9 Technicolor

9.1 Basic idea

- Fund. scalar (Higgs) has quadratically divergent mass correction $\Rightarrow$ hierarchy problem
- Can we avoid fund. scalars?
- Idea: Let the scalar (the VEV of which breaks $SU_2 \times U_1$) arise dynamically, like the pion of QCD ("$\langle \psi\psi \rangle$") (recall that $\mathcal{L}_{QCD}$ contains no fund. scalars). Furthermore, we would like our scalar to be much lighter than the fund. scale of the underlying strongly interacting theory. This is possible if the scalar is a Goldstone boson of some spont. broken global symm. (which is the case for the pion!).

Preliminary remarks:

- Let G be the symm. group of $\mathcal{L}$ (or H) and let $|0\rangle$ break G to $H \subset G$: $\mathfrak{g} = Lie(G) = Lie(H) \oplus \mathfrak{g}' = \mathfrak{h} \oplus \mathfrak{g}'$.
- For $T' \in \mathfrak{g}'$ we have $T'|0\rangle \neq 0$ or $e^{\alpha T'}|0\rangle \neq |0\rangle$.
- At the same time $[H, T'] = 0 \Rightarrow \langle 0 | e^{-\alpha T'} H e^{\alpha T'} | 0 \rangle$

$$= \langle 0 | H | 0 \rangle$$

$\Rightarrow \exists$ a "flat direction" for each $T' \in \mathfrak{g}'$.

("Goldstone bosons")

The case of QCD

- For simplicity, we focus on $\underbrace{u, d}_{\text{ignoring } m_u, m_d.}$ (treating s, c, b, t as "heavy").

- $q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$; $q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$ (all Weyl fermions)

$$\mathcal{L} = -\frac{1}{2g_3^2}\, \mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} + i\bar{q}_L^T \bar{\sigma}^\mu D_\mu \overset{3}{q_L} + i\bar{q}_R^T \bar{\sigma}^\mu D_\mu \overset{\bar{3} \text{ of } SU_3}{q_R}$$

(color indices suppressed)

$\Rightarrow$ global symm. $SU_{2,L} \times SU_{2,R}$: $q_L \to U_L q_L$; $q_R \to \bar{U}_R q_R$

(the "$-$" is conventional)

- Assume that $SU_{3,\text{color}}$ is unbroken in the vacuum.
- Then the only possible fermion condensate is $\langle q_{Li} q_{Rj} \rangle \neq 0$ (color indices contracted).
- Assume $\langle q_{Li} q_{Rj} \rangle = \lambda \delta_{ij}$ (for symm. reasons)

$SU_{2,L} \times SU_{2,R}$: $\langle q_{Li} q_{Rj} \rangle \longrightarrow (U_L)_{ik} \langle q_{Lk} q_{Rl} \rangle (U_R^+)_{lj} \overset{!}{=} \delta_{ij}$

$\Rightarrow U_L = U_R \Rightarrow$ the diagonal subgroup of $SU_{2,L} \times SU_{2,R}$ remains unbroken

- This surviving global symm. $SU_{2,L+R}$, defined by

$$(U, U) \in SU_{2,L} \times SU_{2,R} \quad ; \quad U \in SU_{2,L+R},$$

is also known as the "vectorial subgroup" since it rotates the whole Dirac spinor $\begin{pmatrix} q_L \\ \bar{q}_R \end{pmatrix}$.

[The word vectorial comes from $\bar{\psi}\gamma^\mu\psi$ (vector) in contrast to $\bar{\psi}\gamma^\mu\gamma^5\psi$ (axial vector, sensitive to the L-R-structure. It the same meaning used previously in the context of gauge mediation, where we characterized the messengers as "vector-like".]

- To characterize the Goldstone-bosons, consider the action of $(\Sigma, \mathbb{1}) \in SU_{2,L} \times SU_{2,R}$. The condensate is not invariant:

$$\langle q_{Li} q_{Rj} \rangle \longrightarrow \Sigma_{ik} \langle q_{Lk} q_{Rj} \rangle = \lambda \Sigma_{ij} .$$

- Thus, Σ parameterizes the flat directions. Writing

$$\Sigma = e^{i\pi^a \sigma^a / f_\pi} ,$$

we identify π^a $(a = 1, 2, 3)$ as the pions.

