4. Exercise Sheet for Theoretical Physics I (Mechanics)

Solutions to be handed in in the tutorials of Week 5 (14th November 2007)

Exercise 15: (5 points)

Consider the variational problem

$$\delta J = \delta \int_{x_1}^{x_2} f(y, y', y''; x)\, dx = 0,$$

where $y' = dy/dx$ and $y'' = d^2y/dx^2$. The end-points should not be varied, i.e. the variations of both y and y' should vanish at x_1 und x_2. What is the corresponding Euler-Lagrange equation?

Exercise 16: (5 points)

Determine the horizontal deflection caused by the Coriolis force of a freely falling particle in the gravitational field of the Earth.

(1) First neglect the forces due to the motion of the Earth around the Sun and due to any variation in the angular velocity of the Earth's rotation ($\dot{\boldsymbol{\omega}} \simeq 0$).

Furthermore suppose that the particle falls from such a small height that the acceleration of the Earth may be locally considered as constant (note that the effective acceleration of the Earth already includes the centrifugal force). Moreover it is assumed that the particle is dropped in a vacuum.

(2) Consider the magnitude of the Earth's rotational angular velocity ω as a small parameter and first determine the horizontal deflection to first order in ω.

(3) Show that a particle which is dropped at latitude 45° North from a height of 100 m will be deflected by 1.55 cm towards the East.

(4) Show that, to second order in ω, a small southerly deflection arises (in the Northern hemisphere), that vanishes at the North Pole und the Equator.

Exercise 17: (5 points)

Little Konrad stands on a turntable rotating with angular velocity ω and shoots a bow and arrow at a target, also standing on the disc and at a distance d. How far does the arrow, travelling with velocity v, deviate from the centre of the target, if he does not take account of the rotation of the turntable?

a) Konrad in the centre of the turntable.

b) Target in the centre of the turntable.

c) Konrad and the target each at a distance d from the centre.

Solve the problem in each of the three cases, both in an inertial system and in the reference frame of the turntable (using the famous formula for fictitious forces). Work to linear approximation in ω, and indicate which (dimensionless!) parameter must be much smaller than 1, in order for this to be a good approximation.