6 MSSM - phenomenology

6.1 Electroweak symm. breaking

- From $\mathcal{L}_{SUSY-SM} + \mathcal{L}_{soft}$, we can work out the scalar potential for the Higgs scalars $H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}$; $H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$.

  (The indices refer to the electric charge, which is different for the lower & upper components of an SU_2-doublet, in analogy to the lepton doublet.)

- The potential reads

$$
\begin{aligned}
V = &(|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\
&+ b\,(H_u^+ H_d^- - H_u^0 H_d^0) + \text{h.c.} \\
&+ \frac{1}{8}(g_1^2 + g_2^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 \\
&+ \frac{1}{2} g_2^2 \, |H_u^+ \overline{H_d^0} + H_u^0 \overline{H_d^-}|^2
\end{aligned}
$$

- Here the last two lines come from the D-term potential induced by the U_1 & SU_2 gauge theories.

- The terms $\sim |\mu|^2$ come from the F-term potential of the chiral SF model

- The remaining terms come from ~~SUSY~~.

Comment: The full scalar potential of the MSSM involves squarks and sleptons. If they acquire non-zero VEVs, we are dealing with charge- and/or color-breaking vacua, which are phenomenologically unacceptable. The absence of such vacua puts certain constraints on the SUSY parameters, which we assume to be fulfilled. We also assume that the masses of squarks & sleptons are large enough so that we can ignore their effect on the Higgs potential (i.e. they are integrated out by simply setting them to zero).

- In analogy to our analysis of the SM Higgs sector, we use SU_2-symmetry to set $H_u^+ = 0$ in the vacuum.

- $$\frac{\partial V}{\partial H_u^+} = (\mu^2 + m_{H_u}^2)\bar{H}_u^+ + b H_d^- + \frac{1}{8}(g_1^2 + g_2^2)\cdot 2\bar{H}_u^+ \cdot$$
$$\cdot\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right) + \frac{1}{2} g_2^2 \bar{H}_d^0 \left(\bar{H}_u^+ H_d^0 + \bar{H}_u^0 H_d^-\right)$$

- $$\left.\frac{\partial V}{\partial H_u^+}\right|_{H_u^+ = 0} = b H_d^- + \frac{1}{2} g_2^2 \bar{H}_d^0 \bar{H}_u^0 H_d^-$$

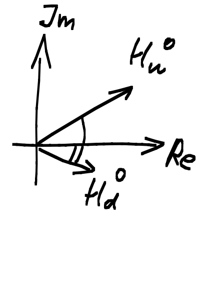
  $\Rightarrow$ There is an extremum at $H_d^- = 0$.
  (Analogously, one can show that also $\partial V / \partial H_d^-$ is zero at this point.)

- Thus, we now set $H_u^+ = H_d^- = 0$ and consider the potential as a fct. of H_u^0 & H_d^0:

$$V = m_2^2 |H_u^0|^2 + m_1^2 |H_d^0|^2 - b H_u^0 H_d^0 + \text{h.c.} \qquad \Big\| \qquad m_2^2 \equiv |\mu|^2 + m_{H_u}^2$$
$$+ \frac{1}{8}(g_1^2 + g_2^2)\left(|H_u^0|^2 - |H_d^0|^2\right)^2 \qquad \Big\| \qquad m_1^2 \equiv |\mu|^2 + m_{H_d}^2$$

- Without loss of generality, we can assume b to be real and positive by a phase redefinition of the Higgs doublets.

- Then, with respect to the phases of H_u^0 & H_d^0, the term $-\mathrm{Re}[b H_u^0 H_d^0]$ is minimal if the phases are opposite.



- Furthermore, using a $U_{1,Y}$ gauge trf. $H_u^0 \to e^{i\alpha} H_u^0$; $H_d^0 \to e^{-i\alpha} H_d^0$, we can set the phase to zero.

  ($\Rightarrow$ CP can not be spontaneously broken since all VEVs can be chosen to be real in the vacuum.)

- Now focus on the potential near $H_u^0 = H_d^0 = 0$ (neglecting the quartic part):

$$V \simeq m_2^2 (H_u^0)^2 + m_1^2 (H_d^0)^2 - 2b\, H_u^0 H_d^0$$

(H_u^0, H_d^0 are now real).

