6 MSSM – phenomenology

6.1 Electroweak symm. breaking

- From $\mathcal{L}_{SUSY-SM} + \mathcal{L}_{soft}$, we can work out the scalar potential for the Higgs scalars $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$; $H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$.

  (The indices refer to the electric charge, which is different for the lower & upper components of an SU_2-doublet, in analogy to the lepton doublet.)

- The potential reads

$$
\begin{aligned}
V = {} & (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\
& + b\,(H_u^+ H_d^- - H_u^0 H_d^0) + \text{h.c.} \\
& + \frac{1}{8}(g_1^2 + g_2^2)\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2 \\
& + \frac{1}{2} g_2^2 \, |H_u^+ \overline{H_d^0} - H_u^0 \overline{H_d^-}|^2
\end{aligned}
$$

- Here the last two lines come from the D-term potential induced by the U_1 & SU_2 gauge theories.
- The terms $\sim |\mu|^2$ come from the F-term potential of the chiral SF model
- The remaining terms come from ~~SUSY~~.

Comment: The full scalar potential of the MSSM involves squarks and sleptons. If they acquire non-zero VEVs, we are dealing with charge- and/or color-breaking vacua, which are phenomenologically unacceptable. The absence of such vacua puts certain constraints on the ~~SUSY~~ parameters, which we assume to be fulfilled. We also assume that the masses of squarks & sleptons are large enough so that we can ignore their effect on the Higgs potential (i.e. they are integrated out by simply setting them to zero).

- In analogy to our analysis of the SM Higgs sector, we use SU_2 – symmetry to set $H_u^+ = 0$ in the vacuum.

- $$\frac{\partial V}{\partial H_u^+} = (\mu^2 + m_{H_u}^2)\bar{H}_u^+ + b H_d^- + \frac{1}{8}(g_1^2 + g_2^2) \cdot 2 \bar{H}_u^+ \cdot$$
$$\cdot \left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2 \right) + \frac{1}{2} g_2^2 \bar{H}_d^0 \left(\bar{H}_u^+ H_d^0 + \bar{H}_u^0 H_d^- \right)$$

- $$\left.\frac{\partial V}{\partial H_u^+}\right|_{H_u^+ = 0} = b H_d^- + \frac{1}{2} g_2^2 \bar{H}_d^0 \bar{H}_u^0 H_d^-$$

$\Rightarrow$ There is an extremum at $H_d^- = 0$.
(Analogously, one can show that also $\partial V / \partial H_d^-$ is zero at this point.)

- Thus, we now set $H_u^+ = H_d^- = 0$ and consider the potential as a fct. of H_u^0 & H_d^0:

$$V = m_2^2 |H_u^0|^2 + m_1^2 |H_d^0|^2 - b H_u^0 H_d^0 + \text{h.c.} + \frac{1}{8}(g_1^2 + g_2^2)\left(|H_u^0|^2 - |H_d^0|^2 \right)^2$$

$$m_2^2 \equiv |\mu|^2 + m_{H_u}^2$$
$$m_1^2 \equiv |\mu|^2 + m_{H_d}^2$$

- Without loss of generality, we can assume b to be real and positive by a phase redefinition of the Higgs doublets.

- Then, with respect to the phases of H_u^0 & H_d^0, the term $-b H_u^0 H_d^0$ is minimal if the phases are opposite.

- Furthermore, using a $U_{1,Y}$ gauge trf. $H_u^0 \to e^{i\alpha} H_u^0$; $H_d^0 \to e^{-i\alpha} H_d^0$, we can set the phase to zero.

  ($\Rightarrow$ CP can not be spontaneously broken since all VEVs can be chosen to be real in the vacuum.)

- Now focus on the potential near $H_u^0 = H_d^0 = 0$ (neglecting the quartic part):

$$V \approx m_2^2 (H_u^0)^2 + m_1^2 (H_d^0)^2 - 2b H_u^0 H_d^0$$

(H_u^0, H_d^0 are now real).

