5. Exercise Sheet for Theoretical Physics I (Mechanics)

Solutions to be handed in in the tutorials of Week 6 (21st. November 2007)

Exercise 21: (5 points)

Consider a simple pendulum of length d, swinging freely in three spatial directions under gravity. The mass of the bob is m, that of the cable is negligible.

a) Establish the Lagrangian in spherical polar coordinates (distance r from the suspension point, polar angle ϕ and azimuthal angle θ).

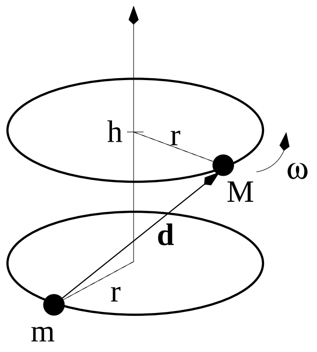
b) Identify the symmetries of the system and the corresponding conserved quantities.

c) Establish the equations of motion. (The solution of the equations of motion is not required.)

Exercise 22: (5 points)

A point mass m can move frictionlessly on a horizontal ring with radius r. A second point mass M moves on an identical horizontal ring, placed above the first ring at a difference in elevation h, with a constant angular velocity (determined by an external constraint) ω. A conservative force, defined by a potential $V(d)$, acts between the two masses (d being the magnitude of the distance vector $\mathbf{d}$ between the two masses, $d = |\mathbf{d}|$).

Give the (explicitly time-dependent!) Lagrangian for the first point mass m; find a symmetry and derive the corresponding conserved quantity by means of Noether's theorem. Show that this corresponds to the energy in the rotating frame of reference.



Exercise 23: (5 points)

Half of a homogeneous (perfectly flexible and inextensible!) cord of length L lies (straight!) on a (horizontal!) table, the other half hangs off the edge of the table. At time $t = 0$ the cord is released and starts to slide down. (Assume that the height of the table above the floor is larger than L!) Let the mass per unit length be μ.

a) Determine the Lagrangian.

b) Set up the equation of motion and integrate it.