Beyond the Standard Model – from Supersymmetry to extra Dimensions

1 Introduction

1.1 Basic Structure of the SM

$$S = \int d^4x \, \mathcal{L} = \int d^4x \left(\mathcal{L}_{gauge} + \mathcal{L}_{fermions} + \mathcal{L}_{scalars} + \mathcal{L}_{Yukawa} \right)$$

$$\mathcal{L}_{gauge} = -\frac{1}{4g_1^2} F^{(1)}_{\mu\nu} F^{(1)\mu\nu} - \frac{1}{2g_2^2} \operatorname{tr} F^{(2)}_{\mu\nu} F^{(2)\mu\nu} - \frac{1}{2g_3^2} \operatorname{tr} F^{(3)}_{\mu\nu} F^{(3)\mu\nu}$$

$$\text{"="} - \sum_{i=1}^{3} \frac{1}{2g_i^2} F^{(i)}_{\mu\nu} F^{(i)\mu\nu}$$

, where i labels the 3 factors of the gauge group

$$G_{SM} = U_1 \times SU_2 \times SU_3$$

$$\mathcal{L}_{fermions} = \sum_j \bar{\psi}_j \, i \not{D} \psi_j \quad ; \quad D_\mu = \partial_\mu + i R_j(A_\mu)$$

l.h. or r.h. Dirac-fermions

representation appropriate to fermion ψ_j (for details see below)

$$j \in \left\{ \{ q_L^a, u_R^a, d_R^a, \ell_L^a, e_R^a \}, \; a = 1,2,3 \right\}$$

families or generations

$$\mathcal{L}_{scalars} = -(D_\mu \phi)^+ (D^\mu \phi) - V(\phi) \quad ; \quad V(\phi) = -\nu \, \phi^+\phi + \lambda (\phi^+\phi)^2$$

$$\phi^+\phi = (\bar{\phi}^1, \bar{\phi}^2) \cdot \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix} = |\phi^1|^2 + |\phi^2|^2$$

$$\mathcal{L}_{Yukawa} = \sum_{\substack{\text{all possible} \\ \text{gauge-inv. terms}}} -\lambda_{jk} \bar{\psi}_j \psi_k \phi + \text{h.c.}$$

contractions of group indices implicit

1.2 Higgs mechanism

For $\nu > 0$, V is minimized by $\phi^+\phi = \nu/(2\lambda) \equiv v^2$ (v real)

By an SU_2 gauge trf., we can always choose $\phi = \begin{pmatrix} 0 \\ v \end{pmatrix}$ in the vacuum.

- Terms of the type $\sim A_\mu A^\mu \phi^+\phi \subset (D_\mu\phi)^+(D^\mu\phi)$ give masses to 3 of the 4 vector bosons in $SU_2\times U_1$

  $[m_{W^\pm} \approx 80\,\text{GeV}\,;\; m_Z \approx 90\,\text{GeV} \Rightarrow v \approx 174\,\text{GeV}]$

  (The convention $v \to v/\sqrt{2}$ with $v \approx 246\,\text{GeV}$ is also widely used.)

- The Yukawa terms give masses to the fermions, in particular

$$m_t \approx \lambda_t v \approx 170\,\text{GeV} \Rightarrow \lambda_t \approx 1.$$

(the largest of the Yukawa couplings)

- Expanding around the vacuum, $\phi = \begin{pmatrix} 0 \\ v + h(x)/\sqrt{2} \end{pmatrix}$,

  we find $\mathcal{L} \supset -\frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \nu h^2$, i.e., $m_h^2 = 2\nu = 4\lambda v^2$.

- From LEP2, we know that $m_h \gtrsim 114\,\text{GeV}$, i.e., $\lambda \gtrsim 0.11$.
- Strong coupling sets in at $\lambda/(16\pi^2) \approx 1$, corresponding to

  $\uparrow$ "loop suppression factor" (see below)

  $m_h \approx 4\,\text{TeV}$. However, electroweak precision data (indirect effect of Higgs in loops) gives the much stronger limit $m_h \lesssim 160\,\text{GeV}$ at 2σ or 95% CL (this bound varies depending on how certain mild discrepancies within the data are treated).

- Crucial conclusion: The only dim.ful parameter $\sqrt{\nu}$ ($[\nu] = [\text{mass}^2]$) in the SM is of the order of the el.-weak scale: $\underline{\underline{\sqrt{\nu} \sim 10^2\,\text{GeV}}}$.

1.3 Hierarchy problem

There are (at least) 3 different issues that might be referred to as "the hierarchy problem":

① The large hierarchy problem: Appearance of very different scales

The "full" fundamental lagrangian: $\mathcal{L} = \frac{1}{2}\bar{M}_p^2 R + \mathcal{L}_{SM}$

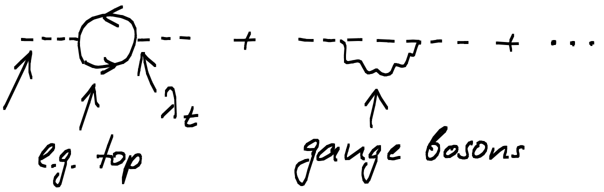
reduced Planck mass; $\sim 10^{18}$ GeV — Ricci scalar

Why is $v \ll \bar{M}_p^2 \equiv M_p^2/8\pi$ (Maybe "just" an aesthetical problem.)

