8 Supergravity & SUSY-Breaking Mediation

8.1 Supergravity (very brief)

- We will take the superspace approach, taking

$$z^M = (x^m, \theta^\mu, \bar{\theta}_{\dot{\mu}})$$

to be coordinates on a (curved) super-manifold.

$$\left[\, m = 0,\dots,3 \;\; ; \;\; \mu = 1,2 \;\; ; \;\; \dot{\mu} = 1,2 \quad (\underline{\text{not}}\ \mu = 0,\dots,3\ !!) \right]$$

- Recall that in GR we can define the geometry on the basis of a vielbein $e_a{}^m$, interpreted as a set of 4 (by definition orthonormal) vectors labelled by "a":

$$e_a{}^m \underbrace{e^b{}_m}_{\text{inverse matrix}} = \delta_a{}^b \quad ; \qquad e^a{}_m e^b{}_n \eta_{ab} = g_{mn} \, .$$

- By analogy, we define a vielbein $E_A{}^M(z) = E_A{}^M(x,\theta,\bar{\theta})$;

$$A = (a, \alpha, \dot{\alpha}) \quad - \text{ Lorentz indices}$$
$$M = (m, \mu, \dot{\mu}) \quad - \text{ Einstein indices.}$$

- In GR, we introduce a constraint (vanishing torsion) and declare local Lorentz rotations & diffeomorphisms to be gauge symmetries. This allows us to express the curvature in terms of a (small number of) phys. d.o.f.s contained in $e_a{}^m$.

- An appropriate generalization of this procedure (most importantly, the constraints become much more complicated) allow one to express the superspace curvature in terms of a (small number of) physical d.o.f.s contained in $E_A{}^M$.

- Roughly speaking $E_A{}^M(z)$ can be expressed through the two superfields $\mathcal{H}^m(z)$ (real) & $\varphi(z)$ (chiral), with component fields

$e_a{}^m(x)$ (vielbein)

$\psi^m{}_\alpha(x)$ (gravitino)

$\left.\begin{array}{l} A^m(x) \\ B(x) \end{array}\right\}$ (auxiliary fields).

- The simplest action is

$$S = \underbrace{\int \underset{\substack{\uparrow \\ d^4x\, d^2\theta\, d^2\bar\theta}}{d^8z}\, E}_{\text{"volume of superspace"}} \cdot (-3\bar{M}_p^2) = \int d^4x \sqrt{-g}\, \frac{\bar{M}_p^2}{2} R + \underbrace{\cdots}_{\substack{\text{terms involving} \\ \text{gravitino \& auxiliaries}}}$$

- Furthermore, a cosmological constant term can be added:

$$S_{c.c.} = \int d^8z\, E \underset{\substack{\uparrow \\ \text{superspace Ricci scalar}}}{\frac{1}{R}} + h.c. = \int \underset{\substack{\uparrow \\ d^4x\, d^2\theta}}{d^6z}\, \varphi^3 \underset{\substack{\uparrow \\ \text{obtained by} \\ y^m \to x^m, \\ \text{as familiar from} \\ \text{flat-case superpotential}}}{(x,\theta)} + h.c. = \int d^4x \sqrt{-g} + \cdots$$

(The cosmol. constant is always negative.)

- Including a physical chiral SF ϕ (not to be confused with φ, which is part of pure supergravity), i.e. coupling a WZ-model to supergravity, we have:

$$S = \int d^8z\, E \underset{\substack{\uparrow \\ \text{real fct. of } \phi \,\&\, \bar\phi\text{, sometimes called} \\ \text{the "superspace kinetic function"}}}{\Omega(\phi,\bar\phi)} + \Big[\int d^6z\, \varphi^3 \underset{\substack{\uparrow \\ \text{supergravity-} \\ \text{superpotential}}}{W(\phi)} + h.c. \Big]$$