- This corresponds to a mass matrix

$$M^2 = \begin{pmatrix} m_2^2 & -b \\ -b & m_1^2 \end{pmatrix}.$$

- Its eigenvalues are the solutions of

$$\det(M^2 - \lambda \mathbb{1}) = 0$$

or $$(m_1^2 - \lambda)(m_2^2 - \lambda) - b^2 = 0$$

$$\lambda_{1,2} = \frac{m_1^2 + m_2^2}{2} \pm \sqrt{\frac{(m_1^2 + m_2^2)^2}{4} + b^2}$$

(m_1^2, m_2^2 are real!).

- Unless $b^2 > m_1^2 \cdot m_2^2$, both eigenvalues are positive and we find a minimum at $H_u^0 = H_d^0 = 0$ (i.e. no electroweak symm. breaking).

- For $b^2 > m_1^2 m_2^2$, the extremum at the origin has an unstable direction, along which a minimum occurs (at the point where the quartic term starts to dominate).

- However, there is a direction in field space along which the quartic part (D-term potential) is identically zero. This "D-flat direction" arises for $H_u^0 = H_d^0$. Along this direction the potential reads

$$V = (m_1^2 + m_2^2 - 2b)(H_u^0)^2 .$$

Thus, the potential is unbounded from below unless

$$m_1^2 + m_2^2 > 2b .$$

- We have thus derived (necessary & sufficient) <u>conditions for electroweak symm. breaking</u>:

$$\left\| \begin{array}{l} b^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2) \\ 2b < (|\mu|^2 + m_{H_u}^2) + (|\mu|^2 + m_{H_d}^2) \end{array} \right\|$$

(Note that $m_{H_u}^2$, $m_{H_d}^2$ are just names, i.e., there is no reason for these quantities to be positive. The first condition is easily realized if m_1^2 or m_2^2 is negative, but this is not necessary.)

- It is customary to call $\langle H_u^0 \rangle \equiv v_u$; $\langle H_d^0 \rangle \equiv v_d$.

- Calculating the resulting Z-boson-mass (in analogy to the SM-case), one finds:

$$v_u^2 + v_d^2 = 2 m_Z^2 / (g_1^2 + g_2^2) .$$

- It is customary to introduce the angle β defined by

$$\tan\beta \equiv v_u / v_d .$$

- Thus, instead of v_u & v_d, we can use m_Z and $\tan\beta$ to characterize minimum of the potential.

- The relations $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$ relate these quantities to the lagrangian parameters m_1^2, m_2^2, b:

$$m_2^2 = b\cot\beta + \frac{m_Z^2}{2}\cos 2\beta$$

$$m_1^2 = b\tan\beta - \frac{m_Z^2}{2}\cos 2\beta .$$

- We see from these formulae that the μ-term can not be extremely large compared to the ~~SUSY~~ parameters $m_{H_u}^2, m_{H_d}^2, b$. This highlights the importance of realizing a non-zero μ-term together with ~~SUSY~~, as in the Giudice-Masiero mechanism.

- It is also useful to derive, by adding the two relations given above:

$$m_1^2 + m_2^2 = b\,(\tan\beta + \cot\beta) = 2b/\sin 2\beta .$$

From this we see that the region of large $\tan\beta$ ($\tan\beta \gtrsim 10$), which is phenomenologically most interesting for reasons to be discussed later, is characterized by small values of b at the lagrangian level.

[We will see that for large $\tan\beta$ it is easier to obtain a large mass for the lightest physical Higgs, which is needed to avoid the experimental lower bound from LEP.]

Summary of the main logical steps:

- $H_u^+ = 0$ in vacuum (gauge choice)
- show that $H_u^+ = 0$ & $H_d^- = 0$ extremizes potential

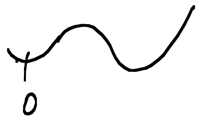
Comment 1: The relevant eq. was $H_d^- \left(b + \frac{1}{2} g_2^2 \overline{H_d^0} \overline{H_u^0}\right) \overset{!}{=} 0$. This can also be solved for $H_d^- \neq 0$, which means that there could also be minima breaking $U_{1,ED}$. It turns out (but we will not demonstrate this) that in cases where we find a proper SM-like minimum no such (dangerous) extra minimum exists.