② The large hierarchy problem: Fine tuning

- Assume some finite theory of quantum gravity (e.g. superstring theory)
- Then v is calculable, including quantum corrections:

$v = v_{tree} + \delta v$; $\delta v \sim$



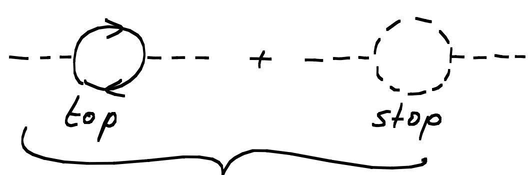
$$\delta v \sim \int^{\Lambda^2} d(k^2) \sim \Lambda^2 \sim M_p^2$$

$\Rightarrow (100\text{ GeV})^2 \sim v_{tree} + (10^{18}\text{ GeV})^2$; This requires an unbelievable fine tuning of v_{tree}.

③ LEP paradox or little hierarchy problem

- Assume that the SM is replaced by some more fund. theory at some scale $\Lambda \ll M_p$.
- Assume there is no quadratic Higgs mass divergence in this theory

(e.g. SUSY: $\delta v =$



cancel exactly if SUSY unbroken.)

$$\Rightarrow \quad \delta v \sim \frac{\lambda_t}{16\pi^2} \int^{\Lambda^2} d(k^2) \quad , \text{ where } \Lambda \text{ is the scale of "new physics".}$$

loop suppression factor : $\displaystyle \int \frac{d^4k}{(2\pi)^4} \cdot \frac{1}{k^2} = \frac{\Omega_3}{(2\pi)^4} \int \frac{|k|^3 d|k|}{|k|^2}$

$$= \frac{2\pi^2}{(2\pi)^4} \cdot \frac{1}{2} \int d(k^2) = \frac{1}{16\pi^2} \int d(k^2)$$

(This method of estimating loop integrals is also known as "naive dimensional analysis" or NDA.)

- With $\lambda_t \approx 1$ and $v \simeq (100\,\text{GeV})^2$ we expect $\Lambda \lesssim 1$ TeV if there is no cancellation between v_{tree} and δv (no fine-tuning).

- The problem arises since, integrating out the new physics at scale Λ, generically induces higher-dim. operators, e.g.

$$\frac{(D^2H)(D^2H)^+}{\Lambda^2} \quad ; \quad \frac{|H^+ D_\mu H|^2}{\Lambda^2} \, ,$$

the effect of which can be seen in precision observables. These are constrained by LEP, requiring $\Lambda \gtrsim 5$ TeV.

- This also explains the name "LEP paradox": LEP has measured precision observables implying $\begin{cases} m_h \lesssim 160 \text{ GeV} & \Rightarrow \quad \Lambda \lesssim 1 \text{ TeV} \\ \text{small deviations from SM} & \Rightarrow \quad \Lambda \gtrsim 5 \text{ TeV} \end{cases}$

Comments:
- In some specific models (in particular SUSY) the corrections from higher-dim. operators are small, allowing for low Λ. However (see below), SUSY turns out to have a (small) fine-tuning problem of its own.
- The small value of Λ required by the low Higgs mass suggested by LEP has a positive implication: The new physics at Λ should be visible at the LHC!

1.4 The SUSY solution to the hierarchy problem

- Basic idea already explained above: Bosonic (fermionic) superpartners for SM fermions (bosons) cancel divergent loop contributions to Higgs mass.
- Problem: Minimal SUSY extension of SM (MSSM) predicts $m_h \leq m_Z$ at tree-level. Loop corrections can lift m_h above 114 GeV, but this requires (in the simplest case) $m_{\tilde{t}} \approx 600$ GeV. Such a large stop mass contributes (again via loop effects) to the (SUSY equivalent of) the quadratic Higgs term $v\phi^+\phi$ and reintroduces a certain fine-tuning.

- Extra advantage: in MSSM, the g_i's unify at $\sim 10^{16}$ GeV in the scheme $U_1 \times SU_2 \times SU_3 \subset SU_5$ ("SUSY GUT")

1.5 The technicolor solution

- fund. Higgs scalar $\longrightarrow$ condensate of "techni-quarks" f confined under technicolor group (like pions in QCD)
- $v \ll M_P$ because technicolor coupling runs from small value at M_P to $O(1)$-value at m_{EW} (analogous to QCD).