- This corresponds to a mass matrix

$$M^2 = \begin{pmatrix} m_2^2 & -b \\ -b & m_1^2 \end{pmatrix}.$$

- Its eigenvalues are the solutions of

$$\det (M^2 - \lambda \mathbb{1}) = 0$$

or
$$(m_1^2 - \lambda)(m_2^2 - \lambda) - b^2 = 0$$

$$\lambda_{1,2} = \frac{m_1^2 + m_2^2}{2} \pm \sqrt{\frac{(m_1^2 + m_2^2)^2}{4} + b^2 - m_1^2 m_2^2}$$

(recall that m_1^2, m_2^2 are real but not necessarily positive)

- If $b^2 < m_1^2 m_2^2$, either both eigenvalues are positive (i.e. no* el. weak symm. breaking since $H_u^0 = H_d^0 = 0$ is a minimum) or both eigenvalues are negative (this is also inacceptable, for reasons which will become apparent in a moment).

  see p. 45/46

- If $b^2 > m_1^2 m_2^2$, the extremum at the origin has an unstable direction along which a minimum occurs (at the point where the quartic term starts to dominate). This is the case of interest!

- However, there is a direction in field space along which the quartic part (D-term potential) is identically zero. This "D-flat direction" arises for $H_u^0 = H_d^0$. Along this direction the potential reads

$$V = (m_1^2 + m_2^2 - 2b)(H_u^0)^2.$$

Thus, the potential is unbounded from below unless

$$m_1^2 + m_2^2 > 2b.$$

- We have thus derived (necessary & sufficient) <u>conditions for electroweak symm. breaking</u>:

$$\left\| \begin{aligned} b^2 &> (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2) \\ 2b &< (|\mu|^2 + m_{H_u}^2) + (|\mu|^2 + m_{H_d}^2) \end{aligned} \right\|$$

(Note that $m_{H_u}^2$, $m_{H_d}^2$ are just names, i.e., there is no reason for these quantities to be positive. The first condition is easily realized if m_1^2 or m_2^2 is negative, but this is not necessary.)

- It is customary to call $\langle H_u^0 \rangle \equiv v_u$; $\langle H_d^0 \rangle \equiv v_d$.

- Calculating the resulting Z-boson-mass (in analogy to the SM-case), one finds:

$$v_u^2 + v_d^2 = 2m_Z^2/(g_1^2 + g_2^2).$$

- It is customary to introduce the angle β defined by

$$\tan\beta \equiv v_u/v_d.$$

- Thus, instead of v_u & v_d, we can use m_Z and $\tan\beta$ to characterize minimum of the potential.

- The relations $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$ relate these quantities to the lagrangian parameters m_1^2, m_2^2, b:

$$m_2^2 = b\cot\beta + \frac{m_Z^2}{2}\cos 2\beta$$
$$m_1^2 = b\tan\beta - \frac{m_Z^2}{2}\cos 2\beta.$$

- We see from these formulae that the μ-term can not be extremely large compared to the ~~SUSY~~ parameters $m_{H_u}^2, m_{H_d}^2, b$. This highlights the importance of realizing a non-zero μ-term together with ~~SUSY~~, as in the Giudice-Masiero mechanism.

- It is also useful to derive, by adding the two relations given above:
$$m_1^2 + m_2^2 = b\,(\tan\beta + \cot\beta) = 2b/\sin 2\beta.$$

From this we see that the region of large $\tan\beta$ ($\tan\beta \gtrsim 10$), which is phenomenologically most interesting for reasons to be discussed later, is characterized by small values of b at the lagrangian level.

[We will see that for large $\tan\beta$ it is easier to obtain a large mass for the lightest physical Higgs, which is needed to avoid the experimental lower bound from LEP.]