- Note: $SU_{2,L} \times SU_{2,R} : \quad \Sigma \longrightarrow U_L \Sigma U_R^+ .$

- The lowest-dimensional invar. Lagrangian is

$$\mathcal{L} = \frac{f_\pi^2}{4} tr \left[(\partial_\mu \Sigma)(\partial^\mu \Sigma) \right] = \frac{1}{2} (\partial_\mu \pi^a)(\partial^\mu \pi^a) + \dots$$

- In fact, $SU_{2,L}$ is gauged by the weak SU_2: $\partial_\mu \rightarrow D_\mu = \partial_\mu + iA_\mu$

$$\Rightarrow \mathcal{L} = \frac{f_\pi^2}{4} tr \left[(D_\mu \Sigma)(D^\mu \Sigma)^+ \right]$$

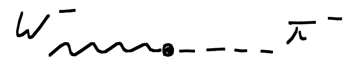
$$= \frac{f_\pi^2}{4} tr \left[(\partial_\mu \Sigma)(\partial^\mu \Sigma)^+ + \underbrace{\{ i g_2 A_\mu \Sigma (\partial^\mu \Sigma^+) + h.c. \}} - g_2^2 A_\mu \Sigma A^\mu \Sigma^+ \right]$$

$$\downarrow$$

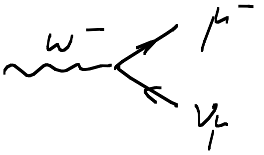
$$\frac{f_\pi}{2} g A_\mu \partial^\mu \pi$$

$$\downarrow$$

e.g.



- Combining this with the vertex



,

we can calculate $\Gamma(\pi^- \rightarrow \mu^- + \nu_\mu)$ and determine f_π experimentally: $f_\pi = 83$ MeV (Note: Sometimes $\sqrt{2} f_\pi = 140$ MeV is called f_π)

- Now imagine there were no Higgs and $W^\pm, Z$ would still be massless at ~ 1 GeV. The term

$$\left(\frac{g f_\pi}{2}\right)^2 tr(A_\mu A^\mu) ,$$

which is also contained in the above $\mathcal{L}$, would provide a mass

$$m_W = \frac{g f_\pi}{2} .$$

(The pions would provide the longitudinal d.o.f.'s of the SU_L gauge bosons and disappear from the spectrum.)

- We can now state the <u>main idea of technicolor</u>:

  Assume that, in addition to $SU(3)_{QCD}$ with $\Lambda_{QCD} \sim 0.2$ GeV there is a technicolor group $SU(N)_{TC}$ with $\Lambda_{TC} \sim 1$ TeV.

- The condensate $\langle Q_{Li} Q_{Rj} \rangle$ of the techniquarks Q_L, Q_R

  will break $SU_2 \times U_{1,Y}$ (we ignored $U_{1,Y}$ in our discussion above for simplicity)

  to $U_{1,EM}$ and give $m_W = \frac{g \cdot f_{\pi,TC}}{2}$ (and a corresponding m_Z).

- To get $m_W \sim 80$ GeV we need $f_{\pi,TC} \sim 250$ GeV (and we expect techni-hadrons at ~ 1 TeV)

9.2 Towards a realistic model

The simplest models go back to Weinberg ('76) and Susskind ('79).

- gauge group: $SU(N) \times SU(3) \times SU(2) \times U(1)$

- Fermions: SM fermions + two flavor doublets / color singlets

$\begin{pmatrix} T_L \\ B_L \end{pmatrix}$, $\begin{pmatrix} T_R \\ B_R \end{pmatrix}$.

- Chiral symm. group: $$SU_{2,L} \times SU_{2,R} \times \underset{\substack{\uparrow \\ \text{broken by axial anomaly} \\ \text{and by condensate}}}{[U_{1,A}]} \times \overset{\substack{\text{rotates Dirac spinor} \\ \text{\& remains unbroken} \\ \downarrow}}{U_{1,B}}$$

- SM gauge trfs.: $\begin{pmatrix} T_L \\ B_L \end{pmatrix}$ – SU_2 doublet; $U_{1,Y}$ singlet

  $\begin{pmatrix} T_R \\ B_R \end{pmatrix}$ – SU_2 singlet ; $Y = \begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix}$

- anomalies: SU_2 is anomaly free (since $2 = \bar{2}$)

  $U_{1,Y}$ is anomaly free since $+1/2$ & $-1/2$ are present with same # of fermions

  [<u>Note</u>: SU_2 has a global anomaly ("Witten's global anomaly"). It vanishes if N is even.]

- Condensate: $\langle Q_{L\,i}\, Q_{R\,j} \rangle \sim \delta_{ij}$, as in QCD.

- Solution to hierarchy problem / unification

  $\alpha_T(M_{GUT}) = \alpha_3(M_{GUT})$; increase # of doublets to $N_D > 1$

  Standard analysis gives, e.g.:

  assume: $\alpha_3(M_{GUT}) = 1/30$; $N = 4$; $N_D = 4$; $\Lambda_{QCD} = 0.2\,\text{GeV}$

  find: $\Lambda_T \approx 165\,\text{GeV}$; $f_T = 85\,\text{GeV}$; $v \approx 190\,\text{GeV}$

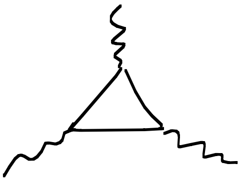
✓

<u>Comment concerning anomalies</u>

In all extensions of the SM, the absence of gauge-anomalies is an essential constraint (see above).