Comment 2: We will have to check in the end that the minimum in the remaining potential for H_u^0, H_d^0 which we will find is also a minimum (rather than a maximum or saddle point) w.r.t. H_u^+, H_d^-. That this is indeed the case will follow from the full mass matrix at the extremum (which we will write down).

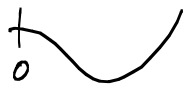
- $\Rightarrow V(H_u^0, H_d^0)$; where H_u^0, H_d^0 can be taken real without loss of generality

Demanding that the quadratic part has an instable direction (but is stable in the direction where the quartic part vanishes) leads to the conditions $b^2 > m_1^2 m_2^2$; $2b < m_1^2 + m_2^2$.

Comment: In cases where the quadratic part is positive definite, there can really be no minimum away from zero. The reason is that, fixing a certain direction in field space and calling x the "distance" in this direction, the potential reads $\alpha x^2 + \beta x^4$. Thus, situations like



are impossible. Only situations like



can occur (but they require a neg. eigenvalue of the quadratic part).

6.2 General 2-Higgs-doublet model

(including more details of the SUSY case)

- The analysis in S. Martin's review (hep-ph/9709356) which we largely follow is not very detailed. Historically, the main references are

  Haber, Kane, Sterling '78 } Nucl. Phys. B
  Gunion, Kane '85 }
  Gunion, Haber, Kane, Dawson: "The Higgs Hunter's Guide" 1990,

  which start with more general (not necessarily SUSY) 2-Higgs-doublet models. These are still the best references!
  (in particular the first two papers)

- Non-SUSY 2HDMs are not very attractive extensions of the SM since in such models all fermion mass can, in principle, come from a combination of both Higgs VEVs. This leads in general to unacceptably large FCNCs.

- Nevertheless, let us follow Gunion, Haber and write

$$V(\phi_1,\phi_2) = \lambda_1(\phi_1^+\phi_1 - v_1^2)^2 + \lambda_2(\phi_2^+\phi_2 - v_2^2)^2$$

two equivalent Higgs-doublets

$$+\lambda_3\left[(\phi_1^+\phi_1 - v_1^2) + (\phi_2^+\phi_2 - v_2^2)\right]^2$$
$$+\lambda_4\left[(\phi_1^+\phi_1)(\phi_2^+\phi_2) - (\phi_1^+\phi_2)(\phi_2^+\phi_1)\right]$$
$$+\lambda_5\left[\mathrm{Re}(\phi_1^+\phi_2) - v_1 v_2\cos\xi\right]^2$$
$$+\lambda_6\left[\mathrm{Im}(\phi_1^+\phi_2) - v_1 v_2\sin\xi\right]^2$$

<u>Comment:</u> In the original paper, it is claimed that this is the most general potential under the requirements of $SU_2\times U_1$ gauge invariance & $\phi_i \to -\phi_i$ symmetry of all mass-dimension-zero operators ["this symmetry is only broken softly" (i.e. not in the UV)]. In an erratum, the authors admit that the term

$$\lambda_7\left[\mathrm{Re}\,\phi_1^+\phi_2 - v_1 v_2\cos\xi\right]\left[\mathrm{Im}\,\phi_1^+\phi_2 - v_1 v_2\sin\xi\right]$$

has been forgotten. However, it turns out that this does not affect the results of the original analysis.

- For $\lambda_i \geq 0$, the minimum is at

$$\langle\phi_1\rangle = \begin{pmatrix}0\\ v_1\end{pmatrix} \;;\; \langle\phi_2\rangle = \begin{pmatrix}0\\ v_2 e^{i\xi}\end{pmatrix}.$$

(Note that here it is natural to ascribe the <u>same</u> $U_{1,Y}$ charge to both doublets, unlike what is conventionally done in the SUSY case.)

<u>Note:</u> $\xi = 0$ corresponds to "no CP violation".

- The SUSY case follows from this by imposing the appropriate relations between the λ_i (Given these constraints, $\xi = 0$ can be realized by a phase redefinition of the doublets).