Summary of the main logical steps:

- $H_u^+ = 0$ in vacuum (gauge choice)
- show that $H_u^+ = 0$ & $H_d^- = 0$ extremizes potential

Comment 1: The relevant eq. was $H_d^- \left(b + \frac{1}{2} g_2^2 \overline{H_d^0} \overline{H_u^0}\right) \stackrel{!}{=} 0$. This can also be solved for $H_d^- \neq 0$, which means that there could also be minima breaking $U_{1,ED}$. It turns out (but we will not demonstrate this) that in cases where we find a proper SM-like minimum no such (dangerous) extra minimum exists.

Comment 2: We will have to check in the end that the minimum in the remaining potential for H_u^0, H_d^0 which we will find is also a minimum (rather than a maximum or saddle point) w.r.t. H_u^+, H_d^-. That this is indeed the case will follow from the full mass matrix at the extremum (which we will write down).

- $\Rightarrow V(H_u^0, H_d^0)$; when H_u^0, H_d^0 can be taken real without loss of generality

Demanding that the quadratic part has an instable direction (but is stable in the direction where the quartic part vanishes) leads to the conditions $b^2 > m_1^2 m_2^2$; $2b < m_1^2 + m_2^2$.

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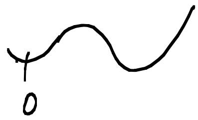
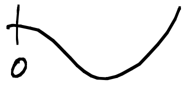
*) Comment: In cases where the quadratic part is positive definite, there can really be no minimum

away from zero. The reason is that, fixing a certain direction in field space and calling x the "distance" in this direction, the potential reads $\alpha x^2 + \beta x^4$. Thus, situations like



are impossible. Only situations like



can occur (but they require a neg. eigenvalue of the quadratic part).

6.2 General 2-Higgs-doublet model

(including more details of the SUSY case)

- We want the masses of all 5 Higgs scalars!
- The analysis in S. Martins review (hep-ph/9709356) which we largely follow is not very detailed. Historically, the main references are

  Haber, Kane, Sterling '78 } Nucl. Phys. B
  Gunion, Kane '85 }
  Gunion, Haber, Kane, Dawson: "The Higgs Hunter's Guide" 1990,

  which start with more general (not necessarily SUSY) 2-Higgs-doublet models. These are still the best references!
  (in particular the first two papers)

- Non-SUSY 2HDMs are possible extensions of SM. In a generic 2HDM, every fermion mass comes from a combination of both

Higgs VEVs. This leads to unacceptably large FCNCs and has to be avoided by postulating extra symmetries.

- Let us follow Gunion/Haber and write (For modern treatment see Maniatis, Nachtmann, ...)

$$
\begin{aligned}
V(\phi_1, \phi_2) = \; & \lambda_1 (\phi_1^+\phi_1 - v_1^2)^2 + \lambda_2 (\phi_2^+\phi_2 - v_2^2)^2 \\
& + \lambda_3 \left[(\phi_1^+\phi_1 - v_1^2) + (\phi_2^+\phi_2 - v_2^2) \right]^2 \\
& + \lambda_4 \left[(\phi_1^+\phi_1)(\phi_2^+\phi_2) - (\phi_1^+\phi_2)(\phi_2^+\phi_1) \right] \\
& + \lambda_5 \left[Re(\phi_1^+\phi_2) - v_1 v_2 \cos\xi \right]^2 \\
& + \lambda_6 \left[Im(\phi_1^+\phi_2) - v_1 v_2 \sin\xi \right]^2
\end{aligned}
$$

two equivalent Higgs-doublets

Comment: In the original paper, it is claimed that this is the most general potential under the requirements of $SU_2 \times U_1$ gauge invariance & $\phi_i \to -\phi_i$ symmetry of all mass-dimension-zero operators ["this symmetry is only broken softly" (i.e. not in the UV)]. In an erratum, the authors admit that the term

$$\lambda_7 \left[Re\, \phi_1^+\phi_2 - v_1 v_2 \cos\xi \right] \left[Im\, \phi_1^+\phi_2 - v_1 v_2 \sin\xi \right]$$

has been forgotten. However, it turns out that this does not affect the results of the original analysis.

- For $\lambda_i \geq 0$, the minimum is at

$$\langle \phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \; ; \; \langle \phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}.$$

(Note that here it is natural to ascribe the <u>same</u> $U_{1,Y}$ charge to both doublets, unlike what is conventionally done in the SUSY case.)