An anomaly is the breaking of a symm. of the classical action by quantum effects. For a global symm., this is acceptable in principle. For a gauge symm., this leads to inconsistencies since gauge-invariance is crucial to prove unitarity (reducing the 4 apparent d.o.f.s of A_μ to the two phys. polarizations).

- Of particular importance is the <u>chiral</u> anomaly, which potentially arises whenever a chiral fermion (Weyl fermion) is gauged. The technical reason is the difficulty to regularize the triangle diagram

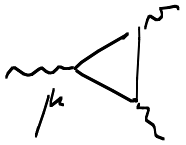


in a gauge inv. way. Specifically in dim. regul. (which is one of the few ways to regularize gauge theory, especially if it's non-abelian, without breaking gauge-inv. directly by the regularization) the problem comes with γ_5:

- We need to use Dirac-spinors (since the relation to $SL(2,\mathbb{C})$ and hence Weyl spinors is a specialty of $d=4$).
- for a chiral theory, we then have to use γ^5 in the trace associated with the fermion loop. It turns out that, insisting on $\{\gamma^\mu, \gamma^5\}=0$ for all γ-matrices (more than 4) makes it impossible to define traces of γ-matrices as meromorphic fcts. of d and get the right results for $d=4$.
- Following 't Hooft & Veltman, one keeps $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$

and accepts that γ^5 will not anticommute with the extra γ-matrices. In the above loop calc. one then gets extra pieces $\sim \epsilon$ from the trace which, when cancelled against the $1/\epsilon$ from the divergence, give a gauge-symm.-violating result:

$$\partial_\mu j^\mu \neq 0 \quad , \quad \text{for}$$



(cf. $j^\mu = \bar{\psi} \gamma^\mu \psi$ classically).

- The only way to keep a chiral gauge theory consistent is then to insist that the triangle diagrams cancel among themselves.

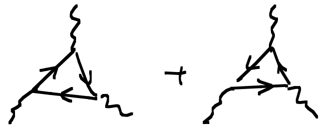
(<u>Note</u>: The problem is not just a technical regularization problem. One can show very generally that the anomaly is there in any consistent regularization. Most clearly this can be demonstrated by showing that the path integral measure $\int D\psi$ is actually not $U(1)$-invariant for a chiral fermion.)

Anomaly cancellation in SM:

As stated: $\quad \operatorname{tr}\left(\{T^a(R), T^b(R)\} T^c(R)\right\} \overset{!}{=} 0 .$

for the full fermion representation R.

[The anticommutator comes from



,

i.e. $\operatorname{tr}(T^a T^b T^c) + \operatorname{tr}(T^c T^b T^a)$

$= \operatorname{tr}(T^a T^b T^c) + \operatorname{tr}(T^b T^a T^c) = \operatorname{tr}(\{T^a, T^b\} T^c)$]

let us check some special cases:

1) if $T^a = T^b = T^c = T_Y$:

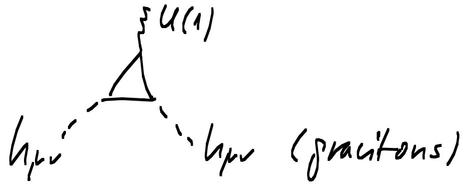
$tr((T^Y)^3) \overset{!}{=} 0$, i.e.

	q_L	u_R	d_R	l_L	e_R
Y:	$1/6$	$-2/3$	$1/3$	$-1/2$	1
Y^3:	$1/6^3$	$-8/3^3$	$1/3^3$	$-1/8$	1

$$\underset{\substack{\uparrow\\ \text{color}\\ \text{\& } SU_2 \text{ doublet}}}{6\cdot\frac{1}{6^3}} - \underset{\substack{\uparrow\\ \text{color}}}{3\frac{8}{3^3}} + 3\frac{1}{3^3} - \underset{\substack{\uparrow\\ \text{doublet}}}{2\frac{1}{8}} + 1 \overset{!}{=} 0$$

$$\frac{1}{36} - \frac{32}{36} + \frac{4}{36} - \frac{9}{36} + \frac{36}{36} = 0 \quad \checkmark$$

<u>also</u>:



$\Rightarrow \ \Sigma Y \overset{!}{=} 0$, i.e.

