10. Exercise Sheet for Theoretical Physics I (Classical Mechanics)

Solutions to be handed in in the tutorials of Week 12 (16th Jan. 2008)
Class exercise to be discussed in Week 11 (9th Jan. 2008)

Exercise 39: (5 points)

A ladder of length L and mass M is leaning against a wall. Let the moment of inertia of the ladder about an axis through its middle rung be I. At the foot of the ladder a static friction force acts with coefficient of static friction μ.

a) Up to what angle of inclination ϕ_0 (from the vertical) can the ladder remain static?

b) The ladder begins to slide at the angle ϕ_0 and the sliding friction force is now negligible. Find the Lagrangian and set up the equations of motion.

Exercise 40: (5 points)

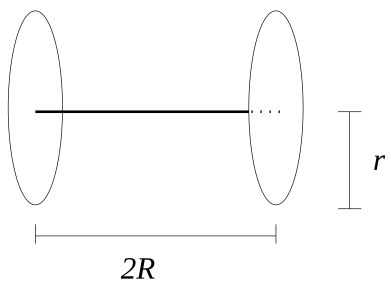
Little Konrad plays the dice game called "Meiern" (or "Mäxchen") with little Cornelia. They play with two cubical dice of constant density ρ whose edges have length a. Konrad tries to predict the throw of the dice purposefully.

a) Determine for Konrad the moment of inertia of one cube about its edge.

b) Determine the moment of inertia of one cube about an axis passing through the centre and one vertex.

Exercise 41: (5 points)

Two wheels of radius r are mounted upon a single axis at a distance $2R$ from each other. The wheels can rotate independently of each other. This contraption rolls on a horizontal plane without sliding and without either rolling resistance or air resistance. Find suitable generalized coordinates to describe the motion and formulate the corresponding (non-holonomic) differential constraints. Calculate the most general motion of the system by means of Lagrange equations of the first kind (with constraints), and describe all the possible "wheel tracks". (The inverted commas relate to the fact that in the absence of dissipation no tracks are actually made.)



Exercise 38 (Class exercise):

Which of the following matrices may represent an inertia(l) tensor in Cartesian coordinates?

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 2 \end{pmatrix}$$