<u>Note:</u> $\xi = 0$ corresponds to "no CP violation".

- The SUSY case follows from this by imposing the appropriate relations between the λ_i (Given these constraints, $\xi = 0$ can be realized by a phase redefinition of the doublets).

- The analysis proceeds by redefining

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} = \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix} \rightarrow \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{pmatrix}$$

$$\phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} = \begin{pmatrix} \varphi_5 + i\varphi_6 \\ \varphi_7 + i\varphi_8 \end{pmatrix} \rightarrow \begin{pmatrix} \varphi_5 \\ \vdots \\ \varphi_8 \end{pmatrix}$$

and analysing the mass matrix

$$M_{ij}^2 = \frac{1}{2} \cdot \frac{\partial^2 V}{\partial\varphi_i \partial\varphi_j}$$

at the minimum. The matrix turns out to have block-diagonal form, with 2×2 blocks:

- <u>Indices 1, 2, 5, 6</u> $\rightarrow$ two 2×2 blocks of the form

$$\lambda_4 \begin{pmatrix} v_1^2 & -v_1 v_2 \\ -v_1 v_2 & v_2^2 \end{pmatrix}.$$

They can be diagonalized by an SO_2 rotation with angle β ($\tan\beta = v_1/v_2$); which explains why our parameter β introduced earlier is really an angle. The eigenvalues of each block are

$$\underset{\substack{\uparrow \\ \text{2 Goldstone bosons}}}{0} \quad \text{and} \quad \underset{\substack{\uparrow \\ \text{charged Higgs particle}}}{m_{H^\pm}^2} = \lambda_4 (v_1^2 + v_2^2) .$$

The relevant fields are $G^\pm = \phi_1^\pm \cos\beta + \phi_2^\pm \sin\beta$ ("Goldstones") ; $\phi_i^- \equiv (\phi_i^+)^*$

and $H^\pm = -\phi_1^\pm \sin\beta + \phi_2^\pm \cos\beta$.

- Indices 4, 8: another 2×2 block of the same form as above, but with $\lambda_4 \to \lambda_6$.

$\Rightarrow$ one further Goldstone + one further massive (but this time neutral) Higgs boson.

- Indices 3, 7:
$$\begin{pmatrix} 4v_1^2(\lambda_1+\lambda_3)+v_2^2\lambda_5 & (4\lambda_3+\lambda_5)v_1v_2 \\ (4\lambda_3+\lambda_5)v_1v_2 & 4v_2^2(\lambda_2+\lambda_3)+v_1^2\lambda_5 \end{pmatrix}$$

$\Rightarrow$ two more massive neutral Higgses

In total: $8 = 3 + 5$

3: Goldstones (from breaking $SU_2 \times U_{1,Y} \to U_{1,ED}$)

5: 1 charged (i.e. 2 d.o.f.) + 3 neutral Higgs bosons.

Summary of spectrum (now returning to the more specific SUSY case; and changing the notation correspondingly)

$$m_{A^0}^2 = 2b/\sin 2\beta$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2$$

$$m_{h^0,H^0}^2 = \frac{1}{2}\left(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2+m_Z^2)^2 - 4m_Z^2 m_{A^0}^2 \cos^2 2\beta}\right)$$

Note: For $m_{A^0}^2 \to \infty$, all masses go to infinity except

$$m_{h^0}^2 \simeq \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{4m_Z^2 m_{A^0}^2\cos^2 2\beta}{m_{A^0}^2+m_Z^2} \simeq m_Z^2\cos^2 2\beta .$$

6.3 $\tan\beta$ and Yukawa couplings

- recall: $\tan\beta = v_u / v_d$; $\mathcal{L} \supset (\lambda_u U Q H_u + \lambda_d D Q H_d)|_{\Theta^2}$

  $(0 \le \beta \le 90°)$ (focus on 3rd generation)

$$\Rightarrow m_t = \lambda_t v_u \; ; \; m_b = \lambda_b v_d$$

- Since $m_W^2 = \frac{1}{2} g_2^2 (v_u^2 + v_d^2)$; $m_Z^2 = \frac{1}{2}(g_1^2 + g_2^2)(v_u^2 + v_d^2)$,

  we have, for large $\tan\beta$, $v_u \approx 175$ GeV, as in SM.