- The analysis the proceeds by redefining

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix} \to \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix}$$

$$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} = \begin{pmatrix} \varphi_5 + i\varphi_6 \\ \varphi_7 + i\varphi_8 \end{pmatrix} \to \begin{pmatrix} \varphi_5 \\ \vdots \\ \varphi_8 \end{pmatrix}$$

and analysing the mass matrix

$$M_{ij}^2 = \frac{1}{2} \cdot \frac{\partial^2 V}{\partial \varphi_i \partial \varphi_j}$$

at the minimum. The matrix turns out to have block-diagonal form, with 2×2 blocks:

- <u>Indices 1, 2, 5, 6</u> $\to$ two 2×2 blocks of the form

$$\lambda_4 \begin{pmatrix} v_1^2 & -v_1 v_2 \\ -v_1 v_2 & v_2^2 \end{pmatrix}.$$

They can be diagonalized by an SO_2 rotation with angle β ($\tan\beta = v_1/v_2$), which explains why our parameter β introduced earlier is really an angle. The eigenvalues of each block are

$$\underset{\substack{\uparrow \\ \text{2 Goldstone bosons}}}{0} \quad \text{and} \quad \underset{\substack{\uparrow \\ \text{charged Higgs particle}}}{m_{H^\pm}^2} = \lambda_4 (v_1^2 + v_2^2)$$

The relevant fields are $G^\pm = \phi_1^\pm \cos\beta + \phi_2^\pm \sin\beta$ ("Goldstones"); $\phi_i^- \equiv (\phi_i^+)^*$

and $H^\pm = -\phi_1^\pm \sin\beta + \phi_2^\pm \cos\beta$.

- Indices 4, 8: another 2×2 block of the same form as above, but with $\lambda_4 \to \lambda_6$.

$\Rightarrow$ one further Goldstone + one further massive (but this time neutral) Higgs boson.

- Indices 3, 7:
$$\begin{pmatrix} 4v_1^2(\lambda_1+\lambda_3)+v_2^2\lambda_5 & (4\lambda_3+\lambda_5)v_1v_2 \\ (4\lambda_3+\lambda_5)v_1v_2 & 4v_2^2(\lambda_2+\lambda_3)+v_1^2\lambda_5 \end{pmatrix}$$

$\Rightarrow$ two more massive neutral Higgses

In total: $8 = \underset{\substack{\uparrow \\ \text{Goldstones} \\ \text{(from breaking} \\ SU_2\times U_{1,Y}\to U_{1,ED})}}{3} + \underset{\substack{\uparrow \\ \text{1 charged (i.e. 2 d.o.f.)} \\ \text{+ 3 neutral Higgs bosons.}}}{5}$

Summary of spectrum (now returning to the more specific SUSY case; and changing the notation correspondingly)

$$m_{A^0}^2 = 2b/\sin 2\beta$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2$$

$$m_{h^0,H^0}^2 = \frac{1}{2}\left(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2+m_Z^2)^2 - 4m_Z^2 m_{A^0}^2\cos^2 2\beta}\right)$$

Note: For $m_{A^0}^2 \to \infty$, all masses go to infinity except

$$m_{h^0}^2 \simeq \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{4m_Z^2 m_{A^0}^2\cos^2 2\beta}{m_{A^0}^2+m_Z^2} \simeq m_Z^2\cos^2 2\beta .$$

6.3 $\tan\beta$ and Yukawa couplings

- recall: $\tan\beta = v_u / v_d$; $\mathcal{L} \supset (\lambda_u U Q H_u + \lambda_d D Q H_d)\big|_{\theta^2}$
  $(0 \le \beta \le 90°)$ (focus on 3rd generation)

$$\Rightarrow m_t = \lambda_t v_u \; ; \; m_b = \lambda_b v_d$$

- Since $m_W^2 = \frac{1}{2} g_2^2 (v_u^2 + v_d^2)$; $m_Z^2 = \frac{1}{2}(g_1^2 + g_2^2)(v_u^2 + v_d^2)$,
  we have, for large $\tan\beta$, $v_u \approx 175$ GeV, as in SM.
  Hence: $\lambda_t \simeq 1$; also: $\frac{m_t}{m_b} = \frac{\lambda_t v_u}{\lambda_b v_d} \Rightarrow \lambda_b = \lambda_t \frac{v_u}{v_d} \cdot \frac{m_b}{m_t}$

$$\text{or } \lambda_b \approx \frac{m_b}{m_t} \cdot \tan\beta \approx \frac{4.2}{175} \cdot \tan\beta \; ;$$

  $\Rightarrow$ $\tan\beta$ can be very large without driving λ_b into the non-perturbative regime ($\lambda_b \gg 1$).

