11. Exercise Sheet for Theoretical Physics I (Classical Mechanics)

Solutions to be handed in in the tutorials of Week 13 (23rd Jan. 2008)

Exercise 40: (5 points)

A spherical gyroscope of mass M rotates in the gravitational field with an angular velocity ω_Z about its symmetry axis Z. The moment of inertia about this axis is Θ_3 and the centre of mass is at a distance s from the fixed point of rotation (foot of the gyroscope). The Z-axis presents a fixed angle θ from the vertical Z'-axis, *i.e.* the gyroscope experiences a constant rate of precession. The gyroscope is situated in a lift; suppose that its foot is fixed to the floor of the lift by a frictionless ball and socket joint. Calculate the speed of precession Ω, stating the direction, in the case that:

a) the lift is stationary,

b) the lift is accelerated downwards at $2g$.

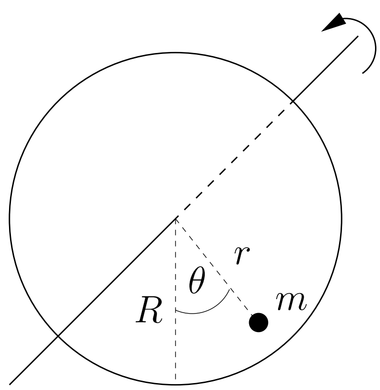
Hint: Use the approximation of a "fast gyroscope" *i.e.* calculate to leading order in $1/\omega_Z$. Which dimensionless parameter must be much larger than unity in order for this approximation to be justified?

Exercise 41: (5 points)

Consider the following pendulum system. A homogeneous circular disc of mass M and radius R is fixed to a (frictionless) horizontal axis of rotation which coincides with the axis of symmetry of the disc. A point mass m is fixed to the disc at a fixed distance r from the axis and is subject to the force of gravity.

a) Determine the Lagrangian and the resulting equations of motion.

b) Determine the Hamiltonian and the resulting canonical equations of motion.



Exercise 42: (5 points)

a) Consider a one-dimensional simple harmonic oscillator with Lagrangian

$$L = \frac{1}{2}m\dot{q}^2 - \frac{m\omega^2}{2}q^2.$$

i) Determine the canonical momentum p and the Hamiltonian. Set up Hamilton's equations and solve them.

ii) Show that the transformation

$$q \to a = \frac{1}{\sqrt{2im\omega}}p + \sqrt{\frac{im\omega}{2}}\,q, \qquad p \to \tilde{a} = \frac{1}{\sqrt{2im\omega}}p - \sqrt{\frac{im\omega}{2}}\,q$$

is canonical, by calculating the Poisson bracket $\{a, \tilde{a}\}$. Write the Hamiltonian as a function of a und $\tilde{a}$, set up Hamilton's equations and solve them.

b) Consider a particle in a central potential with the Lagrangian

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2) - V(r)$$

Find the canonical momenta p_r und p_ϕ associated to r and ϕ respectively, as well as the Hamiltonian. Set up Hamilton's equations and show that a solution is given by $r(t) = R$ and $\phi(t) = \omega t$ with suitable R and ω. What is the relationship between R, ω and $V'(R)$?