  Hence: $\lambda_t \approx 1$; also: $\frac{m_t}{m_b} = \frac{\lambda_t v_u}{\lambda_b v_d} \Rightarrow \lambda_b = \lambda_t \frac{v_u}{v_d} \cdot \frac{m_b}{m_t}$

  or $\lambda_b \approx \frac{m_b}{m_t} \cdot \tan\beta \approx \frac{4.2}{175} \cdot \tan\beta$;

  $\Rightarrow$ $\tan\beta$ can be very large without driving λ_b into the non-perturbative regime ($\lambda_b \gg 1$).

- <u>Note</u>: In certain GUT or heterotic string models at a very high scale, $\lambda_t = \lambda_b$ may be enforced by symmetry. Then large $\tan\beta$ is necessary (and hence particularly attractive) to explain the ratio m_b / m_t.
  (For an actual numerical determination of the relevant $\tan\beta$, the running of λ_t, λ_b with the energy scale must be taken into account.)

- The opposite limit of $\tan\beta \ll 1$ can not so easily be taken: Then $v_d \approx 175$ GeV and $\lambda_b \approx \frac{m_b}{v_d} \approx \frac{4.2}{175}$.

We get $\lambda_t \approx \lambda_b \frac{v_d}{v_u} \cdot \frac{m_t}{m_b} \approx \frac{4.2}{175} \cdot \tan\beta \cdot \frac{175}{4.2}$ $\Rightarrow$ non-perturb.!
(This gets worse at higher scales since λ_t runs "up")

6.4 Some calc. details / large $\tan\beta$ region / limit of one light Higgs

- Let us use our knowledge that we can choose without loss of generality (in the vacuum) $H_u^0 = v_u$, $H_d^0 = v_d$ real and $b > 0$ real.

- Define $\alpha = \frac{1}{4}(g_1^2 + g_2^2)$ and minimize the potential of Sect. 6.1 explicitly:

$$0 = \frac{\partial V}{\partial v_u} = 2m_2^2 v_u - 2b v_d - 2\alpha v_u (v_d^2 - v_u^2)$$

$$0 = \frac{\partial V}{\partial v_d} = 2m_1^2 v_d - 2b v_u + 2\alpha v_d (v_d^2 - v_u^2)$$

$$\Rightarrow \quad m_2^2 = b \frac{v_d}{v_u} + \alpha (v_d^2 - v_u^2)$$

$$m_1^2 = b \frac{v_u}{v_d} - \alpha (v_d^2 - v_u^2)$$

- recall that $m_Z^2 = \frac{1}{2}(g_1^2 + g_2^2)(v_u^2 + v_d^2) = \frac{1}{2}(g_1^2 + g_2^2) v^2$

$$\Rightarrow \quad m_2^2 = b \cot\beta + \frac{m_Z^2}{2}(\cos^2\beta - \sin^2\beta)$$

$$m_1^2 = b \tan\beta - \frac{m_Z^2}{2}\underbrace{(\cos^2\beta - \sin^2\beta)}_{\equiv \cos 2\beta}$$

(These are the formulae which we stated earlier without derivation)

- From this, we have already derived $\sin 2\beta = \dfrac{2b}{m_1^2 + m_2^2}$

or $\tan\beta + \cot\beta = \dfrac{m_1^2 + m_2^2}{2b}$.

