6. Exercise Sheet for Theoretical Physics I (Mechanics)

Solutions to be handed in in the tutorials of Week 9 (5th December 2007).

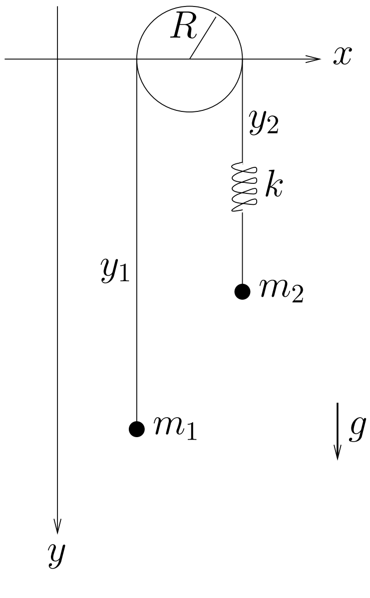
Exercise 24: (5 points)

The Atwood machine is composed of two masses m_1 and m_2 in the gravitational field of the Earth, that are connected by a cable of length l_0 which runs frictionlessly over a pulley of radius R. The cable and pulley are to be assumed massless. An ideal spring with spring constant k is additionally installed in the cable.

a) Determine the Lagrangian of the Atwood machine with additional spring using coordinates x and y (see figure).

b) Consider new coordinates z_1 and z_2, in which the Lagrangian decomposes into a sum of terms that depend only z_1 or z_2. These coordinates are linear combinations of x, y, m_1 and m_2.

c) Set up the Euler-Lagrange equations and explain their physical significance.



Exercise 25: (5 points)

Little Cornelia slides down the back of a giant tortoise (*Geochelone nigra*). At which angle does she first no longer touch the tortoise and instead fall freely? Neglect friction and assume that the tortoise's shell is perfectly spherical. Cornelia starts at the very apex with negligible initial velocity.

Exercise 26: (5 points)

In the Kepler problem $V(r) = -a/r$, in addition to energy and angular momentum, the Lenz vector $\mathbf{M} = \dot{\mathbf{r}} \times \mathbf{L} - a\,\hat{\mathbf{r}}$ is a conserved quantity. (Angular momentum: $\mathbf{L} = m\,\mathbf{r} \times \dot{\mathbf{r}}$).

a) Show using the equations of motion that $\dot{\mathbf{M}} = 0$. In which plane does $\mathbf{M}$ lie?

b) Choose initial conditions $\mathbf{r} = (s, 0, 0)$ and $\dot{\mathbf{r}} = (0, v_0, 0)$. Use the conservation of angular momentum and the Lenz vectors to derive the equation

$$y^2 = \lambda(\lambda - 2)x^2 - 2\lambda(\lambda - 1)sx + \lambda^2 s^2, \qquad \lambda = \frac{msv_0^2}{a},$$

for the trajectory. Which parameters result in the following trajectories: parabola; circle; straight line; hyperbola; ellipse?

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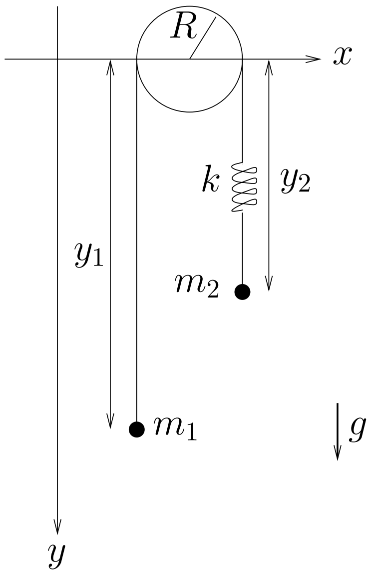
a) Determine the Lagrangian of the Atwood machine with additional spring using coordinates y_1 and y_2 (see figure).

b) Consider new coordinates z_1 and z_2, in which the Lagrangian decomposes into a sum of terms that depend only z_1 or z_2. These coordinates are linear combinations of y_1, y_2 with and without coefficients m_1 and m_2 respectively.

c) Set up the Euler-Lagrange equations and explain their physical significance.



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