<u>Note:</u> $K = -3\ln(-\Omega/3M_p^2)$ is the supergravity-Kähler potential

$$(\Omega = -3M_p^2 e^{-K/3})$$

- Going to flat space $(e_a{}^m = \delta_a{}^m)$ and setting $\psi^m{}_\alpha = 0$; $A^m = 0$ we get

$$S = \int d^8z\, \varphi\bar{\varphi}\, \Omega(\phi,\bar{\phi}) + \left[\int d^6z\, \varphi^3 W(\phi) + h.c.\right]$$

with $\varphi = 1 + \theta^2 F_\varphi$.
↑ related to $B(x)$ above

↓ Integrate out F_φ & F_ϕ

supergravity scalar potential $V_{BD}(\phi,\bar{\phi})$ (where ϕ & $\bar{\phi}$ are the A-terms of the chiral SF ϕ)

<u>Note:</u> If we had kept non-trivial $e_a{}^m$, we would have found an Einstein-Hilbert term

$$\int d^4x \sqrt{-g}\, \frac{1}{2} R \cdot \left(-\Omega(\phi,\bar{\phi})/3\right), \text{ i.e.}$$

we are in a "Brans-Dicke frame". Rescaling the metric to absorb the factor $-\Omega/3$, we find the supergravity scalar potential in the "Einstein frame":

↓ "Weyl rescaling"

$$V = \frac{1}{M_p^2} e^K \Big(K_{\phi\bar{\phi}}^{-1} |\underbrace{W_\phi + K_\phi W}_{\equiv D_\phi W}|^2 - 3|W|^2 \Big)$$

- More generally, for a set of chiral SFs ϕ_a, we have

$$\boxed{V = \frac{1}{M_p^2} e^K \left(K^{a\bar{b}} D_a W\, D_{\bar{b}} \bar{W} - 3|W|^2 \right) \quad \text{with } D_a W = \partial_a W + K_a W}$$

- The first term is the SUGRA-analogue of the F-term potential.

Note: In rigid (or "flat-space") SUSY one usually states

$$\cancel{\text{SUSY}} \iff \text{vac. energy} > 0,$$

based on this positive definite F-term potential (& the pos. def. D-term potential). Here we see how this is violated in SUGRA because $W \neq 0$ is possible in the vacuum, which contributes negatively to the vac. energy (cf. our previous SUGRA cosmol. constant term).

- The fct. W also multiplies a gravitino-bilinear,

$$\mathcal{L} \supset \sim -\overline{W}\, \psi_a \sigma^{ab} \psi_b + \text{h.c.} \quad \text{(befor Weyl rescaling)}$$

such that the gravitino mass is given by

$$m_{3/2} = e^{K_0/2}\, \underset{\substack{\uparrow \\ \text{vacuum value}}}{W_0} / \overline{M}_P^{\,2} \, .$$

- One often sets $\overline{M}_P = 1$ in formulae of this type, writing simply

$$\underline{\underline{m_{3/2} = W e^{K/2}}}$$

Given that the cosm. const. is (practically) zero in our vacuum, we have:

$$\Lambda = 0 \quad \Rightarrow \quad |F| \sim |W| \quad \Rightarrow m_{3/2} \sim |F|$$

(for the dominant F-term in the model).

- An important special case are <u>No-scale models</u>. The simplest version (1-field case) is

$$K = -3 \ln(\phi + \bar{\phi}) + \text{const.} \quad , \quad W = W_0 = \text{const.}$$

One easily checks: $V \equiv 0$, i.e. the value of ϕ and hence the size of $m_{3/2}$ is not fixed ("no scale").

(<u>Note</u>: Loop corrections generically destroy this structure)

8.2 Supergravity mediation

- A further important special case is provided by the Polonyi-model:

$$K = \underbrace{\phi\bar{\phi}}\quad ; \quad W = c_1\phi + c_2$$

such a K is frequently used since it leads to a canonical kinetic term,

$$\mathcal{L} \supset - K_{a\bar{b}}\, \partial_m \phi^a \partial^m \bar{\phi}^{\bar{b}} .$$

[Comment: We were unable to discuss kinetic terms in our simplified analysis since we set $A^m = 0$. However, $\partial_m \phi \neq 0$ induces $A^m = 0$ and we should have used this auxiliary field to also derive kinetic terms.]

- In the Polonyi model, SUSY is broken in the vacuum ($F_\phi \neq 0$) and the constants can be adjusted to ensure $\Lambda = 0$.

