3. Exercise Sheet for Theoretical Physics I (Mechanics)

To be handed in in the exercise groups of the 4th week (7th November, 2007)
Discussion of the class exercis in the 5th week (14th November, 2007)

Exercise 11: (5 points)

a) As shown in the lecture, the Levi-Civita symbol $\epsilon^{i_1 i_2 i_3}$ transforms under rotations $R \in SO(3)$ as follows:

$$\epsilon^{i_1 i_2 i_3} \longrightarrow R^{i_1 j_1} R^{i_2 j_2} R^{i_3 j_3} \epsilon^{j_1 j_2 j_3}.$$

Show that the result of the transformation is totally antisymmetric under exchange of the indices i_1, i_2, i_3.

b) For given vectors $\vec{a}, \vec{b}, \vec{c}$ und $\vec{d}$, prove that:

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) - (\vec{b} \cdot \vec{c})(\vec{a} \cdot \vec{d}).$$

c) Prove also that:

$$(\vec{a} \times \vec{b}) \cdot [(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})] = [\vec{a} \cdot (\vec{b} \times \vec{c})]^2.$$

Exercise 12: (5 points)

A general Galilean transformation is given by

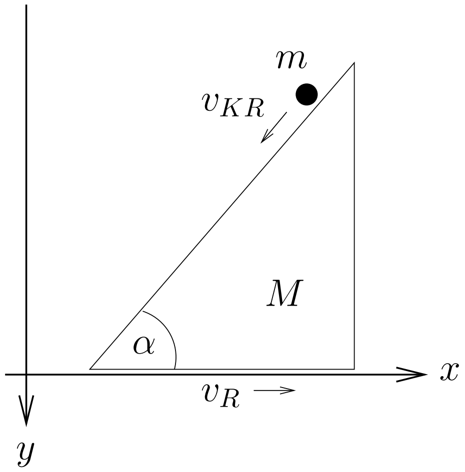
$$\tilde{x}^i = A^i{}_k x^k + v^i t + b^i \ , \quad \tilde{t} = t + c$$

with $A^T = A^{-1}$.

a) Show that the inverse transformation is also a Galilean transformation (thus, can also be written in this form).

b) Also, two consecutively applied Galilean transformations again yield a Galilean transformation. Calculate the parameters of the (resulting) composite transformation.

c) Since the Galilean transformations are also associative in respect of their composition, they form a group, the Galilean group. Determine the number of parameters by which the Galilean group is described. Find (at least) three nontrivial sub-groups of the Galilean group.



Exercise 13: (5 points)

Little Konrad (who can be regarded as a point mass of mass m) slides frictionlessly down a wedge-shaped slide of mass M with basis angle α, while playing on a frozen lake. The slide slides frictionlessly over the ice, and is initially at rest.

a) Let $\vec{v}_K$ be the velocity of Konrad and v_R be the speed of the slide in the x-direction, in a stationary xy-coordinate system. Furthermore let v_{KR} be the velocity with which Konrad slides down the slide (in the rest frame of the slide), and $|\vec{F}|$ be the force between Konrad and the slide. Find a differential equation of first order for $v_R(t)$ giving the dependence on m, M und α. (*Hint:* Put together sufficient equations by means of Newton's laws and elementary geometry.)

b) Discuss both the limiting cases $m \ll M$ and $m \gg M$.

Exercise 14 (class exercise):

A man comes home after a business trip. His wife immediately remarks that his new suitcase is scratched. He excuses himself with an outlandish story: at baggage reclaim in the airport, while his suitcase was approaching on the conveyor belt, a cat jumped onto it from one side and made the scratches while "braking" itself.
His wife rejects this with the argument that, if this were true, the scratches should clearly be curved – but in fact they are straight.

Is the suspicion of his wife justified? (Give a convincing qualitative argument.)