Exercise 2:

I)
Determine $\pi/4$ in two different ways using Monte Carlo:

a)

$$f(x) = \sqrt{1-x^2} \quad x \in [0,1) \tag{1}$$

Compute the integral

$$I = \int_0^1 \mathrm{d}x \; f(x) \tag{2}$$

using

$$I \approx \frac{1}{N} \sum_{i=1}^{N} f(x_i) \tag{3}$$

where the random numbers x_i are uniformly in the interval $[0,1)$. Chose N such that the computation takes about 5 min on the computer. Determine the statistical error.

b) Consider the square $[0,1) \times [0,1)$:
Define the function $f(x_1, x_2)$ as

$$\begin{aligned} f(x_1,x_2) &= 1 \quad \text{if} \quad x_1^2 + x_2^2 < 1 \\ f(x_1,x_2) &= 0 \quad \text{else} \end{aligned}$$

Compute the integral

$$I = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \; f(x_1,x_2) \tag{4}$$

Generate a 2 component random number that is uniformly distributed in $[0,1) \times [0,1)$ using a pair (x_1, x_2) of two random numbers. Choose N as above. Determine the statistical error.

II)
Compute the integral

$$I = \int_0^\infty \mathrm{d}x \; \exp(-x)x \tag{5}$$

using Monte Carlo integration. Choose $w(x) = \exp(-x)$ as distribution of the random number. Determone the statistical error.