- The simplest version of gravity mediation is then to take

S – hidden sector

Q – SM SFs (e.g. quarks)

$$K = S\bar{S} + Q\bar{Q} \quad ; \quad W = c_1 S + c_2$$

$\Rightarrow F_S \neq 0$ (size governed by c_1, c_2)

$$\Omega = -3e^{-K/3} = -3e^{-(S\bar{S}+Q\bar{Q})/3} = -3(1 + \cdots + S\bar{S}Q\bar{Q}/18 + \cdots)$$

$$\int d^8z\, \varphi\bar{\varphi}\, \Omega \supset \sim S\bar{S}Q\bar{Q}\Big|_{\theta^2\bar{\theta}^2}$$

$F_S \neq 0 \Rightarrow m_0^2 \sim F_S^2$ (in Planck units)

From our previous discussion, $m_{3/2} \sim F_s$, i.e.
the gravitino mass is comparable to the scale of soft terms.
(reintroducing $M_p \equiv M$: $\quad F_s = M_s^2/M \sim m_{3/2}$;

$$\frac{1}{M^2} S\bar{S} Q \bar{Q} \Rightarrow m_0^2 \sim \frac{F_s^2}{M^2} \sim \left(\frac{M_s^2}{M}\right)^2 \sim m_{3/2} \Big)$$

Note: The special structure $K = S\bar{S} + Q\bar{Q}$ has no justification so far. However, very similar structures appear generically in string models. Unfortunately, "flavor blindness" is not generically realized in such cases:

$$K \neq S\bar{S} + \sum_{\text{flavors}} Q^i \bar{Q}^i .$$

It is nevertheless used in what people call "MSUGRA" ("minimal SUGRA"), defined by

m_0 (all scalars)

$m_{1/2}$ (all gauginos)

A (all trilinear terms, up to Yukawa-coupling-prefactor)

$m_{3/2}$, μ, $B\mu$.

(minimal set of parameters).

8.3 Anomaly mediation

- in conventional (non-SUSY) GR, one can consider local scale trfs. (Weyl rescalings):

$$g_{\mu\nu}(x) \longrightarrow \omega^2(x)\, g_{\mu\nu}(x).$$

- A theory of GR invariant under such trfs. would be called "conformal gravity".

- The experimentally observed GR is not invariant under such trfs.:

$$\sqrt{-g}\,R \;\longrightarrow\; \omega^2 \sqrt{-g}\,R + \underbrace{\cdots}_{\substack{\text{terms involving}\\ \text{derivatives of } \omega}}$$

[This also clear from the fact that the coefficient of $\sqrt{-g}\,R$, $\frac{1}{2}\bar{M}_p^2$, has mass dim. 2 and thus breaks rescaling invariance explicitly.]

- However, conventional GR can be viewed as spontaneously broken conformal gravity (at the price of introducing an extra massless scalar):

$$\mathcal{L} = \tfrac{1}{2}\sqrt{-g}\,\varphi^2 R - \tfrac{1}{2}(\partial\varphi)^2 \qquad ; \qquad \langle\varphi\rangle = \bar{M}_p$$

[φ compensates for scale trfs. by $\varphi \to \varphi/\omega$, making $\mathcal{L}$ invariant. Hence one may call φ a "conformal compensator".]

- The connection to usual gravity can be made manifest by absorbing the φ^2-coefficient in the metric:

$$g^{\mu\nu} \;\longrightarrow\; g^{\mu\nu}\cdot\frac{\bar{M}_p^2}{\varphi^2}$$

$$\Rightarrow\; \mathcal{L} = \frac{1}{2}\sqrt{-g}\,\bar{M}_p^2 R + \text{modified kinetic term for } \varphi$$

<u>Return to SUSY:</u>

- A more elegant way of deriving the SUGRA action starts from conf. supergravity, spontaneously broken by $\langle\varphi\rangle$. (φ is now again a chiral SF, as before).