- <u>Note</u>: In certain GUT or heterotic string models at a very high scale, $\lambda_t = \lambda_b$ may be enforced by symmetry. Then large $\tan\beta$ is necessary (and hence particularly attractive) to explain the ratio m_b / m_t.
  (For an actual numerical determination of the relevant $\tan\beta$, the running of λ_t, λ_b with the energy scale must be taken into account.)

- The opposite limit of $\tan\beta \ll 1$ can not so easily be taken:
  Then $v_d \approx 175$ GeV and $\lambda_b \approx \frac{m_b}{v_d} = \frac{4.2}{175}$.
  We get $\lambda_t \approx \lambda_b \frac{v_d}{v_u} \cdot \frac{m_t}{m_b} = \frac{4.2}{175} \cdot \tan\beta \cdot \frac{175}{4.2}$

$$\Rightarrow \lambda_t = 1/\tan\beta \gg 1,$$ which would make the theory non-perturbative.

[This gets worse at high scales through running of λ_t.]

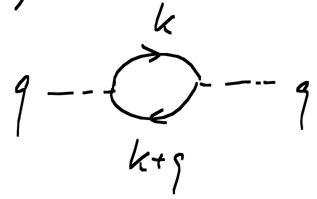
<u>A brief excursion:</u> One can be more precise about what it means to be in the non-perturbative regime:

Consider a Yukawa coupling, $\mathcal{L} \supset \lambda \varphi \bar{\psi} \psi$, and consider some (UV-finite, for simplicity) loop correction:

$$\frac{\Delta m_\varphi^2}{m_\varphi^2} \approx \frac{\quad \cdots \quad}{m_\varphi^2}$$

- The loop contribution reads, very roughly,

$$\lambda^2 \int \frac{d^4k}{(2\pi)^4} \cdot \frac{m^2}{(k^2-m^2)((k+q)^2-m^2)}$$



$$\sim \frac{\lambda^2 m^2}{16\pi^4} \int_{S^3} \int^{m} k^3 dk / (k^2)^2$$

$$\sim \frac{\lambda^2 m^2}{16\pi^4} 2\pi^2 \int^{m^2} (k^2)\, dk^2 \cdot \frac{1}{2} \sim \frac{\lambda^2}{16\pi^2} \cdot m^2$$

- Assuming all relevant mass scales to be roughly equal, i.e. $m_\varphi^2 / m^2 = O(1)$, we find

$$\frac{\Delta m_\varphi^2}{m_\varphi^2} \sim \frac{\lambda^2}{16\pi^2},$$ i.e. perturbation theory is reliable if $\frac{\lambda^2}{16\pi^2} \ll 1$, not $\lambda \ll 1$.

- In practice, such factors $16\pi^2$ can be very important. For example, they are the reason why $\alpha_s = \frac{g_3^2}{16\pi^2}$ is the relevant expansion parameter for QCD. In fact, even at relatively high energies, like m_Z, $\alpha_s \simeq 0.1$. The expansion in g_3 ($g_3 > 1$!) would obviously not work.
- This line of reasoning concerning the boundary of perturbation theory (and hence about the maximal values of couplings at which a perturbative treatment makes sense) is called "<u>naive dimensional analysis</u>". It is particularly important for theories with more than 4-dimensions. For details see e.g. Chacko, Luty, Ponton, hep-ph/9909248.

(The "loop suppression factor in general dimension d is

$$\frac{1}{\ell_d} = \frac{1}{2^d \pi^{d/2} \Gamma(d/2)} \quad \Rightarrow \quad \ell_4 = 16\pi^2 .)$$

End of excursion.

<u>Jargon:</u> "large $\tan\beta$" means $\tan\beta \gtrsim 10$; up to ~ 50
"small $\tan\beta$" means $\tan\beta = O(1)$.