$$6\cdot\frac{1}{6} - 3\frac{2}{3} + 3\cdot\frac{1}{3} - 2\frac{1}{2} + 1 \overset{!}{=} 0$$

$$1 - 2 + 1 - 1 + 1 = 0 \quad \checkmark$$

2) SU(2):

$$tr(\{\sigma^i, \sigma^j\}\sigma^k) = tr(2\cdot\delta^{ij}\cdot\mathbb{1}\cdot\sigma^k) = 0$$

$\Rightarrow$ SU(2) is anomaly free

3) ... similarly for SU(3) & "mixed anomalies"

Note: pro GUT:

$$SO_{10} \supset SU_5 \supset SU_3 \times SU_2 \times U_1$$

$$\underset{\substack{\downarrow \\ \text{anomaly free!}}}{16} = \underset{\substack{\uparrow \\ (5\times 5)_A}}{10} + \bar{5} + 1 = [1 \text{ SM generation}] + \text{r.h. neutrino}$$

contra GUT:

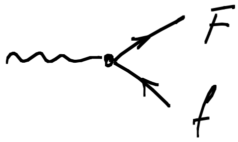
just fixing the SU_3 & SU_2 repres. content of the SM, the anomaly-free $U(1)$-charge-assignment is (essentially) unique.

- Our specific "minimal" technicolor model is too simple to be realistic (especially for Yukawa couplings, see below)
- In more complicated models, one usually has more than 3 Goldstone bosons, such that even after $W^\pm, Z$ have absorbed 3 of them, some massless fields remain. Their mass is generated by higher-order effects ("Pseudo-Goldstone-Bosons).
- To get Yukawa couplings, one needs to appeal to higher-dim. operators:
$$\mathcal{L} \supset \frac{1}{M^2}(\underset{\substack{\uparrow \\ \text{SM-}}}{f \cdot f})(\underset{\substack{\uparrow \\ \text{techni-fermions}}}{F \cdot F})$$
- To generate such non-renormalizable operators in a renormalizable theory at higher energy scale, one appeals to "Extended technicolor" ("ETC").

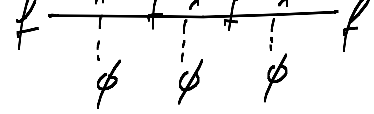
- Start with large gauge group G_{ETC}:

$$G_{ETC} \supset G_{TC} \times G_{SM}$$

$\Rightarrow$ V_{Ff}

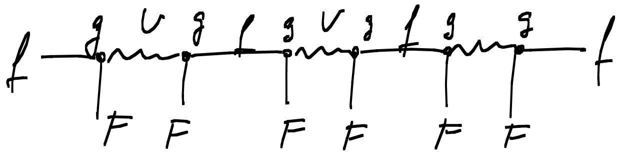


- To understand fermion mass generation in this context, read usual Higgs-induced masses as effect of propagation in Higgs-background:



$\Rightarrow$ $m_f \sim \lambda \langle \phi \rangle$

- In ETC:



$$\Rightarrow m_f \sim \frac{g_{ETC}^2}{m_V^2} \underbrace{\langle FF \rangle}_{\sim \Lambda_{TC}^3}$$

- m_V can be generated by dynamical symm. breaking (as in TC), just at higher scale

- To create a mass hierarchy, one can start with very large G_{ETC} and break it stepwise ("tumbling").

$$G_{ETC} \longrightarrow G_{ETC'} \longrightarrow G_{ETC''} \longrightarrow G_{TC}$$

$$m_{V_1} \gg m_{V_2} \gg m_{V_3}$$

e.g. 10^3 TeV ; 30 TeV ; 1 TeV

- A further possibility is "Walking technicolor". It adresses the following problem (where, for simplicity, we return to an ETC scenario without tumbling, i.e. $G_{ETC} \to G_{TC}$):

- On the one hand, we want Λ_{ETC} to be high to avoid FCNC's (induced by operators like $\psi_{SM}^4/\Lambda_{ETC}^2$)

- On the other hand, we need large masses for at least some of the SM fermions, which conflicts with formula $m_f \sim \langle FF \rangle / m_V^2 \sim \Lambda_{TC}^3 / \Lambda_{ETC}^2$ for large Λ_{ETC}.

- The basic idea of Walking Technicolor is to assume that α_{TC} remains large between Λ_{TC} & Λ_{ETC} (unlike QCD, where α_s falls rapidly above Λ_{QCD} because of asymptotic freedom).

  [This can be realized if the TC-gauge theory is near a regime with a "non-trivial IR fixed point".]

- As a result, quantum corrections can significantly enhance the coefficient of the operator $(\bar{f}f) \cdot (\bar{F}F)/\Lambda_{ETC}^2$ when going from the scale Λ_{ETC} to the scale Λ_{TC} (allowing for larger m_f).

  [The term "walking" is used as an antonym to the term "running" used for α_s.]