_\mu e)$

↓

NSI bounds to type-II bounds

↳ EW precision:

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta} = 1.\dots \pm 0.\dots \quad (SM = 1)$$

→ sensitive to contributions to gauge boson mass $(D_\mu \Delta)^\dagger (D^\mu \Delta) \to \dots W_\mu^+ W^{\mu -} + \dots Z_\mu Z^\mu$

$\Rightarrow v' \leq 4.3$ GeV (natural for light neutrino mass)

## → type-III seesaw mechanism*

↳ adding a neutral, fermionic $SU(2)_L$ triplet $\Sigma = (\Sigma^1, \Sigma^2, \Sigma^3) \sim (3,0)$

$$\mathcal{L}_{III} = \mathrm{tr}[\bar{\Sigma} i \not{D} \Sigma] - \frac{1}{2}\mathrm{tr}[\bar{\Sigma} M_\Sigma \Sigma^c + \bar{\Sigma}^c M_\Sigma^* \Sigma]$$
$$- H^+ \bar{\Sigma} \sqrt{2}\, Y_\Sigma L + h.c. \quad \text{with } \Sigma = \frac{\sigma_i}{\sqrt{2}} \Sigma_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -\Sigma^0 \end{pmatrix}$$

↳ neutrino mass generation similar to type-I case:

replace $M_R \to M_\Sigma$

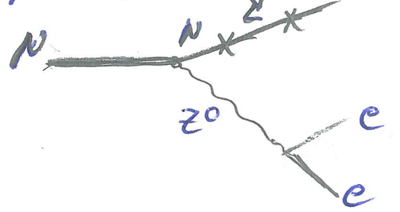
$$m_{III} = \frac{Y_\Sigma^2 v^2}{M_\Sigma}$$

$\boxed{\text{type-III seesaw formula} = \text{type-I seesaw formula}}$

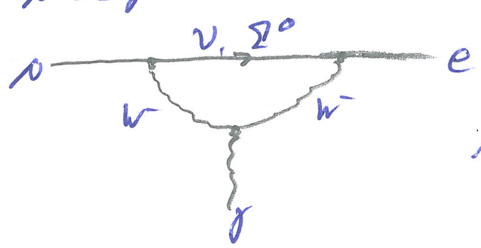
↳ in contrast to type-I case: more signatures

- kinetically coupled to gauge bosons, like type-II → direct production
- Yukawa interaction induces $(\Sigma^\pm - e^\pm)$-mixing and $(\Sigma^0 - \nu)$-mixing: mixing $\sim \mathcal{O}\left(\frac{Y_\Sigma v}{M_\Sigma}\right)$

$\mu \to eee$




$\mu \to e\gamma$




$M_\Sigma > 200$ TeV

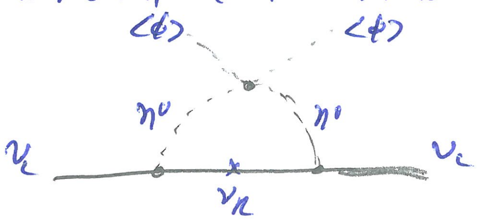
## → Radiative neutrino mass models*

↳ realize Weinberg operator @ 1-loop level → natural suppression by loop factors

### Inert doublet model / Scotogenic model

↳ 3 $\nu_R$'s + scalar doublet $\begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix}$

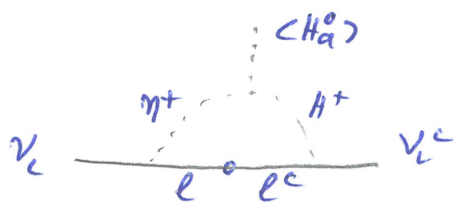
↳ interaction: $(\nu\eta^0 - \ell\eta^+)\nu_R$




### Zee model

↳ add scalar doublet $(H_a)$ + one singlet charged scalar $(\eta^+)$

↳ $\mathcal{L} = f L L \eta^+ + \bar{L}(Y^{SM} H + Y' H_a) e + \mu H^T H_a \eta^+ + h.c.$




↳ also models @ higher loop level: Zee-Babu model (2-loop)

→ Summary: "Something must be small or large"!

1) Dirac neutrino mass: $m_\nu = Y \frac{v}{\sqrt{2}}$ → small Yukawa

2) type-I/III seesaw: $m_\nu = \frac{Y^2 v^2}{M_{N/\Sigma}}$ → heavy sterile neutrino/fermionic triplet
(Majorana mass)

3) type-II seesaw: $m_\nu = Y' v' = Y \frac{\mu_\Delta v^2}{m_\Delta^2}$ → small VEV
(Majorana mass)

↳ seesaw model = Weinberg operator @ tree level

→ zillions of neutrino mass models on the market, but general principles easy to grasp