- Let us also derive the value of m_Z^2 by writing

$$m_1^2 - m_2^2 \tan^2\beta = \frac{m_Z^2}{2}(\sin^2\beta - \cos^2\beta)(1 + \tan^2\beta)$$

$$\Rightarrow m_Z^2 = 2 \cdot \frac{m_1^2 - m_2^2 \tan^2\beta}{(\tan^2\beta - 1)\cos^2\beta\,(1+\tan^2\beta)}$$

$$m_Z^2 = 2\,\frac{m_1^2 - m_2^2\tan^2\beta}{\tan^2\beta - 1}$$

Summary

$$m_2^2 = b/\tan\beta + (m_Z^2/2)\cos 2\beta$$
$$m_1^2 = b\cdot\tan\beta - (m_Z^2/2)\cos 2\beta$$

also:

$$2b < m_1^2 + m_2^2$$
$$b^2 > m_1^2 m_2^2$$

$$\Updownarrow$$

$$m_Z^2/2 = (m_1^2 - m_2^2\tan^2\beta)/(\tan^2\beta - 1)$$
$$\tan\beta + 1/\tan\beta = (m_1^2 + m_2^2)/2b$$

- To develop some feeling for the parameters, take $\tan\beta \gg 1$ (specifically, e.g., $\tan\beta \sim 10$):

- The last two eqs. now read:

$$m_Z^2/2 = m_1^2/\tan^2\beta - m_2^2$$
$$\tan\beta = (m_1^2 + m_2^2)/2b$$

a) $m_1^2, m_2^2 > 0$: $\quad m_Z$-formula $\Rightarrow$ $m_2^2 \ll m_1^2$

$\tan\beta$-formula $\Rightarrow$ $b \simeq m_1^2/2\tan\beta$

$b^2 > m_1^2 m_2^2 \Rightarrow m_2^2 < b^2/m_1^2 \simeq b/2\tan\beta$

hence:

$$\underbrace{m_2^2 \ll b}_{\text{factor} \gtrsim 2\tan\beta} \underbrace{\ll m_1^2}_{\text{factor} \simeq 2\tan\beta}$$

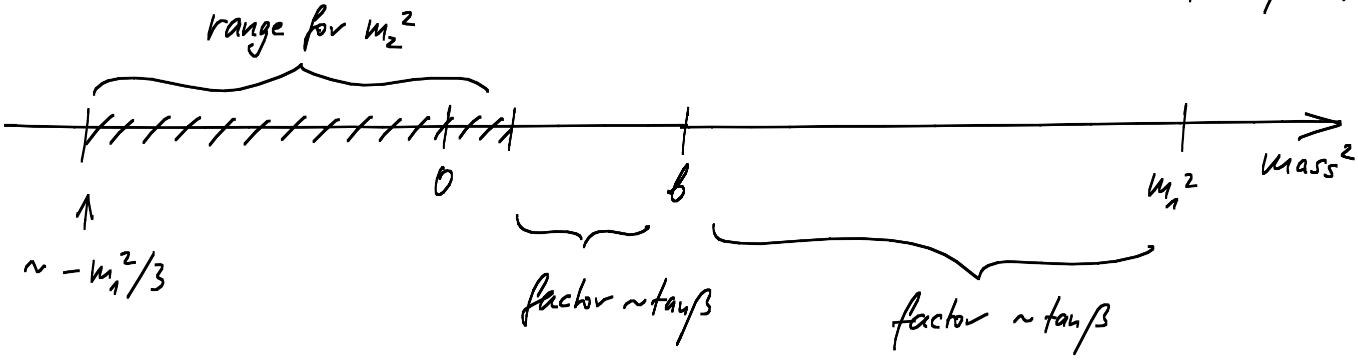
b) $m_1^2 > 0,\ m_2^2 < 0$: We are only interested in the lower bound on m_2^2.

If $|m_2^2| \gg m_1^2/\tan^2\beta$, then the m_Z-formula implies $|m_2^2| \simeq m_Z^2/2$

Recall also: $m_{A_0}^2 = m_1^2 + m_2^2$

Thus: $m_{A_0}^2 \gtrsim m_Z^2 \Rightarrow m_1^2 - m_Z^2/2 \gtrsim m_Z^2 \Rightarrow m_1^2 > 3|m_2^2|$

Combining a) & b), we arrive at the following overall picture:
(still for $\tan\beta \gg 1$)



<u>Problem:</u> Derive the Higgs sector of the SM as the low-energy limit of the Higgs sector of the MSSM (and determine the conditions on the MSSM parameters for which this is realized).
Beware: This is not as simple as it sounds!