- In our presentation, this role of φ as a "chiral (conformal) compensator" is not obvious since we have chosen a gauge where $\varphi = 1 + F_\varphi \theta^2$. One may, however, understand (at least heuristically) the way in which φ appears in the action as follows:

– Ignore φ for the moment and write the action in terms of $M = \overline{M}_p$ and dim. less couplings, i.e.

$$S = \int d^8z \left(-3M^2 e^{-K(\phi/M,\,\bar\Phi/M)}\right) + \int d^6z\, M^3\, \tilde{W}(\phi/M) + \text{h.c.}$$

dim. less version of W

dim. less fcts. of the dim. less quantity ϕ/M.

– φ really appears because its non-zero VEV breaks conf. invariance and hence induces M: $M \to \varphi$!

– It is convenient to rescale φ: $\varphi \to M\varphi$, i.e.

$$S = \int d^8z \left(-3M^2 \varphi\bar\varphi\, e^{-K(\phi/M\varphi,\,\bar\Phi/M\bar\varphi)}\right) + \int d^6z\, M^3\varphi^3\, \tilde{W}(\phi/M\varphi) + \text{h.c.}$$

or, after $\phi \to \phi\varphi$,

$$S = \int d^8z\, \varphi\bar\varphi\, \Omega + \int d^6z\, \varphi^3 W + \text{h.c.}\,,$$ as before.

– The important point to remember is that, when the action is written only in terms of M & dim. less couplings, φ simply always appears together with M, in the combination $\underline{\underline{M\varphi}}$.

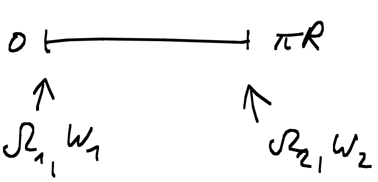
Application

- Consider a model with two sectors:

$$\Omega = \underset{\substack{\uparrow \\ \text{hidden}}}{\Omega_1(S,\bar{S})} + \underset{\substack{\uparrow \\ \text{"MSSM"}}}{\Omega_2(Q,\bar{Q})} \; ; \; W = W_1(S) + W_2(Q) .$$

[Here gravity mediation terms $\sim S\bar{S}Q\bar{Q}|_{\theta^2\bar{\theta}^2}$ are not present by assumption ("sequestering").]

[<u>Concretely</u>: 5d-models on $S^1/\mathbb{Z}_2$:



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- Let us furthermore assume that

$$\int d^4\theta \, \varphi\bar{\varphi}\,\Omega_1 + \int d^2\theta \, \varphi^3 W_1 + \text{h.c.}$$

has a ~~SUSY~~ vacuum with $F_S \neq 0$; $F_\varphi \neq 0$ (which is generically the case) and that this is not disturbed by adding the MSSM-sector.

- How does φ (or F_φ) affect the "MSSM sector"

$$\int d^4\theta \, \varphi\bar{\varphi}\,\Omega_2(Q,\bar{Q}) + \int d^2\theta \, \varphi^3 W_2(Q) + \text{h.c.} \quad ?$$

- Let's assume the "MSSM sector" has no dimensionful parameters (as e.g. for the NMSSM):

$$\int d^4\theta \, \varphi\bar{\varphi}\, Q\bar{Q} + \int d^2\theta \, \varphi^3 Q^3 + \text{h.c.}$$

- By the redefinition $Q = \hat{Q}/\varphi$, φ can be completely removed, as expected for a "conformal compensator", which should indeed disappear if the theory is conformal.

- If, however, a mass-parameter is present,

$$W \supset \mu Q^2 ,$$

the above redefinition leads to $W \supset \overbrace{\mu \cdot \varphi}^{\varepsilon M \cdot \varphi \text{ as explained}} Q^2$, i.e. φ appears as a coefficient of any mass-parameter.

- Applying this to the MSSM, we get

$$\mu \varphi H_u H_d \big|_{\theta^2} \longrightarrow \underbrace{\mu F_\varphi}_{\hat{=} B\mu} H_u H_d ,$$

i.e. ~~SUSY~~ dominated by the $B\mu$-term, which is not acceptable phenomenologically. (This is anyway not a good example since $\mu \ll M$ should result from some dynamics!)

