2. Exercise Sheet for Theoretical Physics I (Mechanics)

To be handed in in the exercise groups of the 3rd week (31th October, 2007)

Exercise 8: (5 Points)

a) Is the following force field defined on $\mathbb{R}^3 \setminus \{0\}$ conservative? If so, find a potential (c, μ are constants.)

$$\mathbf{F} = \frac{c\, e^{-\mu r}}{r^2}(1 + \mu r)\, \hat{\mathbf{r}}$$

b) Is the following force field defined on $\mathbb{R}^3 \setminus$ (x-axis) conservative? If so, find a potential. (*Hint:* Consider the work along a path that encloses the x-axis.)

$$\mathbf{F} = \frac{c}{y^2 + z^2}\begin{pmatrix} 0 \\ -z \\ y \end{pmatrix}$$

c) Let $\mathbf{a}, \mathbf{b}$ be constant vectors. What condition do they have to fullfil in order to make the force field $\mathbf{F} = \mathbf{a}\,(\mathbf{b} \cdot \mathbf{r})$ conservative?

Exercise 9: (5 Points)

a) Give an explicit expression for the matrices $R_1, R_2, R_3 \in \mathrm{SO}(3)$ that describe a rotation of an angle φ around the coordinate axes $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ of a right handed system. (The sign of φ is defined in the sense of a right-hand screw.)

b) Give the explicit expression of the matrix that describes the rotation of an angle φ around the axis $\mathbf{e} = (0, \sin\alpha, \cos\alpha)$. (*Hint:* First transfer $\mathbf{e}$ into $\mathbf{e}_3$ by rotating around $\mathbf{e}_1$. Then rotate around the resulting axis by an angle φ and finally invert the first rotation.)

Exercise 10: (5 Points)

N people, each of mass m, stand on a platform of train carriage of mass M, which is initially at rest. They jump off the end of the carriage with a horizontal velocity u (i.e. after the drop the relative velocity between person and carriage is u.) The carriage rolls in the opposite direction without friction.

a) What is the final velocity of the carriage when all people jump at the same time?

b) What is the final velocity of the carriage when all people jump one by one?

c) Show that in case b) the final velocity of the carriage is larger than in case a).