6.4 The limits of a single light Higgs / large $\tan\beta$

- Recall our basic potential

$$V = m_2^2 |H_u^0|^2 + m_1^2 |H_d^0|^2 - b H_u^0 H_d^0 + \text{h.c.} + \frac{1}{8}(g_1^2+g_2^2)\left(|H_d^0|^2 - |H_u^0|^2\right)^2$$

- Let us use our knowledge that we can choose, without loss of generality, $\underbrace{H_u^0 = v_u \,,\ H_d^0 = v_d \text{ real}}_{\text{in the vacuum}}$ and $b > 0$ real.

- Define $\alpha = \frac{1}{4}(g_1^2 + g_2^2)$ and minimize the potential explicitly:

$$0 = \frac{\partial V}{\partial v_u} = 2 m_2^2 v_u - 2 b v_d - 2 \alpha v_u (v_d^2 - v_u^2)$$

$$0 = \frac{\partial V}{\partial v_d} = 2 m_1^2 v_d - 2 b v_u + 2 \alpha v_d (v_d^2 - v_u^2)$$

$$\Rightarrow \quad m_2^2 = b \frac{v_d}{v_u} + \alpha (v_d^2 - v_u^2)$$

$$m_1^2 = b \frac{v_u}{v_d} - \alpha (v_d^2 - v_u^2)$$

- recall that $m_Z^2 = \frac{1}{2}(g_1^2 + g_2^2)(v_u^2 + v_d^2) = \frac{1}{2}(g_1^2 + g_2^2) v^2$

$$\Rightarrow \quad m_2^2 = b \cot\beta + \frac{m_Z^2}{2}(\cos^2\beta - \sin^2\beta)$$

$$m_1^2 = b \tan\beta - \frac{m_Z^2}{2} \underbrace{(\cos^2\beta - \sin^2\beta)}_{\equiv \cos 2\beta}$$

(These are the formulae which we stated earlier without derivation)

- From this, we have already derived $\sin 2\beta = \dfrac{2b}{m_1^2 + m_2^2} = \dfrac{2b}{m_{A^0}^2}$

or $\quad \tan\beta + \cot\beta = \dfrac{m_1^2 + m_2^2}{2b}$.

- Let us also derive the value of m_Z^2 by writing

$$m_1^2 - m_2^2 \tan^2\beta = \frac{m_Z^2}{2}(\sin^2\beta - \cos^2\beta)(1 + \tan^2\beta)$$

$$\Rightarrow m_Z^2 = 2 \cdot \frac{m_1^2 - m_2^2 \tan^2\beta}{(\tan^2\beta - 1)\cos^2\beta\,(1+\tan^2\beta)}$$

$$m_Z^2 = 2\, \frac{m_1^2 - m_2^2 \tan^2\beta}{\tan^2\beta - 1}$$

- Since $2b < m_1^2 m_2^2$; m_1^2 & m_2^2 can not be both negative. For $\tan\beta > 1$ (which, motivated by the LEP Higgs mass bound, we will always assume), the formulae for $m_{1,2}^2(b, m_Z, \beta)$ imply $m_1^2 > m_2^2$, i.e. either $m_1^2 > m_2^2 > 0$
  or $m_1^2 > 0$; $m_2^2 < 0$.

- To analyse the various limits, let us now write our lagrangian parameters as

$$m_1^2 \; ; \; m_2^2 = y m_1^2 \; ; \; b = x m_1^2 \qquad (x > 0 \; ; \; y < 1)$$

(in which case the electroweak-breaking conditions read

$$2x < 1 + y \quad \& \quad x^2 > y).$$

- Let us first take the $(\tan\beta \gg 1)$ - limit most seriously. This corresponds to

$$\tan\beta + \frac{1}{\tan\beta} = \frac{1+y}{x} \gg 1 ,$$

while keeping $y = O(1)$ (pos. or neg.). $\tan\beta \gg 1$ corresponds to $x \ll 1$ and $\tan\beta \approx \frac{1+y}{x}$.