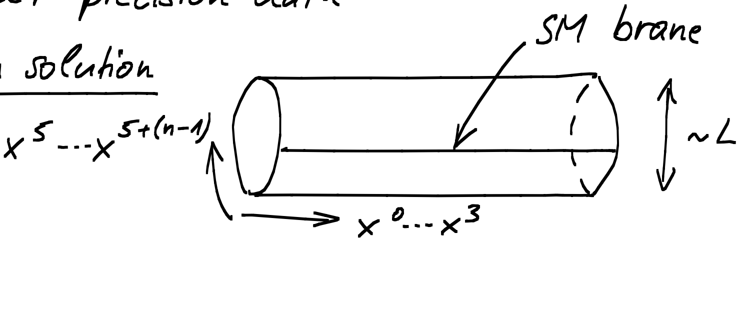
Problems: Yukawas come from operators $\frac{1}{\Lambda^2}(\bar{f}f)(\bar{\psi}\psi) \to \frac{1}{\Lambda^2}\langle\bar{f}f\rangle\bar{\psi}\psi$

large $m_t \Rightarrow$ small Λ; <u>but</u>: operators $\frac{(\bar{\psi}\psi)^2}{\Lambda^2}$ are strongly constrained

+ problems with LEP precision data

1.6 The large extra dimension solution

("ADD")



- SM particles on "brane" (4d) ; gravity in (4+n)d

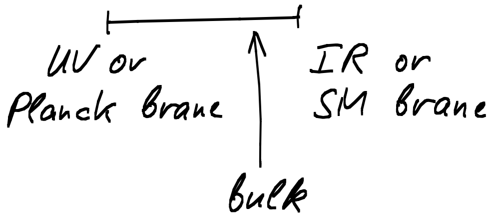
$$\int d^{4+n}x \; M_{P,4+n}^{2+n} \, R_{(4+n)} \rightarrow \int d^4x \; \underbrace{M_{P,4+n}^{2+n} \, L^n}_{\equiv M_{P,4}^2} \, R_{(4)}$$

- For large L, $M_{P,4}$ can be large although $M_{P,4+n} \sim$ TeV solving problem (2) (and (1) if L can be dynamically stabilized). Problem (3) is not addressed.

Note: $n = 1$ excluded ; $n = 2$ disfavored ; $n \geq 3$ ok

1.7 The warped extra dimension solution ("RS" or "RSI")

$d = 5$; compact space = interval:



$$ds^2 = \underbrace{e^{-2ky}}_{(\text{"warp factor"})^2} \eta_{\mu\nu} \, dx^\mu dx^\nu + dy^2$$

- This metric solves Einstein equations if $\Lambda_{5d} < 0$, $\Lambda_{IR} = -\Lambda_{UV} < 0$ (and appropriately tuned) ; $\Lambda_{4d,\,eff} = 0$ (or very small).
- Smallness of m_{EW} follows from suppression of IR-brane-metric by $\exp(-2k(y_{IR} - y_{UV}))$.

- Since this effect is exponential, problem (1) is nicely solved.
- Problem (2) is solved as in ADD.
- Problem (3) is naively not solved, but more elaborate modern versions (with some of SM particles in bulk) are promising.

Note: This is related to technicolor via AdS/CFT.

1.8 Gauge-Higgs unification (in TeV-scale extra dims., i.e. $L \sim TeV^{-1}$)

$$\phi \;\rightarrow\; A_5 \text{ or } A_5 + iA_6 \quad (\text{from some higher-dim. gauge field } A_\mu)$$

- 5d or 6d gauge invariance forbids Higgs mass $\Rightarrow$ no UV divergences ; a finite loop effect nevertheless produces a non-vanishing Higgs potential

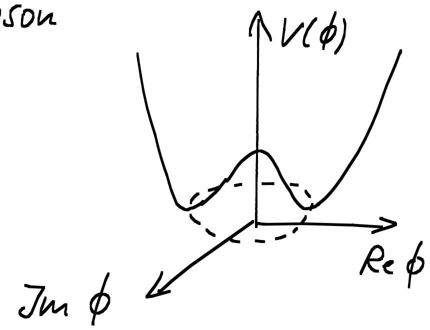
Problem: A sufficiently large quartic coupling & top Yukawa term are hard to get

1.9 The Little-Higgs solution

The Higgs is light (or "little") compared to fundamental lagrangian parameters because it is a "pseudo Goldstone boson":

Recall: spont. broken global symm. $\Rightarrow$ (exactly massless) Goldstone Boson

- If potential respects symm., but full lagrangian doesn't, a mass is induced "at loop level". $\Rightarrow$ "pseudo" Goldstone boson.



- The Higgs is "little" because of loop suppression.
- This is closely related to 1.8, where $A_5 \rightarrow A_5 + \text{const.}$ is the relevant global symmetry.

- Both 1.8 & 1.9 only address the little hierarchy problem.

1.10 Plan:

- The above "solutions" in detail, with focus on SUSY
- Grand Unification
- Strong CP problem
- String phenomenology