- However, φ also appears through radiative corrections, which break conf. symm. (conf. anomaly, hence: "anomaly mediation"):

$$Q\bar{Q}\big|_{\theta^4} \longrightarrow Q\bar{Q}\left(1 + \underbrace{\lambda^2}_{\text{some generic coupling}} \ln \frac{\overbrace{\Lambda^2}^{\text{cutoff}} \varphi\bar{\varphi}}{p^2}\right)\Big|_{\theta^4}$$

p: renormalization scale, to be thought of as, e.g., the energy scale of a collider experiment

- After $\ln \Lambda^2$ is absorbed in the renormalization of Q, one is left with $(\varphi\bar{\varphi})^{-1}$ accompanying "p^2".

- Expanding in $F_\varphi \theta^2$, one finds a term

$$\sim \lambda^2 Q\bar{Q}|F_\varphi|^2 \theta^2\bar{\theta}^2 \big|_{\theta^2\bar{\theta}^2} ,$$

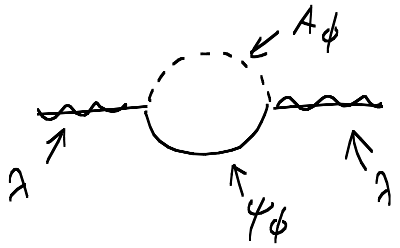
i.e., ~~SUSY~~ scalar masses!

(analogously, gaugino masses come from the renormalization of the coupling in $\frac{1}{g^2} W^2 \big|_{\theta^2}$.)

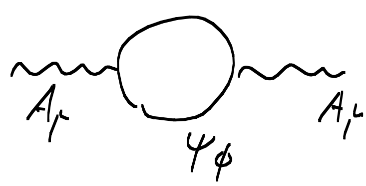
[Pure anomaly mediation leads to the "tachyonic slepton problem", which can be overcome at the price of extra fields.]

8.4 Gauge mediation

- Consider an "O'Raifeartaigh-type" sector with some chiral SF S and $F_S \neq 0$ in the vacuum.
- S can not be charged under G_{SM} since this would break the gauge symmetry.
- However S can couple to messenger fields $\phi, \tilde{\phi}$ (vector-like) via
$$S\phi\tilde{\phi}\big|_{\theta^2} ,$$
leading to a mass-splitting between bosons & fermions in $\phi, \tilde{\phi}$.
- Let $\phi, \tilde{\phi}$ be charged under G_{SM}. Then loop corrections induce gaugino masses via



(The SUSY analogue of

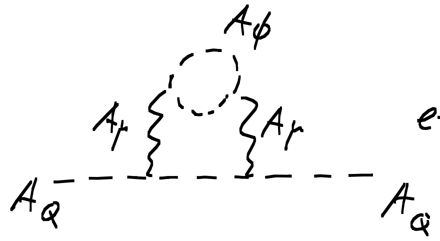


).

$$\Rightarrow \quad m_{1/2} \sim \frac{g^2}{16\pi^2} \cdot \frac{F_S}{M_\phi}$$

M_ϕ = mass of $\phi, \tilde{\phi}$.

$$\& \quad m_0^2 \sim \left(\frac{g^2}{16\pi^2}\right)^2 \left|\frac{F_S}{M_\phi}\right|^2$$ at two-loop-level:



etc.

- Interesting fact:

Since $m_{3/2} \sim \frac{W}{\bar{M}_P^2}$ & $W \sim F_S \cdot \bar{M}_P$

(vanishing cosmol. const.),

we find

$$m_{3/2} \sim \frac{F_S}{M_P}, \quad \text{as usual.}$$

However, in contrast to gravity mediation $\frac{F_S}{M_P} \neq M_{soft}$.

$m_{1/2}, m_0$, etc.

Instead:

$$F_S \sim \frac{m_{1/2} \cdot M_\phi}{g^2/16\pi^2}$$

$$\& \quad m_{3/2} \sim m_{1/2} \cdot \underbrace{\frac{M_\phi}{M_P \cdot (g^2/16\pi^2)}}$$

Since M_ϕ can be as small as, say, 10 TeV, the gravitino can, in principle, be extremely light!