- Furthermore, $m_Z^2 \approx 2m_1^2 \dfrac{x^2 - y(1+y)^2}{(1+y)^2 - x^2} \simeq -2 y m_1^2 = 2|y| m_1^2$, which enforces $y < 0$. For the physical Higgs masses are: $m_{A^0}^2 \simeq m_1^2(1+y) \simeq \frac{m_Z^2}{2}\left(\frac{1}{|y|}-1\right)$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2 \quad \text{as always};$$

$$m_{H^0,h^0}^2 \simeq \frac{1}{2}\left[(m_{A^0}^2+m_Z^2) \pm \sqrt{(m_{A^0}^2+m_Z^2)^2 - 4m_{A^0}^2 m_Z^2 \cos^2 2\beta}\right]$$

$$\simeq \frac{1}{2}\left[(m_{A^0}^2+m_Z^2) \pm |m_{A^0}^2 - m_Z^2|\right] \quad (\text{for } \tan\beta \gg 1)$$

$$\simeq \begin{cases} m_{A^0}^2 \\ m_Z^2 \end{cases} \text{or} \quad \begin{matrix} m_Z^2 \\ m_{A^0}^2 \end{matrix}, \text{ if } m_{A^0} < m_Z .$$

($m_{A^0} < m_Z$ arises if $m_{A^0}^2 - m_Z^2 = \frac{m_Z^2}{2}\left(\frac{1}{|y|}-1-2\right) < 0$,
i.e. if $|y| < \frac{1}{3}$, which is probably ruled out anyway in spite of loop corrections)

- We see that for $x \ll 1$ (large $\tan\beta$) and $y = O(1)$ ($|m_1|^2 \sim |m_2^2|$), a reasonable phenomenology may emerge if $|y|$ is "smallish". The "soft masses" m_1^2 & m_2^2 are $O(m_Z)$, which is <u>problematic</u>:

  We will need large $m_{\tilde{t}}^2$ ($O(\text{TeV})$) to lift m_{h^0} to 115 GeV. Loop corrections based on $m_{\tilde{t}}^2$ will affect m_1^2, m_2^2 and make them generically $O(\text{TeV})$ as well. Keeping <u>both</u> of them small is quite unnatural.

  (For b, which is also small in this setting, the situation is not as clear since loop corrections to b are $\sim b$ or $\sim \mu$, <u>not</u> $\sim m_{\tilde{t}}^2$.)

- We take this (and the desire to have a simple toy model where the extra Higgses decouple) as a motivation to look more specifically into the situation

$$x \ll 1 \quad \& \quad |y| \ll 1 .$$

- As before: $\tan\beta \approx \frac{1+y}{x}$; even simpler: $\underline{\underline{\tan\beta \approx \frac{1}{x}}}$.

Also: $m_Z^2 \approx 2m_1^2(x^2-y)$ (where we now can't conclude $y<0$. <u>However</u>: if $y>0$, then $|y| < x^2$.)

$\Rightarrow \quad m_Z \ll m_1$.

Physical masses: $m_{A^0} \approx m_{H^\pm} \approx m_{H^0} \approx m_1$; $m_{h^0} \approx m_Z$

- We can view this as follows $1/\tan\beta$ (or x) is small but not extremely small. y is extremely small, which corresponds to a fine-tuning (details see below), enforced by the need to have

$$\left| \begin{array}{l} m_{h^0} > 115\,\text{GeV} \Rightarrow m_{\tilde{t}} \text{ large} \Rightarrow m_1^2, m_2^2 \text{ large} \\ \Rightarrow m_1^2 \text{ large}, m_2^2 \text{ small by fine-tuning.} \end{array} \right|$$

- Furthermore, if we take limit of small x more seriously than of small y ($x^2 \ll y$), this region of "<u>small $\tan\beta$ & single light Higgs</u>", allows for the following interpretation:

By choosing $m_2^2 < 0$; $m_1^2 > 0$ and $|m_1^2| \gg |m_2^2|$ at $\theta = 0$ we decouple H_d. This would give us just the SM Higgs sector with an extra constraint linking the quartic coupling to the gauge coupling and thus enforcing $m_Z^2 \approx m_{h^0}^2$. This would mean $\tan\beta = \infty$

and $\lambda_b = \infty$, which is unacceptable. Thus, we have to allow for at least a small $b \neq 0$, giving

$\tan\beta \simeq \frac{m_1^2}{b}$.

<u>Final figure:</u>